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# Trade, Environment and Income Inequality: An Optimal Taxation Approach

Philippe Bontems\* and Estelle Gozlan†

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## Abstract

In a small open economy, how should a government pursuing both environmental and redistributive objectives design domestic taxes when redistribution is costly? And how does trade liberalization affect the economy's levels of pollution and inequalities, when taxes are optimally and endogenously adjusted? Using a general equilibrium model under asymmetric information with two goods, two factors (skilled and unskilled labor) and pollution, this paper characterizes the optimal mixed tax system (nonlinear income tax and linear commodity and production taxes/subsidies) with both production and consumption externalities. While optimal income taxes are not directly affected by environmental externalities, conditions are derived under which under- or over-internalization of social marginal damage is optimal for redistributive considerations. Assuming that redistribution operates in favor of the unskilled workers and that the dirty sector is intensive in unskilled labor, simulations suggest that trade liberalization involves a clear trade-off between the reduction of inequalities and the control of pollution when the source of externality is only production; this is not necessarily true with a consumption externality. Finally, an increase in the willingness to redistribute income towards the unskilled results paradoxically in less pollution and more income inequalities.

**JEL:** H21, H23, F13, F18

**Keywords:** Optimal tax theory; International trade; Pollution; Production efficiency

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# 1 Introduction

The social and environmental consequences of trade liberalization have received considerable attention both in civil society and in the economic literature. Governments in developed countries regularly face anti-globalization movements, due to concerns about job losses, growing inequalities, increased pressure on the environment, and national sovereignty losses.<sup>1</sup> Conversely, environmental protection is often seen as a threat to international competitiveness in political debates.

It is sometimes argued that many negative outcomes imputed to trade liberalization are rather the consequence of missing or badly designed environmental and redistributive policies. Among trade economists, there is wide agreement that domestic distortions should be addressed through domestic policy intervention, and that trade restrictions for domestic policy objectives can at best be second best (Bhagwati, 1994). In particular, when local environmental externalities are the only market failure, a move towards free trade with optimal environmental policy is always welfare-improving, even though it may involve an increase in pollution : with endogenous policy, any increase/decrease in pollution then reflects an optimal trade-off between pollution and income at the country level (Copeland & Taylor, 2003). However, these results are obtained within a representative-agent framework. Yet the view that the efficiency gains brought by trade liberalization allow governments to leave everyone better off with well-targeted redistributive policies has been objected in modern normative economic theory: because individual characteristics are not directly observable by governments, individualized lump-sum transfers are not feasible in practice, which challenges the separation of efficiency and redistributive considerations (Naito, 1996; Guesnerie, 2001; Tuomala, 2016).

On the other hand, environmental policy itself can have important distributive effects

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<sup>1</sup>The success of protectionist parties in the 2014 elections for the European parliament, and activism against the US-EU negotiations on the Transatlantic Free Trade Area, are recent illustrations of such concerns. Recently, Donald Trump's election has been seen by many analysts as the result of political claims resurgence from losers of globalization.

across heterogeneous households, whether it be through the regressive effect of consumption taxes (if low-income individuals are affected more heavily), or be it through changes in relative returns to factors resulting from production taxes.<sup>2</sup> In other words, the internalization of both consumption and production externalities might interfere with any given redistribution objective.

Keeping this in mind calls for a fresh look at the optimal tax system as a whole in an open economy (i.e. where the pattern of specialization matters for environmental and inequality-related outcomes), when the government pursues both environmental and redistributive objectives. In particular, one would like to know under which conditions reducing pollution and redistributing wealth are conflicting tasks if any. This issue actually translates into asking whether the government should over- or under-internalize social marginal environmental damage by means of taxes on polluting goods. Also, one would like to know how the government should adapt its fiscal system including environmental taxes when facing exogenous shocks like increasing globalization or changes in social preferences with respect to wealth redistribution.

To investigate these issues, we consider a Heckscher-Ohlin model of a small open economy with two sectors (clean and polluting) and two factors (skilled and unskilled labor). Assuming that emissions may arise either from the production or from the consumption of the polluting good, we characterize the optimal tax system when policy is constrained only by the information available to the regulator. The government maximizes a weighted sum of skilled and unskilled agents' utility, using non-linear income taxes (because skills are not observable) and linear taxes/subsidies on consumption and production of the polluting good (because purchases are anonymous). Income redistribution is socially costly because of incentive compatibility. While optimal income taxes are not directly affected by environmental externalities, we show that optimal distortion of the consumer and the producer

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<sup>2</sup>See Fullerton (2011) for a complete review of the mechanisms at stake, including the possibility that different types of agents might face different damages from emissions (a possibility not considered in this paper).

prices in the polluting sector consist in a pigovian term and a redistributive term. It is then optimal to over- or under-internalize the social marginal environmental damage whenever it helps decreasing the cost of wealth redistribution due to incentive compatibility. From Pirttilä & Tuomala (1997), we know that, for consumption externalities, this depends on the way pollution, consumption and leisure interact within individual preferences. In particular, assuming that preferences are not separable, there is accordance (conflict) between environmental and redistributive concerns when the polluting good and leisure are complements (substitutes). In the present paper, because the non linear technology with imperfect substitutability between skilled and unskilled labor types makes wages endogenous, it is also optimal to distort producer prices for redistributive considerations, and whenever the sector making intensive use of unskilled labor is also characterized by production externalities, then the pigovian and redistributive components of the production tax/subsidy are of opposite signs (under-internalization).

We then explore the consequences on the level of pollution and inequality of exogenous shocks on this economy (a move towards freer trade; a change in the government priorities in terms of redistribution), when taxes are optimally and endogenously adjusted. We show that (i) under openness to trade, the source of the externality (consumption or production) matters for redistribution, while it is not the case in autarky; (ii) whether the economy specializes into the clean or the dirty sector, trade liberalization involves a clear trade-off between the control of income inequalities and the control of pollution when the source of externality is production; this is not necessarily true with a consumption externality; (iii) an increasing willingness to redistribute income corresponds to a shrinking economy, where consumption and production decreases, and therefore results in less pollution; however, the level of after-tax income inequalities paradoxically increases because of an increasing cost of redistribution.

The paper is organized as follows. The related literatures and our original contributions are exposed within the next subsection. Section 2 presents the basic framework in a general

formulation that will be used to characterize the optimal structure of taxation in a situation of trade (section 3) and in autarky (section 3.5). Section 4 is devoted to the simulation-based analysis of how increasing globalization and policy changes impact the extent of labor income inequality and pollution. Section 5 draws conclusions.

## Related literature and our contributions

This paper borrows from and contributes to several strands of the literature. First, the tax incidence literature addresses the general equilibrium effects of environmental policy in economies with heterogenous households, highlighting in particular the importance of the substitutability between factors, and on the relative factor intensity in the polluting sector (Fullerton & Heutel, 2007), a feature that will prove also to be important in our analysis.<sup>3</sup> Within the “green tax reform” literature, redistribution and environmental considerations are addressed through the recycling of environmental tax revenues in the vein of the double-dividend literature.<sup>4</sup> These papers take the initial tax system as given, and the scope for improvement from tax reform is also limited by the usual revenue-neutrality assumption.<sup>5</sup> In contrast, we focus on the welfare-maximizing design of the tax system as a whole, following an optimal taxation approach.

The second-best analysis of interactions between environmental policy and public finance has initially combined the Ramsey and Pigovian objectives of taxation (Bovenberg & van der

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<sup>3</sup>Using a GE model of the incidence of an environmental tax that allows for differential factor intensities and general forms of substitution among inputs of labor, capital and pollution, Fullerton & Heutel (2007) characterize the effects on prices and quantities resulting from an exogenous change in the pollution tax. While labor is homogenous in their model, it can be easily adapted with heterogenous skills (Fullerton & Monti, 2013).

<sup>4</sup>In particular, Fullerton & Monti (2013) study the conditions under which tax rebates can protect low wage earners, while Chiroleu-Assouline & Fodha (2014) characterize the tax-revenue recycling mechanisms allowing for Pareto-improving environmental tax reforms.

<sup>5</sup>The first wave of literature on the double-dividend was on the potential efficiency gains brought by environmental taxes if the revenue was used to cut other distortionary taxes (Goulder, 1995), without any equity considerations; however, it has been argued that an important reason why the preexisting tax system should be distortionary was redistributive considerations (Bovenberg, 1999; Sandmo, 1998) : this is an argument in favor of optimal taxation models, where distributive and environmental policies are designed simultaneously - which can be understood as an endogenous optimization over environmental tax revenue recycling.

Ploeg, 1994), focusing on the optimal linear tax structure to internalize externalities and meet an exogenous revenue target. Sandmo (1975) introduced distributional considerations among unequally productive households.<sup>6</sup> However, imposing the linearity of tax instruments puts unnecessary restrictions on the optimum, while “a truly optimal tax system must implement a Pareto-efficient allocation constrained only by the information structure in the economy” as suggested by Cremer et al. (2015).

Because one objective of this paper is to disentangle the role of trade liberalization and of policy on pollution and inequalities, we need to design domestic taxes as optimally as possible, under realistic constraints of asymmetric information (unobservable skills, anonymous purchases of commodities): therefore our analysis follows the mirrleesian tradition to optimal taxation (Mirrlees, 1971), and builds on Atkinson & Stiglitz (1976)’s mixed taxation model, allowing for nonlinear income taxes, with linear commodity taxes. Most related to this paper are the contributions of Pirttilä & Tuomala (1997); Cremer et al. (1998); Micheletto (2008); Cremer et al. (2010) and Jacobs & de Mooij (2015). All these papers address the trade-off between environmental and redistributive goals within optimal taxation models.<sup>7</sup>

Importantly, while these papers assume that agents differ in their labor productivity (which determines their earning abilities), the representation of the productive sector usually adopted relies on a linear technology with all types of labour being perfectly substitutable. This implies (like in Mirrlees’s model) that wages are fixed, and there is no need to distort production efficiency. Cremer et al. (2010) overcome this shortcoming in studying optimal

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<sup>6</sup>He finds scope for taxing externality-generating commodities according to the Pigovian principle, even in a second best world where the allocative function of the price system cannot be separated from its effect on distribution.

<sup>7</sup>Pirttilä & Tuomala (1997) highlight the role that pollution can play on the individual incentive compatibility constraints that originate from the unobservability of skills and that make wealth redistribution socially costly. In a setting where agents differ along two dimensions (skills and tastes), Cremer et al. (1998) compare the way externalities affect the optimal mixed taxation system when the government can observe individual consumption choices (allowing for non-linear commodity taxes as a tool for redistribution), and when only aggregate purchases can be observed (linear commodity taxes). Micheletto (2008) provides a quite general framework, for a closed economy with fixed wages, to design taxes optimally with asymmetric information and externalities, allowing in particular for pollution to affect agents in a differentiated way. See also Jacobs & de Mooij (2015) using an alternative definition for the cost of public funds, and linking the results from the non-linear optimal taxation and the double-dividend literature.

environmental and income taxes when wages are endogenous.<sup>8</sup> Yet, they assume that labor is homogenous in efficiency units (although different agents have different endowments) and show that consequently the Diamond & Mirrlees (1971) production efficiency result still holds despite endogenous wages. We extend this literature by allowing for endogenous wage formation with different types of labor acting as imperfect substitutes within non-linear technologies - finding scope for optimal departure from production efficiency.

Our representation of the productive sector (using a standard 2-factors, 2-goods Heckscher-Ohlin model) bridges the gap between the aforementioned literature and the traditional trade theory of factor pricing. In particular, a related literature examines the labor-income redistribution issue in open economies, when government intervention is constrained by the unobservability of labor types.<sup>9</sup> When different types of labor are imperfect substitutes in production, Naito (1996, 2006) finds that tariffs or production taxes/subsidies are optimal because it reduces the incentive cost of the nonlinear income tax system.<sup>10</sup> In a related paper, Spector (2001) shows that trade, by making prices exogenous, reduces the government's set of redistributive policies, and as a result that opening borders may decrease welfare when the use of subsidies/tariffs is prohibited, even if income taxes can be optimally adjusted. Environmental issues are not considered in these contributions. The present paper extends this literature in several directions: we consider the pollution and the trade issues simultaneously; we allow for both production and consumption externalities, and examine their respective impact on incentive compatibility constraints under different assumptions regarding the separability of the utility function in consumption, leisure and pollution; and we provide a full characterization of the optimal mixed tax system (nonlinear income tax and linear commodity and production taxes/subsidies) in two situations of interest, autarky and international trade.

