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Dynamic Modeling, Simulation and PID Controller of Unmanned Aerial Vehicle UAV

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Abstract

This paper is focused on modeling and control of quadrotor; first modeling of moments and torque, second is of rotor, the result is the complete dynamics of UAV. mathematical model is presented for a general study with disturbance and we take account all parameters of control. the PID controller is presented without disturbance in dynamics equation for a linear model, where we can use in the control of a group of quadrotors for obstacle avoidance, and a group of quadrotors with a rigid and flexible links for transporting payloads in the free environment. finally our results are simulated in the simulink and the virtual reality environment.

keywords: *Quadrotor, UAVs, Modeling, Control, PID Controller, Virtual Reality*

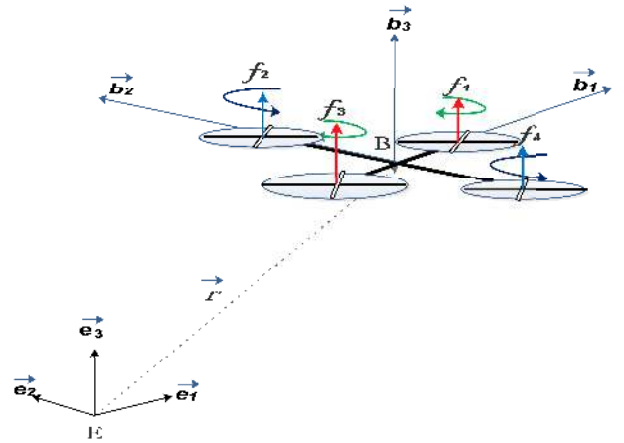


Figure 1: Quadrotor Model in $SO(3)$

1 INTRODUCTION

The quadrotor, UAV (Unmanned aerial vehicles) is the autonomous unmanned flying machines offering us an additional degree of freedom and an unlimited workspace, many researchers are facing this new device, and several studies are led to discover the areas of application of quadrotors. Modeling and control of quadrotor has been studied in several papers; mathematical and dynamic model in [8, 9] linear and nonlinear controller in [2] and for a quadrotor control with PID controller [10, 1, 3, 11] but the most of them is not a complete modeling in all cases of quadrotor situation; the recent research is mainly based on the fact that the quadrotor is an element of agent set to schedule a mission that requires cooperation and the control of a group of quadrotor, several research are based on the complete modeling with more mathematical details [4, 7]. This paper presents a complete modeling that is required for a projects that use the quadrotor as an elementary agent for cooperation. this paper is organized as follows: first, a mathematical model of quadrotor is presented.

2 DYNAMIC MODEL

the quadrotor uav model is defined in the two frames, an inertial reference frame $\vec{E} : \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ and a body fixed frame $\vec{B} : \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$

Two vectors associated to quadrotor, \mathbf{q} for position and rotation in the \vec{E} frame and \mathbf{V} for translational and rotational velocity in the body-fixed frame \vec{B} . $\mathbf{q} = [X \ \omega]^T = [x \ y \ z \ \phi \ \theta \ \psi]^T$

$\mathbf{V} = [V \ \Omega]^T = [v_x \ v_y \ v_z \ p \ q \ r]^T$ where $X = [x \ y \ z]^T$ is the relative position of the center of mass of the UAV and $\omega = [\phi \ \theta \ \psi]^T = [Roll \ Pitch \ Yaw]^T$ the orientation of the quadrotor in \vec{E} frame, with the constraint that:

Assumption 1: it is assumed that roll, pitch and yaw angles are defined in bounded intervals as:

$$\left(-\frac{\pi}{2} < \phi < \frac{\pi}{2}\right), \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right) \text{ and } (-\pi < \psi < \pi)$$

the Roll ϕ motion is given by the thrust imbalance between f_2 and f_4 forces around \vec{b}_1 , we can write the rotation matrix R_ϕ as:

$$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

the Pitch θ motion is given by the thrust imbalance between f_1 and f_3 around \vec{b}_2 , the R_θ is given by:

$$R_\theta = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

and the Yaw ψ motion is given by the thrust imbalance of all moments around the z axis, the rotation matrix is:

$$R_\psi = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we can now define the global rotation matrix R of quadrotor in the inertial frame E with the product of three matrices as $R = R_\psi R_\theta R_\phi$:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

