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To cite this version:

Tien Hoang, Denis Duhamel, Gilles Forêt, Jean-Luc Pochet. Computational method for the dynamics of railway tracks on a non-uniform viscoelastic foundation. X International Conference on Structural Dynamics (EURODYN 2017), Sep 2017, Rome, Italy. pp.354-359. hal-01698476v2

HAL Id: hal-01698476

https://hal.science/hal-01698476v2

Submitted on 30 Jan 2018

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Procedia Engineering 00 (2017) 000–000

Procedia Engineering

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X International Conference on Structural Dynamics, EURODYN 2017

Computational method for the dynamics of railway tracks on a non-uniform viscoelastic foundation

Tien Hoang[a,*], Denis Duhamel[a], Gilles Foret[a], Jean-Luc Pochet[b]

[a]*Laboratoire Navier, UMR 8205, Ecole des Ponts, IFSTTAR, CNRS, UPE, Champs-sur-Marne, 77455 Marne La Vallée cedex 2, France*
[b]*Eurotunnel Group, BP69, 62904 Coquelles cedex, France*

Abstract

The foundation of a railway track may be non-uniform due to a number of reasons and this area is currently the subject of significant research. In this article, a new method has been developed for computing the responses of railway tracks based on a non-uniform foundation subjected to moving forces. This method is a coupling of an analytical model for the rail together with the sleepers and a finite element method for the foundation. In steady-state, it is supposed that the responses are unchanged when the moving forces come and go away from a larger interval of the railway track which contains a non-uniform zone. The dynamical stiffness matrix (DSM) of the foundation is computed by the finite element method and it is transformed to meet the steady state boundary condition. On the other hand, the rail together with the sleepers and rail pads are modelled by a periodically supported beam subjected to moving forces. This analytical model leads to a relation between the reactions forces and the displacements of the sleepers. This relation describes also the degrees of freedom (DOFs) of the nodes of the foundation at the contact surfaces with the sleepers. Then, a transformation technique has been developed in order to substitute the analytical relation into the DSM. Finally, the responses have been computed by using the transformed DSM. This method is a coupling of the analytical and numerical methods. Therefore, it has reduced all DOFs of the track components (sleepers, rail, and rail pads) which gives a significant advantage in computational time.
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Keywords: Railway track; transition zone; non-uniform foundation; periodically supported beam; structural dynamics.

1. Introduction

The influence of a non-uniform foundation on the response of a railway track has been studied by different methods including [1–6]. The most pressing difficulty of the numerical methods is that the rail with its supports (sleepers) and the foundation are not of the same scale (the dimensions of foundation is much larger than ones of sleepers and rails) which increases the degrees of freedom (DOF) and costs computing time. Recently, some authors have developed different techniques to reduce the number of DOF.

* Corresponding author.
E-mail address: tien.hoang@enpc.fr

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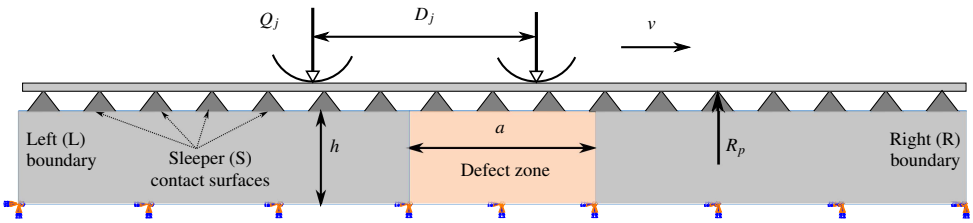

Fig. 1. Coupling of analytical and numerical models

This article presents a new model which is a coupling of analytical and numerical methods. When the rail together with its supports is considered as a periodically supported beam, Hoang et al. [1,7] proved a relation between the support displacements and reaction forces in the steady state and this relation holds for all types of foundation. By using this analytical model, we can write this relation for all DOF at the contact surfaces between the sleepers and the foundation. Then, this relation is substituted in the finite element model of the foundation in order to get the dynamical response.

2. Formulations

Let us consider a railway track based on a visco-elastic foundation which contains a defect zone as shown in Figure 1. In this model, we consider the rail as an infinite beam subjected to moving forces and the sleepers are concentrated supports which are distributed periodically along the beam (the rail together with its supports is called a periodically supported beam subjected to moving forces). Otherwise, the foundation is a 2D visco-elastic mater which is modelled by the finite element method.

Nomenclature

$\mathbf{u}$	vector of nodal displacements
$\mathbf{f}$	vector of nodal forces
$\mathbf{D}$	dynamical stiffness matrix of the foundation
$_S$	denotes the foundation DOF at the contact surfaces with the sleepers
$_L$	denotes the foundation DOF at the left boundary
$_R$	denotes the foundation DOF at the right boundary
$_I$	denotes the other foundation DOF
ω	angular frequency
$\mathbf{Q}_e$	equivalent force of the periodically supported beam
$\mathbf{K}_e$	equivalent stiffness of the periodically supported beam

By using the finite element method we can obtain the following results from the dynamic equation of the foundation

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the foundation, and $\mathbf{u}(t), \mathbf{F}(t)$ are the nodal displacements and forces. We can write the aforementioned equation in the frequency domain

$$\left(-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K}\right)\mathbf{u}(\omega) = \mathbf{F}(\omega) \tag{2}$$

or

$$\mathbf{D}(\omega)\mathbf{u}(\omega) = \mathbf{F}(\omega) \tag{3}$$

where $\mathbf{D}(\omega) = -\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K}$ is the dynamic stiffness matrix of the foundation.

