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# Inventory Routing with Explicit Energy Consumption: A Mass-Flow Formulation and First Experimentation

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**Abstract.** Energy efficiency is becoming an important criteria for the inventory systems. Our aim is to explicitly integrate the energy into the existing Inventory Routing Problem (IRP). The problem is based on a multi-period single-vehicle IRP with one depot and several customers. An energy estimation model is proposed based on vehicle dynamics. A mass-flow based Mixed Integer Linear Programming (MILP) formulation is presented. Instead of minimizing the distance or inventory cost, energy minimization is taken as an objective. Benchmark instances for inventory routing are adapted for energy estimation and experiments are conducted. The results are compared with those of the distance/inventory cost minimization.

## 1 Introduction

The Inventory Routing Problem (IRP) is developed under the Vendor Managed Inventory (VMI) management model, where the supplier monitors the inventory level of each retailer and acts as a central decision maker for the long-term replenishment policy of the whole system. With respect to the traditional Retailer Managed Inventory (RMI), the VMI results in a more efficient resource utilization: on the one hand, the supplier can reduce its inventories while maintaining the same level of service, or can increase the level of service while reducing the transportation cost; on the other hand, the retailers can devote less resources to monitoring their inventories while having the guarantee that no stock-out will occur [7].

Nowadays, the inventory management is faced with a new challenge—the sustainability. As one of the three bottom lines of sustainable supply chain management, environmental sustainability is the most recognized dimension [21]. As shown in [24], energy costs account for about 60% of the total cost of a unit of cargo transported on road. Since the traditional IRP concentrates solely on the economic benefits such as transportation costs and inventory costs, there is definitely a need to study the IRP under the energy perspective.

Under the VMI management model, the IRP combines the inventory management, vehicle routing and scheduling. There are three simultaneous decisions to make [13]:

1. when to serve a customer;
2. how much to deliver when serving a customer;
3. how to route the vehicle among the customers to be served.

These three decisions can be transformed for energy optimization.

1. The visiting time to a customer is adaptable. We can choose a delivery time that is both convenient for the customers and that can also avoid rush hours, as congestion is one of the main causes of high energy consumption and  $CO_2$  emissions.
2. Under the VMI policy, the customer demands are flexible and can be distributed in different combinations. This property allows us to determine an optimal set of delivery quantities that is the most effective for energy use while making sure that stock-out never happens.
3. The order of visit and the vehicle routes are to be determined. It is thus possible to design a routing strategy that takes the roads with the least energy costs.

Our purpose is to explicitly incorporate energy issue into the IRP. We introduce an energy estimation method and propose a Mixed Integer Linear Programming (MILP) optimization model that integrates energy cost into the objective function. Our study concentrates on Decisions 2 and 3 presented above. We discuss the possible influence of distribution and routing strategy to the energy consumption of the inventory system. The main contributions of this paper are: (i) to propose an approach to estimate the energy consumed in the transportation activities of inventory routing; (ii) to reformulate the IRP to explicitly incorporate the energy; (iii) to analyse the possible energy savings and the trade-offs between energy savings, travelled distances and inventory costs.

The remainder of this paper is organized as follows: A brief literature review is provided in Sect. 2. Section 3 gives a description of the energy estimation method, defines the problem and presents the mathematical model. After that, experimentation and results are given in Sect. 4, followed by the conclusion in Sect. 5.

## 2 Literature Review

In the literature, there are a lot of studies on the IRP since its origin in the year 1980s. There are also an emerging number of papers on the environmental-related routing problems these years. However, few researchers have paid attention to the energy IRP. In the remainder of the section, we start from a general literature review of the IRPs and the Green Vehicle Routing Problems(GVRPs), then we discuss the incorporation of these two categories of problems.

### 2.1 Inventory Routing Problem

The IRP was first studied under the context of the distribution of industrial gases [10]. Early studies concentrate on the impact of short-term decisions to

long-term inventory management and the combination of inventory management and vehicle routing [12,19,5]. Later on, various versions of IRP come out but there is no standard version. The IRP can be generally classified by seven criteria as shown in Table 1 [15]. For the inventory policy, under Maximum Level (ML) inventory policy, the replenishment level is flexible but bounded by the capacity available at each customer. while under Order-up-to Level (OU) policy, whenever a customer is visited, the quantity delivered is that to fill its inventory capacity. The IRP can also be considered as deterministic, stochastic or dynamic according to the availability of information on the demands.

Table 1: Classification of IRPs

<b>Criteria</b>	<b>Possible options</b>		
Time horizon	Finite	Infinite	
Structure	One-to-one	One-to-many	Many-to-many
Routing	Direct	Multiple	Continuous
Inventory policy	Order-up-to level (OU)	Maximum level (ML)	
Inventory decisions	Lost sales	Back-order	Non-negative
Fleet composition	Homogeneous	Heterogeneous	
Fleet size	Single	Multiple	Unconstrained

Both exact and approximative methods have been studied to solve the IRP. In [7], an MILP formulation of the IRP is proposed and the first branch-and-cut algorithm is developed. [17] extended the previous formulation to cases with heterogeneous multiple vehicles, with transshipment and also with consistency constraints. They also proposed a branch-and-cut algorithm. Heuristic algorithms are widely applied in early papers, such as assignment heuristic [19], clustering heuristic [5], and trade-off based heuristic [12]. A randomized greedy algorithm is developed in [26] for the inventory routing with continuous moves which contains both pick-ups and deliveries. Later papers applied a variety of metaheuristics, such as Greedy Randomized Adaptive Search Procedure (GRASP) [14], tabu search [6], Adapted Large Neighbourhood Search (ALNS) [16], etc. Recent solution methods combine heuristic and mathematical programming, yielding the so-called "matheuristic" algorithms. For example, based on the formulation in [7], new formulations are proposed in [8]. The authors used a branch-and-cut algorithm that adds cuts heuristically and compared the new formulations with existing ones using a large set of benchmark instances.

Two literature reviews are worth mentioning here. A survey of the industrial aspects of the problem can be found in [3], and the typologies of the problem as well as their solution methods is reviewed in [15].

## 2.2 Green Freight Routing

In the literature, there are a growing number of papers about the green logistics and sustainable supply chain management.

