Dynamic modeling, simulation and PID controller of unmanned aerial vehicle UAV
Kamel Bouzgou, Yasmina Bestaoui, Laredj Benchikh, Benoumeur Ibari, Zoubir Ahmed-Foitih

To cite this version:
Kamel Bouzgou, Yasmina Bestaoui, Laredj Benchikh, Benoumeur Ibari, Zoubir Ahmed-Foitih. Dynamic modeling, simulation and PID controller of unmanned aerial vehicle UAV. 7th International Conference on Innovative Computing Technology (INTECH 2017), Aug 2017, Luton, United Kingdom. pp.64–69, 10.1109/INTECH.2017.8102445. hal-01700501

HAL Id: hal-01700501
https://hal.science/hal-01700501
Submitted on 25 Feb 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Dynamic Modeling, Simulation and PID Controller of Unmanned Aerial Vehicle UAV

BOUZGOU kamel¹,², BESTAOUI Yasmina¹, BENCHIKH laredj ¹, IBARI benaoumeur¹,²,³, and AHMED-FOITIH Zoubir²

¹Laboratoire Informatique, Biologie Intégrative et Systèmes Complexes (IBISC). Université d’Évry-Val-d’Essonne (UEVE), France.
²Laboratoire d’Électronique de Puissance, d’Énergie Solaire et d’Automatique (LEPESA). Université des sciences et de la technologie d’Oran Mohamed Boudiaf, Algérie.
³Département d’électrotechnique, Université de mustapha stambouli, Mascara, Algérie

Abstract

This paper is focused on modeling and control of quadrotor; first modeling of moments and torque, second is of rotor, the result is the complete dynamics of UAV. Mathematical model is presented for a general study with disturbance and we take account all parameters of control. The PID controller is presented without disturbance in dynamics equation for a linear model, where we can use in the control of a group of quadrotors for obstacle avoidance, and a group of quadrotors with a rigid and flexible links for transporting payloads in the free environment. Finally our results is simulated in the simulink and the virtual reality environment.

keywords: Quadrotor, UAVs, Modeling, Control, PID Controller, Virtual Reality

1 INTRODUCTION

The quadrotor, UAV (Unmanned aerial vehicles) is the autonomous unmanned flying machines offering us an additional degree of freedom and an unlimited workspace, many researchers are facing this new device, and several studies are led to discover the areas of application of quadrotors modeling and control of quadrotor has been studied in several papers; mathematical and dynamic model in [8, 9] linear and nonlinear controller in [2] and for a quadrotor control with PID controller [10, 1, 3, 11] but the most of them is not a complete modeling in all cases of quadrotor situation; the recent research is mainly based on the fact that the quadrotor is an element of agent set to schedule a mission that requires cooperation and the control of a group of quadrotors, several research are based on the complete modeling with more mathematical details [4, 7] This paper presents a complete modeling that is required for a projects that use the quadrotor as an elementary agent for cooperation. This paper is organized as follows: first, a mathematical model of quadrotor is presented.

2 DYNAMIC MODEL

The quadrotor uav model is defined in the two frames, an inertial reference frame \( \mathbf{E} : \{e_1, e_2, e_3\} \) and a body fixed frame \( \mathbf{B} : \{b_1, b_2, b_3\} \). Two vectors associated to quadrotor, \( \mathbf{q} \) for position and rotation in the \( \mathbf{E} \) frame and \( \mathbf{V} \) for translational and rotational velocity in the body-fixed frame \( \mathbf{B} \).

\[
\mathbf{q} = [X \ \omega]^T = [x \ y \ z \ \phi \ \theta \ \psi]^T
\]

\[
\mathbf{V} = [V \ \Omega]^T = [v_x \ v_y \ v_z \ p \ q \ r]^T
\]

where \( X = [x \ y \ z]^T \) is the relative position of the center of mass of the UAV and \( \omega = [\phi \ \theta \ \psi]^T = [Roll \ Pitch \ Yaw]^T \) the orientaion of the quadrotor in \( \mathbf{E} \) frame, with the constraint that:

**Assumption 1**: it is assumed that roll, pitch and yaw angles are defined in bounded intervals as:

\( -\frac{\pi}{2} < \phi < \frac{\pi}{2} \), \( -\frac{\pi}{2} < \theta < \frac{\pi}{2} \) and \( -\pi < \psi < \pi \)
the Roll $\phi$ motion is given by the thrust imbalance between $f_2$ and $f_4$ forces around $b_1$ , we can write the rotation matrix $R_\phi$ as:

$$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

the Pitch $\theta$ motion is given by the thrust imbalance between $f_1$ and $f_3$ around $b'_2$ , the $R_\theta$ is given by:

$$R_\theta = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

and the Yaw $\psi$ motion is given by the thrust imbalance of all moments around the $x$ axis, the rotation matrix is:

$$R_\psi = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we can now define the global rotation matrix $R$ of quadrotor in the inertial frame $E$ with the product of three matrices as $R = R_\psi R_\theta R_\phi$:

$$R = \begin{bmatrix} c\theta c\psi & c\theta s\psi & c\phi s\psi + s\phi c\psi \\ c\theta s\psi & c\theta c\psi & c\phi c\psi - s\phi s\psi \\ -s\theta & 0 & c\phi \end{bmatrix}$$

Where $R_\phi, R_\theta, R_\psi$ and $R$ matrices of rotation is defined in the special orthogonal group $SO(3)$ , which have the following properties:

$$SO(3) = \{ R \in \mathbb{R}^{3\times3} | R^T R = I, \det(R) = 1 \}$$

Where: $c\pm x = \cos x$ and $s\pm x = \sin x$

The relation between $\omega$ and $\Omega$ is given by the following equation:

$$\dot{R} = R\hat{\Omega},$$

where the hat map:

$$:\hat{\cdot} : R^3 \to SO(3)$$

$$\hat{x}y = x \times y,$$

where $x, y \in \mathbb{R}^3$ and:

$$\hat{x} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

with $x = [x_1 \ x_2 \ x_3]^T$

To express the angular velocity of quadrotor in the fixed frame, we can use:

$$\Omega = \left( R^T \dot{R} \right)^\vee$$

where $\vee$ map is the inverse of hat map:

$$(\cdot)^\vee : SO(3) \to \mathbb{R}^3$$

$$\dot{R} = \frac{\theta}{m} (R_\psi R_\theta R_\phi).$$

$$\dot{R}_\psi = R_\psi \left( \begin{bmatrix} 0 & 0 & \psi \end{bmatrix}^T \right)^\vee$$

$$\dot{R}_{\theta} = R_{\theta} \left( \begin{bmatrix} 0 & \dot{\theta} & 0 \end{bmatrix}^T \right)^\vee$$

$$\dot{R}_{\phi} = R_{\phi} \left( \begin{bmatrix} \dot{\phi} & 0 & 0 \end{bmatrix}^T \right)^\vee$$

$$R^T = (R_\psi R_\theta R_\phi)^T = R_\phi^T R_\theta^T R_\psi^T.$$

we replace in the equation (4) becomes:

$$\Omega = \left( R_\psi^T R_\theta^T \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right)^\vee + \left( R_\psi^T \begin{bmatrix} 0 & 0 \end{bmatrix} \right)^\vee \left( \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix} \right)^\vee$$

we can use this properties of the hat map:

$$\dot{\Omega} = M \omega \implies \omega = M^{-1} \Omega \ , \ M \in \mathbb{R}^{3\times3}.$$

now we can rewrite the relation between $\dot{q}$ and $\dot{V}$ in the matrix form as:

$$\dot{q} = T.V$$

and

$$T = \begin{bmatrix} R & 0_{3\times3} \\ 0_{3\times3} & M^{-1} \end{bmatrix}$$

**Assumption 2**: we assume that the quadrotor is near of the nominal state, where we can rewrite the angular velocity in inertial frame as:

$$M = I_{3\times3}$$

therefore: ($\dot{\psi} = p \ , \ \dot{\theta} = q \ , \ \dot{\psi} = r$)

the dynamics equation of the unmanned aerial vehicle (UAV) are as:

$$m \dot{v} = f Re_3 - mg e_3$$

$$M = J\Omega + \Omega \times J\Omega.$$
the translational and rotational kinetic energy of quadrotor is given by:
\[ T_{trans} = \frac{1}{2}m\dot{x}^T \dot{x} \]  
\[ T_{rot} = \frac{1}{2}\Omega^T J \Omega \]  
and \( J = \text{diag} \{ J_x, J_y, J_z \} \in \mathbb{R}^{3 \times 3} \) is the inertia matrix of the quadrotor in the body-fixed frame we assume symmetric.

