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A ROBUST ASYMPTOTIC OBSERVER FOR BATCH PROCESSES WITH SINGLE BIOGAS MEASUREMENT

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ABSTRACT. In this paper we propose an observer for a dynamical system for which the boundary of its domain is not observable, and trajectories converge to this boundary. The proposed case study, a bioreactor in batch operating conditions with a single microbial reaction and gas production, is standard and largely encountered in practical situations. We show also how to extend this observer to obtain an observer in higher dimension that is robust with respect to unbiased noise.

KEY-WORDS. State observer, observability, biological systems.

1. INTRODUCTION

The state observation of nonlinear systems is a wide and active research area resulting in a large scientific publications on the subject. Let us point out a few (e.g. [3] [5], [6]) as well as the interesting and nicely survey paper by Krener [7]. In the present paper, we would like to address a state observation issue that, as far as we know, has not yet been addressed, i.e. the problem of the on-line reconstruction of the variables of a nonlinear system for which there is a loss of observability on the boundary of its domain. We shall indeed concentrate on a biological model with one biomass x , one substrate s and gaseous outflow rate y measured on-line when the initial condition (x_0, s_0) is unknown. Incidentally the design of the state estimate takes advantage of the fundamental reaction invariant property of reaction systems ([4]).

The paper is organized as follows. Section 2 introduces the dynamical model of the biological system. Section 3 provides an analysis of the observability properties of the system under study. An asymptotic observer is derived in Section 4 while Section 5 gives a Luenberger observer for the purpose of comparison. Section 5 provides numerical simulations to illustrate the performance of the proposed observer as well as its comparison with the Luenberger observer designed in Section 5.

2. DYNAMICAL MODEL

Let us consider the dynamical model of a simple microbial growth reaction in a batch reactor (see e.g. [1]):

$$(1) \quad \begin{cases} \dot{x} &= \mu(s)x \\ \dot{s} &= -\mu(s)x \end{cases}$$

where x and s stand for the concentrations in biomass and substrate, respectively. Without loss of generality, we assume that the yield coefficient of the transformation of the substrate into biomass is equal to 1. The specific growth rate function $\mu(\cdot)$ satisfies the usual assumption:

Hypothesis 1. $\mu(\cdot)$ is an analytic function which is increasing and concave on \mathbb{R}_+ and such that $\mu(0) = 0$.

For convenience, we denote the number

$$\bar{\mu} = \max_{s>0} \mu(s)$$

(that could be finite or not). In the present work, we consider that a gaseous by-product flow rate, such as bio-gas (e.g. [2]), is measured on-line as a quantity proportional to the output variable

$$y = \mu(s)x.$$

Typically, it often happens in batch bio-processes that the initial quantities of reactants (x_0, s_0) are unknown and the bio-gas production is the only available measurement during the reaction. As the solutions of (1) clearly satisfy $\lim_{t \rightarrow +\infty} s(t) = 0$, the initial quantity of substrate can be recovered as

$$s_0 = \lim_{t \rightarrow +\infty} \int_0^t y(\tau) d\tau$$

However the main interest is usually to estimate the total production of biomass, that is $\lim_{t \rightarrow +\infty} x(t)$. The purpose of the present work is to present a simple and reliable methodology to estimate the x concentration.

3. OBSERVABILITY ANALYSIS

Equivalently, one can consider the dynamics in the (z, s) coordinates with $z = x + s$

$$(2) \quad \frac{d}{dt} \begin{bmatrix} z \\ s \end{bmatrix} = f(z, s) = \begin{bmatrix} 0 \\ -\mu(s)(z - s) \end{bmatrix}$$

$$(3) \quad y = h(z, s) = \mu(s)(z - s)$$

on the positive cone

$$\mathcal{C} := \{(z, s) \in \mathbb{R}^2 \mid z > s > 0\}$$

Lemma 1. *Under Hypothesis 1, the system (2)-(3) is differentially observable on \mathcal{C} , but not on its boundary.*

Proof. On the open cone \mathcal{C} , one has $\mu(s) > 0$ and can write

$$(4) \quad \begin{aligned} L_f h(z, s) &= -[\mu'(s)(z - s) - \mu(s)]\mu(s)(z - s) \\ &= -\left(\frac{\mu'(s)}{\mu(s)}h(z, s) - \mu(s)\right)h(z, s). \end{aligned}$$