On the transportation side, the eco-driving mechanism is developed to guide the driver to perform the most fuel efficient operation [1], and the eco-routing navigation systems aim to identify the most energy-efficient route for a vehicle to travel between two points in real-time [11].

More and more researchers pay attention to the incorporation of energy into Vehicle Routing Problem (VRP). In [22], the Capacitated Vehicle Routing Problem (CVRP) is extended with a new cost function that depends on both the distance travelled and the load of the vehicle and this problem is defined as the Energy Minimizing Vehicle Routing Problem (EMVRP). In [29], the Fuel Consumption Rate (FCR), a factor depending on load, is added to the CVRP with the objective of minimizing fuel consumption. The authors proposed a String-model-based simulated annealing algorithm to solve the problem. They discovered that the difference of the FCR induced by vehicle loads, the diverse demands and uneven geographical positions of the customers can all influence fuel cost savings. Focusing on the pollution and  $CO_2$  emission generated by the road transport sector, the Pollution Routing Problem (PRP) is proposed to explicitly control the Greenhouse Gas (GHG) emission of the transportation [9]. In this paper, the authors discussed the trade-offs between distance, load and energy minimizing objectives and the influence of parameters such as vehicle speed and load as well as customers time windows.

A detailed literature review of the GVRP can be found in [23]. In this review, the environmental sensitive Vehicle Routing Problem is divided into three groups: the Green-VRP for the optimization of energy consumption [22,29]; the PRP for the reduction of pollution, especially GHG emissions [9]; and the Vehicle Routing in Reverse Logistics for the collection of wastes and end-of-life product. It is pointed out that incorporating inventory models with PRP models can be promising.

The vehicle emission models and their applications to road transportation planning are reviewed in [18]. Different factors affecting fuel consumption or vehicle emission are discussed in this paper. However, few models focus on the energy consumed.

The first paper that incorporates environmental aspects in the IRP is a case study from the petrochemical industry [27]. They proposed a mathematical model that integrate  $CO_2$  cost into the objective function. A multi-vehicle inventory routing with  $CO_2$  emission is also studied in [2], where a penalty is added if the  $CO_2$  emission of the planning route is higher than a predefined capacity of the road. However, neither of them consider the energy estimation, nor are the influential factors clearly exposed.

### 3 Problem Statement

The problem in our study is based on a *multi-period single-vehicle deterministic* IRP with *one depot and several customers*. The planning horizon is defined by a set of periods. The vehicle can leave the depot only once per period. In each period, it makes a tour around the customers that need to be refilled and returns

to the depot. Stock-out and back-orders are not allowed in this model. Instead of the distance and inventory minimization, we take energy minimization as objective. Both the ML and the OU policy are applied to see the influence of different replenishment strategies to the energy consumption.

In this section, we start with the energy estimation model that explains how to estimate the energy cost of a road segment with vehicle speed variation. Then we present the parameters and decisions variables of the problem. Finally we give the mathematical formulation of the problem.

### 3.1 Energy Estimation Model

In most of the literature related to IRP, transportation cost is represented by the distance travelled. This is not suitable for the energy minimization because energy is influenced by various factors as shown in Sect. 2.2. In the literature related to energy issue, most models focus on the fuel consumption or  $CO_2$  emission and most of them depend on vehicle type. Nevertheless, with the emergence of electric and hybrid vehicles, we find it more appropriate to estimate the energy used directly. In addition, for the generality of the problem, it is important that the energy estimation model would apply for every type of vehicles.

According to [25], travelling kinematic variation (accelerations, idle duration, etc.) obviously affects engine load and by turns the energy consumption. Thus, in this paper, we propose a general simple model based on vehicle dynamics. This model would be applicable to European suburban transportation network with short or medium distances and potentially high traffic intensity. It can give us a gross estimation of the energy required by a vehicle on a road segment with speed variation, independent of vehicle type or energy source.

**General Parameters.** Suppose a vehicle travelling from one location to another. The path of the vehicle between two locations is supposed to be predefined with an average stop rate  $\tau$ , and the total distance travelled is  $s$ . So the vehicle stops  $\tau \cdot s$  times during the trip. The coefficient of friction is a fixed parameter  $\mu = 0.01$ . The gravitational acceleration is  $g = 9.81m/s^2$ . The environmental effects of the road (wind, temperature etc.) as well as the viscosity of air are ignored. Road slopes, denoted by the rate  $\theta$ , is deduced from the difference of altitude between the origin and the destination ( $\sin \theta = \frac{\Delta h}{s}$  where  $\Delta h = h_{\text{end}} - h_{\text{begin}}$ ). We consider that on the same segment of road, the vehicle only climb or descend once. If the vehicle climbs (the destination is higher than the origin), then  $\theta$  is positive. If the vehicle descends, then  $\theta$  is negative. The only forces exerted on the vehicle are the gravity, the rolling resistance and the traction force of the engine.

In our model, the stop rate  $\tau$ , i.e., the number of stops per unit of distance is used to model the dynamics of the vehicle on a fixed segment of road. This parameter can also represent the traffic condition on the road. More precisely, with a traffic near free flow,  $\tau$  takes a value near 0, which means that the vehicle goes through the road fluently without any stops; however, with congestion, this

number corresponds to a higher value to indicate a frequent speed variation. Usually  $\tau$  takes a value between 0 and 4 depending on road types [4]. Moreover, there exists an interrelationship between the distance travelled, the stop rate and the speed and acceleration of the vehicle, which is generally explained in Sect. 4.1.

Between every two stops, the vehicle speed is supposed to follow a fixed pattern of variation—acceleration, uniform speed movement and stop. Each time, the vehicle speeds up from 0 to the target speed  $V$  with a fixed acceleration  $a_{acc}$ . It goes on at this speed for a while and then stops. The stop is supposed to be instantaneous. This pattern is repeated  $\tau \cdot s$  times supposing that the vehicle has no speed at both the starting and the ending point. After each stop, it speeds up again to the same target speed. This speed profile is shown in Fig. 1.

