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"Wavelet Analysis of the Cosmic Velocity Field"

Abstract

We present a method, based on the properties of wavelets transforms, for inferring a 3-dimensional and irrotational velocity field from its observed radial component. Our method is comparable in its objective to POTENT but the use of the wavelet analysis offers in addition a robust tool in order to smooth the sparsely sampled cosmic velocity field. The application of our method to simulations permits us to study the influence of the sparse sampling as well as the distance measurement errors. Finally, the potential velocity field within a cube of size 10000 km.s^{-1} and centered on our galaxy is derived from the redshift-distance catalog MARK II compiled by D. Burstein.

I. Introduction

In a previous paper (Rauzy, Lachièze-Rey and Henriksen (1993), hereafter RLH I), we devised a method based on the properties of the wavelet transforms, for inferring an irrotational velocity field from its observed radial component. Our method, applied on simulated velocity fields sampled on an ideal 3-dimensional grid, offers a natural way for smoothing the velocity field and for separating its contributions at different scales. Unfortunately, galaxies with measured radial peculiar velocity are sparsely distributed throughout the space, leaving large regions of missing information on the velocity field. Thus, one has to first smooth the observed velocity field before applying the reconstruction procedure on the velocity field. The POTENT method (Dekel et *al.* (1990)) has given impressive results in this way. However, we see two limitations in the smoothing scheme used by the POTENT method. Firstly, the size of the smoothing window function doesn't vary throughout the space, and so fictitious information is thus added in undersampled regions and a significant part of the signal is lost in oversampled regions. Secondly, the errors interverning during the smoothing procedure of the POTENT method are difficult to control, i.e : the output smoothed velocity field is not linked with a computable theoretical quantity. In section II, we summarize a new way to smooth a field sampled on a support distributed inhomogeneously throughout the space (the complete presentation of our method will be found in Rauzy, Lachièze-Rey and Henriksen, hereafter RLH II). The variation in size of the smoothing window function permits us to realize a minimal smoothing procedure (i.e : without loss of information).

Moreover, we prove that our smoothed output field matches with a theoretical quantity (defined by the wavelet analysis formalism). In section III, we analyse the operation involved in the reconstruction of the velocity field from its smoothed radial component only. We show that the operation of reconstruction doesn't commute with the preliminary smoothing operation. This creates difficulties already at the 'a priori' theoretical level when attempting to compare the reconstructed velocity field with the cosmic velocity field obtained from other studies. Finally, we present the application of our method on the compiled catalogue MARK II of D. Burstein. Measurement errors involved in the determination of the distances to galaxies are taken into accounts.

II. Our smoothing procedure

II.1 The philosophy

Dekel and Bertschinger (1989) have pointed out that if the cosmic velocity field $\mathbf{v}(\mathbf{x})$ derives from a potential ($\mathbf{v}(\mathbf{x}) = \nabla\,\Phi(\mathbf{x})$, i.e : $\mathbf{v}$ is curl-free), this kinematical potential Φ can be extracted by integrating the radial component of the velocity field $v_r(\mathbf{x})$ along the line-of-sight :

$$\Phi(\mathbf{x}) \;=\; (Pv_r)(\mathbf{x}) \;=\; \int_0^1 dl\, v_r(l\,\mathbf{x}) \tag{1}$$

But the observed radial velocity field is sampled on a spatial support defined by the positions of galaxies throughout the space. Thus, in order to evaluate $v_r(\mathbf{x})$ all along the line-of-sight, one needs to first smooth the observed radial velocity field.

This smoothing procedure is simple if the spatial support (the positions of galaxies) of the field is an ideal 3-dimensional grid. We have shown in RLH I that, thanks to the wavelet reconstruction theorem, the radial velocity field can be decomposed as follows :

$$v_r(\mathbf{x}) \;=\; (Wv_r)(\mathbf{x}) \;=\; \int_0^\infty \frac{ds}{s}\; v_r^{(s)}(\mathbf{x}) \tag{2}$$

where the integral is performed over the scales s and $v_r^{(s)}(\mathbf{x})$ is equal to the spatial convolution of the radial velocity field $v_r(\mathbf{x})$ with the "reproducing kernel" $K(s,\mathbf{x},\mathbf{y})$, centered on $\mathbf{x}$ and of spatial extension s :

$$v_r^{(s)}(\mathbf{x}) \;=\; \int_0^\infty \int_0^\infty \int_0^\infty d\mathbf{y}\; K(s,\mathbf{x},\mathbf{y})\, v_r(\mathbf{y}) \tag{3}$$