Finally, the Heckscher-Ohlin model with pollution adopted here borrows from the trade-

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<sup>8</sup>See also Tenhunen (2007b).

<sup>9</sup>See Verdier (2005) for a survey.

<sup>10</sup>This is because lower costs of redistribution outweigh the inefficiencies created in production



and-environment framework a la Copeland and Taylor (2003). While a limitation of the present paper is that we do not take into account the possibility of abatement in polluting emissions, as Copeland and Taylor do, we extend their model by making factor supply endogenous (a departure from the fixed endowments assumption in many trade models), by letting comparative advantage be influenced by both environmental and redistributive policies (whereas endowments and environmental policy determine the economy's comparative advantage in Copeland and Taylor), and by addressing both production and consumption externalities.

## 2 The model

We consider a two agent types, two factors and two goods model of a small, perfectly competitive economy, facing fixed world prices. The government designs and implements the whole fiscal system, composed of income taxes, commodity and production subsidies/taxes, and consumer-workers react by choosing their consumption of private goods and labour supply. There are two types of consumer-workers, skilled or unskilled (denoted respectively by superscript  $s$  and  $u$ ), in respective proportions  $\pi^s$  and  $\pi^u$  (with  $\pi^s + \pi^u = 1$ ).

One of the two goods (say good 1) is chosen as the numeraire and without loss of generality goes untaxed so that the design of the indirect tax system is reduced to the choice of the commodity tax and production subsidy on good 2. We denote  $p_k^*$  the international price of good  $k$ ,  $k = 1, 2$  whereas  $p_k$  and  $q_k$  denote respectively the producer and consumer price in the country. As good 1 is the numeraire, we also normalize for convenience the international price of good 1 to 1. This implies that  $p_1^* = p_1 = q_1 = 1$ . We assume that the good 2 produced at home and in the rest of the world are perfect substitutes and hence it follows that the government does not need to impose a tariff on good 2 as this instrument would be redundant given the production subsidy and the commodity tax on good 2. To save on notations, we denote hereafter  $p^*$ ,  $p$  and  $q$  the international, production and consumer price for good 2. Hence, the production subsidy on good 2 is given by  $p - p^*$  and the commodity

tax on good 2 is given by  $q - p^*$ .

Polluting emissions  $e$  stem from both consumption and production of good 2. More precisely, denoting the consumption of good 2 by agent of type  $i$  as  $c_2^i$ , then the corresponding aggregate consumption  $c_2$  is:

$$c_2 = \pi^s c_2^s + \pi^u c_2^u$$

and denoting the production level of good 2 as  $y_2$ , we define the aggregate level  $e$  of emissions by:

$$e = E(c_2, y_2). \tag{1}$$

We assume that the function  $E(.,.)$  is increasing in each argument. The reason why we model a situation with externalities coming from both production and consumption is that, as it will become clear, the source of externality matters for the features of the open economy equilibrium, especially with respect to the relationship between income inequality and pollution.

## 2.1 The consumers

Workers have preferences over two consumption goods, labor supply  $l$  and the public bad (pollution)  $e$ , given by a strictly quasi-concave utility function  $U(c_1, c_2, l, e)$ . We assume that  $U(.)$  is strictly increasing in consumption of the two goods and decreasing in labor supply and public bad. We assume that both private goods and leisure are normal goods. Also we assume, as in most of the literature, that all types of agents face the same pollution level  $e$  (the externality is “atmospheric”).<sup>11</sup>

At this stage, we do not make any specific assumption regarding the separability of pollution, leisure and consumption, in order to take into account the impact of pollution on both consumption and labor supply. Indeed, a change in the pollution level may well influence the pattern of consumption: for instance an improvement in environmental quality may

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<sup>11</sup>However, it is straightforward to extend the model to the case where workers are not equally vulnerable to the externality (see footnote 16). See also Hotte & Winer (2012) for a model where individuals are not equally vulnerable to pollution and can privately mitigate its impact.

increase the marginal utility of consuming outdoor activities. Furthermore, pollution and leisure may be complements or substitutes as well: improving the quality of the environment may increase the disutility of labor (because one would want to consume more of leisure activities) or may decrease it because of reduced pollution-related health problems.

As it is usual in the optimal taxation literature, we break the individual's optimization problem into two parts: (i) given disposable income, choose the optimal commodity basket and obtain the conditional indirect utility function; (ii) choose hours of work.

Let  $I^i$  denoting the disposable (or after tax) income for consumer  $i = s, u$ . We define the conditional indirect utility function as follows:

$$V(q, I^i, l^i, e) = \max_{c_1^i, c_2^i} U(c_1^i, c_2^i, l^i, e) \text{ s.t. } c_1^i + qc_2^i = I^i$$

and the optimal  $c_1^i$  and  $c_2^i$  of this maximization program are the conditional demand functions, denoted respectively  $c_1(q, I^i, l^i, e)$  and  $c_2(q, I^i, l^i, e)$ . Note that the influence of pollution on the consumption pattern depends essentially on how the marginal rate of substitution between the two goods ( $(\partial U^i / \partial c_2) / (\partial U^i / \partial c_1)$ ) evolves with pollution.

At the second stage of utility maximization, the number of hours worked  $l^i$  is chosen to maximize indirect utility subject to the relationship between primary (or before tax) income and disposable income as implied by the income tax schedule:

$$l^i(q, e) = \arg \max V(q, I^i, l^i, e) \text{ s.t. } I^i = w^i l^i - T(w^i l^i) \quad (2)$$

where  $w^i$  denotes the wage rate for workers of type  $i$  and  $T(w^i l^i)$  the income tax.

Finally, we assume that the agent monotonicity condition hold, meaning that the marginal rate of substitution between primary income  $wl$  and after-tax income  $I$ , i.e.  $MRS = -(1/w)(\partial V / \partial l) / (\partial V / \partial I)$ , is lower for the skilled worker compared to the unskilled one. This means that in space  $(wl, I)$  the curves for  $V$  are flatter for the skilled worker as this worker can transform labor in consumption more easily.

## 2.2 The producers

Concerning production, we assume the standard Heckscher-Ohlin framework. There are two industries that each exhibits constant returns to scale and a concave production function,  $y_k = F_k(L_k^u, L_k^s)$ ,  $k = 1, 2$ . Hence, production  $y_k$  is made from both skilled labor  $L_k^s$  and unskilled labor  $L_k^u$ . Each industry maximizes its profit taking as given the price of goods and wages. Let  $w^u$  and  $w^s$  denote wages for unskilled and skilled workers respectively. We assume that at equilibrium the production is diversified and two goods are always produced. We assume also that one of the two industries is always skilled-labour intensive for each pair of wages.

Let  $C_k(w^s, w^u)$  denote the cost function for one unit of good  $k$ , then perfect competition and constant returns to scale imply (when production is diversified):

$$C_1(w^s, w^u) = 1$$

$$C_2(w^s, w^u) = p.$$

Hence, given  $p$ , these two equations determine the wages uniquely and so the wage ratio  $\frac{w^u}{w^s}$  as a function of  $p$ . According to the Stolper-Samuelson theorem, if the producer price  $p$  increases, then the wage of labour force intensively used in sector 2 will increase while the other wage will decrease. In particular, if the polluting sector is intensive in unskilled labor then we have  $\partial(w^u/w^s)/\partial p > 0$ .

Finally, from Shephard's lemma, factor demands are:

$$L_k^i = y_k \frac{\partial C_k(w^i, w^j)}{\partial w^i} \quad (3)$$

for  $i, j = s, u$ ,  $i \neq j$  and  $k = 1, 2$ .

## 2.3 Production and equilibrium

Concerning the labor market, we assume that labor is perfectly mobile across sectors within the small economy, but not internationally. Hence, labor market equilibrium conditions are:

$$\pi^i l^i = L_1^i + L_2^i$$

for  $i = s, u$ . Using the labor demands given by (3) and labor supplies, it follows that we can write production  $y_k$  as a function of  $p$ ,  $\pi^s l^s$  and  $\pi^u l^u$ :  $y_k(p, \pi^s l^s, \pi^u l^u)$ .

Goods market equilibrium conditions are:

$$\pi^s c_1^s + \pi^u c_1^u = y_1 + m_1 \quad (4)$$

$$\pi^s c_2^s + \pi^u c_2^u = y_2 + m_2 \quad (5)$$

where  $m_k$  represents the amount of good  $k$  that is imported/exported when the economy is open to trade, while balanced trade implies that:

$$m_1 + p^* m_2 = 0$$

Hence, replacing the expressions of  $m_1$  and  $m_2$  using (4) and (5), we have

$$\pi^s c_1^s + \pi^u c_1^u - y_1 + p^* (\pi^s c_2^s + \pi^u c_2^u - y_2) = 0. \quad (6)$$

## 2.4 The government

The objective of the government is to design a tax system to maximize a weighted sum of the utility of unskilled and skilled workers. Let  $\lambda^s$  and  $\lambda^u$  represent the weights of the skilled and unskilled workers in the government objective ( $\lambda^s + \lambda^u = 1$ ), which may differ from the actual proportions  $\pi^s$  and  $\pi^u$ .

Since the government cannot differentiate taxes by skills, because it observes only primary income and not the agent's type, it can design a non-linear income tax constrained by the incentive compatibility constraints: each type should weakly prefer and select the bundle of disposable income-primary income  $(I, wl)$  intended for it instead of mimicking the one intended for the other. Formally, the incentive compatibility constraint for type  $i$  writes as follows:

$$V(q, I^i, l^i, e) \geq V(q, I^j, \frac{w^j l^j}{w^i}, e)$$

for any  $i \neq j, i, j = s, u$ . Indeed, a type- $i$  worker whose wage is  $w^i$  is obliged to work  $\frac{w^j l^j}{w^i}$  to mimic the primary income  $w^j l^j$  get by worker  $j$ . To save on notations, we will denote in the

following  $V^{ij} = V(q, I^j, \frac{w^j l^j}{w^i}, e)$  for  $i, j = s, u$ . Following most of the literature (e.g. Cremer et al. 2010 or Pirttilä and Tuomala 1997), we will consider the redistributive case where only the incentive compatibility constraint for the skilled workers is binding at the optimum, which is the interesting case with an utilitarian government. As will be clear below, the cost of redistribution originates from the government's incentive to distort downward the labor supply of the unskilled to prevent skilled workers from mimicking.