the Lagrangian of the quadrotor is given:
\[ \delta L = F_\tau \]  
\[ \frac{d}{dt} \left( \frac{\delta L}{\delta \dot{q}} \right) - \frac{\delta L}{\delta q} = \begin{bmatrix} F \end{bmatrix} \]  
\( F \) is the external force applied to the quadrotor and \( \tau \) is the generalized moments around the UAV.

the Lagrangian of the quadrotor is given:
\[ L(q, \dot{q}) = T_{trans} + T_{rot} - U \]
\[ L(q, \dot{q}) = \frac{1}{2}m\dot{x}^T \dot{x} + \frac{1}{2}\Omega^T J \Omega - mgz \]
\[ \frac{d}{dt} \left( \frac{\delta L}{\delta \dot{q}} \right) \]  
\[ \frac{\delta L}{\delta q} = \begin{bmatrix} F \end{bmatrix} \]  
\( F \) is the force of translational movement generated by propellers and \( \tau \) is the drag forces applied to the quadrotor along \( \{ 0, \dot{b}_3, \dot{b}_3 \} \) vectors, \( f = \sum_{i=1}^{4} f_i \) and \( F_X = f R e_3 \)

\[ F_X = f \begin{bmatrix} \cos \theta \phi s \psi + \sin \phi s \theta \psi \\ \cos \phi \theta s \psi - \sin \phi s \theta \psi \\ \cos \theta \phi c \psi \end{bmatrix} \]  
\[ F_i = \begin{bmatrix} -K_{fx} & 0 & 0 \\ 0 & -K_{fy} & 0 \\ 0 & 0 & -K_{fz} \end{bmatrix} \dot{x} \]  
where \( K_{fx}, K_{fy}, K_{fz} \) are drag coefficients.

\[ \dot{m} = f R e_3 + F_i - m g e_3 \]  
\[ m \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = f \begin{bmatrix} \cos \phi \theta c \psi + \sin \phi s \theta \psi \\ \cos \phi \theta s \psi - \sin \phi s \theta \psi \\ \cos \theta \phi c \psi \end{bmatrix} + \begin{bmatrix} -K_{fx} & 0 & 0 \\ 0 & -K_{fy} & 0 \\ 0 & 0 & -K_{fz} \end{bmatrix} \dot{x} - m g e_3 \]  
\[ J \dot{\Omega} = \Omega \times J \Omega + \sum_{i=1}^{4} J_i (\Omega \times e_3) \Omega_i + \tau \]  
\[ J_i \dot{\phi} = \begin{bmatrix} -\dot{\phi}_1 (f_1 + f_2) \\ -\dot{\phi}_2 (f_1 + f_2) \\ -\dot{\phi}_1 (f_1 + f_2) \end{bmatrix} + J_r \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} \Omega_r + \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \]

where:
\[ \Omega_r = \omega_2 + \omega_1 - \omega_1 - \omega_3 \]

the thrust generated by \( i \)-th propeller along \( \dot{b}_3 \) is:
\[ f_i = b \omega_i^2 \]
\( b \) is the thrust factor, and \( \omega_i \) is the speed of \( i- \text{th} \) propeller.

the rotor \( i \) also produce a moment given by:
\[ \tau_i = \pm d \omega_i^2 \]
and the total torque is:
\[ \tau_M = \sum_{i=1}^{4} (-1)^i d \omega_i^2. \]
the determinant of the above $4 \times 4$ matrix is $-8db^3$, so it is invertible when $d \neq 0$ and $b \neq 0$.

