Solving the Flight Radius Problem

Assia Kamal Idrissi, Arnaud Malapert, Rémi Jolin

Solving the Flight Radius Problem

Assia Kamal Idrissi[1], Arnaud Malapert[2] and Rémi Jolin[1]
[1] *Milanamos, 1047 route des Dolines, Sophia Antipolis, France*
[2] *Université Côte d'Azur, CNRS, I3S, France*
{*assia.elafouani, remi.jolin*}*@milanamos.com, arnaud.malapert@unice.fr*

Keywords:
Flight Radius Problem, Airline Schedule Design, Shortest Path Algorithms, Regret.

Abstract:
In this article, we present the flight radius problem on the condensed network. This problem consists of locating in the network what routes represent business opportunities that are attractive regarding time or cost criteria, and passing through a specific flight. We introduce a regret function to model the regret compared to the optimal value of such criteria. We work with a startup company specialized in air transportation. The company has developed a decision tool for airline managers to analyze and simulate a new market. Our problem is derived from this application. Thus, we start by formulating the problem as finding a maximal sub-graph such as for each node, there exists a valid path by the regret function. Then, we propose two methods for solving the problem. One using procedures of the graph database `Neo4j` where the condensed network is stored. The second one is a new algorithm based on `Dijkstra` algorithm. Finally, we have been able to report results on a set of real-world instances, based on different (OD) pairs and various values of the regret, studying the impact of considering different combinaison of node's type: Hub & Spoke.

1 INTRODUCTION

The air transportation industry has evolved rapidly over the last years. The growth of air passenger demands has pushed airlines to enhance their quality of service. The airlines should focus on the route network development which is considered as the initial problem addressed by the airlines. It aims to determine a set of routes to be operated in an airline's network. It takes passengers demand, airport, aircraft characteristics, and then generate a set of origin-destination pairs (OD) to serve, the schedule design problem aims to define the frequency and departure time of each flight.

Airlines have the choice to create a new route to serve a new destination, this new route provides connecting traffic to other flights, or to increase/decrease the frequency of existing routes. The first requires the route network development problem and the schedule design problem which demand a lot of investment. While in the second, the airline network already exists. The problem of allocating a new flight is related to these problems. The problem consists of determining a set of (OD) pairs to serve and then choose flight schedules with respect to the quality of service index (QSI) model. QSI is a market share model used by most of airlines to estimate their part of the market (Jacobs et al., 2012). We define a *flight* by three attributes: Origin-Destination (OD) pair (an OD pair is a couple of airports), arrival/departure time and aircraft type. We distinguish between three different types of flights: A *non-stop flight* is a single flight with no intermediate stops. In the absence of such flights, passengers must take either a direct flight or a connecting flight. A *direct flight* is operated by the same aircraft and includes at least one stop. A *connecting flight* is a flight where passengers have to change of the aircraft in a hub. A *route* is a sequence of flights with unique flight numbers that begins at the origin airport and ends at the destination airport (Hall, 2012).

The problem of allocating a new flight evokes design and visualization of the airline's network. However, the network is so large that it cannot be visualized. Thus, we proposed the flight ra-

dius problem which is related to the route network development problem. This problem consists in showing only interesting airports with respect to a specific flight regarding the QSI criteria. Cost and time are the major QSI criteria of this model. Besides, the regret criterion is compared to the optimal time or optimal cost. The main idea of the flight radius problem is to locate in the network what routes, passing through a specific flight, and represent business opportunities that are attractive to the passengers according to different preferences. The choice depends on the passengers since they have different preferences over the criteria, and type of flight is one of these criteria. For this reason, these preferences should be taken into consideration in this problem; it is modeled by the regret function. The function aims to model the regret compared to the optimal value of cost or time. The visualization is a simple way to remove the irrelevant routes. Since the airline network exists already, our aim is to filter the network and keep only important routes passing through this flight. For this reason, we omit the schedule design by working on the condensed network (see section 4). In such network, we just consider the transfer time without checking if the route is viable. Hence, for an airline managers, what is the relevant subnetwork related to a given flight? What are the passengers origins and destinations?

