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# Performance of a condensation procedure based on projection on propagating waves

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*In this paper, we aim at reducing the cost of resolution of a system composed of an acoustic cavity coupled to a damping treatment. We use the characteristics of the waves propagating in the medium to condense the dynamics of the acoustic treatment on the surrounding cavity. These waves are predicted by using the Wave Finite Element method, and take into account lateral boundary conditions, spatial inhomogeneities and non-local reaction of poroelastic materials. The condensation procedure can be coupled with Component Mode Synthesis for the external medium. The efficiency of the method is discussed. It is shown that interesting reduction of computation times can be obtained, especially by selecting the waves which contribute to the coupling with the external media.*

## 1 Introduction

Dissipative interfaces in vibroacoustics are often used in transport industry. They are classically composed of poroelastic layers, presenting very good sound absorbing properties-to-mass ratio. To predict the response of such materials, the Biot-Allard theory [1] is usually used. However, the discretization by Finite Elements (FE) of a poroelastic domain leads to large and frequency dependent matrices. In order to reduce the numerical cost associated to the presence of poroelastic layers, several strategies have been adopted. Some attempts of building models for equivalent plates have been proposed (e.g. [2]). Normal modes were used to reduce the number of unknowns in the layer [5, 3]. Impedance modelling is the fastest approach today, but even though accurate in a large number of cases, this approach is limited to samples of large areas when lateral boundary conditions have no influence on the response of the layer.

The Wave Finite Element Method (WFE, e.g. [4]) is a technique allowing to compute the waves propagating in an arbitrary medium, provided that periodicity in one direction is achieved. We show here that the modes computed by this technique can be used to condense the dissipative part on the acoustic part and thus to reduce the problem to solve. The speedup of the algorithm with respect to the direct FE resolution is then discussed.

## 2 Condensation procedure of the interface

We consider first a waveguide of constant section along thickness. This waveguide is in the same material and has the same section that the dissipative layer. Only a cell of length  $L$  is modelled, using one element in thickness. The wave modes are defined by a wave number  $k$ , a section deform  $\Phi$ , and can be computed from the FE dynamic stiffness matrix  $\mathbf{D}$  of the cell (see [4]).

Once the wave modes of the medium are known, it is possible to express the virtual work exerted by the interface on the acoustic cavity as a function of the wave modes and of the nodal pressure. The discretization by FE makes it possible to express the influence of the dissipative treatment by an additional coupling matrix  $\mathbf{D}_{\text{coupl}}$ , so that the pressure in the acoustic cavity is given at the angular frequency  $\omega$  by:

$$\left[ \mathbf{D}_a(\omega) + \mathbf{D}_{\text{coupl}}(L_p, \mathbf{k}(\omega), \Phi(\omega)) \right] \mathbf{p}_a(\omega) = \mathbf{F}_a(\omega) \quad (1)$$

where  $\mathbf{D}_a(\omega)$  is the dynamic stiffness matrix of the acoustic cavity with rigid walls,  $\mathbf{p}_a$  the vector of nodal acoustic pressures and  $\mathbf{F}_a$  the vector of nodal acoustic loads. The coupling matrix  $\mathbf{D}_{\text{coupl}}$  is derived from the resolution of a linear problem in terms of the amplitude of the travelling waves, written by projecting the boundary conditions in both ends of the waveguide on the propagating waves. It depends on the propagating waves ( $\mathbf{k}(\omega), \Phi(\omega)$ ) computed at the frequency  $\omega$ , and on the thickness of the medium  $L_p$ . The method can thus be seen as a generalized impedance approach, extending the local impedance to the case of non-local fields.

### 3 Performance of the approach and selection of waves

An acoustic cavity of dimensions (40cm × 30cm × 70cm) Figure 1 is covered on the bottom face by a layer of poroelastic material (thickness 5cm). The mesh consists in (20 × 15 × 40) and (20 × 15 × 10) linear acoustic elements in the cavity and in the poroelastic layer (limp assumption) respectively. For this mesh, WFE results in a set of 336 incident waves and 336 recessive waves, among which only a few really influence the acoustic response. 250 modes with fixed interface are used for CMS in the acoustic domain. Figure 2 presents the sound pressure level in the cavity, as well as the speedup factor (SU) with respect to the direct resolution by FE, when FE or CMS is used for the acoustic cavity, and when FE or wave condensation is used for the poroelastic layer. Good superposition is obtained between all

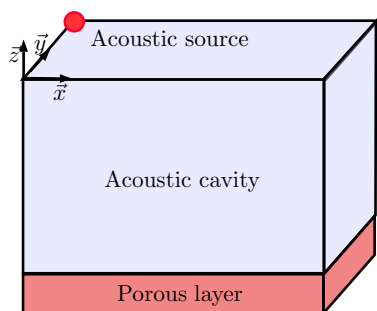


Figure 1: Schema of the test case.

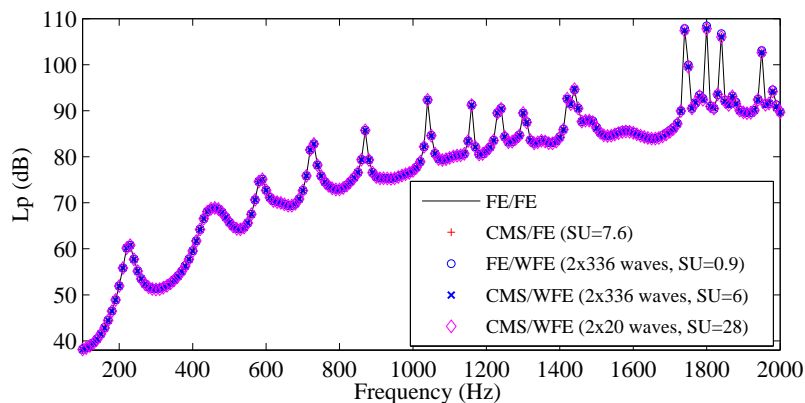


Figure 2: Sound pressure level in the acoustic cavity.

the curves. CMS is quite efficient, because the number of unknowns in the poroelastic domain is quite small (3696 dofs) compared to the FE model (13776 dofs). The efficiency of the proposed procedure is penalized by the high area-to-thickness ratio of the interface, which leads to a large number of elements in the section compared to the number of elements in the thickness. However, very good speedup factors can be obtained by selecting carefully the wave modes, without losing in accuracy. In particular, even larger speedups can be expected in the case of thicker interfaces.

### 4 Conclusion

In this paper, the efficiency of a FE approach is discussed. This approach is based on condensation on interfaces by wave projection. The method is limited by the area-to-thickness ratio of the interface. Nevertheless, the approach can be combined with CMS, and a proper selection of contributing waves leads to large speedup with respect to direct FE.

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