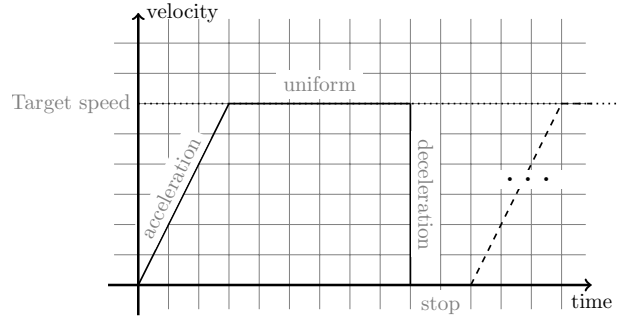


Fig. 1: The speed variation of the vehicle with time

**Energy Consumption Formula.** According to knowledge of physics and energy conservation, under the hypothesis of speed variation presented above, the energy consumption on a segment between two locations with a distance  $s$  can be calculated as follows:

*Acceleration Phase.* In this phase, the speed of the vehicle increases from 0 to the target speed  $V$  with a constant acceleration  $a_{acc}$ . If we note  $F_{acc}$  the traction force of the engine,  $s_{acc}$  the distance travelled on the slope  $\theta$ ,  $E_{acc}$  the energy consumed, and  $P_{acc}(t)$  the engine power at instant  $t$ , we have:

$$\begin{aligned}
 v(t) &= a_{acc}t; \\
 V &= a_{acc}t_{acc}; \\
 s_{acc}(t) &= \frac{1}{2}a_{acc}t^2; \\
 F_{acc} - mg\mu \cos \theta - mg \sin \theta &= ma_{acc}; \\
 P_{acc}(t) = F_{acc}v(t) &= m(a_{acc} + g\mu \cos \theta + g \sin \theta)a_{acc}t;
 \end{aligned}$$

$$E_{\text{acc}} = \int_0^{t_{\text{acc}}} P_{\text{acc}}(t) dt.$$

At the end of this phase, the engine power is

$$P_{\text{acc}}^{\text{end}} = m(a_{\text{acc}} + g\mu \cos \theta + g \sin \theta)V;$$

the total distance travelled is

$$s_{\text{acc}} = \frac{V^2}{2a_{\text{acc}}}.$$

The total energy cost per unit of mass is:

$$\begin{aligned} c_{\text{acc}} &= \frac{1}{2}(a_{\text{acc}} + g\mu \cos \theta + g \sin \theta)a_{\text{acc}}t_{\text{acc}}^2 \\ &= \frac{1}{2}V^2 + g(\mu \cos \theta + \sin \theta)\frac{V^2}{2a_{\text{acc}}}. \end{aligned}$$

*Uniform-Speed Phase.* In this phase, the vehicle travels in uniform speed for a distance  $s_u$ . This distance is computed as the difference between the total distance  $s$  and the total distance travelled in acceleration and deceleration. Since the deceleration is considered to be instantaneous ( $s_{\text{dec}} = 0$ ), the total distance travelled at uniform speed is calculated as:

$$s_u = s - \tau s (s_{\text{acc}} + s_{\text{dec}}) = s - \tau s s_{\text{acc}}$$

with  $\tau \cdot s$  the total number of stops. The engine force is constant

$$F_u = \mu mg \cos \theta + mg \sin \theta = mg(\mu \cos \theta + \sin \theta).$$

The engine power is also constant

$$P_u(t) = F_u V = mg\mu V.$$

The total energy cost per unit of mass in the uniform phase is:

$$c_u = g(\mu \cos \theta + \sin \theta)(s - \tau s s_{\text{acc}}).$$

*Deceleration Phase.* In this phase, since we consider an instantaneous stop, the distance  $s_{\text{dec}}$  and the engine power  $P_{\text{dec}}$  is 0, the energy is lost immediately:  $E_{\text{dec}} = \frac{1}{2}mV^2$ . The energy cost per unit of mass is

$$c_{\text{dec}} = \frac{1}{2}V^2.$$



*Summary of Energy Cost.* According to the three-phase analysis of the vehicle energy, the total energy cost per unit of mass when distance  $s$  is travelled with stop rate  $\tau$  is then:

$$c = c_u + \tau \cdot s (c_{\text{acc}} + c_{\text{dec}}).$$

Finally, we get:

$$c = g(\mu \cos \theta + \sin \theta)s + \tau s V^2$$

Since  $\theta$  is usually a small value, we can take  $\cos \theta \simeq 1$  and  $\sin = \frac{\Delta h}{s}$ , then the energy cost would be

$$c = g\mu s + g\Delta h + \tau s V^2, \quad (1)$$

which is the same result as obtained by the law of conservation of energy.

Figure 2 shows the power variation of the vehicle under the previous speed variation. We can see that each time the vehicle speeds up, there appears a "peak" of engine power which corresponds to a potentially high energy consumption. This is also reflected by (1)—the more the vehicle stops on a road segment ( $\tau$  takes a bigger value), the higher the energy would cost. In addition, if the vehicle climbs a mountain ( $\Delta h$  positive), more energy would be used.

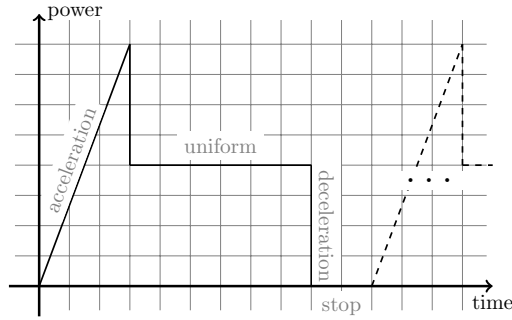


Fig. 2: The power variation with time

In this way, we define  $c_{ij} = g\mu s_{ij} + g\Delta h_{ij} + \tau_{ij} s_{ij} V_{ij}^2$  the energy cost per unit of mass from location  $i$  to location  $j$ . It is related to the distance travelled  $s_{ij}$ , the variation of altitude  $\Delta h_{ij}$  and the dynamics of the vehicle on the road as expressed by the stop rate  $\tau_{ij}$  and target speed  $V_{ij}$ .

### 3.2 Problem Definition

To facilitate the energy estimation, two units are used to measure inventory—the number of components and the weight in kilograms (kg). The number of components is used by the customers to represent their inventory levels and to count the number of packages of delivered goods. The weight is used by the transporters. It is the physical mass of the components transported by the vehicle.