As the scale s decreases, more and more detailed information is available concerning the radial velocity field. If the observed galaxies are distributed on a grid of elementary length scale s_c, we introduce the 2 operators W_{s_c} and W^{s_c} acting on v_r as follows :

$$v_r \;=\; Wv_r \;=\; W_{s_c}v_r \;+\; W^{s_c}v_r \tag{4}$$

$$(W_{s_c} v_r)(\mathbf{x}) = \int_{s_c}^{\infty} \frac{ds}{s} \, v_r^{(s)}(\mathbf{x}) \tag{5}$$

$$(W^{s_c} v_r)(\mathbf{x}) = \int_{0}^{s_c} \frac{ds}{s} \, v_r^{(s)}(\mathbf{x}) \tag{6}$$

$(W^{s_c} v_r)(\mathbf{x})$ contains the information about $v_r(\mathbf{x})$ at all scales smaller than s_c. As a matter of fact, it is not possible to evaluate the function $v_r^{(s)}(\mathbf{x})$ intervening in equation 6 for scales smaller than s_c because then the spatial convolution (equation 3) is performed with the reproducing kernel having spatial extension smaller tha n the elementary length scale of the grid.

On the other hand, $v_r^{(s)}(\mathbf{x})$ is a well-defined quantity for scales larger than s_c. It can be evaluated with no prejudice by replacing the integral over the space involved in equation 3 by its associated discrete riemannian sum over the grid (because the spatial extension of the kernel is indeed greater than s_c). Thus the function $(W_{s_c} v_r)(\mathbf{x})$, i.e : the wavelet reconstruction of the radial velocity field stopped at the cut-off scale s_c, can be evaluated and contains all the the information that can be extracted from the radial velocity field sampled on a grid of elementary length scale s_c. $(W_{s_c} v_r)(\mathbf{x})$ may be roughly compared with a smoothed version of the radial velocity field with a smoothing window function of size s_c (but not with the complete radial velocity field $v_r = W_{s_c} v_r + W^{s_c} v_r$, because $W^{s_c} v_r$ is unknown).

Unfortunately, real catalogs of galaxies for which radial peculiar velocities are measured are sparsely sampled throughout space. It thus becomes impossible to define an elementary lenght scale s_c. Indeed, the separation between neighbouring galaxies varies from place to place : it is large in the undersampled regions of the catalog and small in the oversampled regions. After several tests, Bertshinger et *al.* (1990) in the POTENT method chose to smooth the cosmic radial velocity field with a smoothing length scale constant throughout the space. Fictitious information is thus added where voids larger than the constant smoothing length scale are present, and information is lost in oversampled regions. This loss of information has to be avoided, especially because actual peculiar velocity catalogs don't possess a lot of data points. By permitting the smoothing length scale to vary from place to place, our goal is to extract a smoothed velocity field in output that contains all the information present in the catalog of observed radial velocity of galaxies.

Moreover, we wish that our smoothed velocity field should be comparable to a theoretical quantity, directly linked with the real radial velocity field. For instance, this is not the case for the smoothing scheme used in the POTENT method (the smoothed field derived from velocities sampled on an inhomogeneous support doesn't coincide with the

smoothed field derived from the same velocity field sampled on a grid). This point is particularly important in order to compare the output smoothed radial velocity field with any other smoothed field obtained from different studies.

II.2 The smoothed velocity field in output

Because galaxies of real catalogs are distributed inhomogeneously in space, it is not possible to define a cut-off scale s_c common to all the regions of space sampled by the catalog and to apply the operator W_{s_c} on the observed radial velocity field. However, if we first restore homogeneity to the spatial support of the field, we can afterwards apply the operator W_{s_c} without prejudice. Our smoothing scheme explores just this possibility. We call $E_{\mathbf{x}}$ the real space where the spatial support $\{\mathbf{x}_i\}_{i=1,N}$ of the catalog is inhomogeneously distributed and we define by $\rho(\mathbf{x})$ the spatial distribution of the support in this space. We introduce a mapping μ from this real space $E_{\mathbf{x}}$ into a fictitious space E_μ such that the image $\{\mu_i = \mu(\mathbf{x}_i)\}_{i=1,N}$ of the support by the mapping μ is uniformly distributed in the space E_μ :

$$\mu \ : \ \begin{cases} E_{\mathbf{x}} \ \longrightarrow \ E_\mu \\ \\ \mathbf{x} \ \longmapsto \ \mu(\mathbf{x}) \end{cases} \qquad J_\mu(\mathbf{x}) \ = \ \left| det \left[\frac{\partial \mu_j}{\partial x_k}(\mathbf{x}) \right] \right| \ = \ \rho(\mathbf{x}) \tag{7}$$

In practice, we evaluate the mapping μ using an algorithm. The fact that $\rho(\mathbf{x})$ is a density distribution function ensures us that the inverse mapping μ^{-1} is a well-defined function.