The flight radius problem is formulated as a problem of finding a maximal subgraph in terms of nodes. We constructed the condensed flight network from the company flight database using the time-independent approach and stored it in `Neo4j` the graph database since the current database presents some limits (Neo4j, 2017). The problem can be solved using the shortest path algorithms to find the maximal subgraph of the graph. The output subgraph contains paths that are longer than the shortest path with a certain regret, and passing through the specific arc. In this paper, we propose an algorithm to solve the problem of flight radius. This problem is derived from the application developed by the company that has developed a decision tool for airline managers to analyze and simulate a new market using QSI models. We are interested to simulate a new market. Given a specific flight, the process starts by finding important airports whose routes pass by the specific flight in terms of QSI criteria, and then estimate market share for each route. The solution proposed is to reduce the number of routes before applying QSI models.

This paper is organized as follows. Section 2 introduces some definitions of graph theory. Then, introduction of flight timetables. In Section 3, we review related work of the air scheduling problems, transportation networks, and shortest path algorithms. Section 4 describes the condensed network. Section 5 gives our formulation of the flight radius problem, and its properties. Section 6 describes methods proposed to solve the flight radius problem. Section 7 is dedicated to experiments.

2 PRELIMINARIES

Graph Theory. A *graph* G is a tuple $G = (V, E)$ consisting of a finite set V of nodes or vertices and a set $E \subseteq V \times V$ of arcs which are ordered pairs (u, v) if the graph is directed. The node u is called the *tail* of the edge, and v is called the *head*. Each arc $(u, v) \in E$ has an associated non-negative weight $w(u, v)$. The reversed graph $\vec{\mathbf{G}} = (V, \vec{\mathbf{E}})$ is the graph obtained from G by substituting each edge $(u, v) \in E$ by (v, u). We define $|V| = n$, the order of the graph as the number of nodes meanwhile $|E| = m$ its size. In a directed graph, the arcs point from one node to another. For instance, airline networks are weighted directed graphs where the weights represent the prices or the duration of the flight. A direct flight from one city to another does not necessarily imply that there is also a direct return flight. A *sub-graph* $G' = (V', E')$ of a graph G where V' is a subset of V and E' is a subset of E. A *path* is a sequence of nodes $\{v_1, v_2, ..., v_k\}$ such that for each $1 \leq i < k$ condition $(v_i, v_{i+1}) \in E$ holds. If additionally $v_1 = v_k$, then the path is a *cycle*. The length of a path is the sum of its edge weights along the path and is denoted by:

$$l(P) := \sum_{i=1}^{k-1} w(v_i, v_{i+1}).$$

By extension, we define $l^\star(s, t)$ for a given pair of vertices, the length of the shortest path starting at s and ending at t. A path in G is called *elementary* if no vertex occurs more than once. A graph G is *connected* if there exists a path joining any two vertices. A transportation network should be a connected graph.

Flight Timetables. In this study we restrict to flight networks that rely on timetables. A *flight timetable* is defined by a tuple $(\mathcal{C}, \mathcal{A}, \mathcal{F}, \mathcal{T})$ where

$\mathcal{A}$ is a set of airports, $\mathcal{F}$ is a set of flights, $\mathcal{T}$ is the periodicity of the timetable, and $\mathcal{C}$ is a set of elementary connections. An *elementary connection* $c \in \mathcal{C}$ is a tuple $c = (f, o, d, t_s, t_e)$ which represents *flight* $f \in \mathcal{F}$ departing from the airport $o \in \mathcal{A}$ at $t_s < \mathcal{T}$ and arriving at the airport $d \in \mathcal{A}$ in time $t_e < \mathcal{T}$. Concretely, an elementary connection corresponds to an event in the timetable. A *passenger trip* $(c_1, c_2, \ldots, c_{n-1}, c_n)$ is a sequence of elementary connections, with the origin of an elementary connection the same as the destination of its predecessor in the sequence, and the elapsed time between two successive connections at least as great as the minimum connecting time:

$$o(c_{i+1}) = d(c_i) \wedge t_e(c_i) + MCT(d) \leq t_s(c_{i+1}) \quad \forall 1 \leq i \leq n-1$$

Where MCT is the minimum connecting time at the destination airport $d(c_i)$.