Where, R_ϕ, R_θ, R_ψ and R matrices of rotation is defined in the special orthogonal group $SO(3)$, which have the following properties:

$$SO(3) = \{R \in \mathbb{R}^{3 \times 3} \mid R^T R = I, \det(R) = 1\}$$

Where: $c x \triangleq \cos x$ and $s x \triangleq \sin x$

The relation between $\dot{\omega}$ and Ω is given by the following equation:

$$\dot{R} = R\hat{\Omega}, \quad (2)$$

where the hat map:

$$\hat{\cdot} : \mathbb{R}^3 \rightarrow SO(3) \quad (3)$$

$$\hat{x}y = x \times y, \quad (4)$$

where $x, y \in \mathbb{R}^3$ and:

$$\hat{x} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \quad (5)$$

with $x = [x_1 \ x_2 \ x_3]^T$

to express the angular velocity of quadrotor in the fixed frame, we can use:

$$\Omega = \left(R^T \dot{R}\right)^\vee \quad (6)$$

where \vee map is the inverse of $\hat{\cdot}$ map:

$$(\cdot)^\vee : SO(3) \rightarrow \mathbb{R}^3$$

$$\begin{aligned} \dot{R} &= \frac{d}{dt} (R_\psi R_\theta R_\phi) \\ &= \dot{R}_\psi R_\theta R_\phi + R_\psi \dot{R}_\theta R_\phi + R_\psi R_\theta \dot{R}_\phi. \end{aligned}$$

$$\dot{R}_\psi = R_\psi \left(\begin{bmatrix} 0 & 0 & \dot{\psi} \end{bmatrix}^T \right)^\wedge$$

$$\dot{R}_\theta = R_\theta \left(\begin{bmatrix} 0 & \dot{\theta} & 0 \end{bmatrix}^T \right)^\wedge$$

$$\dot{R}_\phi = R_\phi \left(\begin{bmatrix} \dot{\phi} & 0 & 0 \end{bmatrix}^T \right)^\wedge$$

$$R^T = (R_\psi R_\theta R_\phi)^T = R_\phi^T R_\theta^T R_\psi^T.$$

we replace in the equation (4) becomes:

$$\Omega = \left(R_\phi^T R_\theta^T \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}^\wedge R_\theta R_\phi + R_\phi^T \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}^\wedge R_\phi + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}^\wedge \right)^\vee$$

we can use this properties of the $\hat{\cdot}$ map:

$$R\hat{x}R^T = (\hat{R}x)^\wedge$$

the equation (4) becomes:

$$\Omega = \left(\left(R_\phi^T R_\theta^T \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \right)^\wedge + \left(R_\phi^T \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} \right)^\wedge + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}^\wedge \right)^\vee$$

$$\Omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (7)$$

$$\Omega = M\dot{\omega} \implies \dot{\omega} = M^{-1}\Omega, \quad M \in \mathbb{R}^{3 \times 3}.$$

now we can rewrite the relation between $\dot{\mathbf{q}}$ and \mathbf{V} in the matrix form as:

$$\dot{\mathbf{q}} = T \cdot \mathbf{V} \quad (8)$$

and

$$T = \begin{bmatrix} R & 0_{3 \times 3} \\ 0_{3 \times 3} & M^{-1} \end{bmatrix}$$

Assumption 2: we assume that the quadrotor as near of the nominal state, where we can rewrite the angular velocity in inertial frame as:

$$M = I_{3 \times 3} \text{ therefore: } (\dot{\phi} = p, \dot{\theta} = q, \dot{\psi} = r)$$

the dynamics equation of the unmanned aerial vehicle UAV are as:

$$m\dot{v} = fRe_3 - mge_3 \quad (9)$$

$$M = J\dot{\Omega} + \Omega \times J\Omega. \quad (10)$$

the translational and rotational kinetic energy of quadrotor is given by:

$$T_{trans} = \frac{1}{2}m\dot{X}^T\dot{X} \quad (11)$$

$$T_{rot} = \frac{1}{2}\dot{\Omega}^T J \dot{\Omega} \quad (12)$$

and $J = \text{diag}\{J_x \ J_y \ J_z\} \in \mathbb{R}^{3 \times 3}$ is the inertia matrix of the quadrotor in the body-fixed frame we assume symmetric.

the potential energy of body is:

$$U = mgz \quad (13)$$

the Lagrangian of the quadrotor is given:

$$L(q, \dot{q}) = T_{trans} + T_{rot} - U$$

$$L(q, \dot{q}) = \frac{1}{2}m\dot{X}^T\dot{X} + \frac{1}{2}\dot{\Omega}^T J \dot{\Omega} - mgz$$

$$\frac{d}{dt} \left(\frac{\delta L}{\delta \dot{q}} \right) - \frac{\delta L}{\delta q} = \begin{bmatrix} F \\ \tau \end{bmatrix}$$