For a given non-negative number y , we define the function

$$\varphi_y(s) = \mu(s) - \frac{\mu'(s)}{\mu(s)}y$$

whose derivative is

$$\varphi'_y(s) = \mu'(s) - \left(\frac{\mu''(s)}{\mu(s)} - \frac{\mu'(s)^2}{\mu(s)^2} \right) y.$$

Under Hypothesis 1, $\varphi'_y(s)$ is positive for any s , and therefore the inverse φ_y^{-1} is well defined from $\varphi(\mathbb{R}_+)$ to \mathbb{R}_+ . As $h(z, s) > 0$ on \mathcal{C} , one can write

$$\begin{cases} s &= \varphi_{h(z,s)}^{-1} \left(\frac{L_f h(z, s)}{h(z, s)} \right) \\ z &= \frac{h(z, s)}{\mu(s)} + s \end{cases}$$

and conclude that the map

$$\begin{bmatrix} z \\ s \end{bmatrix} \mapsto \begin{bmatrix} h(z, s) \\ L_f h(z, s) \end{bmatrix}$$

is injective on \mathcal{C} , that is the system is differentially observable on \mathcal{C} .

On the boundary of \mathcal{C} , one has $L_{f^i} h \equiv 0$ for any integer i and therefore the system is not differentially observable on $\partial\mathcal{C}$. \square

One can note that the boundary $z = s$ (that is $x = 0$) of the cone \mathcal{C} is invariant but repulsive for the dynamics, while the boundary $s = 0$ is attractive. Therefore for any initial condition with $z_0 > s_0$ (that is $x_0 > 0$), the solution converges to the boundary $s = 0$ where the system is no longer observable. This feature prevents classical constructions of nonlinear observers, such as the high-gain observer, that requires the (differential) observability on a compact invariant set (which here has to contain $s = 0$).

4. AN ASYMPTOTIC OBSERVER

We first show that the system is detectable from any positive initial condition.

Proposition 1. *Assume that one as $y(0) > 0$, then one has*

$$\lim_{t \rightarrow +\infty} s(t) = 0, \quad \lim_{t \rightarrow +\infty} x(t) = z$$

where

$$(5) \quad z = \frac{y(t)}{\mu \left(\int_t^{+\infty} y(\tau) d\tau \right)} + \int_t^{+\infty} y(\tau) d\tau$$

for any $t \geq 0$.

Proof. When $y(0) > 0$, one has $x_0 > 0$ and $s_0 > 0$. Clearly, the solution of (1) fulfills $(x(t), s(t)) \rightarrow (z, 0)$ when t tends to $+\infty$, where $z = x_0 + s_0$. Therefore, one can write

$$s(t) = \int_t^{+\infty} y(\tau) d\tau$$

at any time $t \geq 0$, and then one also

$$x(t) = \frac{y(t)}{\mu \left(\int_t^{+\infty} y(\tau) d\tau \right)}.$$

Finally, one obtains

$$z = x(t) + s(t) = \frac{y(t)}{\mu \left(\int_t^{+\infty} y(\tau) d\tau \right)} + \int_t^{+\infty} y(\tau) d\tau$$

□

One can then consider the following observer.

Proposition 2. *For any initial condition (x_0, x_0) with $x_0 > 0$ and $s_0 > 0$, the following observer*

$$(6) \quad \begin{cases} \dot{v} &= y(t), & v(0) = 0 \\ \dot{\hat{x}} &= \frac{y(0)}{\mu(v)} + v \end{cases}$$

fulfills

$$\lim_{t \rightarrow +\infty} \hat{x}(t) - x(t) = 0$$

Moreover, when the function μ is increasing, the error $\hat{x} - x$ is decreasing with time.