The condensed network is generated from the flight timetable where nodes represent airports meanwhile the presence of an arc indicates that there exists at least one elementary connection between two airports. Each arc is constructed by aggregating all elementary connections between each pair of airports (see section 4).

3 RELATED WORK

Air Scheduling Problems. The air scheduling development problem has been broken, in practice, into several subproblems (Barnhart and Cohn, 2004). This is due to its very large-scale nature. Thereby, the route network development, schedule design, fleet assignment, aircraft routing, and crew scheduling are the five facets of the air scheduling development optimization problems (Rebetanety, 2006).

Route network development : deciding which set of origin-destination pairs to serve.

Schedule design : defining the frequency of each flight.

Fleet assignment : specifying the type and the size of aircraft serving each flight in a given schedule.

Aircraft routing : determining feasible aircraft routes under maintenance and time constraints.

Crew scheduling : assigning crews to flights.

Shortest Path Algorithms. There are two categories of shortest path algorithms: Setting algorithms and correcting algorithms. Shortest path algorithms are based on labeling method for solving the shortest path problem. For each node v, the method maintains a distance label $d(v)$ which is an upper bound on the shortest path length to the node v, parents $P(v)$, and status $S(v)$. we have three status : unreached, labeled, and scanned. Initially for each node v, $d(v) = inf$, $P(v) = nil$, and $S(v) = unreached$. Then, the algorithm starts by scanning labeled nodes until there does not exist such node. The two types of algorithms differ in the strategy of selecting labeled nodes to be scanned (Cherkassky et al., 1996). `DIJKSTRA's algorithm` is the most know setting algorithm and works with positive weight arcs. In `DIJKSTRA's algorithm`, the principle is to select a node with the minimum weight at each iteration, and then each node is scanned at most once. That leads to a complexity of $\mathcal{O}(n^2)$ as time bound in the worst case (Ahuja et al., 1993) where n is the number of nodes. There are many versions of `DIJKSTRA'S algorithm` with the aim of improving this time bound by trying different data structures and several implementations of the algorithm (Ahuja et al., 1993). In some applications of the shortest path problem, we want uniquely to determine the shortest path between two nodes. `BIDIRECTIONAL DIJKSTRA's algorithm` solves the problem of finding the shortest path between two nodes faster since it eliminates some unnecessary computations. Besides, `BELLMAN-FORD-MOORE` which is known as a correcting shortest path algorithm. It achieves the best currently know bound of time with negative weight arcs $\mathcal{O}(nm)$ where m is the number of edges. The algorithm maintains the set of labeled nodes in a FIFO queue and allows detecting negative cycle in a weighted directed graph. Unlike `DIJKSTRA'S algorithms` where we need to find minimum value of all vertices, in Bellman-Ford, arcs are considered one by one. The next node to be scanned is removed from the head of the queue; a node that becomes labeled is added to the tail of the queue. The algorithm performs at most $n-1$ passes through arcs. Since each pass requires $\mathcal{O}(1)$ computations for each arc, this conclusion implies $\mathcal{O}(nm)$ time bound for the algorithm.