F is the external force applied to the quadrotor .

and τ is the generalized moments around the UAV.

given by $F = F_X + F_t$ and $\tau = [\tau_\phi \ \tau_\theta \ \tau_\psi]^T$ where F_X is the force of translational movement generated by propellers, and F_t is the drag forces applied to the quadrotor along $\{\vec{b}_1 \ \vec{b}_2 \ \vec{b}_3\}$ vectors, $f = \sum_{i=1}^4 f_i$ and $F_X = fRe_3$

$$F_X = f \begin{bmatrix} c\phi s\theta c\psi + s\phi s\psi \\ c\phi s\theta s\psi - s\phi c\psi \\ c\phi c\theta \end{bmatrix} \quad (14)$$

$$F_t = \begin{bmatrix} -K_{fx} & 0 & 0 \\ 0 & -K_{fy} & 0 \\ 0 & 0 & -K_{fz} \end{bmatrix} \dot{X} \quad (15)$$

where K_{fx} , K_{fy} , K_{fz} are drag coefficients.

$$m\dot{v} = fRe_3 + F_t - mge_3 \quad (16)$$

$$m \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = f \begin{bmatrix} c\psi s\theta c\phi + s\psi s\phi \\ s\psi s\theta c\phi - s\phi c\psi \\ c\theta c\phi \end{bmatrix} + \begin{bmatrix} -K_{fx} & 0 & 0 \\ 0 & -K_{fy} & 0 \\ 0 & 0 & -K_{fz} \end{bmatrix} \dot{X} - mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (17)$$

$$J\dot{\Omega} = \Omega \times J\Omega + \sum_{i=1}^4 J_r(\Omega \times e_3)\Omega_i + \tau \quad (18)$$

$$\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\dot{\psi}\theta I_y + \dot{\theta}\psi I_z \\ \dot{\phi}\psi I_x - \dot{\phi}\dot{\psi} I_z \\ -\dot{\theta}\phi I_x + \dot{\phi}\theta I_y \end{bmatrix} + J_r \begin{bmatrix} \dot{\theta} \\ -\dot{\phi} \\ 0 \end{bmatrix} \Omega_r + \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} I_x \ddot{\phi} \\ I_y \ddot{\theta} \\ I_z \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\theta}\dot{\psi}(I_z - I_y) \\ \dot{\phi}\dot{\psi}(I_x - I_z) \\ \dot{\theta}\dot{\phi}(I_y - I_x) \end{bmatrix} + \begin{bmatrix} J_r \dot{\theta} \Omega_r \\ -J_r \dot{\phi} \Omega_r \\ 0 \end{bmatrix} + \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \quad (20)$$

$$m\ddot{x} = (c\phi s\theta c\psi + s\phi s\psi)U_1 - K_{fx}\dot{x} \quad (21)$$

$$m\ddot{y} = (c\phi s\theta s\psi - s\phi c\psi)U_1 - K_{fy}\dot{y} \quad (22)$$

$$m\ddot{z} = (c\phi c\theta)U_1 - mg - K_{fz}\dot{z} \quad (23)$$

$$I_x \ddot{\phi} = \dot{\theta}\dot{\psi}(I_y - I_z) - J_r \dot{\theta} \Omega_r + IU_2 \quad (24)$$

$$I_y \ddot{\theta} = \dot{\phi}\dot{\psi}(I_z - I_x) - J_r \dot{\psi} \Omega_r + IU_3 \quad (25)$$

$$I_z \ddot{\psi} = \dot{\phi}\dot{\theta}(I_x - I_y) + U_4 \quad (26)$$

where:

$$\Omega_r = \omega_2 + \omega_4 - \omega_1 - \omega_3 \quad (27)$$

the thrust generated by i -th propeller along \vec{b}_3 is:

$$f_i = b\omega_i^2 \quad (28)$$

b : is the thrust factor, and ω_i is the speed of i -th propeller.

the rotor i also produce a moment given by:

$\tau_i = \pm d\omega_i^2$ and the total torque is:

$$\tau_{M_i} = \sum_{i=1}^4 (-1)^i d\omega_i^2. \quad (29)$$

d : is the drag factor.