Proof. Note that for $t > 0$, the solution of (6) is given by the expression

$$(7) \quad \hat{x}(t) = \frac{y(0)}{\mu \left(\int_0^t y(\tau) d\tau \right)} + \int_0^t y(\tau) d\tau$$

The convergence of the observer is then a simple consequence of Proposition 1. The time derivative of the x -error is determined straightforwardly as

$$\frac{d}{dt}(\hat{x} - x)(t) = -y(t_0) \frac{\mu' \left(\int_{t_0}^t y(\tau) d\tau \right)}{\left[\mu \left(\int_{t_0}^t y(\tau) d\tau \right) \right]^2} y(t) < 0$$

which shows the monotonic behavior of the error, when the function μ is increasing □

Remark 1. *The fact that the error $\hat{x} - x$ is decreasing with time guarantees that $\hat{x}(t)$ is a guaranteed upper estimation of $x(t)$ at any time t and that the estimator does not oscillate as it could happen with high-gain observers.*

5. A ROBUST ASYMPTOTIC OBSERVER

We consider that the measurement is now given by

$$y_{obs}(t) = y(t) + p(t)$$

where $p(\cdot)$ is a disturbance. The drawback of the observer presented in Proposition 2 is its lack of robustness with respect to the initial error $p(t_0)$ (because the initial measurement $y(t_0)$ is not forgotten, differently to “classical” observers). Even when the disturbance $p(\cdot)$ is unbiased, one can easily check that the observer $\hat{z}(\cdot)$ given in (6) with $y(\cdot)$ replaced by $y_{obs}(\cdot)$ presents an asymptotic bias:

$$(8) \quad \lim_{t \rightarrow +\infty} \hat{x}(t) - x = \frac{p(t_0)}{\mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right)}$$

We consider the following stronger assumption on the bias of the disturbance $p(\cdot)$.

Hypothesis 2. *There exists $T < +\infty$ such that*

$$\frac{1}{T} \int_t^{t+T} p(\tau) d\tau = 0, \quad \forall t > 0$$

We show that under this assumption, it is possible to derive an observer without asymptotic bias.

Proposition 3. *For any $p(\cdot)$ that satisfies Hypothesis 2, the following estimator*

$$(9) \quad \hat{x}(t) = \frac{\int_{t_0=0}^{t_0=T} \left\{ y_{obs}(t_0) + \int_{t_0}^t y_{obs}(\tau) d\tau \cdot \mu \left(\int_{t_0}^t y_{obs}(\tau) d\tau \right) \right\} dt_0}{\int_{t_0=0}^{t_0=T} \mu \left(\int_{t_0}^t y_{obs}(\tau) d\tau \right) dt_0}$$

which is defined for $t > T$, is unbiased.

Proof. First note that, due to Hypothesis 2, one has

$$\int_{t_0=0}^{t_0=T} y_{obs}(t_0) dt_0 = \int_{t_0=0}^{t_0=T} y(t_0) dt_0$$

and

$$\int_{t_0}^{+\infty} y_{obs}(\tau) d\tau = \int_{t_0}^{+\infty} y(\tau) d\tau, \quad \forall t_0 \in [0, T]$$

Moreover, from equation (5), one has

$$\mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right) z = y(t_0) + \int_{t_0}^{+\infty} y(\tau) d\tau \cdot \mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right), \quad \forall t_0 \in [0, T]$$

Integrating this last inequality between $t_0 = 0$ and $t_0 = T$, one obtains

$$\begin{aligned} & \int_{t_0=0}^{t_0=T} \mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right) dt_0 \cdot z \\ &= \int_{t_0=0}^{t_0=T} y(t_0) dt_0 + \int_{t_0=0}^{t_0=T} \int_{t_0}^{+\infty} y(\tau) d\tau \cdot \mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right) dt_0 \end{aligned}$$

or equivalently

$$z = \frac{\int_{t_0=0}^{t_0=T} y(t_0) dt_0 + \int_{t_0=0}^{t_0=T} \int_{t_0}^{+\infty} y(\tau) d\tau \cdot \mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right) dt_0}{\int_{t_0=0}^{t_0=T} \mu \left(\int_{t_0}^{+\infty} y(\tau) d\tau \right) dt_0}$$

which proves that one has

$$\lim_{t \rightarrow +\infty} \hat{x}(t) = z$$

and we conclude by Proposition (1). \square

Remark 2. *The observer (9) is not an averaging of the observer proposed in Proposition 2, which can be equivalently written in the integral form (7). It is neither the expression (7) evaluated on an average of the measurements.*