\[
\begin{bmatrix}
\frac{1}{J_t^2} & 0 & -\frac{1}{\eta_r J_t^2} & -\frac{1}{\eta_r J_t^2} \\
0 & \frac{1}{\eta_r J_t} & 0 & \frac{1}{\eta_r J_t} \\
-\frac{1}{\eta_r J_t^2} & 0 & \frac{1}{\eta_r J_t} & 0 \\
-\frac{1}{\eta_r J_t^2} & 0 & 0 & \frac{1}{\eta_r J_t}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{J_t^2} & 0 & -\frac{1}{\eta_r J_t^2} & -\frac{1}{\eta_r J_t^2} \\
0 & \frac{1}{\eta_r J_t} & 0 & \frac{1}{\eta_r J_t} \\
-\frac{1}{\eta_r J_t^2} & 0 & \frac{1}{\eta_r J_t} & 0 \\
-\frac{1}{\eta_r J_t^2} & 0 & 0 & \frac{1}{\eta_r J_t}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{J_t^2} & 0 & -\frac{1}{\eta_r J_t^2} & -\frac{1}{\eta_r J_t^2} \\
0 & \frac{1}{\eta_r J_t} & 0 & \frac{1}{\eta_r J_t} \\
-\frac{1}{\eta_r J_t^2} & 0 & \frac{1}{\eta_r J_t} & 0 \\
-\frac{1}{\eta_r J_t^2} & 0 & 0 & \frac{1}{\eta_r J_t}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix}
(36)

2.1 Rotor dynamics

The quadrotor thrust and moments are produced by four DC-motors, the dynamics modeling consist the electrical and mechanical equations of motor and propeller.

the motor is controlled by an electric voltage input; in order to have the induced voltage (BEMF) to generate the desired angular velocity; the equation of electrical model is:

\[
v = Ri + L \frac{di}{dt} + e
\]

with $e = K_e \omega_m$, in the next, the differential term was neglected because the most motors used are a very small inductance, therefore:

\[
v = Ri + K_e \omega_m
\]

the mechanical model of motor is given by:

\[
J_m \dot{\omega}_m = M_m - M_f
\]

where $M_m = K_m i$ we used the propeller with a gearbox, then the friction torque of the motor is given by:

\[
M_f = \frac{\eta_r^2}{\eta_r} \omega_m - \frac{d}{\eta_r} \omega_m^2 + \frac{K_m}{\eta_r} \omega_m v
\]

therefore the equation (41) becomes:

\[
(\frac{J_p}{\eta_r^2} + J_m) \omega_m = -\frac{K_m K_e}{\eta_r} \omega_m - \frac{d}{\eta_r^2} \omega_m^2 + \frac{K_m}{\eta_r} \omega_m v
\]

we have the electric and mechanic power of motor are equals, therefore $K_m = K_e$, and we replace $\frac{J_p}{\eta_r^2} + J_m$ by $J_t$ we obtain:

\[
\dot{\omega}_m = \frac{K_m^2}{RJ_t} \omega_m - \frac{d}{\eta_r^3 J_t} \omega_m^2 + \frac{K_m}{RJ_t} \omega_m v
\]

this equation of second degree is too difficult for control, we used the taylor series in the first order at $\omega_0$ value to linearize it : $f(\omega_m) = f(\omega_0) + f(\omega_0)(\omega_m - \omega_0)$

where $f(\omega_m) = \dot{\omega}_m$, the linear equation of motor dynamic is written as:

\[
\dot{\omega}_m = \frac{2d \omega_0}{\eta_r^3 J_t} + \frac{K_m^2}{RJ_t} \omega_m - \frac{K_m}{RJ_t} \omega_m + \frac{d \omega_0^2}{\eta_r^3 J_t}
\]

the equation has the linear form as: $\dot{\omega}_m = A \omega_m + B v + C$, the position of each term is shown with simulink model in fig.3.