The first step of our smoothing scheme is to associate to the set of data $\{v_r(\mathbf{x}_i)\}_{i=1,N}$ of the real space $E_{\mathbf{x}}$, the set $\{v'_r(\mu_i) \ = \ v'_r(\mu(\mathbf{x}_i))\}_{1,N}$ in the fictitious E_μ space. This operation is illustrated figures 1 and 2. We have simulated a cosmic radial velocity field sampled on the support defined by the real positions (expressed in cartesian supergalactic coordinates) of the galaxies of MARK II catalog compiled by D. Burstein. The figures show the radial velocity field on nine cuts passing trough a cube of size 10000 km.s^{-1} centered on our galaxy. Figure 1 shows $\{v_r(\mathbf{x}_i)\}_{i=1,N}$ in the real space $E_{\mathbf{x}}$ and figure 2 $v'_r(\mu(\mathbf{x}_i))\}_{i=1,N}$ in the fictitious E_μ space.

We remark that the function v'_r is sampled on an homogeneous suppor t $\{\mu_i\}_{i=1,N}$ in E_μ. It is thus possible to define an elementary length scale s_μ and to perform in the E_μ space the wavelet reconstruction $W_{s_\mu} v'_r$ of v'_r stopped at the cut-off scale s_μ. This is done figure 3. Note that $W_{s_\mu} v'_r(\mu)$ is defined for every μ of E_μ.

The last step of our smoothing procedure is to come back to the real space $E_{\mathbf{x}}$ through the inverse mapping μ^{-1}. Our output smoothed velocity field $(Mv_r)(\mathbf{x})$ is finally the field corresponding to $W_{s_\mu} v'_r$ in the real space $E_{\mathbf{x}}$:

$$(Mv_r)(\mathbf{x}) \ = \ (W_{s_\mu} v'_r)(\mu(\mathbf{x})) \tag{8}$$

Note that $W_{s_\mu} v'_r(\mu)$ contains all the information which can be extracted from the data in the E_μ space. Thus, because the mapping μ establishes a one-to-one correspondance between $E_{\mathbf{x}}$ and E_μ, our smoothing procedure is minimal (no loss of information).

II.3 Link with a theoretical quantity

Thanks to the scaling properties of the wavelet transforms, our output smoothed radial velocity field $(Mv_r)(\mathbf{x})$ can be linked with a theoretical quantity. We prove in RLH II that, as long as the mapping μ verifies a validity condition (see below equation 10), the following equality holds :

$$(Mv_r)(\mathbf{x}) \;=\; (W_{s_c(\mathbf{x})} v_r)(\mathbf{x}) \quad with \quad s_c(\mathbf{x}) \;=\; \frac{s_\mu}{\rho(\mathbf{x})^{1/3}} \tag{9}$$

Thus our output smoothed radial velocity field is equal to the wavelet reconstruction of v_r stopped at the cut-off scale map $s_c(\mathbf{x})$ which varies with $\mathbf{x}$. We show in figure 5 the cut-off scale map associated to the spatial distribution previously presented in figure 1. The value of $s_c(\mathbf{x})$ is derived from the jacobian associated with the mapping μ (see equation 6). The lower the density at the position $\mathbf{x}$, the larger is its associated cut-off scale. We present in figure 6 the wavelet reconstruction of the previous simulated radial velocity field stopped at the cut-off scale map $s_c(\mathbf{x})$. We notice that even if the main features remain, our smoothed radial velocity field $(Mv_r)(\mathbf{x})$ of figure 4 shows some differences with $(W_{s_c(\mathbf{x})} v_r)(\mathbf{x})$. The reason for this discrepancy is that the mapping μ doesn't verify the validity condition which stipulates that for every $\mathbf{x}$ and vector $\mathbf{h}$:

$$\text{if } \|\mathbf{h}\| \;\leq\; s_c(\mathbf{x})\,, \qquad \left\| \left[\frac{\partial \mu_j}{\partial x_k}(\mathbf{x})\right] .[\mathbf{h}] \right\| \;\approx\; \left| det \left[\frac{\partial \mu_j}{\partial x_k}(\mathbf{x})\right] \right|^{1/3} \times \|\mathbf{h}\| \tag{10}$$

or in other words that the mapping μ is locally equivalent to a rotation-dilation transformation (see RLH II).