4 CONDENSED NETWORK

The condensed network is generated from a `NoSQL` database and stored it in `Neo4j` the graph database using a time-independent approach. It

is one of the most popular graph databases where queries can easily be expressed through *Cypher* query language. `Neo4j` is used in many use cases, typically recommendation systems and complex networks like transportation network. In `Neo4j`, data are represented in nodes, relationships, and properties. Both nodes and relationships contain properties (Robinson et al., 2015). A relationship connects a pair of nodes, it has a direction, type, a start node, and an end node. In the company's database, data are collected and queried monthly then it makes sense to create a relationship per period which is a month of the year. the relationship represents flight information. We dispose of historical data about the last fifteen years. The current database used is a `MongoDB` database that stored data in a disconnected way. This database does not use a graph structure. Therefore, we opt for the graph database `Neo4j` as another alternative database to overcome the limits of the existing database (Neo4j, 2017). `Neo4j` uses a graph structure that regroups data and allows visualizing what happens in the network when creating a new route or deleting an existing route. Furthermore, this graph database performed well on the graph traversal since our study is based on such algorithms (Holzschuher and Peinl, 2014). The graph database `Neo4j` offers the possibility to implement algorithms as user defined procedures to call in *Cypher* query. That is can be easy to use it by the final user. Indeed, `Neo4j` proposed *APOC* (Awesome Procedures on Cypher) as stored procedures that regroup a list of procedures. Graph algorithms are part of these procedures namely some shortest path algorithms. Once the database is chosen, we proceed to model the graph. The model takes into account the transfer time. It is represented by a relationship in the graph. This technique is often used to model the information about transfer since it is important in computing shortest paths. The figure below illustrates the condensed graph in `Neo4j`. The graph contains four airports and four flight arcs. Nodes in thin style represent departures (origin nodes), dashed nodes for arrival nodes (destination nodes). Double edge is transferring time meanwhile bold edges model flight time. Besides, dotted edges for departures.

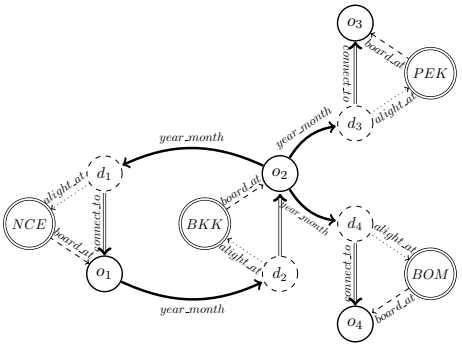

Figure 1: Model of condensed flight network in `Neo4j`.

Thus, the condensed graph was generated for 1 year and has 33,901 nodes and 562,294 relationships.

5 PROBLEM FORMULATION

The flight radius problem consists in retrieving only relevant routes passing through a specific flight, and satisfying the regret function. The flight considered is represented by an arc (o,d) in the graph where $o,d \in A$. Then, we are interested in retrieving paths passing by the arc (o,d) that could be relevant regarding the regret defined for the time and cost criteria. In other words, traveling from $o_1 \in A$ to $d_1 \in A$ by passing through the arc (o,d) may be interesting if and only if the path $\{o_1,..,o,d,..,d_1\}$ between o_1 and d_1 is accepted by the regret function. This function depends on the shortest path between o_1 and d_1. Let R be a Boolean regret function defined on paths of the graph G. Therefore, the problem consists in finding a maximal sub-graph, in terms of nodes, such that each node supports a path accepted by the regret function R.

Hence, the problem is formulated as follows:

Input a graph $G=(V,E)$, the arc (o,d), and the regret function R

Output a maximal subset $E' \subseteq E$ such that $G'=(V',E')$ is a sub-graph of G and that each node supports a path passing through the arc (o,d) accepted by the regret function.

In this paper, we use the regret function R to identify what paths are supported. Let's define what the regret function R is. Let $w(i,j)$ be the weight of the arc (i,j) and let $l^\star(i,j)$ be the length of the shortest path from i to j. Let $l(i,j)$ the length of a path passing through the arc (o,d), and let consider the following regret function defined for each criterion:

$$R^+_{od}(i,j) = l(i,j) \leq l^\star(i,j) + K$$

Where $K \geq 0$. Each node must support at least a valid path by the regret function. Then, we are looking for retrieving paths that satisfied at least one criterion.