3 Quadrotor Control

3.1 position control

from the dynamic equations, we have a decoupled system where the translational accelerations don’t depend on angular acceleration, from the first three equations we can extract the pitch and roll angles when we introduce the desired position and attitude.

from equations (21)(22)(23) we can calculate $\theta$ and $\phi$ depended of position vector and attitude $[x, y, z, \psi]$:

\[
\tan \theta = \frac{x}{x^2 + \frac{K_f x}{m} \dot{x}} \psi + \frac{y}{y^2 + \frac{K_f y}{m} \dot{y}} \psi
\]

\[
\sin \phi = \sqrt{\left(\frac{x}{x^2 + \frac{K_f x}{m} \dot{x}} \psi + \frac{y}{y^2 + \frac{K_f y}{m} \dot{y}} \psi \right)^2 + \left(\frac{z + \frac{K_f z}{m} \dot{z}}{z^2 + \frac{K_f z}{m} \dot{z}} \psi \right)^2}
\]

in our case, we assumed that the disturbance is neglected, then $K_{fx} = K_{fy} = K_{fz} = 0$ we study that in other
papers, therefore:

\[
\tan \theta = \frac{\ddot{x} \psi + \ddot{y} s \psi}{\ddot{z} + g}
\]

(44)

\[
\sin \phi = \frac{\ddot{x} s \psi + \ddot{y} c \psi}{\sqrt{\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g)^2}}
\]

(45)

### 3.2 Rotation Control

After we can inject the angle values founded to calculate the quadrotor position, the following figure shows the strategy of control structure.

![Figure 3: Block diagram of the used control structure](image)

### 3.3 PID Controller

In the continuous time, the parallel PID controller is defined as:

\[
U(t) = K_{pc} e(t) + K_{ic} \int_0^t e(\tau) d\tau + K_{dc} \frac{de(t)}{dt}
\]

(46)

\(\zeta\) is the variable to be controlled. \(\zeta \in \{x, y, z, \phi, \theta, \psi\}\)

Where:

- \(e(t) = \zeta_d - \zeta\), is the error between the desired and measured value.
- \(K_{pc}\): Proportional gain.
- \(K_{ic}\): Integral gain.
- \(K_{dc}\): Derivative gain.

### 4 Simulation results

We turn the PID parameters until we find values that converge to a stability situation, from a desired position and attitude of quadrotor, the values obtained are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>(K_{pc})</th>
<th>(K_{ic})</th>
<th>(K_{dc})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X)</td>
<td>4.5</td>
<td>0.001</td>
<td>11.8</td>
</tr>
<tr>
<td>(Y)</td>
<td>3.0</td>
<td>0.005</td>
<td>14</td>
</tr>
<tr>
<td>(Z)</td>
<td>6.7</td>
<td>0.001</td>
<td>12.8</td>
</tr>
<tr>
<td>(Phi)</td>
<td>8.3</td>
<td>0.09</td>
<td>13</td>
</tr>
<tr>
<td>(Theta)</td>
<td>9</td>
<td>0.07</td>
<td>13</td>
</tr>
<tr>
<td>(Psi)</td>
<td>13</td>
<td>0.001</td>
<td>19.5</td>
</tr>
</tbody>
</table>

![Figure 4: Position regulation signals of Quadrotor.](image)

![Figure 5: Attitude regulation signals of Quadrotor.](image)

It is necessary to define the four outputs (we have four con-
trols) these outputs are chosen to be the translational position $x, y$ and $z$ and the yaw angle $\psi$.

Singularity in (42), can only appear when $K_{fz} = 0$ and $\ddot{z} = -g$, so is when the quadrotor is in free fall without wind disturbances.

Figure 6: Trajectory of Quadrotor in 3D.

more simulation work VRML used for validation of results [5, 7, 6]; our work is validated in a platform for virtual reality, why we designed an application that has the control interface and a simulation in VRML to show the appearance of the quadrotor in the 3D space. The IHM is shown in figure 7.

Figure 7: VR interface of Quadrotor in 3D.

5 Conclusion

This paper includes the complete modeling of quadrotor, mathematical and dynamics model in the contraint environment, where is the strong point of that paper. Simulation results have been presented to show the powerful of the proposed controllers in an equilibrium point, these results will able to solve some problems of agents cooperative with a simple manipulation of position and attitude.

References