6. NUMERICAL COMPUTATION OF THE ROBUST OBSERVER

The robust observer given in Proposition 3 requires an integration on t_0 and therefore cannot be written as a solution of dynamical system in finite dimension. However, the integrals

$$I(t_0, t) = \int_{t_0}^t y_{obs}(\tau) d\tau, \quad t > t_0$$

can be determined for N discrete values of $t_0 = 0, h, 2h, \dots, T - h$ with $h = T/N$, by integrating the variables v_i solutions of the system of N differential equations

$$\dot{v}_i = y_{obs}(t) \mathbb{1}_{\{t > (i-1)h\}}(t), \quad v_i(0) = 0 \quad (i = 1 \dots N)$$

where $\mathbb{1}_A(\cdot)$ denotes the indicator function of a set A . Clearly, one has

$$v_i(t) = I((i-1)h, t), \quad t > (i-1)h \quad (i = 1 \dots N)$$

Then, the integrals of $\gamma(I(t_0, t))$ from $t_0 = 0$ to $t_0 = T$, where $\gamma(\cdot)$ is any smooth function are approximated by

$$\int_{t_0=0}^{t_0=T} \phi(I(t_0, t)) \simeq h \sum_{i=1}^N \gamma(v_i(t)), \quad t > T$$

and we obtain the following approximation of the observer (9) in a form of a finite dimensional system:

$$(10) \quad \begin{cases} \dot{v}_i &= y_{obs}(t) \mathbb{1}_{\{t > (i-1)h\}}(t), \quad v_i(0) = 0 \quad (i = 1 \dots N) \\ \hat{x}(t) &= \frac{\frac{N}{T} v_1(T) + \sum_{i=1}^N v_i(t) \mu(v_i(t))}{\sum_{i=1}^N \mu(v_i(t))}, \quad t > T \end{cases}$$

Remark 3. *In practice, one can choose T large enough compared to the inverse of the fundamental frequency and first harmonics of the Fourier decomposition of the noise.*

7. CONSIDERATION OF AN LUENBERGER OBSERVER

As we aim at reconstructing the biomass x , we write the system (1) in (x, y) coordinates as follows

$$(11) \quad \begin{cases} \dot{x} &= y \\ \dot{y} &= \phi(x, y)y \end{cases}$$

with

$$\phi(x, y) = -\mu' \circ \mu^{-1} \left(\frac{y}{x} \right) x + \frac{y}{x}$$

Notice that μ^{-1} is well defined on $[0, \bar{\mu}]$ under the Hypothesis 1. However, the map ϕ has a singularity at $x = 0$, but solutions of (1) satisfy $x(t) \geq x_0$ at any $t > 0$ and therefore avoid this singularity for any initial condition with $x_0 > 0$. For the construction of an observer, one has to extend this dynamics for values of x that are non positive or such that y/x is larger than $\bar{\mu}$. Let ϵ be a positive number, and consider the map

$$\tilde{\phi}(x, y) = \phi(\max(x, \epsilon, y/\bar{\mu}), y)$$

which coincides with ϕ along any solution of (11) with $x_0 \geq \epsilon$. Moreover $\tilde{\phi}$ is Lipschitz continuous on $\mathbb{R} \times \mathbb{R}_+$. We can then consider a Luenberger observer [8] in the (x, y) coordinates for the dynamics with ϕ replaced by $\tilde{\phi}$:

$$(12) \quad \begin{cases} \dot{\hat{x}} &= y + G_1(\hat{y} - y(t)) \\ \dot{\hat{y}} &= \tilde{\phi}(\hat{x}, y(t))y(t) + G_2(\hat{y} - y(t)) \end{cases}$$

Let us write the error equation as follows:

$$\frac{d}{dt} \begin{bmatrix} e_x \\ e_y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G_1 \\ \delta(t)y(t) & G_2 \end{bmatrix}}_{A(t)} \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

where

$$\delta(t) = \begin{cases} \frac{\tilde{\phi}(\hat{x}(t), y(t)) - \phi(x(t), y(t))}{\hat{x}(t) - x(t)} & \text{if } \hat{x}(t) \neq x(t) \\ \partial_x \phi(x(t), y(t)) & \text{if } \hat{x}(t) = x(t) \end{cases}$$

Under Hypothesis 1, the derivative μ' is bounded on \mathbb{R}_+ and then the map $x \mapsto \tilde{\phi}(x, y)$ has linear growth for any $y \in \mathbb{R}_+$. As $y(\cdot)$ is bounded, $\delta(\cdot)$ is thus bounded whatever is the solution $\hat{x}(\cdot)$.