Furthermore, because only aggregate consumption or production levels are observable, the government cannot do better than taxing/subsidizing linearly the production and consumption of good 2. Hence, the government's budget constraint writes as follows:

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) - (p - p^*) y_2 + \sum_{i=s,u} (q - p^*) \pi^i c_2^i = 0$$

recalling that sector 1 goes untaxed, that  $w^i l^i - I^i$  is the income tax for type- $i$  agents and that sector 2 is subject to a production subsidy  $p - p^*$  per unit and a consumption tax  $q - p^*$  per unit.<sup>12</sup>

Note first, that by Walras' law, the balanced trade equation and the budget constraint are necessarily equivalent. Also, as noted earlier, a tariff would be redundant with our mix of production subsidy and consumption tax. Nevertheless, it is possible to reinterpret the model by specifying an indirect tax policy based on a tariff equal to  $q - p^*$  and a tax on production equal to  $q - p$ . Indeed, using (5) and rearranging, the budget constraint becomes now

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) + (q - p) y_2 + (q - p^*) m_2 = 0.$$

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<sup>12</sup>Under autarky ( $y_2 = c_2$ ), the budget constraint would simply be:

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) + (q - p) y_2 = 0.$$

### 3 Optimal tax policy in a small open economy

The program of the government can be written as follows:<sup>13</sup>

$$\begin{aligned}
& \max_{I^i, l^i, q, p, e} \sum_{i=s, u} \lambda^i V(q, I^i, l^i, e) \\
& \text{s.t.} \\
& V(q, I^i, l^i, e) \geq V(q, I^j, \frac{w^j l^j}{w^i}, e) \quad \forall i, j = s, u, i \neq j \\
& \sum_{i=s, u} \pi^i (w^i l^i - I^i) + \sum_{i=s, u} (q - p^*) \pi^i c_2^i(q, I^i, l^i, e) - (p - p^*) y_2(p, \pi^s l^s, \pi^u l^u) = 0 \\
& e = E(\pi^s c_2^s(q, I^s, l^s, e) + \pi^u c_2^u(q, I^u, l^u, e), y_2(p, \pi^s l^s, \pi^u l^u)) \quad \forall i, j = s, u
\end{aligned}$$

Let  $\mu^i$ ,  $\nu$  and  $\rho$  be the Lagrange multipliers of the type- $i$  workers' incentive-compatibility constraint, the budget constraint (balanced trade condition) and the pollution definition, respectively. The corresponding Lagrangian is given by:

$$\begin{aligned}
\mathcal{L} = & \sum_{i=s, u} \lambda^i V(q, I^i, l^i, e) + \sum_{\substack{i, j=s, u \\ i \neq j}} \mu^i \left[ V(q, I^i, l^i, e) - V(q, I^j, \frac{w^j l^j}{w^i}, e) \right] \\
& + \nu \left[ \sum_{i=s, u} \pi^i [(w^i l^i - I^i) + (q - p^*) c_2^i(q, I^i, l^i, e)] - (p - p^*) y_2(p, \pi^s l^s, \pi^u l^u) \right] \\
& + \rho [E(\pi^s c_2^s(q, I^s, l^s, e) + \pi^u c_2^u(q, I^u, l^u, e), y_2(p, \pi^s l^s, \pi^u l^u)) - e] \tag{7}
\end{aligned}$$

The set of equations corresponding to the first-order conditions characterizes the Pareto-efficient allocations constrained by self-selection in this small open economy with pollution from both production and consumption.

As explained above, we focus in the analysis on the redistributive case where actually only the incentive compatibility constraint for the skilled workers is binding at the optimum (hence  $\mu^s > 0$  and  $\mu^u = 0$ ). In the following subsections, we will derive and discuss the optimal commodity tax ( $q - p^*$ ), the harmfulness of pollution ( $\rho$ ), the optimal production subsidy ( $p - p^*$ ) and finally we will characterize the marginal income tax ( $T^i$ ).

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<sup>13</sup>As explained before, we formulate the government's problem as the search for the optimal allocation in terms of pollution, labor supply and disposable income for each type, which is equivalent to search for the optimal indirect mechanism, namely the optimal tax schedule (see footnote 12).

### 3.1 Optimal commodity tax

The first issue is to see whether there is an interest for the planner to introduce distortions on consumption (i.e. by having consumption price  $q$  being different from the world price  $p^*$  in the polluting sector 2). This is established in the following proposition.

**Proposition 1** *The optimal consumption tax structure satisfies:*

$$q - p^* = -\frac{\rho}{\nu} \frac{\partial E}{\partial c_2} + t_{NS} \quad (8)$$

with  $t_{NS} = \frac{\mu^s (c_2^u - c_2^{su}) \frac{\partial V^{su}}{\partial I}}{\nu \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial q}}$ , where  $\tilde{c}_2^i$  denotes individual  $i$ 's compensated demand of good 2

**Proof.** See Appendix B. ■

The optimal distortion of the consumption price with regard to the world price consists of two terms: the first part of the right hand side of equation (8) represents the pigovian term ( $-\frac{\rho}{\nu} \frac{\partial E}{\partial c_2}$ ) while the second one  $t_{NS}$  originates from the incentive compatibility constraints where  $NS$  stands for Non Separability (as it will be clear below).

The latter component  $t_{NS}$  depends on the difference between the demand for good 2 by a mimicker and a truly unskilled type. Why would a mimicker with the same income than an unskilled worker select a different consumption basket? This is because their hours of work/leisure differ: let  $H$  represent individuals total endowment in hours, and  $h_{leisure}^i$  the number of leisure hours for type  $i$ . As a skilled mimicker works  $\frac{w^u l^u}{w^s}$  hours to get the gross income of an unskilled worker  $w^u l^u$ , his time available for leisure is  $h_{leisure}^{su} = H - \frac{w^u l^u}{w^s} > H - l^u = h_{leisure}^u$ . In other words, unless the preferences are weakly separable over consumption and leisure, a mimicker would not pick the same consumption basket. This opens the way for the planner to distort consumption as this helps to relax the binding incentive constraints. Indeed, in the redistributive case, if the skilled mimicker consumes more (less) than the unskilled worker (i.e.  $c_2^{su} - c_2^u > (<)0$  or equivalently, good 2 and leisure are complementary (substitute) goods), then it is efficient to tax (subsidize) consumption of good 2 in order to deter the skilled worker from mimicking. When leisure and consumption



are weakly separable in the utility function, this distortion naturally vanishes and only the pigovian term remains in (8).

Note also that the tax/subsidy  $t_{NS}$  is larger ceteris paribus when the social marginal cost of incentive compatibility ( $\mu^s/\nu$ ) is increasing. This is because weakening the incentive compatibility constraint by distorting consumption is now more needed as incentive compatibility is more socially costly. Conversely, a larger sensitivity of compensated demand to its own price implies a lower size for the tax/subsidy.

Finally, the pigovian term reflects the marginal social cost of consumption externalities ( $\rho \frac{\partial E}{\partial c_2}$ ) weighted by the marginal cost of public funds. Its sign is clearly related to the sign of  $\rho$  because both  $\frac{\partial E}{\partial c_2}$  and  $\nu$  are positive. We will show that despite the fact that consumers are willing to pay for a reduction of pollution, it is not guaranteed that  $\rho$  is always negative due to some general equilibrium effects. We may as well have a commodity subsidy for the polluting good as we explain in the next section.

We sum up our discussion on optimal over- or under-internalization of social marginal damages coming from consumption emissions in the following corollary.

**Corollary 2** *Assume that pollution is socially harmful to the country ( $\rho < 0$ ). Concerning the tax policy on consumption, there is accordance (conflict) between environmental and redistributive concerns when the polluting good and leisure are complements (substitutes).*

### 3.2 The harmfulness of pollution

Differentiating (7) with respect to  $e$  gives the following first-order condition with respect to pollution:

$$\frac{\partial \mathcal{L}}{\partial e} = \sum_{i=s,u} \lambda^i \frac{\partial V^i}{\partial e} + \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left( \frac{\partial V^i}{\partial e} - \frac{\partial V^{ij}}{\partial e} \right) + \nu(q-p^*) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} - \rho = 0. \quad (9)$$

Using (9) with  $\mu^u = 0$ , we follow here the analysis of Pirttilä & Tuomala (1997) and we indicate in C how to get the following expression of the shadow cost of pollution measured

in terms of government's revenue:

$$\frac{\rho}{\nu} = - \sum_{i=s,u} \pi^i MWP^i - \frac{\mu^s}{\nu} \frac{\partial V^{su}}{\partial I} (MWP^u - MWP^{su}) + t_{NS} \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial e} \quad (10)$$

where  $MWP^i = -\frac{\partial V^i}{\partial e} / \frac{\partial V^i}{\partial I} > 0$  represents the marginal willingness to pay for a pollution reduction for  $i = s, u$  or  $su$ .

There are three components in the RHS of (10) that we examine in turn. The first term reflects the direct negative impact of pollution on the workers' utility and represents the social marginal willingness to pay to avoid pollution. It corresponds to the standard pigovian rule of externality internalization.

The second term represents the impact of pollution on the binding incentive compatibility constraint for skilled workers. Indeed, the planner recognizes that by manipulating the tax system, pollution will change and will impact the cost of redistributing labor incomes. The sign of this term depends on the difference between the marginal willingness to pay for a pollution reduction by a skilled mimicker ( $MWP^{su}$ ) and that one by a truthful revealing unskilled worker ( $MWP^u$ ). Since the mimicker is a skilled worker, he needs to work less to mimic the primary income of an unskilled worker and hence he has more leisure time than a truthful revealing skilled agent. If  $MWP$  is increasing in leisure time, then it follows that  $MWP^u < MWP^{su}$ . Hence, if environmental quality and leisure time are complements, the second term is positive and it counterbalances the first term. The intuition is that here increasing pollution diminishes the desirability of mimicking and in this case there is some conflict between protecting the environment and redistributing labor incomes. If on the contrary,  $MWP$  is decreasing in leisure time, then  $MWP^u > MWP^{su}$ . Environmental quality and leisure time are substitutes, the second term is negative and it adds to the first term. Here decreasing pollution diminishes the desirability of mimicking. The environmental objective is in accordance with the redistributive objective in that case.

Finally, the third term in the RHS of equation (10) is a tax revenue effect. When considering a marginal change in pollution, one has to anticipate that this will modify the

consumption of the polluting good and in turn the money collected from consumers through the redistributive part of the consumer tax (or transferred to them in case of a subsidy). Assume that  $t_{NS}$  is positive at the optimum, which means that one has to tax good 2 in order to alleviate incentive compatibility constraints. It remains to ascertain whether pollution encourages or discourages the (compensated) consumption of good 2. If the former holds, then an increase in pollution will increase the tax collected on goods which in turn is socially beneficial in terms of budget. This impact is to be taken into account in estimating  $\rho/\nu$ .

Overall, it is clear that the sign of  $\rho/\nu$  is generally ambiguous. In particular, it would be possible to have a *positive* sign if the last two terms are sufficiently positive, that is if the impact of pollution on the self-selection constraints is such that one would want to increase pollution to save on redistribution cost and if pollution stimulates sufficiently the demand for good 2.

### 3.3 Optimal production subsidy

We now turn to the issue of fixing the production price. The next Proposition describes the optimal distortion brought to the production sector.

**Proposition 3** *The optimal production subsidy structure satisfies:*

$$p - p^* = \frac{\rho}{\nu} \frac{\partial E}{\partial y_2} + t_{EW} \quad (11)$$

with  $t_{EW} = -\frac{\mu^s}{\nu} l^u \frac{\partial V^{su}}{\partial l} \frac{\partial(\frac{w^u}{w^s})}{\frac{\partial y_2}{\partial p}} > 0$  where  $EW$  stands for endogenous wages and  $sign(t_{EW}) = sign(\frac{\partial(\frac{w^u}{w^s})}{\partial p})$

**Proof.** See Appendix C. ■

Let us assume first that sector 2 is intensive in unskilled labor. As  $\mu^s > 0$ , the term  $t_{EW}$  of the right hand side of (11) is positive because of the Stolper-Samuelson effect (and because  $\frac{\partial y_2}{\partial p} > 0$ ) while the first (pigovian) term is negative if  $\rho < 0$ . This means that on the one hand, one would want to subsidize production because this allows to relax the incentive

compatibility constraint (due to the subsidy impact on the wage ratio,  $\frac{\partial(\frac{w^u}{w^s})}{\partial p}$ ) and on the other hand one would want to tax production because of harmful pollution. Whether there is a subsidy or a tax on production (i.e. whether  $p > p^*$  or not) depends on the trade-off between these two forces. In any case, the production system is (in general) not efficient.

Another way to interpret the result is as follows. Introducing a pure pigovian tax on an otherwise untaxed good may decrease welfare. It is only if the environmental tax takes into account the redistributive concern (i.e. the impact on the distribution of wages) that there is a Pareto-improvement. In other words, the optimal production tax is non pigovian and there is under-internalization of the social marginal environmental damage.

Note that we can deduce from the expression of  $t_{EW}$  that, *ceteris paribus*, a larger production subsidy is implemented when the social marginal cost of incentive compatibility ( $\mu^s/\nu$ ) is increasing. This is because redistributing income through distorting production is now relatively less costly. A larger subsidy is also needed when the wage ratio is more sensitive to output price, because this implies that the subsidy is a more efficient way to redistribute. Conversely, a larger sensitivity of output to its own price implies a lower size for the production subsidy.

Now, suppose on the contrary that the polluting sector is intensive in skilled labor, then the redistributive term is negative (because the Stolper-Samuelson effect goes in the other way, i.e.  $\frac{\partial(\frac{w^u}{w^s})}{\partial p} < 0$ ). It is then optimal to over-internalize the social marginal environmental damage.