The flight radius problem consists in finding valid paths by the regret function R. These paths depend on finding shortest path. Most traditional path finding are based on shortest path finding:

$$l(i,j) \geq l^\star(i,o) + w(o,d) + l^\star(d,j) \qquad (1)$$

In other words, following the shortest path from i to o, passing by the arc (o,d), and then following the shortest path from d to j is always a valid path if it exists. The subpath from o to j of a valid path is also valid.

$$\begin{aligned} l^\star(i,o) + w(o,d) + l^\star(d,j) &\leq l^\star(i,j) + K \\ &\leq l^\star(i,o) + l^\star(o,j) + K \\ w(o,d) + l^\star(d,j) &\leq l^\star(o,j) + K \end{aligned}$$

Reciprocally, the subpath from i to d is valid.

$$\begin{aligned} l^\star(i,o) + w(o,d) + l^\star(d,j) &\leq l^\star(i,j) + K \\ &\leq l^\star(i,d) + l^\star(d,j) + K \\ l^\star(i,o) + w(o,d) &\leq l^\star(i,d) + K \end{aligned}$$

Finally, the search can be restricted to shortest valid paths starting from o or ending at d.

Lemma 5.1. *Let p be a valid path, all the nodes belong to G'. For any shortest path p from o to j in G. If it passes through by d then it is a valid path and consequently j is going in the subgraph G'.*

The subpath of the shortest path is also a shortest path (Ahuja et al., 1993). Consequently, nodes j represent set of nodes that support paths accepted by the regret function.

In the following, we focus on solving the part of finding the subpath from o to each vertex j.

6 SOLVING METHODS

6.1 A Query-Based Solution

In the earlier section, we proved that the search of valid paths can be restricted to finding valid shortest paths. The problem was solved in *Cypher* query using the algorithm `BIDIRECTIONAL DIJKSTRA` implemented in `Neo4j` as a procedure in *APOC* (Larsson, 2008). In `Neo4j`, we use a parametrized query. The parameters are: *o_code* and *d_code* to specify o and d, *rel* to identify type of relationship to traverse, `criterion` for time or cost, and K determines the regret.

The query described in 1 contains three major blocks. The first block of the query includes the first three lines. The `MATCH` clause is used to match the graph pattern which is the arc (o,d) using the supplied parameters. The second block contains the call of the algorithm. Then, the procedure `DIJKSTRA` is called from the origin o to all other airports A in the graph in order to find the shortest path in terms of time, and finally gets the shortest paths from the destination d. The second `WITH` clause is to aggregate outputs of the first procedure. . Thus, calling the second procedure `DIJKSTRA` in the second block would execute the procedure for every row. The final block begins by the `UNWIND` clause to disaggregate previous aggregate outputs. Meanwhile, the last `WITH` clause filters the set of paths according to the regret function.