The quadrotor is defined as a six degrees of freedom, three for the position of center of mass, and three for rotation, which represents the output variables and also the final state of quadrotor in three dimensional space, but the system's input are just four, one thrust force and torque, we named by U_i $i = 1, \dots, 4$ variable inputs of UAV controller.

$$U_1 = \sum_{i=1}^4 f_i = f_1 + f_2 + f_3 + f_4 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (30)$$

$$U_2 = f_2 - f_4 = b(\omega_2^2 - \omega_4^2) \quad (31)$$

$$U_3 = f_1 - f_3 = b(\omega_1^2 - \omega_3^2) \quad (32)$$

$$U_4 = \sum_{i=1}^4 \tau_{M_i} = d(-\omega_2^2 - \omega_4^2 + \omega_1^2 + \omega_3^2) \quad (33)$$

$$\tau = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} IU_2 \\ IU_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} lb(\omega_4^2 - \omega_2^2) \\ lb(\omega_3^2 - \omega_1^2) \\ d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) \end{bmatrix} \quad (34)$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -b & 0 & b \\ -b & 0 & b & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (35)$$

the determinant of the above 4×4 matrix is $-8db^3$,so it is invertible when $d \neq 0$ and $b \neq 0$.

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4b} & 0 & -\frac{1}{2b} & -\frac{1}{4d} \\ \frac{1}{4b} & -\frac{1}{2b} & 0 & -\frac{1}{4d} \\ \frac{1}{4b} & 0 & \frac{1}{2b} & -\frac{1}{4d} \\ \frac{1}{4b} & \frac{1}{2b} & 0 & \frac{1}{4d} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad (36)$$

2.1 Rotor dynamics

The quadrotor thrust and moments are produced by four DC-motors, the dynamics modeling consist the electrical and mechanical equations of motor and propeller.

the motor is controlled by an electric voltage input;in order to have the induced voltage (BEMF) to generate the desired angular velocity,the equation of electrical model is:

$$v = Ri + L \frac{di}{dt} + e \quad (37)$$

with $e = K_e \omega_m$, in the next, the differential term was neglected because the most motors used are a very small inductance, therefore:

$$v = Ri + K_e \omega_m \quad (38)$$

the mechanical model of motor is given by:

$$J_m \dot{\omega}_m = M_m - M_f \quad (39)$$

where $M_m = K_m i$

we used the propeller with a gearbox,then the friction torque of the motor is given by:

$$M_f = \frac{d\omega_m^2}{\eta r^3} \quad \text{and} \quad \omega_p = \frac{\omega_m}{r}, J_{pm} = \frac{J_p}{\eta r^2}$$

therefore the equation (41) becomes:

$$\left(\frac{J_p}{\eta r^2} + J_m \right) \dot{\omega}_m = -\frac{K_m K_e}{R} \omega_m - \frac{d}{\eta r^3} \omega_m^2 + \frac{K_m}{R} v$$

we have the electric and mechanic power of motor are equals, therefore $K_m = K_e$, and we replace $\left(\frac{J_p}{\eta r^2} + J_m \right)$ by J_t we obtain:

$$\dot{\omega}_m = -\frac{K_m^2}{R J_t} \omega_m - \frac{d}{\eta r^3 J_t} \omega_m^2 + \frac{K_m}{R J_t} v \quad (40)$$

this equation of second degree is too difficult for control, we used the taylor series in the first order at ω_0 value to linearize it : $f(\omega_m) = f(\omega_0) + f'(\omega_0)(\omega_m - \omega_0)$

where $f(\omega_m) = \dot{\omega}_m$, the linear equation of motor dynamic is written as:

$$\dot{\omega}_m = -\left(\frac{2d\omega_0}{\eta r^3 J_t} + \frac{K_m^2}{R J_t} \right) \omega_m + \frac{K_m}{R J_t} v + \frac{d\omega_0^2}{\eta r^3 J_t} \quad (41)$$

the equation has the linear form as: $\dot{\omega}_m = A\omega_m + Bv + C$, the position of each term is shown with simulink model in fig.3.