We sum up our results on over or under-internalization of damages coming from production emissions as follows.

**Corollary 4** *Assume that pollution is socially harmful to the country ( $\rho < 0$ ). Concerning the tax/subsidy policy on production, there is conflict (resp. accordance) between environmental and redistributive concerns when the polluting sector is intensive in unskilled (resp. skilled) labor.*

### 3.4 Marginal Income tax rates

Let us turn to the optimal non-linear income tax in this small open economy. Recall that the primary income is  $w^i l^i$  while the disposable income is  $w^i l^i - T(w^i l^i)$  where  $T(\cdot)$  denotes the income tax. The second step of consumer optimization, i.e. when the worker maximizes his utility w.r.t. his labor supply subject to a given tax schedule, enables the marginal income tax rate to be expressed in terms of the utility function:

$$T'^i \equiv T'(w^i l^i) = 1 + \frac{\frac{\partial V^i}{\partial l}}{w^i \frac{\partial V^i}{\partial I}}. \quad (12)$$

The next Proposition characterizes the optimal non-linear income tax schedule for this small open economy.

**Proposition 5** *The optimal income tax schedule is such that marginal rates are:*

$$T'^u = \underbrace{\frac{t_{EW}}{w^u} \frac{\partial y_2}{\partial L^u}}_{\text{Production efficiency}} + \underbrace{\frac{\mu^s}{\nu \pi^u} \frac{\partial V^{su}}{\partial I} \left[ \frac{1}{w^s} \frac{\partial V^{su}}{\partial I} - \frac{1}{w^u} \frac{\partial V^u}{\partial I} \right]}_{\text{Incentive compatibility}} - \underbrace{\frac{t_{NS}}{w^u} \frac{dc_2^u}{dl}}_{\text{Consumption}} \Big|_{dV^u=0} \quad (13)$$

$$T'^s = \frac{t_{EW}}{w^s} \frac{\partial y_2}{\partial L^s} - \frac{t_{NS}}{w^s} \frac{dc_2^s}{dl} \Big|_{dV^s=0} \quad (14)$$

**Proof.** See Appendix D. ■

From (14) and (13), note that the presence of externalities does not influence directly the marginal rate of income taxation. This is consistent with the *Principle of Targeting*: it does not pay to distort labor supply in order to reduce pollution in this model. Taxing the consumption and the production of the polluting good is sufficient to internalize the externalities, even if wage rates are endogenous.

Concerning the skilled workers, the marginal rate of income tax is the sum of two terms (see (14)). The first term  $\frac{t_{EW}}{w^s} \frac{\partial y_2}{\partial L^s}$  comes from the endogeneity of wages and is always negative whatever the sector intensive in skilled labor. Indeed, if sector 2 is intensive in unskilled labor, then  $t_{EW} > 0$  and  $\frac{\partial y_2}{\partial L^s} < 0$  (because of the Rybczynski theorem) and vice versa if sector 2 is intensive in skilled labor. Intuitively, it always pays for a government to distort upwards the labor supply from skilled workers in order to diminish their equilibrium wage

and thereby to increase the wage of unskilled workers. This is why the first term tends to make the marginal income tax negative. The second term is due to the non separability between consumption of the polluting good and labor.

Concerning the unskilled workers, the marginal income tax is the sum of three terms (see (13)). The first term is positive: the government has interest to tax the labor supply of unskilled workers in order to increase their equilibrium wage which in turn allows to reduce the wages inequality. The second term corresponds to the usual Mirrleesian distortion and is positive thanks to the agent monotonicity condition: by taxing the labor supply of unskilled workers, one increases the cost of mimicking for skilled workers which then allows to reduce the overall cost of redistribution under asymmetric information. Finally, the third term represents the incentives to distort labor supply because of the non separability between consumption and leisure.

### 3.5 Optimal fiscal policy in autarky

For completeness, we now describe what would happen in an otherwise identical but closed economy.<sup>14</sup> First, under autarky, the optimal consumption tax naturally aggregates the four terms identified in the optimal indirect tax system for an open economy. Indeed, the optimal consumption is composed of the two pigovian taxes (one for the consumption externality and the other one for the production externality), plus the tax due to the non separability between consumption and labor in the individuals' preferences and the subsidy (when the polluting sector is intensive in unskilled labor and for the redistributive case) due to the endogeneity of wages:

$$q - p = -\frac{\rho}{\nu} \left( \frac{\partial E}{\partial c_2} + \frac{\partial E}{\partial y_2} \right) + t_{NS} - t_{EW}.$$

Moreover, the marginal income tax rate remains unchanged, so that the comparison of the optimal mixed tax system under autarky and trade is straightforward. In the simulations presented in the next section, the autarky situation will be replicated for an open economy

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<sup>14</sup>All the results are straightforward and are established in an appendix available upon request.

by the specific value of the (relative) world price  $p^*$  that entails no trade with the rest of the world.

## 4 Simulating the impacts of trade liberalization and changes in social preferences

Section 3 characterized the optimal tax system in a small open economy with pollution and labor income inequality, when policy is only constrained by the available information : this maximizes domestic welfare for a given economic and policy situation, namely the relative price in the world market  $p^*$ , and the government's preference for redistribution (vector of welfare weights  $\lambda^i$ ).

We examine below the effects of an exogenous move towards freer trade under alternative assumptions regarding the source of the externality (in Section 4.2). Because tenants of free trade sometimes argue that many negative outcomes imputed to trade liberalization are rather due to badly designed (environmental and redistributive) policies, our goal here is to see how the pollution-inequality nexus evolves with trade liberalization, when the tax system is adjusted optimally. In particular, how does it compare to a laissez-faire situation (no taxes at all)? and how does the unobservability of skills weight on the government's ability to redistribute income? Finally, because the level of taxes is optimally set for a given vector of welfare weights, we are also interested in investigating how a change in unskilled welfare weight  $\lambda^u$  does affect income inequalities and emissions (in Section 4.3).

To illustrate these questions, we need to put more structure on the model, and turn to simulations.<sup>15</sup> In the next section, we present all the details regarding the chosen specification and the resulting features of the optimal fiscal system.

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<sup>15</sup>Simulations are widely used for the analysis of optimal tax systems (see e.g. Cremer et al. (2010)). The reason is that analytical results are usually difficult to obtain unless one imposes the quasi-linearity of preferences with respect to leisure and perfect substitution between different labor types in the production function (see Weymark (1987); Brett & Weymark (2013, 2008a, 2011, 2008b)). Extending the results obtained by this literature to our setting with imperfect substitutability between labor types and non quasi-linearity of preferences is outside the scope of this paper and is devoted to future research.

## 4.1 Model specification and implications for the optimal fiscal policy

Consider the following additive separable form of the utility function:

$$U(c_1, c_2, l, e) = \ln [c_1^\alpha c_2^{1-\alpha}] - \gamma \frac{l^{1+1/\xi}}{1+1/\xi} - \omega \frac{e^{1+1/\phi}}{1+1/\phi}$$

where  $\alpha$  is a positive share parameter in the (homothetic) consumption sub-utility and  $\gamma$  and  $\omega$  are positive scale parameters of the (isoelastic) labor and pollution disutilities respectively.<sup>16</sup> Parameter  $\xi$  is the Frisch elasticity of labor supply and is usually estimated between 0.1 and 1 in econometric studies, whereas parameter  $\phi > 0$  plays a similar role in the pollution disutility function.

Regarding production of good  $k = 1, 2$ , we assume a Cobb-Douglas technology with constant returns to scale,  $F_k(L_k^u, L_k^s) = A_k [L_k^s]^{\sigma_k} [L_k^u]^{1-\sigma_k}$ , and we concentrate on the situation where the dirty sector is intensive in unskilled labour, i.e.  $\sigma_2 < \sigma_1$ .

Polluting emissions are given by the following linear specification:

$$e = \beta [\delta c_2 + (1 - \delta)y_2] \quad (15)$$

Parameter  $\beta$  is a positive scale parameter representing emission intensity while  $\delta$  (resp.  $(1 - \delta)$ ) represents the share of consumption (resp. production) of the polluting good in total emissions, which allows to represent in a simple manner the case of a pure production externality ( $\delta = 0$ ), of a pure consumption externality ( $\delta = 1$ ), and any intermediary situation.<sup>17</sup> Table 1 indicates the parameters values we hold constant in all sets of simulations.<sup>18</sup>

[Table 1 about here.]

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<sup>16</sup>For the clarity of exposition, we have assumed that pollution hurts the skilled and the unskilled workers in the same manner. Actually, introducing a differentiated impact of pollution on consumers (through some type-dependant scale parameters  $\omega^s \neq \omega^u$ ) would not change the optimal policy in this setting as long as the "global" scale parameter  $\omega = \omega^s + \omega^u$  is kept constant, because welfare is the sum of individual indirect utilities.

<sup>17</sup>As we assume that the emission intensity in the dirty sector is fixed, we do not take into consideration the possibility for polluting firms to abate emissions. For this, one could consider an augmented production function with three inputs, pollution and the two types of labor (see the concluding remarks in section 5).

<sup>18</sup>Although our parameters values are arbitrary, we have conducted simulations using alternative values for  $\alpha$ ,  $\beta$ ,  $\xi$ ,  $\gamma$  and  $\omega$ . Results do not differ drastically. In addition, we believe that calibrating such a model with only two sectors would not be relevant.



The above specification generates several implications for the economy and its fiscal system. First, assuming homothetic preferences for consumption implies that  $c_2$  and thus emissions from consumption are a function of the *aggregate* income (and not of the *distribution* of type-specific incomes) which simplifies the analysis.<sup>19</sup>

Second, separability between labor and consumption decisions implies that there is no need to distort consumption in order to alleviate the cost of incentive compatibility constraints because a skilled mimicker and a truthful revealing unskilled consume the same quantity. Therefore, the term  $t_{NS}$  vanishes in the expression of the optimal consumption tax in (8), so that  $q - p^* = -\frac{\rho}{\nu}\beta\delta$  and consumption externalities are fully internalized. However, production externalities are under-internalized as, following Proposition 3, the dirty sector is at the same time subsidized for redistributive reason and taxed for environmental purpose. Regarding marginal income tax rates given in Proposition 5, since  $t_{NS} = 0$ , skilled workers will always face a negative marginal rate while unskilled workers will face a positive marginal rate consisting in a (positive) endogenous wages term and a (positive) *mirrleesian* term. This means that a government concerned with costly redistribution in favor of the unskilled does optimally distort upward the labour supply from the skilled, and does distort downward the labour supply from the unskilled, in order to increase their equilibrium wages  $w^u$ .

Third, the separability between pollution and consumption/labor decisions implies that pollution is socially harmful: because  $t_{NS} = 0$  and because the marginal willingness to pay for a pollution reduction by a skilled mimicker ( $MWP^{su}$ ) is the same as for a truthful revealing unskilled worker ( $MWP^u$ ), from (10) we have  $\frac{\rho}{\nu} = -\sum_{i=s,u} \pi^i MWP^i < 0$  and the social cost of pollution only reflects the sum of consumers' marginal willingness to pay to avoid pollution.

Fourth, solving the first order conditions with regard to  $I^s, I^u$  and  $q$  yields a simple ex-

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<sup>19</sup>Total consumption of good 2 is  $c_2(q, I^s, I^u) = \frac{(1-\alpha)}{q} \sum_i \pi^i I^i$ . Also, with homothetic preferences, the indirect utility function  $V^i$  is increasing in real income, defined as the ratio of net income  $I^i$  to a consumer price index  $F(q) : V^i(I^i, l^i, q, e) = \ln \frac{I^i}{F(q)} - \gamma \frac{l^{1+1/\xi}}{1+1/\xi} - \omega \frac{e^{1+1/\phi}}{1+1/\phi}$  where  $F(q) = \frac{q^{1-\alpha}}{\alpha^\alpha \cdot (1-\alpha)^{1-\alpha}}$ .

pression for the optimal levels of after-tax incomes:  $I^s = \frac{\lambda^s + \mu^s}{\nu \pi^s}$  and  $I^u = \frac{\lambda^u - \mu^s}{\nu \pi^u}$ . It follows that the aggregate income is  $\sum_i \pi^i I^i = \frac{1}{\nu}$ . This allows to write the share of national income for each type of workers as  $\frac{\pi^s I^s}{\sum_i \pi^i I^i} = \lambda^s + \mu^s$  and  $\frac{\pi^u I^u}{\sum_i \pi^i I^i} = \lambda^u - \mu^s$ . Hence, a larger welfare weight for a given type of worker tends to increase its share of national income. However, the cost of redistribution impacts negatively (resp. positively) the share of unskilled (resp. skilled) workers. Indeed, any incentive compatibility policy implies here that an informational rent has to be paid to skilled workers, at the expense of unskilled workers.