The next parts present in details the parameters and variables of the problem. In particular, mass flow variables are introduced to link the energy estimation and the inventory management.

**General Settings for Routing.** The problem is constructed on a *complete undirected* graph  $\mathcal{G} = \{V, E\}$ .  $V = \{0, \dots, n\}$  is the vertex set. It includes one depot denoted by 0 and the customers to visit denoted by the set  $V_c = \{1, \dots, n\}$ .  $E = \{(i, j) \mid i, j \in V \text{ and } i < j\}$  is the set of undirected edges. There are  $T$  replenishment planning periods. Each period can be a day, a week or even a month according to the real situation. In each period, only one tour can be performed. If a tour is presented in a period, the vehicle starts from the depot, travels through all the customers who need to be served at this period and returns back to the depot at the end of the period.

Three sets of decision variables  $z_i^t$ ,  $x_{ij}^t$ , and  $y_{ij}^t$  correspond to routing. For each  $i \in V_c$ ,  $t \in T$ ,  $z_i^t$  is a binary variable indicating whether customer  $i$  is served at period  $t$ . It equals 1 if customer  $i$  is served and 0 otherwise. Particularly,  $z_0^t$  indicates whether the tour at period  $t$  is performed (equals 1) or not (0). For each edge  $(i, j) \in E$  and each period  $t \in T$ ,  $x_{ij}^t$  is an integer variable indicating the number of times that edge  $(i, j)$  is used in the tour of period  $t$ .  $x_{ij}^t \in \{0, 1\}$  if  $i, j \in V_c$  since a customer can be visited only once per period and  $x_{0j} \in \{0, 1, 2\} \forall j \in V_c$  because direct shipping is allowed between the depot and a customer. For each arc  $(i, j) \in V \times V$  and each period  $t \in T$ , variable  $y_{ij}^t$  is a binary variable to indicate the direction of the vehicle route. It equals 1 if the vehicle travels from  $i$  to  $j$  at period  $t$ .

The vehicle has a capacity  $Q$  expressed in numbers of components and a mass limit  $M$ . The empty vehicle mass, or curb weight of the vehicle is  $W$  (in kg).

**Inventory Characteristics.** Inventory levels at customers and depot are monitored during the whole planning time horizon. They are summarised at the end of each replenishment period. The customer demands are described as demand rates per period. In each period,  $r_i$  is the number of units of components demanded by the customer  $i \in V_c$ . In particular,  $r_0$  is the number of components made available at the depot in each period. Each customer  $i \in V_c$  has a stocking capacity  $C_i$ , while the depot is supposed to have an unlimited stocking capacity.  $h_i$  is the inventory storage cost per unit of component per period at customer  $i$  or the depot.

Two variables are defined for the inventory management. The variable  $I_i^t$  is the inventory level in number of components at the depot 0 or at the customer  $i \in V_c$  at the end of period  $t$ . The variable  $q_i^t$  is the number of components delivered to customer  $i \in V_c$  during period  $t \in T$ .

**Commodity Mass Flow.** The energy cost per unit of mass for each arc  $(i, j) \in V \times V$  is defined at the end of Sect. 3.1. If  $m_{ij}$  is the mass (kg) loaded on the

vehicle when traversing from vertex  $i$  to vertex  $j$  and  $W$  the vehicle weight(kg), the energy cost of the vehicle travelling from  $i$  to  $j$  is thus:

$$c_{ij}(m_{ij} + W) \quad (2)$$

As we can see from (2), the total energy cost is a linear function of mass. Meanwhile, the mass or the quantity of products is also an important element in the inventory management. It is a measurement of the inventory levels. In fact, there exists a mass flow inside the transportation network and it can serve as a bridge linking the inventory routing and the energy optimization.

In the traditional IRP formulations presented in [8], a flow formulation exists to model the inventory flows inside the transportation network. Our model takes advantage of this formulation. Instead of thinking the flows in terms of number of components, the mass of the shipped components is considered. In each period, once we decide the mass transported on each edge of the network, we can deduce the number of components left at each customer vertex. Or inversely, if we know how many units of components are delivered to each customer, we can decide the order of visits and get a mass flow in the transportation network that minimizes the energy consumed.

In our model, variables  $m_{ij}^t$  are defined as the mass transported by the vehicle from  $i$  to  $j$  at period  $t$ . They are linked with the vehicle flow variables  $y_{ij}^t$ . If the vehicle does not go from  $i$  to  $j$  at period  $t$  ( $y_{ij}^t = 0$ ),  $m_{ij}^t$  is equal to 0.

Figure 3 details the various flows traversing customer  $i$  at period  $t$ . The inventory flow  $I_i^t$  and the demand  $r_i$ , expressed in number of components, are associated with the dotted arcs. They describe the variation of the inventory level of  $i$  with time periods. The solid arcs stand for the mass of the incoming and outgoing products ( $m_{ji}^t$  and  $m_{ij}^t$  respectively). They are used to estimate the potential energy consumption, with  $c_{ij}^t$  the energy cost per unit of mass on edge  $(i, j)$ . The weight of one unit of component in kilograms at a customer  $i \in V_c$  is denoted by  $m_i$ . The difference  $\frac{1}{m_i}(\sum_{j \in V \setminus \{i\}} m_{ji}^t - \sum_{j \in V \setminus \{i\}} m_{ij}^t)$  gives the number of components  $q_i^t$  delivered to customer  $i$  during period  $t$ .

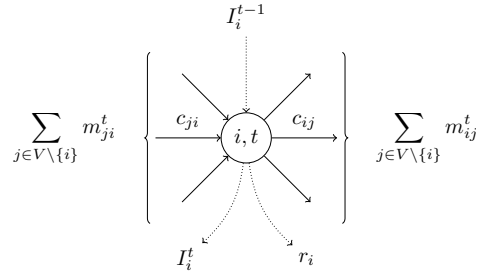


Fig. 3: The flows passing through customer  $i$  at period  $t$

### 3.3 Mathematical Model

With the parameters and the variables defined in Sect. 3, the mathematical model is explained here. The complete model can be found in the annexe.