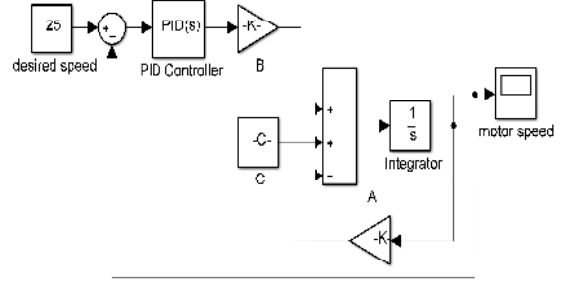


Figure 2: Simulink model of DC-motor

symbol	definition
v	motor voltage input
R	motor resistance
M_m	motor torque
M_f	the load torque
ω_m	motor angular speed
ω_p	propeller angular speed
K_e	the motor constant
K_m	the motor torque constant
J_m	the motor moment of inertia
J_p	the rotor moment in the propeller axis
η	the gear box efficiency
r	reduction ratio

3 Quadrotor Control

3.1 position control

from the dynamic equations , we have a decoupled system where the translational accelerations don't depend on angular acceleration, from the first three equations we can extract the pitch and roll angles when we introduce the desired position and attitude.

from equations (21)(22)(23) we can calculate θ and ϕ depended of position vector and attitude $[x, y, z, \psi]$:

$$\tan \theta = \frac{\left(\ddot{x} + \frac{K_{fx}}{m} \dot{x} \right) c\psi + \left(\ddot{y} + \frac{K_{fy}}{m} \dot{y} \right) s\psi}{\ddot{z} + g + \frac{K_{fz}}{m} \dot{z}} \quad (42)$$

$$\sin \phi = \frac{\left(\ddot{x} + \frac{K_{fx}}{m} \dot{x} \right) s\psi + \left(\ddot{y} + \frac{K_{fy}}{m} \dot{y} \right) c\psi}{\sqrt{\left(\ddot{x} + \frac{K_{fx}}{m} \dot{x} \right)^2 + \left(\ddot{y} + \frac{K_{fy}}{m} \dot{y} \right)^2 + \left(\ddot{z} + g + \frac{K_{fz}}{m} \dot{z} \right)^2}} \quad (43)$$

in our case, we assumed that the disturbance is neglected,then $K_{fx} = K_{fy} = K_{fz} = 0$ we study that in other

papers,therefore:

$$\tan \theta = \frac{\ddot{x}c\psi + \ddot{y}s\psi}{\ddot{z} + g} \quad (44)$$

$$\sin \phi = \frac{\ddot{x}s\psi + \ddot{y}c\psi}{\sqrt{\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g)^2}} \quad (45)$$

3.2 Rotation Control

after we can inject the angle values founded to calculate the quadrotor position, the following figure shows the strategy of control structure.

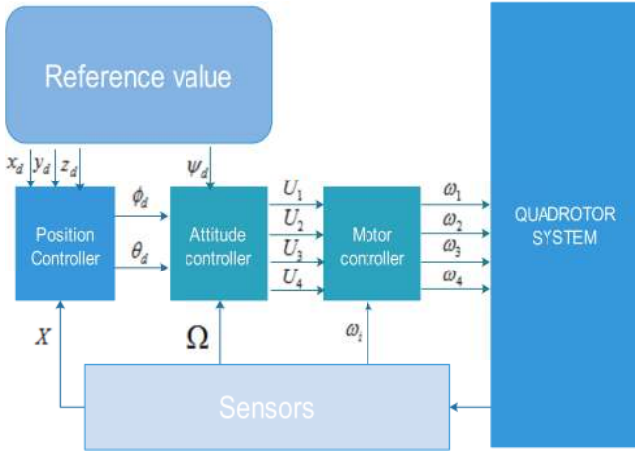


Figure 3: Block diagram of the used control structure

3.3 PID Controller

in the continuous time,the parallel PID controller is defined as:

$$U(t) = K_{p\zeta}e(t) + K_{i\zeta} \int_0^t e(\tau)d\tau + K_{d\zeta} \frac{de(t)}{dt} \quad (46)$$

ζ is the variable to be controlled. $\zeta \in \{x, y, z, \phi, \theta, \psi\}$
Where:

$e(t) = \zeta_d - \zeta$, is the error between the desired and measured value.

$K_{p\zeta}$: Proportional gain.

$K_{i\zeta}$: Integral gain.

$K_{d\zeta}$: Derivative gain.