We also get a measure of after-tax income inequalities through the ratio:

$$\frac{I^u}{I^s} = \frac{(\lambda^u - \mu^s)/\pi^u}{(\lambda^s + \mu^s)/\pi^s} \quad (16)$$

which is a decreasing function of  $\mu^s$ . The degree of after-tax income inequalities is fully determined by the equilibrium value of the shadow cost  $\mu^s$ . We will use some of these features of the optimal policy when interpreting the general equilibrium outcomes obtained by simulations in the next two sections.

## 4.2 How does the pollution-inequality nexus evolves with freer trade?

We model trade liberalization as a reduction in trading costs of the “iceberg” type introduced by Samuelson (1954).<sup>20</sup> This allows to abstract from government revenue effects (as would be required with tariffs) and to focus in a simple manner on the effects of increased trade opportunities. If the economy has a comparative advantage for the good 2 (exports the polluting good), then trade liberalization is equivalent to an increase in the relative world price  $p^*$ . Conversely, if it has a comparative advantage for the clean good (imports the polluting good), then the impact of trade liberalization is captured in a decrease in  $p^*$ .

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<sup>20</sup>As it is common in the literature (e.g. Copeland and Taylor, 2003), we assume that there is no trade friction for the numeraire good.

## Production externality

We start by solving numerically the system for a pure production externality ( $\delta = 0$ ), for the whole range of world prices such that the output of both goods remains positive (partial specialization of the economy).<sup>21</sup> For each level of  $p^*$ , we derive the optimal level of taxes assuming that the weight put on unskilled workers' welfare weight is greater than their demographic weight:  $\lambda^u = 0.6 > \pi^u = 0.5$ , which ensures that incomes redistribution operates from the skilled towards the unskilled. We also obtain the optimal level of emissions  $e$  and of the after-tax income ratio ( $I^u/I^s$ ) for the whole range of the relevant values of  $p^*$ . These results are plotted on the curve labelled "Optimal Taxation (2nd Best)" in Figure 1, with the income ratio  $I^u/I^s$  on the X-axis (inequalities decreasing to the right), the level of emissions on the Y-axis, and each point on the curve resulting from the implemented optimum for a given degree of specialization of the economy (level of the world price  $p^*$ ).

We have also simulated the outcomes of trade liberalization in a situation of *laissez-faire*, i.e. without any taxes aiming to internalize externalities or redistribute income, in order to see the role of optimal policy (our 2nd best fiscal system) on the social and environmental consequences of trade liberalization. Moreover, in order to visualize the weight of incentive compatibility on the degree of inequalities and of pollution reached with our optimal mixed taxation system, we have also simulated the effects of trade liberalization with optimal taxation under perfect information. These two benchmarks are plotted respectively on the curves labelled "*laissez-faire*" and "Optimal taxation (1st best)" in Figure 1.

[Figure 1 about here.]

On each of the three curves, the Autarky point corresponds to the value of the world price such that no trade occurs: for any initial situation above Autarky, the country has a comparative advantage in the production of the polluting good (sector 2), and trade liberalization results in a further specialization in good 2 (move along the dashed arrow). Conversely, any

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<sup>21</sup>The world price  $p^*$  has to be taken in the range  $[1, 1.31]$  for partial specialization to hold. The Mathematica notebooks with the detailed calculations and resolution algorithms are available upon request.

initial situation below Autarky means the country has a comparative advantage in the clean good (sector 1), and trade liberalization involves further specialization in good 1 (move along the plain arrow).

Figure 1 shows that with a pure production externality, the pattern of specialization unambiguously determines the effects of trade liberalization in terms of polluting emissions (under the 3 regimes) and of income inequality (under *laissez-faire* and 2nd best policies). Freer trade results in an increase in pollution but a decrease in after-tax income inequality for an exporter of the dirty good, while for an importer, the decrease in pollution comes with an increase in after-tax income inequality: in other words, there is no “free lunch”.

At first sight, such an outcome may appear disappointing as social and environmental consequences of trade liberalization seem quite similar with and without an optimal realistic (2nd best) government regulation. Indeed, under *laissez-faire*, the pattern of specialization drives directly the level of emissions (due to the composition effect)<sup>22</sup>, and labor income inequalities decrease with an increasing specialization in the dirty good (the production of which is intensive in unskilled labor) - a classical Stolper-Samuelson mechanism.

However, the direction of specialization is determined by the economy’s comparative advantage, which is modified by environmental and redistributive policies. This can be seen in Figure 1. The environmental and social benefits of optimal regulation can be visualized by following the dotted lines, linking a *laissez-faire* equilibrium to the corresponding 2nd-best optimum, for a given value of the world price  $p^*$ . Clearly, not only does our mixed taxation system result in lower emissions and less income inequalities as compared to *laissez-faire*, but if the economy had a comparative advantage in the polluting sector under *laissez-faire*, in these simulations it is reversed under optimal taxation, meaning that a decrease in trade costs will have opposite consequences.

Now let us turn to the importance of incentive compatibility on both the inequalities and

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<sup>22</sup>In a 2 goods, 2 sectors H-O-S model, it can be shown that composition effects always dominate over scale effects, see Copeland and Taylor (2003); and here there is no technique effect in the absence of abatement possibilities.

emissions. This can be seen by comparing the curves under 1st best and 2nd best optimal taxation. If the government were able to observe individual types directly instead of gross incomes, it would implement directly, through personalized lump sum transfers, the desired ratio of after-tax incomes reflecting each type's weight, regardless the economy's pattern of specialization.<sup>23</sup> In our more realistic 2nd best setting, redistribution is constrained by the information rent, the value of which reflects the attractiveness of the mimicking strategy, and therefore varies with the economic conditions. We know from (16) that the impact of trade liberalization on after tax income inequalities is determined completely by the variations in  $\mu^s$ . The shadow cost of the binding incentive compatibility constraint is strictly decreasing in  $p^*$ , because of the Stolper-Samuelson effect: the wage ratio  $\frac{w^u}{w^s}$  increases when trade stimulates the sector making intensive use of unskilled labour, which decreases the information rent necessary to prevent skilled workers from mimicking. A decreasing value for  $\mu^s$  thus involves an increasing ratio of after tax incomes  $\frac{I^u}{I^s}$ : the pattern of specialization matters for redistribution when redistribution is costly.

Moreover, comparing the level of pollution under 1st best and 2nd best (following the dotted line for a given value of  $p^*$ , e.g.  $p^* = 1.09$ ), it is easy to see that 2nd best involves optimally more emissions, but still less than under *laissez-faire*. To explain this, recall that emissions are proportional to production,  $e = \beta y_2$ , and that 2nd best puts an additional subsidy on top of the pigovian tax compared to the 1st best where only the pigovian tax appears. In other words, under 2nd best, the cost of incentive compatibility makes it optimal to distort upwards the size of the polluting sector compared to 1st best, in order to indirectly favor unskilled workers. Of course, this distortion comes at a cost in terms of increased emissions.

Last, Figure 2 indicates that both the pigovian term and the production subsidy  $t_{EW}$  are increasing (in absolute value) with  $p^*$ .

[Figure 2 about here.]

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<sup>23</sup>Which is given in (16) with  $\mu^s = 0$ . Note that here we end up with a ratio larger than 1, due to our assumption of both skills' equal demographic weights and a larger welfare weight for unskilled workers.

Unsurprisingly, the shape of the pigovian term reflects the increasing social cost of pollution in the economy when production of the dirty good increases. However, the shape of  $t_{EW}$  suggests that there is an increasing tendency to under-internalize the production externality when  $p^*$  raises, which seems counter-intuitive at first sight. Indeed, when the relative price of the polluting good increases, the wage gap between the unskilled and the skilled decreases ( $\frac{w^u}{w^s}$  increases), so why should the regulator increase a subsidy that aims precisely at reducing the before-tax wage gap? The intuition can be deduced from the expression of the optimal production subsidy that can be obtained from Proposition 3 with our specification:

$$t_{EW} = \frac{\mu^s}{\nu} \frac{\left[\frac{w^u}{w^s} l^u\right]^{1+1/\xi}}{\frac{\partial y_2}{\partial p}} \frac{\partial \left(\frac{w^u}{w^s}\right)}{\partial p} > 0.$$

Two mechanisms are at stake: first, even though it appears from our simulations that  $\mu^s$  decreases with  $p^*$ , so does the cost of public funds  $\nu$ . The variation of the ratio  $\frac{\mu^s}{\nu}$  with  $p^*$  is not monotonic and is actually U-shaped, meaning that above a threshold level in prices, the decrease in the cost of public funds dominates the one in redistribution cost. In other words, the social cost of distorting production becomes relatively lower. Second, it appears that the marginal increase in the wage ratio  $\frac{\partial \left(\frac{w^u}{w^s}\right)}{\partial p}$  is also an increasing function of  $p^*$ . To sum up, even though an increase in the relative price of the polluting good makes redistribution less necessary, the optimal subsidy to the polluting sector increases because, when  $p^*$  increases, it becomes less distorting and a more powerful instrument to reduce the wage gap.

### Consumption externality

Let us turn to the case where emissions stem only from consumption of good 2 ( $\delta = 1$ ). Clearly, the source of the externality, production or consumption, does not matter in autarky, whatever the policy regime implemented (this can be seen by comparing the autarky levels of pollution and inequality in fig. 1 and 3). However, it matters in an open economy. Looking at the possible outcomes of trade liberalization in terms of both pollution and inequalities leads to the following results, as represented by the curve labelled ‘‘Optimal taxation (2nd best)’’ in Figure 3. Again, we contrast the situation under our optimal mixed taxation

system against a *laissez-faire* situation and a *1st best* regulation.

[Figure 3 about here.]

While the previous results regarding the level of inequality still hold, with income inequality declining when the economy gets more specialized in good 2, with the same mechanisms involved, the effect of trade liberalization on emissions is different and ambiguous. Focusing on the 2nd best curve in Figure 3, one can see that an increasing specialization in good 2 (i.e. a move downward along the curve) now translates most of the time into decreasing emissions from its consumption, except for high values of  $p^*$ . The expression of aggregate polluting consumption is

$$e = \beta c_2 = \beta \frac{(1 - \alpha)}{q\nu}$$

and the driving force behind our result is that consumption price  $q$  is strictly increasing in  $p^*$ . However, the national income  $1/\nu$  is also increasing when, starting from autarky, the economy increasingly specializes in the polluting good. This income effect counterbalances the price effect and actually outweighs it only when the economy is sufficiently specialized in good 2, so that consumption and pollution are increasing in  $p^*$  for high values of  $p^*$ . By contrast, starting from autarky, an increasing specialization in the clean good (due to a decrease in  $p^*$ ) raises pollution: although the national income decreases, the decrease in the consumer price of the polluting good is sufficient to drive the result.

Last, one can see that for a given price  $p^*$ , the pollution level tends to be smaller in the 2nd best compared to the 1st best (see, e.g., the dashed line for  $p^* = 0.985$  in Figure 3). This can be explained by the fact that the national income  $1/\nu$  is larger under 1st best as there are no longer any distortions in the economy. However, consumer price is higher under 1st best because of a larger pigovian tax. The income effect outweighs the price effect to explain the larger consumption and pollution level under 1st best.

### 4.3 Simulating a change in social preference with respect to income redistribution

Because optimal policy is defined for a given set of welfare weights in the government's objective, it is interesting to look at the consequences of a political shock (e.g. a change in the government's objective after elections) in this small open economy. We focus on a change that expresses an increasing willingness of the government to redistribute from the skilled to the unskilled. In this simulation set, we fix the world price and we then vary the welfare weight  $\lambda^u$  from 0.5 to 0.9.<sup>24</sup>

[Figure 4 about here.]