Therefore, as one has $\lim_{t \rightarrow +\infty} y(t) = 0$, we obtain

$$\lim_{t \rightarrow +\infty} A(t) = \bar{A} = \begin{bmatrix} 0 & G_1 \\ 0 & G_2 \end{bmatrix}$$

which is not a Hurwitz matrix. The asymptotic convergence of the error towards 0 is thus not guaranteed and the choice the gains G_1, G_2 does not allow the assign the speed of convergence, as this will be shown with numerical simulations in the next section.

8. NUMERICAL SIMULATIONS

We consider first the asymptotic observer (2).

8.1. Under Hypothesis 1. We consider the Monod function

$$\mu(s) = \frac{1}{1 + s}$$

which satisfies Hypothesis 1 and is often used in microbiology (see Fig. 1).

We have considered the initial condition $(s_0, b_0) = (1.1, 1.5)$ and have compared the asymptotic observer with the Luenberger one initialized with $(\hat{x}, \hat{y}) = (1, y(0))$ for various gains G_1, G_2 . We found a systematic bias on the asymptotic error of the Luenberger observer, as depicted on Fig. 2 (for $G_1 = G_2 = -20$ that gave the best result). These simulations show that the innovation $\hat{y} - y$ of the Luenberger observer reaches zero while the x error has not yet converged to zero, which explains its non null asymptotic error.

We have also performed simulations with measurements randomly disturbed by a white noise proportionate up to 10% of the signal (see Fig. 3). It shows the good behavior of the asymptotic observer with respect to measurement noise, most probably because it does not require a multiplicative gain on the measurement as for the Luenberger observer or any ‘‘classical’’ observers. The asymptotic observer is mainly affected by the initial error on the measurement.

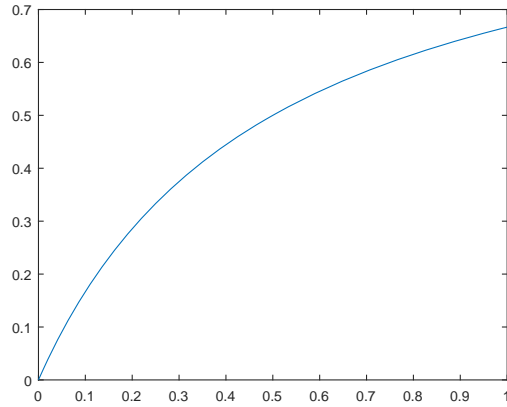


FIGURE 1. Graph of the Monod function

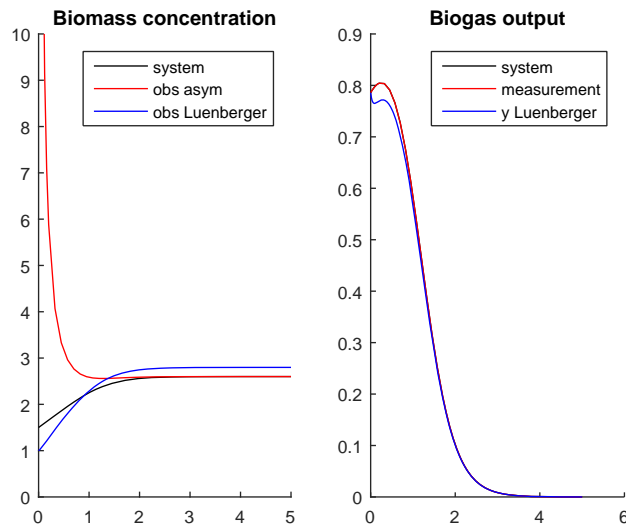


FIGURE 2. Comparison of our proposed observer with the Luenberger observer in absence of noise (with Monod function)

8.2. In absence of Hypothesis 1. It should be underlined that the asymptotic observer of Proposition 2 does not require Hypothesis 1 to be fulfilled. In this section, we consider two different functions $\mu(\cdot)$ that do not satisfy Hypothesis 1, so that the observability analysis of Section 3 cannot be conducted.