However, we can discard the regions of space where the validity condition doesn't hold by evaluating and then comparing the two terms of the equation 10. Moreover, we can improve this validity condition by using anisotropic wavelets (see RLH II).

III. The kinematical potential $\Phi(\mathbf{x}) \;=\; (Pv_r)(\mathbf{x})$

We have shown in section II that, for the regions of space where the validity condition is verified, our smoothed radial velocity field $(Mv_r)(\mathbf{x})$ matches $(W_{s_c(\mathbf{x})} v_r)(\mathbf{x})$. If the cosmic velocity field is irrotational, it is thus possible to inferr the kinematical potential from the radial velocity field only (see equation 1). Unfortunatly, the operator P involved in this operation presents a non-local character. Hence, the potential derived from the smoothed

radial velocity field $P \circ W_{s_c(\mathbf{x})} v_r$ differs from the smoothed potential of the velocity field $W_{s_c(\mathbf{x})} \circ P v_r$ (and of course from the non-smoothed kinematical potential $\Phi = P v_r$) (see RLH II). We illustrate this discrepancy by plotting in figure 7 the potential derived from $(W_{s_c(\mathbf{x})} v_r)(\mathbf{x})$ and in figure 8 the smoothed simulated potential $(W_{s_c(\mathbf{x})} \Phi)(\mathbf{x})$. We want to emphasize that this behaviour is not due to the way we smooth the radial velocity field but is intrinsically linked to the nature of the operator P. The point has its importance since the kinematical potential (or the reconstructed 3-dimensional velocity field) derived from catalogs of the radial peculiar velocity field is often considered as data input for other studies (). For example, the mass density perturbation field $\delta(\mathbf{x})$ is linked to the kinematical potential $\Phi(\mathbf{x})$ through the Poisson equation ($\delta(\mathbf{x}) \propto \nabla^2 \Phi(\mathbf{x})$). A smoothed mass density field is extracted from the analysis of the spatial distribution of galaxies. Unfortunatly :

$$(W_{s_c} \delta)(\mathbf{x}) = (\nabla^2 (W_{s_c} \circ P v_r))(\mathbf{x}) \neq (\nabla^2 (P \circ W_{s_c} v_r))(\mathbf{x}) \tag{11}$$

We thus have to be very cautious when comparing the kinematical potential derived from observed radial peculiar velocity catalogs with quantities obtained from other studies such as those based on number counts.

IV. Application to a real catalog

Finally we present the application of our method on the MARK II catalog of D. Burstein (the same which is used in Bertshinger et *al.* (1990)) within a cube of size 10000 km.s^{-1} centered on our galaxy (416 independent objects are sampled). Our smoothed output radial velocity field $(M v_r)(\mathbf{x})$ (figure 9) is the wavelet reconstruction of $v_r(\mathbf{x})$ stopped at the cut-off scale of figure 5 in the regions of space where the validity condition (equation 9) is satisfied. From this smoothed radial velocity field, we have derived the kinematical potential $(P \circ M v_r)(\mathbf{x})$ (figure 10). The "Great attractor" flow appears clearly in the supergalactic plane. Figure 11 and 12 show the effects of measurement errors involved in the determination of the distances of galaxies. From the original redshift-distance sample, we have created 10 samples with perturbed distances (with a lognormal distribution of errors and $\Delta = 20\%$). Distance as well as peculiar velocity of galaxies are thus modified for each sample. Figure 11 shows the potential of the average over the 10 samples of the smoothed radial velocity field $(P \circ \overline{M v_r})(\mathbf{x})$. This field differs from $(P \circ M v_r)(\mathbf{x})$ because the distance and the peculiar velocity for each galaxy are correlated when the original sample is perturbed. We present in figure 12 the standard dispersion due to measurement errors in distances. We see that this dispersion is sufficiently small for a large region of the sampled volume.

IV. Conclusion

We have presented a method, based on the properties of the wavelet transforms, for smoothing a field sampled on a support inhomogeneously distributed throughout the space. Our smoothing scheme is minimal (no loss of information) and our output smoothed field can be compared with a well-defined theoretical quantity, as long as the spatial support of the field verifies some criteria. The application of this smoothing scheme to the observed cosmic radial velocity field reveals some limitations concerning the reconstruction of the kinematic potential from the smoothed radial velocity field. Indeed, we prove that this potential can't be generally compared without errors with quantities obtained from other cosmological studies. The influence of this kind of errors has to be seriously taken into account.