This scenario corresponds to a shrinking economy. Indeed, all productions (and consumptions for both skilled and unskilled) are decreasing, as well as pollution. It is also worth noting that the polluting sector size is decreasing although the subsidy  $t_{EW}$  actually increases with  $\lambda^u$ , which indicates that the government increasingly seeks to stimulate the polluting sector in order to favor the unskilled wage level.

The intuition is that in this economy, redistribution is socially costly and this inefficiency increases with the willingness to redistribute. The cost of redistribution as measured by  $\mu^s$  unambiguously increases in  $\lambda^u$ , as well as the cost of public funds  $\nu$  (meaning that the national income decreases). Also, from (16), it is easy to see that the after-tax income ratio is decreasing in  $\lambda^u$  whenever  $\mu^s$  grows at a rate larger than unity.<sup>25</sup> This is what is described in Figure 4 where an increase in the government's care for the unskilled workers does paradoxically increase the level of after-tax income inequalities, while pollution is simultaneously decreasing as the economy shrinks.<sup>26</sup>

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<sup>24</sup>We run two sets of simulations to compare the situation where the economy is a net importer of good 2 ( $p^* = 1$ ) or an exporter ( $p^* = 1.14$ ). For the clarity of exposition, we represent the pure production externality case. However the results also hold when pollution stems only from consumption.

<sup>25</sup>We have  $\frac{d(I^u/I^s)}{d\lambda^u} = \frac{\pi^s}{\pi^u} \frac{1-d\mu^s/d\lambda^u}{(\lambda^s+\mu^s)^2}$ .

<sup>26</sup>Even though an increase in  $\lambda^u$  does paradoxically increase the level of after-tax income inequalities, low-skilled agents should in equilibrium be better off when  $\lambda^u$  increases (both due to the reduction in pollution and an increased consumption of leisure).



## 5 Conclusion

Several points deserve particular attention in our analysis. First, we have shown that the optimal mixed tax system does not differ fundamentally under a situation of autarky and of open trade: the formula are similar, even though the equilibrium levels naturally differ. Second, the targeting principle holds true: the level of the externality does not modify the expression of marginal income tax rates. Third, the externality is optimally addressed using indirect taxes on production and consumption (with a pigovian term reflecting the marginal damage arising from either source). Both producer and consumer taxes may include a redistributive term aimed at alleviating the cost of redistribution. These terms may conflict (or not) with pigovian terms: for instance, when the polluting sector is intensive in unskilled labour, a public regulator will optimally under-internalize social marginal damages from production. Also, when externality arises from consumption, it is optimal to under-internalize social marginal damages when the polluting good and leisure are substitutes.

Simulations based on a simple specification (additive separable preferences, Cobb-Douglas technologies, linear externality function) shed some light on several recurrent questions in the literature. First, results suggest that optimal redistributive policies do not cancel the polarization of skilled vs. unskilled interests to trade liberalization: inequalities increase when the sector using unskilled labour intensively suffers from increased imports, and conversely, just as predicted in the standard Stolper-Samuelson framework. However, optimal taxes may reverse the economy's comparative advantage. In such a case, the direction of changes from freer trade is also reversed, as the economy gets increasingly specialized in the sector that would have shrunk without policy.

Second, regarding the trade-pollution-inequalities nexus, our simulations show that the relative contribution of production and consumption to the externality is a crucial determinant. With a pure production externality, trade will result in an increase in emissions and a decrease in after tax income inequalities for an exporter of the polluting good (and conversely for an importer). On the contrary, with a pure consumption externality, trade

liberalization may decrease both pollution and inequalities for an exporter of a dirty good (and conversely for an importer). Hence, even in this simple two goods and two factors general equilibrium model, the impact of trade on pollution and inequalities is not trivial as it depends on several crucial variables like the share of consumption in total emissions, the policy-influenced pattern of specialization, and the importance of income effects.

Last, our simulations also suggest that trade can exacerbate the polarization between environmental and redistributive objectives, because the tendency to under-internalize social marginal damages from production is even greater when the economy becomes an exporter of the polluting good after trade liberalization - even though the wages of the unskilled workers naturally tend to increase.

There are several ways to extend the present analysis. For instance, the intensity of emissions  $\beta$  is fixed, as we do not model the possibility of abatement. Making  $\beta$  endogenous, as e.g. Copeland and Taylor do, and investigating the interactions between the tax system, pollution, inequalities and technology choices could be an interesting extension. In particular, suppose that, following Copeland and Taylor, it is possible to reduce gross pollution by using an abatement technology that employs both types of labor. It follows that the wage ratio now depends not only on the output price but also on the net pollution level (or equivalently on the tax on net pollution). Hence, a government concerned with labor income inequalities may want to distort pollution control for an additional reason: this allows to manipulate the wage ratio before redistributing wealth through the income tax schedule.

Also, we have focused on the situation of a small open economy trying to design optimally its environmental and redistributive policy, taking world prices as given. It would be interesting to investigate the linkages between a country's characteristics in terms of preference for redistribution and environmental technology, and its endogenous comparative advantage in a model of bilateral trade, when the foreign country's characteristics are taken into account for the design of the optimal domestic mixed taxation system. We leave these questions to future research.

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## Appendix

### A Proof that $y_2 = \sum_{i=s,u} \pi^i l^i \frac{\partial w^i}{\partial p}$

The zero profits conditions for both sectors are

$$p = C_2(w^s, w^u)$$

$$1 = C_1(w^s, w^u)$$

Hence,  $y_1 + py_2 = C_2(w^s, w^u)y_2 + C_1(w^s, w^u)y_1$ . Differentiating totally w.r.t.  $p$ , we obtain

$$\begin{aligned} \left( y_2 + p \frac{\partial y_2}{\partial p} + \frac{\partial y_1}{\partial p} \right) dp &= \left( \frac{\partial C_2}{\partial w^s} \frac{\partial w^s}{\partial p} y_2 + \frac{\partial C_2}{\partial w^u} \frac{\partial w^u}{\partial p} y_2 + C_2 \frac{\partial y_2}{\partial p} + \frac{\partial C_1}{\partial w^s} \frac{\partial w^s}{\partial p} y_1 + \frac{\partial C_1}{\partial w^u} \frac{\partial w^u}{\partial p} y_1 + C_1 \frac{\partial y_1}{\partial p} \right) dp \\ &= \left( \frac{\partial w^s}{\partial p} (L_2^s + L_1^s) + \frac{\partial w^u}{\partial p} (L_2^u + L_1^u) + C_2 \frac{\partial y_2}{\partial p} + C_1 \frac{\partial y_1}{\partial p} \right) dp \end{aligned}$$

because  $y_k \frac{\partial C_k}{\partial w^i} = L_k^i$  following Sheppard's lemma. Also from the envelope theorem, we have

$$p \frac{\partial y_2}{\partial p} + \frac{\partial y_1}{\partial p} = C_2 \frac{\partial y_2}{\partial p} + C_1 \frac{\partial y_1}{\partial p} = 0.$$

Hence, we have finally for any  $dp$ ,

$$\left( y_2 - \sum_{i=s,u} L^i \frac{\partial w^i}{\partial p} \right) dp = 0.$$

### B Proof of Proposition 1

Derivating the Lagrangean (7) with respect to labor income yields to:

$$\frac{\partial \mathcal{L}}{\partial I^i} = (\lambda^i + \mu^i) \frac{\partial V^i}{\partial I} - \mu^j \frac{\partial V^{ji}}{\partial I} - \nu \pi^i + \nu \pi^i (q - p^*) \frac{\partial c_2^i}{\partial I} + \rho \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial I} = 0 \quad (17)$$

with  $i, j = s, u$  and  $i \neq j$ . Also the first-order condition w.r.t. the consumption price  $q$  is:

$$\frac{\partial \mathcal{L}}{\partial q} = \sum_{i=s,u} (\lambda^i + \mu^i) \frac{\partial V^i}{\partial q} - \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \frac{\partial V^{ij}}{\partial q} + \nu \sum_{i=s,u} \pi^i c_2^i + \nu \sum_{i=s,u} \pi^i (q - p^*) \frac{\partial c_2^i}{\partial q} + \rho \sum_{i=s,u} \pi^i \left[ \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial q} \right] = 0 \quad (18)$$

By Roy's identity, we have

$$\begin{aligned}\frac{\partial V^i}{\partial q} &= -c_2^i \frac{\partial V^i}{\partial I} \\ \frac{\partial V^{ij}}{\partial q} &= -c_2^{ij} \frac{\partial V^{ij}}{\partial I}.\end{aligned}$$

Introducing these terms in (18), we finally obtain:

$$-\sum_{i=s,u} (\lambda^i + \mu^i) c_2^i \frac{\partial V^i}{\partial I} + \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i c_2^{ij} \frac{\partial V^{ij}}{\partial I} + \nu \sum_{i=s,u} \pi^i c_2^i + \nu \sum_{i=s,u} \pi^i (q-p^*) \frac{\partial c_2^i}{\partial q} + \rho \sum_{i=s,u} \left[ \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial q} \right] = 0 \quad (19)$$

Now, using the FOC w.r.t.  $I$  given in equation (17), we replace the terms  $\frac{\partial V^i}{\partial I}$  in (19) and finally obtain after rearranging:

$$\sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i (c_2^{ij} - c_2^j) \frac{\partial V^{ij}}{\partial I} + \nu (q-p^*) \sum_{i=s,u} \pi^i \left( \frac{\partial c_2^i}{\partial q} + c_2^i \frac{\partial c_2^i}{\partial I} \right) + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \left( \frac{\partial c_2^i}{\partial q} + c_2^i \frac{\partial c_2^i}{\partial I} \right) = 0 \quad (20)$$

Denote  $\tilde{c}_2^i$  the compensated demand (or hicksian demand) for consumer  $i$  and good 2, then the Slutsky equation tells us that

$$\frac{\partial c_2^i}{\partial q} + c_2^i \frac{\partial c_2^i}{\partial I} = \frac{\partial \tilde{c}_2^i}{\partial q}.$$

Replacing in equation (20), we finally obtain

$$\sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i (c_2^{ij} - c_2^j) \frac{\partial V^{ij}}{\partial I} + \nu (q-p^*) \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial q} + \rho \sum_i \pi^i \frac{\partial E}{\partial c_2} \frac{\partial \tilde{c}_2^i}{\partial q} = 0. \quad (21)$$

which establishes the expression (8) in Proposition 1 when taking  $\mu^u = 0$  and  $\mu^s > 0$ .