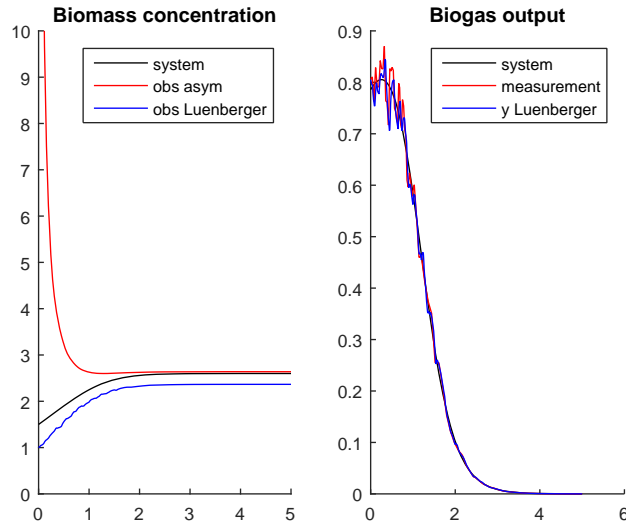


FIGURE 3. Comparison of our proposed observer with the Luenberger observer with noise (with Monod function)

The Hill function for the specific growth rate writes as follows (see Fig. 4):

$$\mu(s) = \frac{\mu_{max} s^\alpha}{K_s^\alpha + s^\alpha} \quad (\alpha > 1)$$

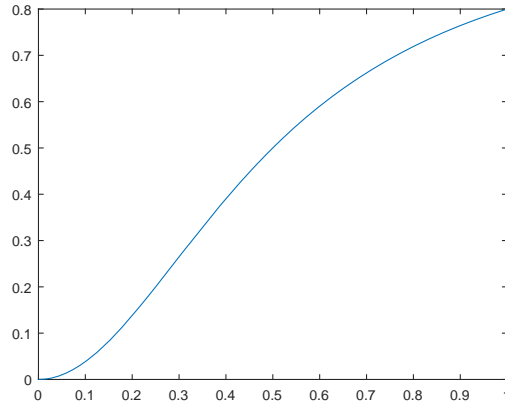


FIGURE 4. Graph of the Hill function

This function is increasing but not concave (see Fig. 4). Simulations of Fig. 5 have been ran for $\mu_{max} = 1$, $K_s = 0.5$ and $\alpha = 2$.

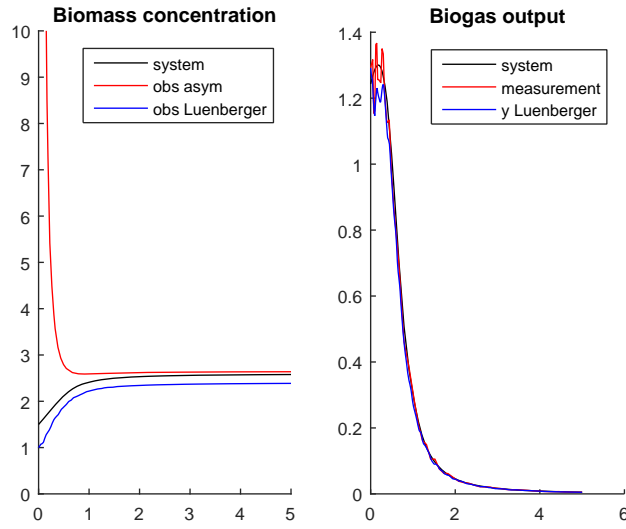


FIGURE 5. Comparison of our proposed observer with the Luenberger observer (with Hill function)