6.2 An Algorithmic Solution

As the problem deals with two criteria, the algorithm starts by considering one criterion and then moves to the second one using at each step information from the previous step. The flight radius algorithm starts by computing the shortest path tree from o and checks if the arc (o,d) exists in the shortest path tree of o (lemma 5.1). After finding the shortest path tree (line 2 of Algorithm 1), we check valid shortest paths passed via d (line 3 of Algorithm 1). This step corresponds to the step (1) in Figure 2. The next step (2) is to compute the shortest path from d. In this step, we get two information: length of the shortest path for one criterion $l_1^\star(d,j)$ and an upper bound for the second criterion $\overline{l_2}(d,j)$. Once we retrieve supported paths for the first criterion. We move to the second criterion and repeat the same process. The third step (step (3) in Figure 2) is more similar to the first. The another case where paths do not pass through d, we can use the upper bound computed in the previous step. We check if the regret function is satisfied for all nodes $i \in V_s$, set of non supported nodes (line 16 of Algorithm 1). We applied the same process for the remaining non supported nodes to get the shortest path tree from d. To do this, we use a second algorithm, called `RevisitedDijkstra`. The algorithm represents the `Dijkstra's algorithm` (Ahuja et al., 1993)

```
MATCH p=(Td:Destination)-[:ALIGHT_AT]->(o:Airport{code:{o_code}})-[:'BOARD_AT']->(To:Origin)-[r
    ]->(d:Destination{code:{d_code}}),(A:Destination)
WHERE NOT A IN [d,Td] AND type(r) = {rel}
WITH r.duration_min-{K} AS LB,To,d,A
CALL apoc.algo.dijkstra(To,A,{rel}+'>|CONNECT_TO>',{criterion}) YIELD path AS p1,weight AS w1
WITH DISTINCT A,collect(w1) AS W1,LB,d
CALL apoc.algo.dijkstra(d,A,{rel}+'>|CONNECT_TO>',{criterion}) YIELD path AS p2, weight AS w2
UNWIND W1 AS w1
WITH w1-w2 AS diff,LB,A WHERE diff>=LB
RETURN DISTINCT A
```

Listing 1: CYPHER query

including the regret function. The algorithm at each iteration scans the node with the minimum label and then relax its neighbors. So, we check before if it satisfies the regret function otherwise we move to the next. Since all arc weights are nonnegative then `Dijkstra's algorithm` finds the shortest path in order of increasing distance. For this reason, we use this manner to quickly remove non supported nodes. Figure 2 describes the steps of the flight radius algorithm.

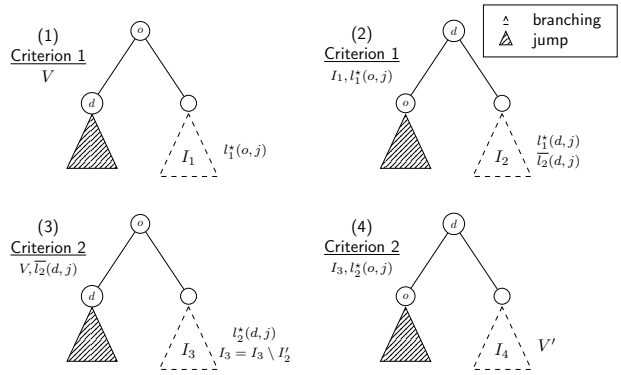

Figure 2: Flight radius algorithm steps.

7 EXPERIMENTS

In this section, we describe experiments on the flight radius problem. We start by evaluating the performance for solving the flight radius problem using a query. After that, we compare the result with those obtained using the algorithm in the case of one criterion. We measure information of the order of the output subgraph and the percentage of nodes filtered with various value of the parameter K. Those values are chosen randomly according to different statistic metrics. Specifically, we address the following questions: How sensitive is flight radius algorithm's performance on the real graph to the choice of parameter K? How does an algorithm's performance when adding a second criterion? How does the choice of one parameter K influence the order of the subgraph?

All the experiments were led on a computer running on Ubuntu 16.04.2 with 32 GB of RAM and one Intel Core i7-3930K 3.20GHz processors (6 cores). The implementation is based on `Neo4j` and *APOC* version 3.2.0.1.

Test Instances. Tests on real-world data were realized on the database of the company. To test the method based on a query, we use 6 instances for the problem, each one of them represents a flight with a different type of airport: hub & Spoke and using different value of the parameter K of one criterion. This parameter is chosen according to the minimum connection time MCT, and the median of each criterion. We compute the value of the parameter K according to the median, the first quartile, and the third quartile. In this way, we can measure the spread to describe the variability in each criterion with conjunction with the median as a measure of central tendency. In the case of criterion cost, K_2 is chosen independently to the minimum connection time MCT. Setting K to zero, for example, means that the subgraph contains all the shortest paths passing through the arc studied (o,d). On the contrary, setting K to a high value implies that the subgraph contains all nodes of the condensed graph. Note that the MCT is set to 120 minutes. Besides, we generate 100 instances that include for each pair of (OD) generated randomly, 10 tests with different classes of two criteria.