## C Shadow price of pollution

With  $\mu^u = 0$ , (9) becomes:

$$\frac{\partial \mathcal{L}}{\partial e} = \sum_{i=s,u} \lambda^i \frac{\partial V^i}{\partial e} + \mu^s \left( \frac{\partial V^s}{\partial e} - \frac{\partial V^{su}}{\partial e} \right) + \nu (q-p^*) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} - \rho \quad (22)$$



Let us introduce the term  $\mu^s \frac{\partial V^{su}}{\partial I} \frac{\partial V^u}{\frac{\partial e}{\partial I}}$  in (22) and rearrange to get:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e} &= \mu^s \left( \frac{\partial V^{su}}{\partial I} \frac{\partial V^u}{\frac{\partial e}{\partial I}} - \frac{\partial V^{su}}{\partial e} \right) + \left( \lambda^u \frac{\partial V^u}{\partial I} - \mu^s \frac{\partial V^{su}}{\partial I} \right) \frac{\partial V^u}{\frac{\partial e}{\partial I}} \\ &\quad + (\lambda^s + \mu^s) \frac{\partial V^s}{\partial e} + \nu(q - p^*) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} - \rho \end{aligned}$$

Let us denote  $MWP^i = -\frac{\partial V^i}{\partial e} / \frac{\partial V^i}{\partial I}$  so that we get:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e} &= -\mu^s \frac{\partial V^{su}}{\partial I} (MWP^u - MWP^{su}) - \left( \lambda^u \frac{\partial V^u}{\partial I} - \mu^s \frac{\partial V^{su}}{\partial I} \right) MWP^u \quad (23) \\ &\quad - (\lambda^s + \mu^s) \frac{\partial V^s}{\partial I} MWP^s + \nu(q - p^*) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} - \rho \end{aligned}$$

Now derivating the lagrangian with respect to incomes and equating with 0 leads to the two following equations:

$$\begin{aligned} \lambda^u \frac{\partial V^u}{\partial I} - \mu^s \frac{\partial V^{su}}{\partial I} &= \nu \pi^u - \nu \pi^u (q - p^*) \frac{\partial c_2^u}{\partial I} - \rho \pi^u \frac{\partial E}{\partial c_2} \frac{\partial c_2^u}{\partial I} \\ (\lambda^s + \mu^s) \frac{\partial V^s}{\partial I} &= \nu \pi^s - \nu \pi^s (q - p^*) \frac{\partial c_2^s}{\partial I} - \rho \pi^s \frac{\partial E}{\partial c_2} \frac{\partial c_2^s}{\partial I} \end{aligned}$$

Introducing these two expressions into  $\frac{\partial \mathcal{L}}{\partial e}$  given by (23) leads to:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e} &= -\mu^s \frac{\partial V^{su}}{\partial I} (MWP^u - MWP^{su}) - \left( \nu \pi^u - \nu \pi^u (q - p^*) \frac{\partial c_2^u}{\partial I} - \rho \pi^u \frac{\partial E}{\partial c_2} \frac{\partial c_2^u}{\partial I} \right) MWP^u \\ &\quad - \left( \nu \pi^s - \nu \pi^s (q - p^*) \frac{\partial c_2^s}{\partial I} - \rho \pi^s \frac{\partial E}{\partial c_2} \frac{\partial c_2^s}{\partial I} \right) MWP^s + \nu(q - p^*) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} - \rho \end{aligned}$$

Rearranging the terms, we get:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e} &= -\mu^s \frac{\partial V^{su}}{\partial I} (MWP^u - MWP^{su}) - \nu \pi^u MWP^u - \nu \pi^s MWP^s \quad (24) \\ &\quad + \nu(q - p^*) \left( \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial I} MWP^i \right) \\ &\quad + \rho \left( \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} + \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial I} MWP^i - 1 \right) \end{aligned}$$

Following Pirttilä & Tuomala (1997), it is possible to state a Slutsky-like equation that determines the marginal impact of pollution on compensated consumption:

$$\frac{\partial c_2^i}{\partial e} = \frac{\partial \bar{c}_2^i}{\partial e} - \frac{\partial c_2^i}{\partial I} MWP^i. \quad (25)$$

Using (25) into (24), we get

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e} = & -\mu^s \frac{\partial V^{su}}{\partial I} (MWP^u - MWP^{su}) - \nu \sum_{i=s,u} \pi^i MWP^i \\ & + \nu(q - p^*) \left( \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial e} \right) + \rho \left( \frac{\partial E}{\partial c_2} \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial e} - 1 \right) \end{aligned}$$

Dividing by  $\nu$  and replacing  $q - p^*$  by its equilibrium value  $-\frac{\rho}{\nu} \frac{\partial E}{\partial c_2} + t_{NS}$ , we get:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e} = & -\frac{\mu^s}{\nu} \frac{\partial V^{su}}{\partial I} (MWP^u - MWP^{su}) - \sum_{i=s,u} \pi^i MWP^i \\ & + \left( -\frac{\rho}{\nu} \frac{\partial E}{\partial c_2} + t_{NS} \right) \left( \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial e} \right) + \frac{\rho}{\nu} \left( \frac{\partial E}{\partial c_2} \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial e} - 1 \right) \end{aligned}$$

At the optimum,  $\frac{\partial \mathcal{L}}{\partial e} = 0$  and we obtain the expected expression of  $\frac{\rho}{\nu}$  contained in (10).

## D Proof of Proposition 3

Derivating the Lagrangean (7) with respect to the production price  $p$  yields to:

$$\frac{\partial \mathcal{L}}{\partial p} = - \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i l^j \frac{\partial V^{ij}}{\partial l} \frac{\partial \left( \frac{w^j}{w^i} \right)}{\partial p} + \nu \left[ \sum_{i=s,u} \pi^i l^i \frac{\partial w^i}{\partial p} - (p - p^*) \frac{\partial y_2}{\partial p} - y_2 \right] + \rho \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial p} = 0 \quad (26)$$

Using the fact that  $\sum_{i=s,u} \pi^i l^i \frac{\partial w^i}{\partial p} = y_2$  as established in A, together with  $\mu^u = 0$  and  $\mu^s > 0$ , then equation (26) yields to the result contained in Proposition 3.

## E Proof of Proposition 5

First, by derivating the Lagrangean (7) with respect to labor supply  $l^i$  yields to:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l^i} = & (\lambda^i + \mu^i) \frac{\partial V^i}{\partial l} - \mu^j \frac{\partial V^{ji}}{\partial l} \frac{w^i}{w^j} + \nu \left[ \pi^i w^i + (q - p^*) \pi^i \frac{\partial c_2^i}{\partial l} - (p - p^*) \pi^i \frac{\partial y_2}{\partial l^i} \right] \\ & + \rho \pi^i \left[ \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial l} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial l^i} \right] = 0 \end{aligned} \quad (27)$$

Recall that  $p - p^* = \frac{\rho}{\nu} \frac{\partial E}{\partial y_2} + t_{EW}$  and that  $q - p^* = t_{NS} - \frac{\rho}{\nu} \frac{\partial E}{\partial c_2}$ . Replacing in (27), we get

$$(\lambda^i + \mu^i) \frac{\partial V^i}{\partial l} = \mu^j \frac{\partial V^{ji}}{\partial l} \frac{w^i}{w^j} - \nu \left[ \pi^i w^i + \pi^i t_{NS} \frac{\partial c_2^i}{\partial l} - \pi^i t_{EW} \frac{\partial y_2}{\partial l^i} \right]$$

Second, applying the same treatment to the first-order conditions with respect to  $I^i$  (17), we obtain:

$$(\lambda^i + \mu^i) \frac{\partial V^i}{\partial I} = \mu^j \frac{\partial V^{ji}}{\partial I} + \nu \pi^i \left( 1 - t_{NS} \frac{\partial c_2^i}{\partial I} \right)$$

Hence,

$$\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} \left[ \mu^j \frac{\partial V^{ji}}{\partial I} + \nu \pi^i \left( 1 - t_{NS} \frac{\partial c_2^i}{\partial I} \right) \right] = \mu^j \frac{\partial V^{ji}}{\partial l} \frac{w^i}{w^j} - \nu \left[ \pi^i w^i + \pi^i t_{NS} \frac{\partial c_2^i}{\partial l} - \pi^i t_{EW} \frac{\partial y_2}{\partial L^i} \right].$$

Rearranging, we get

$$\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} = -w^i + t_{EW} \frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu \pi^i} \left[ \frac{\partial V^{ji}}{\partial l} \frac{w^i}{w^j} - \frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} \frac{\partial V^{ji}}{\partial I} \right] + t_{NS} \left[ \frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} \frac{\partial c_2^i}{\partial I} - \frac{\partial c_2^i}{\partial l} \right]. \quad (28)$$

Denoting<sup>27</sup>

$$\frac{dc_2^i}{dl} \Big|_{dV^i=0} = -\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} \frac{\partial c_2^i}{\partial I} + \frac{\partial c_2^i}{\partial l},$$

and recalling that, from (12), we have

$$\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} = w^i (T^{ii} - 1),$$

we can rewrite (28) as follows:

$$T^{ii} = \frac{t_{EW}}{w^i} \frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu \pi^i} \frac{\partial V^{ji}}{\partial I} \left[ \frac{1}{w^j} \frac{\frac{\partial V^{ji}}{\partial l}}{\frac{\partial V^{ji}}{\partial I}} - \frac{1}{w^i} \frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} \right] - \frac{t_{NS}}{w^i} \frac{dc_2^i}{dl} \Big|_{dV^i=0}$$

which establishes the Proposition, when  $\mu^u = 0$  and  $\mu^s > 0$ .

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<sup>27</sup>Indeed, note that by using (12), we have  $\frac{dc_2^i}{dl} = \frac{\partial c_2^i}{\partial I} w^i (1 - T^{ii}) + \frac{\partial c_2^i}{\partial l} = -\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}} \frac{\partial c_2^i}{\partial I} + \frac{\partial c_2^i}{\partial l}$ .

Demographic weights	$\pi^s = \pi^u = 0.5$
Parameters of the utility function	$\alpha = 0.6; \xi = 0.3; \phi = 0.5; \gamma = 1; \omega = 0.75$
Parameters of the production functions	$A_1 = A_2 = 1; \sigma_1 = 0.8; \sigma_2 = 0.6$
"Emission intensity"	$\beta = 2$

Table 1: Parameters values.

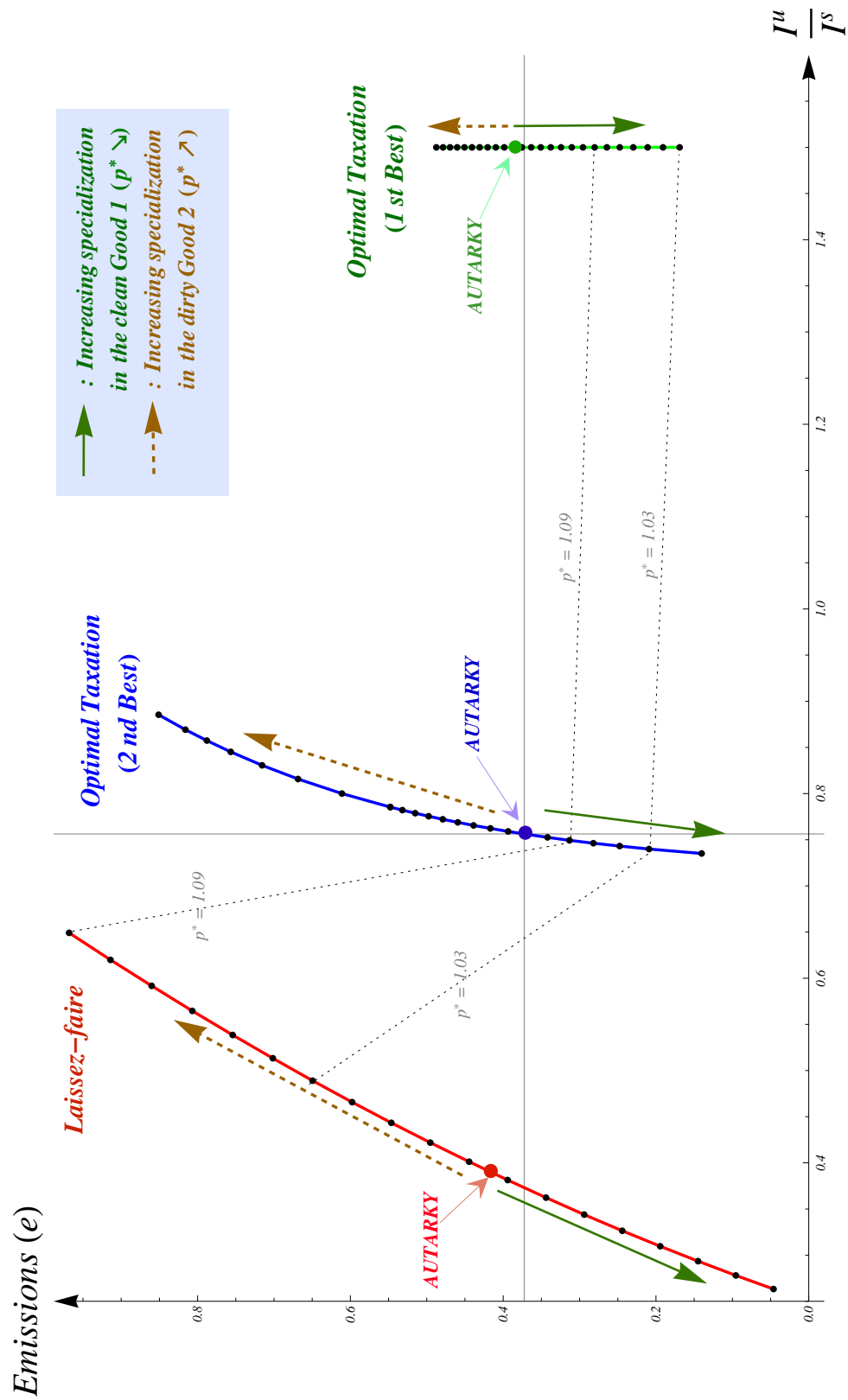


Figure 1: The pollution-inequality effects of trade liberalization : the pure production externality case