The Haldane model for the specific growth rate (see Fig. 6) is given by the following expression:

$$\mu(s) = \frac{\mu_0 s}{K_s + s + \frac{s^2}{K_i}}$$

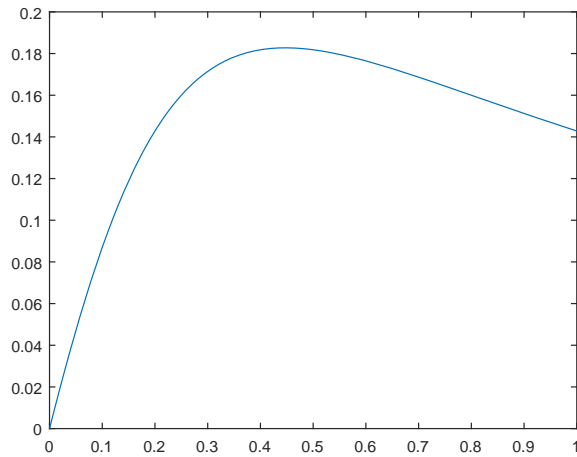


FIGURE 6. Graph of the Haldane function

This function is non monotonic (see Fig. 6). Therefore the inverse of μ is not uniquely defined for the construction of the Luenberger observer in (x, y) coordinates. However, for each positive value m of the function μ , one has $\mu^{-1}(m) = \{s_-(m), s_+(m)\}$ with $0 < s_-(m) \leq \sqrt{K_s K_i} \leq s_+(m)$, and as solutions of system (1) converge to $s = 0$, we have considered $\mu^{-1}(m) = s_-(m)$ in the Luenberger observer (12) (which gives the right inverse as soon as t satisfies $s(t) \leq \sqrt{K_s K_i}$). Simulations of Fig. 7 have been ran for $\mu_0 = 1$, $K_s = 1$ and $K_i = 0.2$.

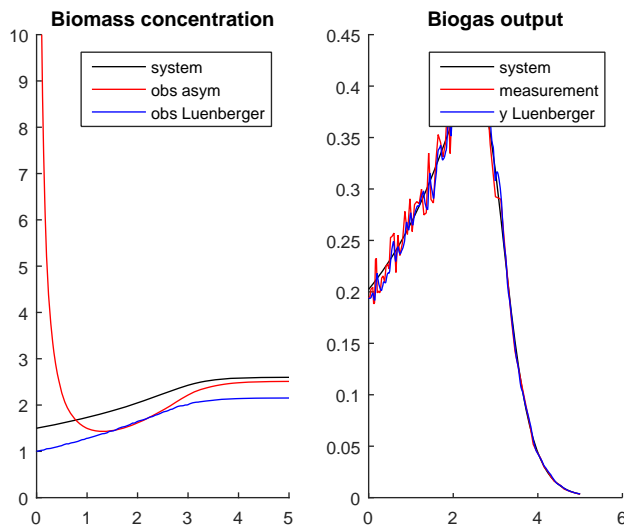


FIGURE 7. Comparison of our proposed observer with the Luenberger observer (with Haldane function)

These simulations show that the proposed observer works satisfactorily in simulation for a large variety of growth functions. One can note that for the Haldane function, the property of having an upper estimation of the biomass is no longer satisfied, in accordance with Remark 1.

8.3. Robust observer. We have consider stronger noise of amplitude 5 time larger. In such cases, the error on the initial measurement $y(0)$ can impact significantly the asymptotic bias of the asymptotic observer (2), as shown in (8). We have compared the robust version of the asymptotic observer proposed in (10) with $T = 0.1$ and $N = 10$, for the same growth functions and parameters of the simulations presented above (see Fig. 8, 9 and 10).

The simulations show that with a relatively small value of N , the approximated robust observer can cope quite well with large deviation of initial measurement and noise.

9. CONCLUSIONS

In this work, we have proposed an observer for a system that is not observable on its set of asymptotic states. To cope with this difficulty, our observer is of a particular structure which does not use the innovation (i.e. the difference between