**Objectives.** Two objectives are defined, one for inventory and distance optimization and the other for energy optimization. Equation (3) is the traditional objective as defined in [8]. It is the sum of the total distance travelled plus the sum of the inventory storage costs over all the periods.

$$\min \sum_{t \in T} \sum_{(i,j) \in V \times V} s_{ij} y_{ij}^t + \sum_{t \in T} \sum_{i \in V} h_i I_i^t \quad (3)$$

Equation (4) is the sum of the total energy consumed in the inventory routing over all the periods. Note that it contains two terms: one is a flexible cost related to the transported mass of the vehicle  $m_{ij}^t$ , and the other is a fixed cost induced by the vehicle curb weight  $W$ .

$$\min \sum_{t \in T} \sum_{(i,j) \in V \times V} c_{ij} m_{ij}^t + W \sum_{t \in T} \sum_{(i,j) \in V \times V} c_{ij} y_{ij}^t \quad (4)$$

**Constraints.** Compared with the basic flow formulation in [8], mass flow variables take place of commodity flow variables.

*Inventory Management.* Constraints (5) to (9) are for monitoring the inventory levels of each location at each period.

$$I_0^t = I_0^{t-1} + r_0 - \sum_{i \in V_c} q_i^t \quad \forall t \in T \quad (5)$$

$$I_i^t = I_i^{t-1} - r_i + q_i^t \quad \forall i \in V_c, t \in T \quad (6)$$

$$q_i^t \geq C_i z_i^t - I_i^{t-1} \quad \forall i \in V_c, t \in T \quad (7)$$

$$q_i^t \leq C_i - I_i^{t-1} \quad \forall i \in V_c, t \in T \quad (8)$$

$$q_i^t \leq C_i z_i^t \quad \forall i \in V_c, t \in T \quad (9)$$

Constraints (5) and (6) ensure that the inventory levels of each station are coherent from one period to another. The OU inventory policy is ensured by constraints (7) and (8)—after each delivery, the inventory level of each visited customer is fulfilled to the maximum. If we delete Constraints (7), the model becomes one under the ML policy, where the replenishment level is flexible but bounded by the stocking capacity of each customer. Constraints (9) ensure that nothing is delivered to a customer  $i$  if he is not visited at a period and that otherwise, the delivered quantity never exceeds the capacity.

*Commodity Mass Flow Management.* Constraints (10) and (11) are the mass flow constraints.

$$\sum_{j \in V_c} m_{0j}^t = \sum_{i \in V_c} q_i^t m_i \quad \forall t \in T \quad (10)$$

$$\sum_{j \in V} m_{ji}^t - \sum_{j \in V} m_{ij}^t = q_i^t m_i \quad \forall i \in V_c, t \in T \quad (11)$$

Constraints (10) ensure that at period  $t$ , the mass out of the depot is equal to the total mass transported to all the customers. Constraints (11) ensure that for each customer  $i$  at each period  $t$ , the quantity received is equal to the difference between the entering and the leaving mass flow.

*Vehicle Routing.* Constraints (12) to Constraints (17) are typical routing constraints.

**Degree Constraints**

$$\sum_{j \in V_c} x_{0j}^t = 2z_0^t \quad \forall t \in T \quad (12)$$

$$\sum_{\substack{j \in V \\ j < i}} x_{ji}^t + \sum_{\substack{j \in V_c \\ j > i}} x_{ij}^t = 2z_i^t \quad \forall i \in V_c, t \in T \quad (13)$$

**Directed vehicle flow**

$$\sum_{j \in V_c} y_{0j}^t = z_0^t \quad \forall t \in T \quad (14)$$

$$\sum_{j \in V} y_{ij}^t = z_i^t \quad \forall t \in T, i \in V_c \quad (15)$$

$$\sum_{j \in V} y_{ji}^t = z_i^t \quad \forall t \in T, i \in V_c \quad (16)$$

$$x_{ij}^t = y_{ij}^t + y_{ji}^t \quad \forall t \in T, (i, j) \in E \quad (17)$$

Constraints (12) and (13) are the degree constraints. They define the route of the vehicle in each period. Constraints (14)–(16) restrain the direction of the vehicle flow. They link  $y$  and  $z$  variables to make sure that in each period at most one tour is performed and that each customer is visited at most once in each period. Constraints (17) link variables  $y$  and  $x$  to ensure that each edge is used at most once in each period.

*Vehicle Capacity.* Constraints (18) and (19) guarantee that the vehicle capacity is never exceeded both in number of components and in unit of mass.

$$\sum_{i \in V_c} q_i^t \leq Qz_0^t \quad \forall t \in T \quad (18)$$

$$m_{ij}^t \leq My_{ij}^t \quad \forall t \in T, (i, j) \in V \times V \quad (19)$$

Constraints (19) also link the mass flow and the vehicle flow on the graph. They make sure that the direction of the vehicle flow is the same as that of the mass flow.

*Variable Domains.* Constraints (20)–(26) are the variable domains.

$$0 \leq I_i^t \leq C_i, I_i^t \in \mathbb{N} \quad \forall i \in V, t \in T \quad (20)$$

$$0 \leq q_i^t \leq Q, q_i^t \in \mathbb{N} \quad \forall i \in V_c, t \in T \quad (21)$$

$$0 \leq m_{ij}^t \leq M, m_{ij}^t \in \mathbb{N} \quad \forall (i, j) \in V \times V, t \in T \quad (22)$$

$$z_i^t \in \{0, 1\} \quad \forall i \in V, t \in T \quad (23)$$

$$x_{ij}^t \in \{0, 1\} \quad \forall (i, j) \in E, i < j, t \in T \quad (24)$$

$$x_{0j}^t \in \{0, 1, 2\} \quad \forall j \in V_c, t \in T \quad (25)$$

$$y_{ij}^t \in \{0, 1\} \quad \forall (i, j) \in V \times V, t \in T \quad (26)$$

All the variables take integer values. Constraints (20) ensure that the inventory level of a customer never exceeds his stocking capacity. Constraints (21) and (22) make sure that the vehicle capacity is never exceeded neither in terms of mass nor in terms of units of components. Note that for variables  $x_{0j}^t$ , since direct routing is possible, they can be assigned with value 2.