4 Simulation results

we tune the PID parameters until we find values that converge to a stability situation, from a desired position and attitude of quadrotor, the values obtained are shown in the following table:

	$K_{p\zeta}$	$K_{i\zeta}$	$K_{d\zeta}$
<i>X</i>	4.5	0.001	11.8
<i>Y</i>	3.0	0.005	14
<i>Z</i>	6.7	0.001	12.8
<i>Phi</i>	8.3	0.09	13
<i>Theta</i>	9	0.07	13
<i>Psi</i>	13	0.001	19.5

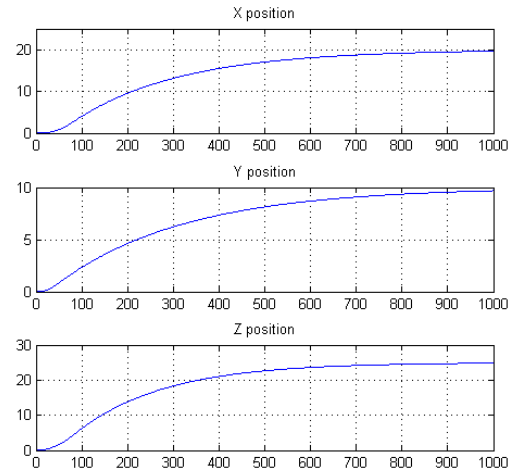


Figure 4: Position regulation signals of Quadrotor.

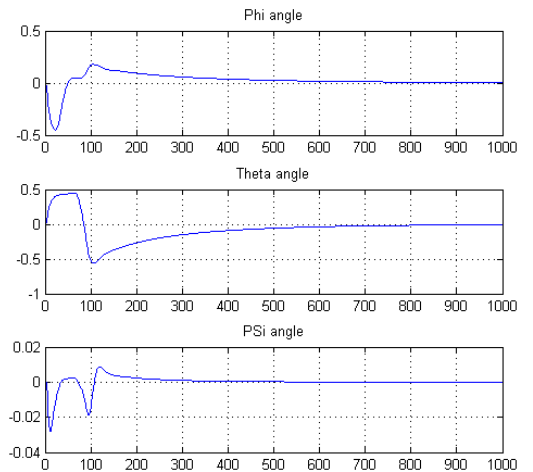


Figure 5: Attitude regulation signals of Quadrotor.

it is necessary to define the four outputs (we have four con-

trols) these outputs are chosen to be the translational position x, y and z and the yaw angle ψ .

singularity in (42), can only appear when:

$K_{fz} = 0$ and $\ddot{z} = -g$, so is when the quadrotor is in free fall without wind disturbances.

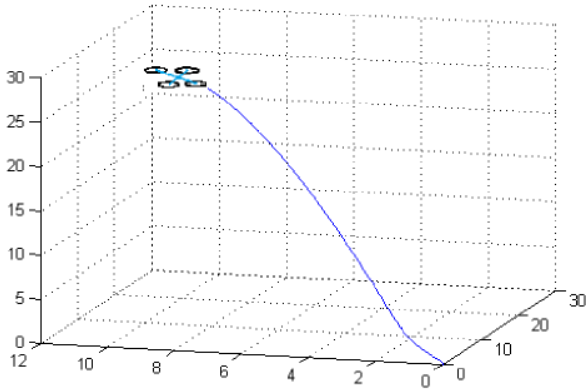


Figure 6: Trajectory of Quadrotor in 3D.

more simulation work VRML used for validation of results [5, 7, 6]; our work is validated in a platform for virtual reality, why we designed an application that has the control interface and a simulation in VRML to show the appearance of the quadrotor in the 3D space. the IHM is shown in figure 7.

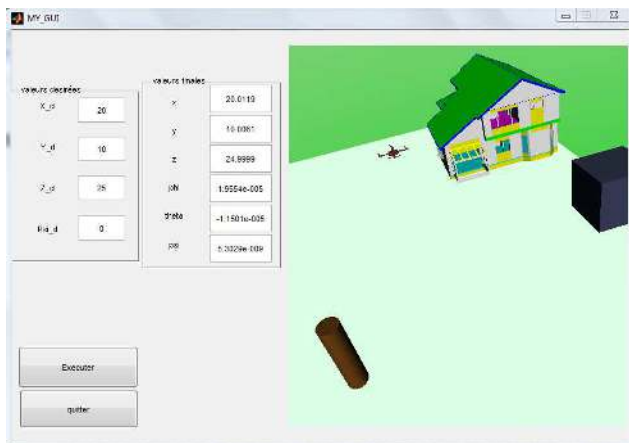


Figure 7: VR interface of Quadrotor in 3D.

5 Conclusion

this paper includes the complete modeling of quadrotor, mathematical and dynamics model in the constraint environment, where is the strong point of that paper, Simulation results have been presented to show the powerful of the proposed controllers in an equilibrium point, these results will able to solve some problems of agents cooperative with a simple manipulation of position and attitude.

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