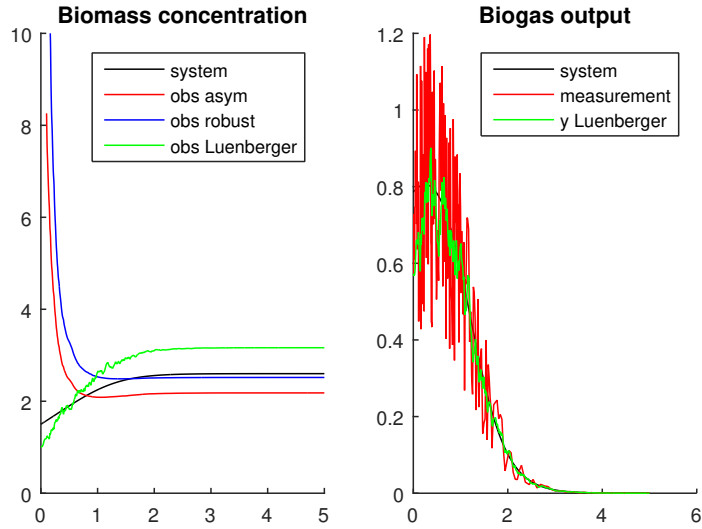


FIGURE 8. Comparison with the robust asymptotic observer under strong noise (with Monod function)

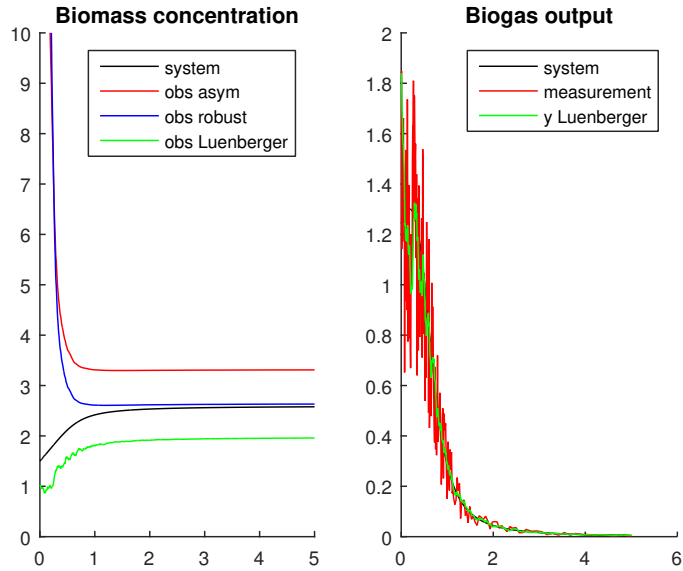


FIGURE 9. Comparison with the robust asymptotic observer under strong noise (with Hill function)

the observation predicted by the observer and the real observation) as the usual observers do. Its convergence speed cannot be freely addressed but we prove its

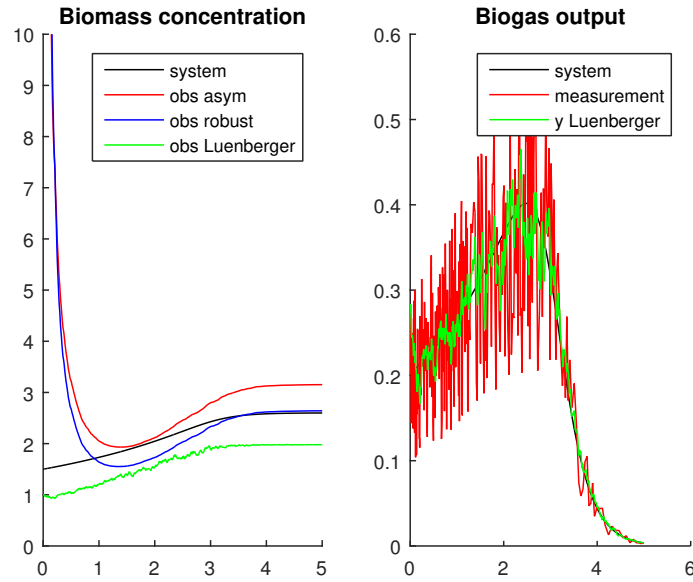


FIGURE 10. Comparison with the robust asymptotic observer under strong noise (with Haldane function)

exact convergence for any initial condition different from a asymptotic state. Moreover we have proposed a robust extension of this observer in higher dimension to deal with measurement disturbances. Numerical simulations have shown the good performances of the proposed observer on a biological system of practical interest, compared to other solutions such as Luenberger observer. Consideration of larger classes of systems with similar lack of observability will be the matter of a future work.

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