## 4 Experimentation and Results

The existing IRP instances proposed in [7] are adapted for energy estimation. The MILP model is constructed and solved using the adapted instances. An analysis of the obtained results is presented.

### 4.1 Data Generation

Information on stop rates  $\tau$  and vehicle target speeds  $V$  relative to the distance is added to the benchmark instances proposed in [7]. The correlation within these parameters is determined based on empirical data of delivery trucks on real routes provided by [28]. The following part explains how the data set is generated.

First, two types of road is considered—highway and national route. For each edge between two locations, the type of road is generated randomly. The target speed and the number of stops for different types of roads are generated using different methods. On a highway, the maximum speed is fixed at  $110 \text{ km/h}$ , and the number of stops is fixed at 2 stops per edge no matter how long is travelled. On a national route, the vehicle speed is fixed at  $80 \text{ km/h}$  and the number of stops is linearly dependent on the distance with a random error. For all types of road, the average acceleration rate is fixed at  $1.01 \text{ m/s}^2$ . The instances generated contain two categories of type proportion: one is with  $\frac{2}{3}$  edges among all the edges defined as highway and  $\frac{1}{3}$  as national route; the other is with  $\frac{1}{3}$  edges among all defined as highway and  $\frac{2}{3}$  as national route.

Then, a random number between 1 and 10 is generated for each customer  $i$  to represent the mass of one unit of components  $m_i$ . Vehicle weight and mass capacity are correlated according to vehicle information provided in [20].

Last, a random number between 0 to 500 is generated as the altitude  $h_i$  of each location  $i$ .

In total, 64 cases are generated. Each case contains 5 instances. The cases are categorized by the number of periods (3 or 6 periods of replenishment planning), the proportion of the inventory storage cost in relation to the transportation cost (high or low), the inventory replenishment policy (OU or ML), the proportion of each type of road in the whole map and the number of customers in the map.

## 4.2 System Settings

The model is realized in C++ with *IBM<sup>®</sup> ILOG<sup>®</sup> CPLEX 12.6.1.0* and solved by the default Branch-and-Bound algorithm with one thread. The operation system is *Ubuntu 14.04 LTS* with *Intel<sup>®</sup> Core<sup>®</sup> i7-4790 3.60GHz* processor and *16 G* memory.

The solution process is divided into two phases. In the first phase, the objective is to minimize the combined cost of transportation and inventory as in objective function (3). In the second phase, starting with the solution of the first phase, the same model is solved to minimize the total energy consumption as computed in objective function (4).

A time limit of 1800 seconds is set for each of the two phases. All the other settings of CPLEX are as default. The results of both of the two phases are compared in the next part.

## 4.3 Result and Analysis

**Performance.** The dimension of an instance is determined by the number of periods and the number of customers. The inventory policy (OU or ML) changes the constraint sets of the model. The combination of these three parameters define a category of instances. Each category contains 20 instances. In Table 2, computation time in seconds of each solution phase ("time1" and "time2") and the solution status within the time limit ("status1" and "status2") are listed for each category. The values for computation time are average values over all the instances of the same category. If all the instances of a category can be solved to optimality by CPLEX, the status is noted "Optimal". If part of the instances of a category can be solved to optimality, then the status is noted "Optimal(*n*)" with a number *n* in parentheses indicating the number of instances solved to optimality in this category. Otherwise, if no optimal solution is found in the time limit by CPLEX, then the average relative gap after 1800 seconds of computation is reported as the status, and the time value is noted 1800.

As we can see from Table 2, energy minimization is much more difficult to solve than inventory and transportation cost minimization (time2  $\gg$  time1). This may result from the large possible combination of the values of the mass flows. The problem is NP since it is an extension of the VRP. It becomes more difficult as the dimension of the instances increases. For both OU and ML policies, instances larger than 20 customers with 3 periods or 15 customers with 6 periods can hardly be solved to optimality for energy optimization within the time limit. The influence of the inventory policy to energy minimization is not as obvious as in the traditional IRP.

Table 2: Solution Status and Solving Time

T	ML policy				OU Policy			
	n	status1	status2	time1 time2	n	status1	status2	time1 time2
3	5	Optimal	Optimal	0.137 0.0892	5	Optimal	Optimal	0.0992 0.0844
	10	Optimal	Optimal	1.78 1.60	10	Optimal	Optimal	1.70 1.34
	15	Optimal	Optimal	12.5 61.5	15	Optimal	Optimal	16.6 35.0
	20	Optimal	Optimal(13)	199 976	20	Optimal	Optimal(17)	65.6 749
	25	Optimal	0.084	67.01 1800	25	Optimal(12)	0.058	787 1800
	30	Optimal	0.12	310 1800	30	Optimal(12)	0.10	1017 1800
	35	Optimal	0.15	183 1800	35	Optimal(8)	0.15	1248 1800
	40	Optimal(16)	0.17	624 1800	40	Optimal(2)	0.19	1714 1800
	45	Optimal(14)	0.18	756 1800	45	0.054	0.22	1800 1800
	50	Optimal(5)	0.23	1649 1800	50	0.10	0.27	1800 1800
6	5	Optimal	Optimal	2.53 0.401	5	Optimal	Optimal	0.489 0.465
	10	Optimal	Optimal	45.0 54.9	10	Optimal	Optimal	29.2 55.3
	15	Optimal	Optimal(1)	429 1790	15	Optimal	Optimal(3)	169 1630
	20	Optimal(4)	0.098	1639 1800	20	Optimal(7)	0.10	1487 1800
	25	Optimal(4)	0.14	1575 1800	25	Optimal(6)	0.16	1515 1800
	30	0.077	0.20	1800 1800	30	0.075	0.21	1800 1800

**Energy Impacting Factors.** Suppose that the energy consumption in Phase 1 is noted  $E_1$  and the consumption in Phase 2 is noted  $E_2$ . The energy reduction in the following paragraphs is defined as the ratio  $r = \frac{E_2 - E_1}{E_1}$ . In general, the energy reduction can achieve 35% in average. It is at least 21% and can reach as high as 46%.