Problem with one criterion. Tests have been run on existing flights between various airports in terms of degree. We apply the query for only one criterion since it takes a lot of time to solve the whole problem. We run tests on the time criterion. Table 1 gives the results of testing both methods. *#_nodes* : the order of output subgraph. *Dur*: flight duration of the arc studied, the parameter K_1 fixed for each test, and the

Algorithm 1: Flight radius algorithm (FRA)

```
procedure : FRA (set of nodes V, node o,node d, parameter K)
input     : A digraph G = (V, A, W), arc (o, d), parameter K_1, parameter K_2
output    : Subgraph G' = (V', E')
V_s ← V;
T_1 ← Dijkstra(o, V, W_1);// Apply Dijkstra algorithm for the first criterion (step (1))
V_s ← V\dChecking(d,T_1);// Check paths passing through d
if V_s == ∅ then
    break
else
    {V_s, T_2} ← RevisitedDijkstra(d, V_s, W_1, W_2, T_1, K_1);
    if V_s == ∅ then
        break
    else
        T_2 ← Dijkstra(o, V_s, W_2);
        V_s ← V\dChecking(d,T_2);
        if V_s == ∅ then
            break
        else
            V_s ← V \ UBChecking {V_s, T_2, T_2}; // Check with upper bound
            if V_s == ∅ then
                break
            else
                {V_s, T_1} ← RevisitedDijkstra(d, V_s, W_1, W_2, T_2, K_2);
            end
        end
    end
end
forall v, w ∈ V \ V_s with e = (v, w) ∈ E do
    E' ← E' ⋃ e;
end
G' = (V' = V \ V_s, E');
```

percentage of nodes filtered. ExecT1 presents the running time of the first method whereas Exect2 is for the second method. The running time of method based on a query is very important. `Neo4j` Implements bidirectional Dijkstra's algorithm. So, the algorithm is repeated for each pair of nodes individually to find the shortest path from a node to all other nodes. Thus, many computations are repeated. However, our problem used the single-source shortest path algorithm. So, we are seeking to return the shortest path tree; that is, the shortest path from source to all nodes. But the result returned is a list of paths. That means, in terms of spatial complexity, the sum of the length of the n paths selected is bound by n^2 in the case of multiple runs of single-source shortest path algorithms rather than n paths in the case of three returned with n the number of nodes in the graph. The time complexity is $\mathcal{O}(n \times (m + n \log n))$ as it runs multiple times as the order of the graph. In the worst case. The query runs in 57 minutes whereas, the algorithm takes only 2321 ms. Therefore, the algorithm outperforms the query.

Problem with two criteria. Table 2 gives the result of running the flight radius algorithm with two criteria. The percentage of supported nodes increases as we add a second criterion. Even with zero regret, the percentage is at least twice than the percentage in one criterion case. For the instance 1 and 6, the subgraph contains all shortest paths passing by these flights: `(NCE, DXB)` and `(AMS,IST)` for both criteria time and cost. In the instance 2, 3, and 4, the regret is chosen respectively to quartiles: Q_1, Q_2, and Q_3.

Table 3 gives the average running time as a function of classes of parameter K_1 and K_2. The average running time increases slightly when both value increase. The algorithm runs in the best case when the parameter K_1 is set to a value greater than the third quartile which represents 75 % of flight duration whereas K_2 is setting to zero. In the worst case, the algorithm runs twice than in the best case. It is achieved when we swap both values. It comes back to the choice of the parameter K_1 since it is computed in relation with the minimum connection time MCT. Then, the algorithm is influenced by the second parameter.

Table 1: Comparaison between two methods.