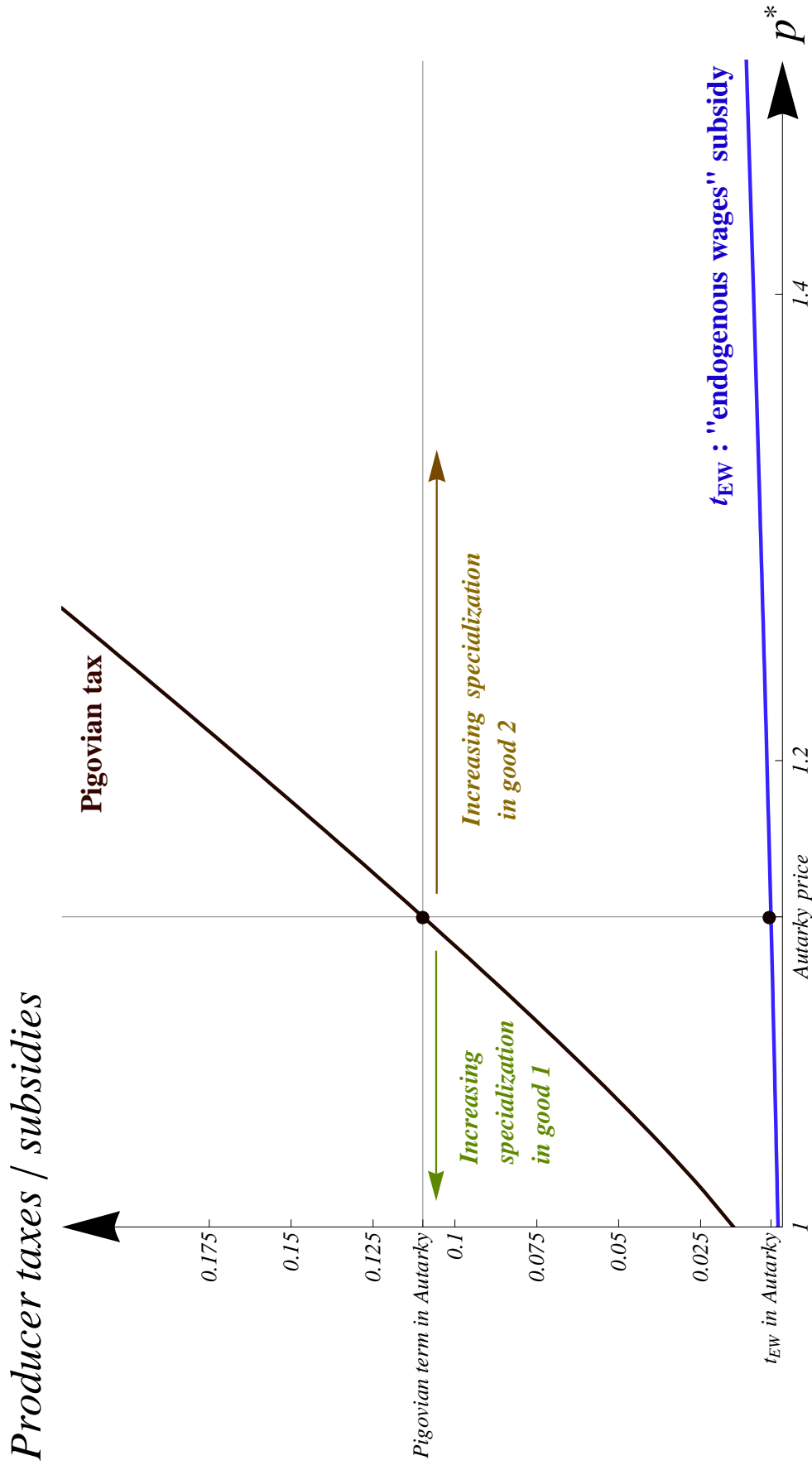


Figure 2: Impact of trade liberalization on the optimal distortion of the producer price

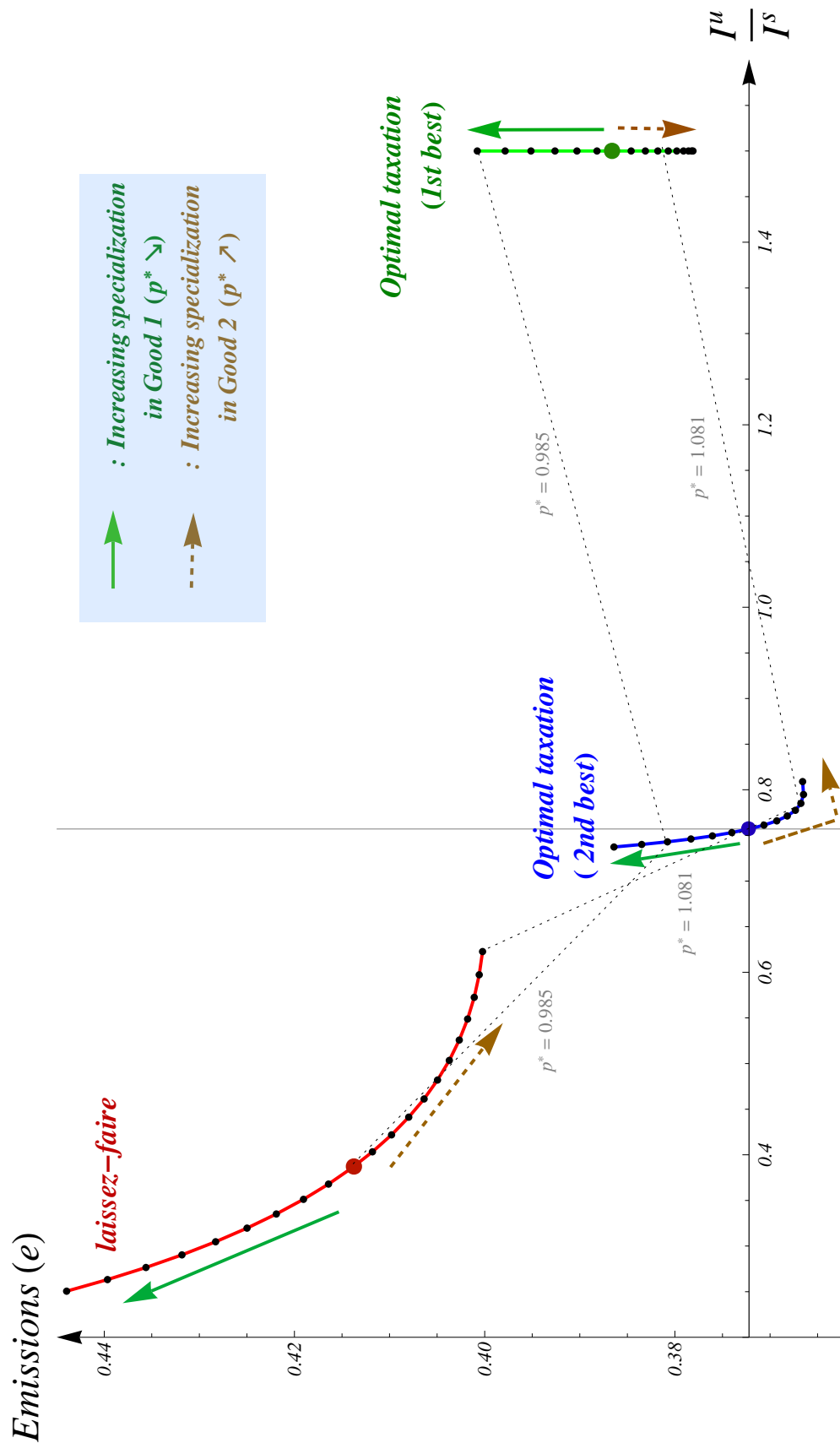


Figure 3: The pollution-inequality effects of trade liberalization : the pure consumption externality case

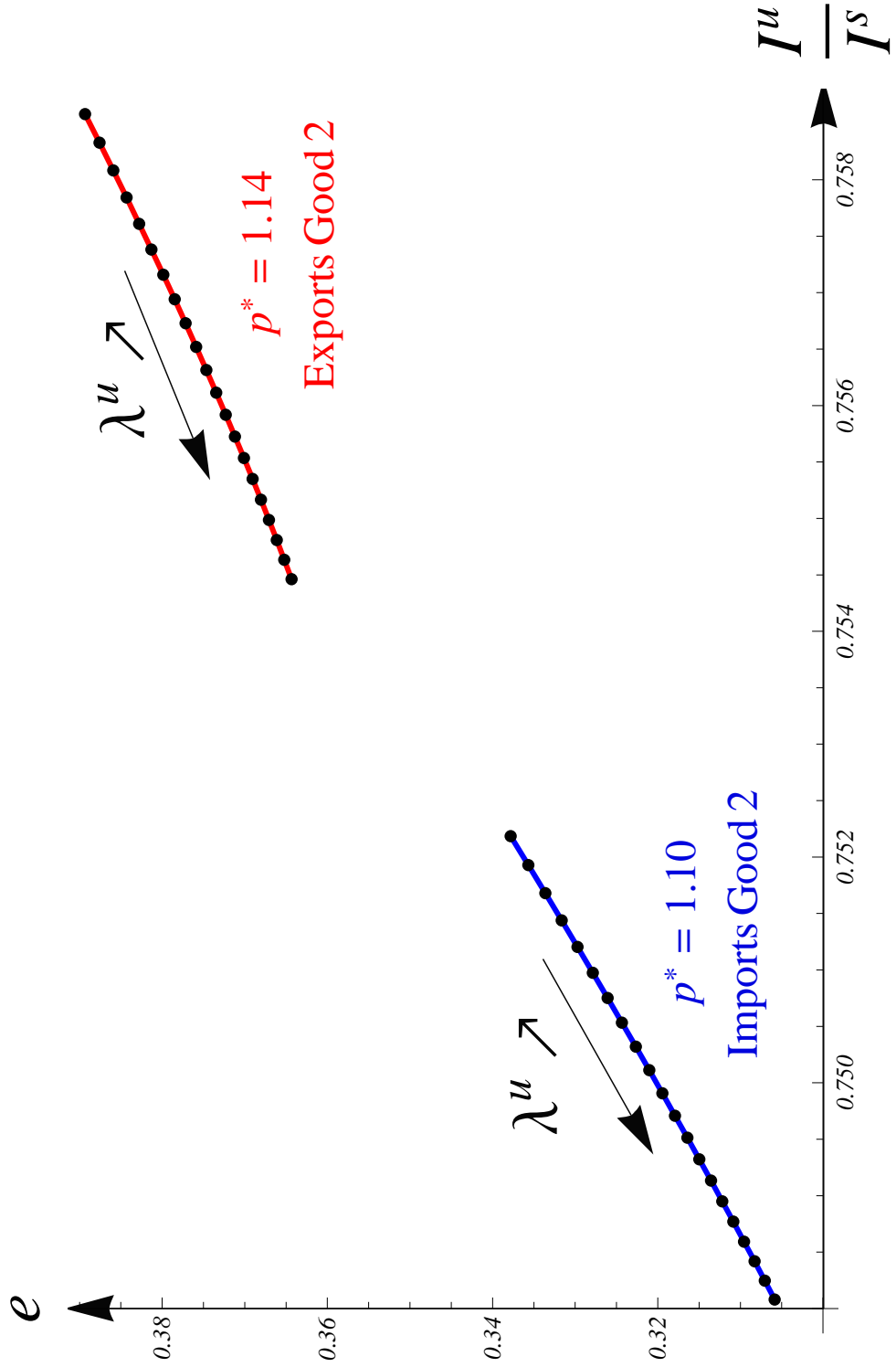


Figure 4: Open Economy with a production externality : increasing weight of the unskilled in the government objective