Several factors have an impact on the energy reduction. First, the size of the instance can influence the potential energy reduction. Larger instances tend to induce higher energy conservation. Figure 4 shows the variation of the energy reduction in relation with the number of customers.

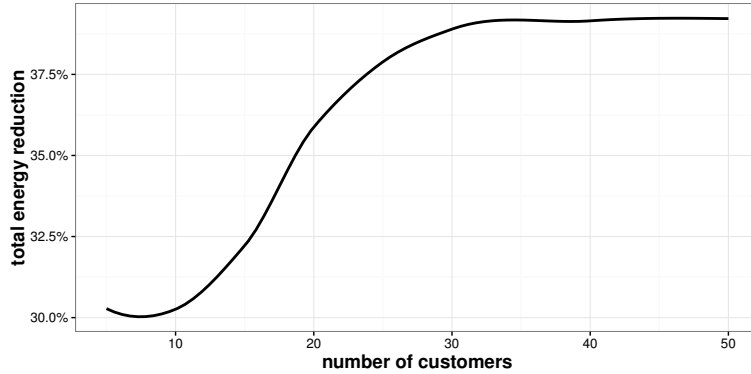


Fig. 4: Number of Customers and Energy Reduction



Second, there exists a compromise between the inventory/distance cost and energy cost since all the energy reduction necessitates an augmentation of distance and inventory costs whatever policy or planning horizon (see Fig. 5). And it seems that under the configuration where there are more national routes (that means a more variation of number of stops), this compromise becomes more obvious.

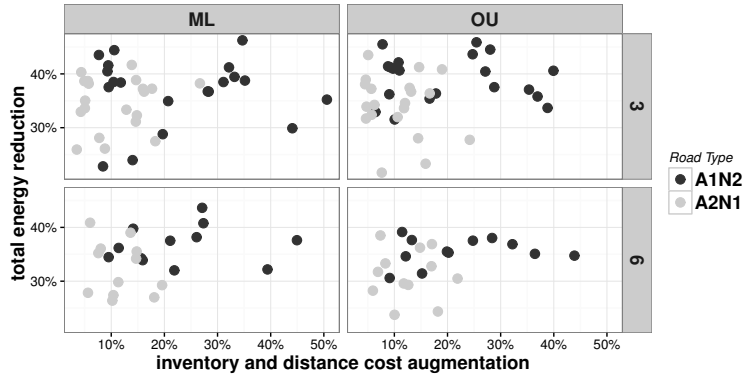


Fig. 5: Distance and Inventory Cost and Energy Cost under Different Configurations

Third, energy reduction does not mean distance minimization. Contrary to the common belief that the shortest route is the one that minimizes the energy, our study shows that both the distance and the vehicle weight and loads are important for the energy consumption. The vehicle with a high load tends to start his journey with the least energy cost road and put to the end the visit to a customer in an area with high energy cost. For example, Fig. 6 shows the route of the vehicle under different objectives with 3 periods and OU policy. 6a is the route obtained with energy minimization. The vehicle serves Customer 1 with 65 kg products in the first period, then makes a tour by visiting customers  $3(1230) \rightarrow 4(766) \rightarrow 5(478) \rightarrow 2(280)$  in the second period (the number in the parentheses is the mass flow on the corresponding arc), and no delivery is done in the third period. With distance and inventory minimization (Fig. 6b), the vehicle serves Customer 3 with 232 kg and 464 kg products in the first and last period respectively, and in the second period, it visits  $4(896) \rightarrow 2(608) \rightarrow 5(328) \rightarrow 1(130)$ . In the route given by energy minimization, only one national route is used (the arc (3,4)) and the maximal mass flow is distributed on arc (0,3) which corresponds to the minimum cost per unit of mass in this instance. In the route given by inventory and distance minimization, however, only one highway is used (the arc (1,0)) and one delivery is planned in addition in period 3, which induces a lot of energy use because the vehicle weight (4000 kg) is important in relation to the payload (464 kg).

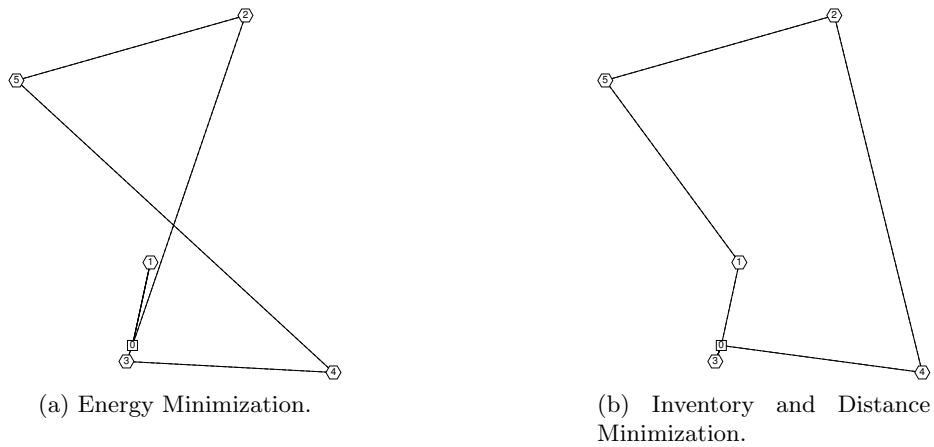


Fig. 6: Vehicle Routes under Different Objectives

The relation between the distance and the energy is influenced by the road type (or vehicle dynamics such as number of stops and vehicle speed). As shown in Fig. 7, in a world with more highways (road type "A2N1" means that 2/3 arcs of all the arcs are highway and 1/3 are national route, similar for "A1N2"), we can achieve 40% of energy reduction with at most 30% of augmentation of distance, whereas in "A1N2" configuration, the augmentation of distance can be as high as 60% to have an energy reduction of 35%. This confirms the fact that a free-flow configuration is better for energy use.