Instance	Flight	Dur (min)	K_1(min)	#_nodes	Percentage of V	ExecT1 (min)	ExecT2 (ms)
1	NCE → DXB	360	0	287	2.5 %	53	**2321**
2	JFK → NCE	490	198	156	1.3 %	51	1127
3	CDG → SCL	870	245	56	0.49 %	55	696
4	LHR → ATL	565	330	771	6.82 %	51	786
5	FRA → PEK	550	198	416	3.68 %	53	736
6	AMS → IST	195	0	65	0.57 %	**57**	693

Table 2: Time needed to solve the flight radius problem with two criteria.

Instance	Dur (min)	K_1(min)	K_2(usd)	#_nodes	Percentage of V	ExecT (ms)
1	360	0	0	1200	10.62 %	3604
2	490	198	17.0	590	5.22 %	1657
3	870	245	42.98	479	4.23 %	1597
4	565	330	96.14	1424	12.60 %	1906
5	550	198	17.0	933	8.25 %	1742
6	195	0	0	477	4.22 %	1554

Table 3: Average running time in function of regret classes.

	Class cost			
Class time	0	1	2	3
0	1573.6	1590.4	1657.2	**2354.8**
1	1783.2	1677.0	1759.4	2246.0
2	1877.8	2007.3	1928.1	2248.0
3	**1367.0**	2271.5	1484.1	1491.5

8 CONCLUSION

This work presents the flight radius problem. We formulated the problem as finding a maximal subgraph, in terms of nodes, such that each node supports a valid path by the regret function. To represent the regret function, we focused in the additive case. In the multiplicative case, the problem seems to be hard to simplify since the regret parameter K depends on the shortest path. Then, we presented two methods to solve the problem. Method using procedures of `Neo4j` the graph database where the condensed graph is stored and, the method based on a new algorithm that relies on `Dijkstra` algorithm. Studied instances in this article were realized on the real-world network. The algorithm outperforms the query method and the choice of the parameter K influences the running time of the algorithm. Latter, we aim to test the algorithm on benchmarks graphs to test the performance when the topology changes. Also, we aim to test another shortest path algorithms which is `Bellman-Ford` since it takes $\mathcal{O}(nm)$ in the worst case and paths in flight network are characterized by small length in terms of number of arcs. Thus, we would like to compare its performance on the flight radius problem compared to `Dijkstra` algorithm.

REFERENCES

Ahuja, R. K., Magnanti, T. L., and Orlin, J. B. (1993). *Network flows: theory, algorithms, and applications*. Prentice hall.

Barnhart, C. and Cohn, A. (2004). Airline schedule planning: Accomplishments and opportunities. *Manufacturing & service operations management*, 6(1):3–22.

Cherkassky, B. V., Goldberg, A. V., and Radzik, T. (1996). Shortest paths algorithms: Theory and experimental evaluation. *Mathematical programming*, 73(2):129–174.

Hall, R. (2012). *Handbook of transportation science*, volume 23. Springer Science & Business Media.

Holzschuher, F. and Peinl, R. (2014). Performance optimization for querying social network data. In *EDBT/ICDT Workshops*, pages 232–239.

Jacobs, T. L., Garrow, L. A., Lohatepanont, M., Koppelman, F. S., Coldren, G. M., and Purnomo, H. (2012). Airline planning and schedule development. In *Quantitative Problem Solving Methods in the Airline Industry*, pages 35–99. Springer.

Larsson, P. (2008). Analyzing and adapting graph algorithms for large persistent graphs. Master's thesis.

Neo4j (2017). `https://www.neo4j.com`.

Rebetanety, A. (2006). *Airline schedule planning integrated flight schedule design and product line design. University Karlsruhe (TH)*. PhD thesis.

Robinson, I., Webber, J., and Eifrem, E. (2015). *Graph databases: new opportunities for connected data*. " O'Reilly Media, Inc.".