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When shop scheduling meets dominoes, eulerian and hamiltonian paths

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1 Introduction

In this paper, we consider no-idle and no-wait shop scheduling problems with some standard configurations namely flow shop, job shop and open shop. We focus on the makespan, referred to as C_{\max} , as a performance measure. More precisely, we tackle four distinct problems which, using the general three-field notation [4], can be denoted as $F2|no-idle, no-wait|C_{\max}$ for the 2-machine flow shop, $J2|no-idle, no-wait|C_{\max}$ for the 2-machine job shop and $O2|no-idle, no-wait|C_{\max}$ for the 2-machine open shop. The m -machine flow shop problem is denoted as $F|no-idle, no-wait|C_{\max}$. We refer first to problem $F2|no-idle, no-wait|C_{\max}$: n jobs are available at time zero; each job j must be processed non-preemptively on two continuously available machines M_1, M_2 with integer processing times $p_{1,j}, p_{2,j}$, respectively. The processing order is $M_1 \rightarrow M_2$ for all jobs. Each machine processes at most one job at a time and operations of each job cannot overlap. For any sequence, $[j]$ denotes the job in position j . The objective is the minimization of the makespan. For the 2-machine jobshop problem, each job is required to strictly follow its own specific processing order and for the 2-machine openshop problem, the operations of a job can be processed in any order.

With respect to the literature, to the best of our knowledge, few works have dealt with the no-idle and no-wait constraints simultaneously. In [1], it is mentioned that both problems $F2|no-idle|\sum C_j$ and $F2|no-wait|\sum C_j$ are NP -hard. Similar consideration holds for problem $F2|no-idle, no-wait|\sum C_j$. The relevant literature includes [3] where it is shown that minimizing the number of interruptions on the last machine is solvable in $O(n^2)$ time on two machines (the problem is denoted as $F2|no-wait|\mathcal{G}$) while it is NP -hard on three machines or more. We remark that problems $F2|no-wait|\mathcal{G}$ and $F2|no-idle, no-wait|C_{\max}$, even though close are not equivalent and an optimal solution with no interruption of problem $F2|no-wait|\mathcal{G}$ may be non-optimal for problem $F2|no-idle, no-wait|C_{\max}$. Consider a 2-job instance with processing times $p_{1,1} = i, p_{2,1} = i, p_{1,2} = i, p_{2,2} = j$, with $j > i$. Then, sequence 1-2 is no-idle, no-wait,

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has makespan $C_{\max}^{1-2} = 2j + i$ and is optimal for problem $F2|no - wait|\mathcal{G}$ as it has no interruption. However, it is not optimal for problem $F2|no - idle, no - wait|C_{\max}$ as sequence 2 - 1 is also no-idle, no-wait and has makespan $C_{\max}^{2-1} = 2i + j < 2j + i$.

In the next section we present some complexity results related to the four tackled shop scheduling problems.

2 Main results

We first point out that the *no - idle, no - wait* constraint is very strong as it forces consecutive jobs to share common processing times, namely, any feasible solution for $F2|no - idle, no - wait|C_{\max}$, requires that $\forall j \in \dots, n - 1, p_{2,[j]} = p_{1,[j+1]}$. Figure 1 provides an example of a feasible no-idle, no-wait schedule for a 2-machine flow shop with four jobs.

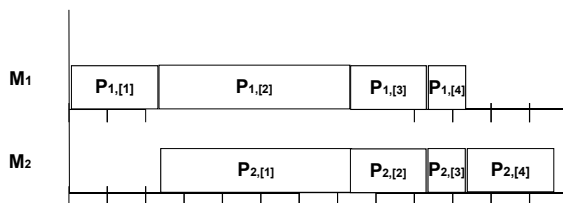


Figure 1: A no-idle no-wait schedule for a 2-machine flow shop

The peculiarity of the *no - idle, no - wait* effect strictly links the above mentioned flow shop problem to the game of dominoes. Dominoes are 1 x 2 rectangular tiles with each 1 x 1 square marked with spots indicating a number. A traditional set of dominoes consists of all 28 unordered pairs of numbers between 0 and 6. We refer here to the generalization of dominoes presented in [2] in which n tiles are present, each of the tiles can have any integer (or symbol) on each end and not necessarily all pairs of numbers are present.

In [2], it is shown that the Single Player Dominoes (*SPD*) problem, where a single player tries to lay down all dominoes in a chain with the numbers matching at each adjacency, is polynomially solvable as it can be seen as the solution of an eulerian path problem on an undirected multigraph. Figure 2 shows the solution of an *SPD* problem with 12 tiles with numbers included between 0 and 6.

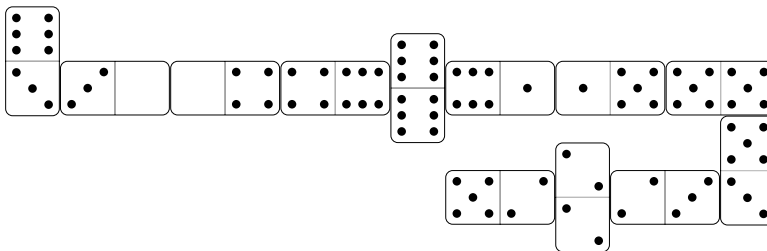


Figure 2: Solution of an SPD problem with 12 dominoes

We refer to the oriented version of *SPD* called *OSPD* where all dominoes have an orientation, e.g. if the numbers are i and j , only the orientation $i \rightarrow j$ is allowed but not viceversa. It is easy to show that also problem *OSPD* is polynomially solvable as it can be seen as the solution of an eulerian path problem on a directed multigraph.

The following proposition holds.

Proposition 1 $F2|no-idle, no-wait|C_{max} \propto OSPD$ and can be solved in $O(n)$ time.

Problem $F2|no-idle, no-wait|C_{max}$ is also linked to a special case of the Hamiltonian Path problem on a connected digraph. Consider a digraph $G(V, A)$ that has the following property: $\forall v_i, v_j \in V$, either $S_i \cap S_j = \emptyset$, or $S_i = S_j$ where we denote by S_i the set of successors of vertex v_i . In other words, each pair of vertices either has no common successor or has all successors in common. Let indicate the Hamiltonian path problem in that graph as the Common/Distinct Successors Hamiltonian Directed Path (CDSHDP) problem.

The following proposition holds.

Proposition 2 $CDSHDP \propto F2|no-idle, no-wait|C_{max}$. Correspondingly, problem *CDSHDP* is polynomially solvable.

The analysis leading to Proposition 1 can be extended also to the m -machine case. The following proposition holds.

Proposition 3 Problem $F|no-idle, no-wait|C_{max}$ is polynomially solvable.

Finally, by reduction from the Numerical Matching with Target Sums (NMTS) problem, the following proposition holds.

Proposition 4 Problems $J2|no-idle, no-wait|C_{max}$ and $O2|no-idle, no-wait|C_{max}$ are NP-Hard in the strong sense.

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