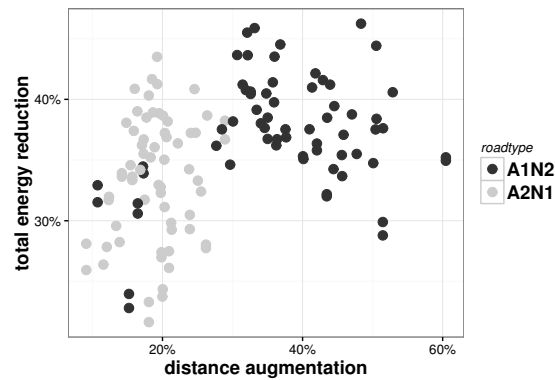


Fig. 7: Distance and Energy Cost under Different Road Types

Last but not least, inventory replenishment strategy can also impact the energy reduction potential of an inventory routing system (See Fig. 8). Under

ML policy, inventory change to save energy is higher than under OU policy, since ML policy is more flexible than OU policy.

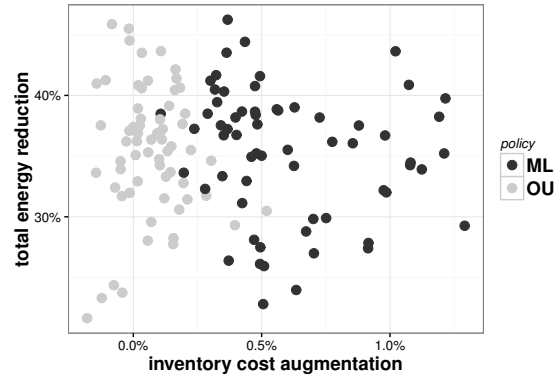


Fig. 8: Inventory and Energy Cost under Different Inventory policies

## 5 Conclusions

Energy consumption is an important aspect in both economical and ecological view. It becomes more and more important with the sustainable requirement of the inventory systems. We address the combination of inventory management, vehicle routing and energy minimization and propose a new mass-flow based formulation of the IRP with explicit energy consumption. This formulation uses an energy estimation methods depending on vehicle dynamics (speed) and road characteristics (stop rate per kilometre and slope). This estimation gives us an energy cost function that is linear to the total mass. In this formulation, the mass is added as a decision variable and the energy cost function is considered as an objective. Our first experimentation shows that there is a great potential in improving the energy efficiency in the inventory routing.

Various parameters can have an impact on the energy consumption. From the transportation aspect, vehicle speed and number of stops are important. The improvement of energy can be higher on a road with congestion. On the inventory management side, inventory strategy can influence the energy consumption. Under the condition that no customer is in stock-out, the ML policy provides much more flexibility for energy minimization than the OU policy.

Further works include modelling of traffic networks, so that different traffic conditions as well as vehicle speed levels could be considered in the decision process. More data are needed from the real world to accomplish this work.

The inventory routing model needs to be improved to better control the time and quantity of each delivery. In fact, the traditional IRP is very aggregated in terms of inventory levels and delivery time since all the inventory monitoring

is summarized in periods. In reality, however, there are two time scales for the routing and inventory management. It is in small scale (minutes or hours) for the vehicle routing because traffic conditions can change in a day, while for the inventory management it is in large scale (days or months) according to the real application. The model should incorporate these two time scales so that there is no loss of information.

Heuristics are being studied to speed up the computation. Especially with realistic data, larger number of customers or longer decision periods is common. The extension of the problem to a multi-objective one is also a promising track of study.

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## Annexe

The energy minimizing IRP mathematical model with OU inventory policy is presented below.

$$\begin{aligned}
\min \sum_{t \in T} \sum_{(i,j) \in V \times V} c_{ij} m_{ij}^t + W \sum_{t \in T} \sum_{(i,j) \in V \times V} c_{ij} y_{ij}^t \\
I_0^t = I_0^{t-1} + r_0 - \sum_{i \in V_c} q_i^t & \quad \forall t \in T \\
I_i^t = I_i^{t-1} - r_i + q_i^t & \quad \forall i \in V_c, t \in T \\
q_i^t \geq C_i z_i^t - I_i^{t-1} & \quad \forall i \in V_c, t \in T \\
q_i^t \leq C_i - I_i^{t-1} & \quad \forall i \in V_c, t \in T \\
q_i^t \leq C_i z_i^t & \quad \forall i \in V_c, t \in T \\
\sum_{j \in V_c} m_{0j}^t = \sum_{i \in V_c} q_i^t m_i & \quad \forall t \in T \\
\sum_{j \in V} m_{ji}^t - \sum_{j \in V} m_{ij}^t = q_i^t m_i & \quad \forall i \in V_c, t \in T \\
\sum_{j \in V_c} x_{0j}^t = 2z_0^t & \quad \forall t \in T \\
\sum_{\substack{j \in V \\ j < i}} x_{ji}^t + \sum_{\substack{j \in V_c \\ j > i}} x_{ij}^t = 2z_i^t & \quad \forall i \in V_c, t \in T \\
\sum_{j \in V_c} y_{0j}^t = z_0^t & \quad \forall t \in T \\
\sum_{j \in V} y_{ij}^t = z_i^t & \quad \forall t \in T, i \in V_c \\
\sum_{j \in V} y_{ji}^t = z_i^t & \quad \forall t \in T, i \in V_c \\
x_{ij}^t = y_{ij}^t + y_{ji}^t & \quad \forall t \in T, (i,j) \in E \\
\sum_{i \in V_c} q_i^t \leq Qz_0^t & \quad \forall t \in T \\
m_{ij}^t \leq M y_{ij}^t & \quad \forall t \in T, (i,j) \in V \times V \\
0 \leq I_i^t \leq C_i, I_i^t \in \mathbb{N} & \quad \forall i \in V, t \in T \\
0 \leq q_i^t \leq Q, q_i^t \in \mathbb{N} & \quad \forall i \in V_c, t \in T \\
0 \leq m_{ij}^t \leq M, m_{ij}^t \in \mathbb{N} & \quad \forall (i,j) \in V \times V, t \in T \\
z_i^t \in \{0, 1\} & \quad \forall i \in V, t \in T \\
x_{ij}^t \in \{0, 1\} & \quad \forall (i,j) \in E, i < j, t \in T \\
x_{0j}^t \in \{0, 1, 2\} & \quad \forall j \in V_c, t \in T \\
y_{ij}^t \in \{0, 1\} & \quad \forall (i,j) \in V \times V, t \in T
\end{aligned}$$