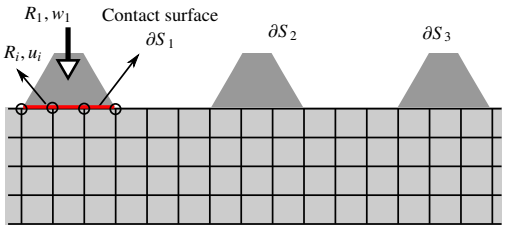

Fig. 2. Nodal forces and displacement at sleeper contact surfaces

If we separate the inner and boundary DOF of the foundation, we can write

$$\mathbf{u} = \begin{pmatrix} \mathbf{u}_S \\ \mathbf{u}_L \\ \mathbf{u}_R \\ \mathbf{u}_I \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \mathbf{f}_S \\ \mathbf{f}_L \\ \mathbf{f}_R \\ \mathbf{0} \end{pmatrix} \tag{4}$$

where $_S$, $_L$, $_R$ and $_I$ denote for the nodes in the different parts of the foundation as shown in Figure 1 (see Table of nomenclature). Then, equation 3 can be rewritten as follows

$$\begin{pmatrix} \mathbf{D}_{SS} & \mathbf{D}_{SL} & \mathbf{D}_{SR} & \mathbf{D}_{SI} \\ \mathbf{D}_{LS} & \mathbf{D}_{LL} & \mathbf{D}_{LR} & \mathbf{D}_{LI} \\ \mathbf{D}_{RS} & \mathbf{D}_{RL} & \mathbf{D}_{RR} & \mathbf{D}_{RI} \\ \mathbf{D}_{IS} & \mathbf{D}_{IL} & \mathbf{D}_{IR} & \mathbf{D}_{II} \end{pmatrix} \begin{pmatrix} \mathbf{u}_S \\ \mathbf{u}_L \\ \mathbf{u}_R \\ \mathbf{u}_I \end{pmatrix} = \begin{pmatrix} \mathbf{f}_S \\ \mathbf{f}_L \\ \mathbf{f}_R \\ \mathbf{0} \end{pmatrix} \tag{5}$$

In the steady state, we suppose that the defect zone is included in a sufficiently large interval of track so that the dynamical responses are unchanged when the moving forces come and leave this interval but with a delay which is equal to the time for the force to cover the length of the track interval (so-called the steady state condition)

$$\mathbf{u}_R = \mathbf{u}_L \mathrm{e}^{\mathrm{i}\frac{\omega L}{v}}, \quad \mathbf{f}_R = -\mathbf{f}_L \mathrm{e}^{\mathrm{i}\frac{\omega L}{v}} \tag{6}$$

where L, v are the length of the interval and the moving force speed respectively.

By substituting equation 6 into equation 5 and transforming the rows and columns of the matrix $\mathbf{D}$ corresponding to $\mathbf{u}_L$ and $\mathbf{u}_R$, we can obtain the following result

$$\begin{pmatrix} \mathbf{D}_{SS} & \tilde{\mathbf{D}}_{SL} & \mathbf{D}_{SI} \\ \tilde{\mathbf{D}}_{LS} & \tilde{\mathbf{D}}_{LL} & \tilde{\mathbf{D}}_{LI} \\ \mathbf{D}_{IS} & \tilde{\mathbf{D}}_{IL} & \mathbf{D}_{II} \end{pmatrix} \begin{pmatrix} \mathbf{u}_S \\ \mathbf{u}_L \\ \mathbf{u}_I \end{pmatrix} = \begin{pmatrix} \mathbf{f}_S \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{7}$$

where

$$\begin{aligned} \tilde{\mathbf{D}}_{LL} &= \mathbf{D}_{LL} + \mathbf{D}_{RR} + \mathrm{e}^{\mathrm{i}\frac{\omega L}{v}} \mathbf{D}_{LR} + \mathrm{e}^{-\mathrm{i}\frac{\omega L}{v}} \mathbf{D}_{RL} \\ \tilde{\mathbf{D}}_{SL} &= \mathbf{D}_{SL} + \mathrm{e}^{\mathrm{i}\frac{\omega L}{v}} \mathbf{D}_{SR}, \qquad \tilde{\mathbf{D}}_{IL} = \mathbf{D}_{IL} + \mathrm{e}^{\mathrm{i}\frac{\omega L}{v}} \mathbf{D}_{IR} \\ \tilde{\mathbf{D}}_{LS} &= \mathbf{D}_{LS} + \mathrm{e}^{-\mathrm{i}\frac{\omega L}{v}} \mathbf{D}_{RS}, \quad \tilde{\mathbf{D}}_{LI} = \mathbf{D}_{LI} + \mathrm{e}^{-\mathrm{i}\frac{\omega L}{v}} \mathbf{D}_{RI} \end{aligned}$$

Equation 7 is a reduced form of the dynamic equation under the steady state condition. We need to calculate the nodal forces $\mathbf{f}_S$ at the contact surface between the sleepers and the foundation. We suppose that the sleeper is rigid with one degree of freedom corresponding to its vertical displacement. Therefore, all DOF of foundation at the contact surface with a sleeper have the same vertical displacement which is equal to the sleeper displacement. For example, we denote w_1, R_1 the displacement and reaction force of the sleeper S_1 as shown in Figure 2, we have

$$\forall i \in \partial S_1 : \quad u_i = w_1, \quad \text{and} \quad \sum_i f_i = -R_1 \tag{8}$$

where ∂S_1 is the contact surface between the sleeper S_1 and the foundation. Therefore, if we denote d_{ik} the rows of DSM corresponding to nodal force f_i and d_{ki} the column of DSM corresponding to DOF u_i, we have

$$f_i = \sum_k d_{ik} u_k \quad \forall i \in \partial S_1 \tag{9}$$

By combining equations 8 and 9, we have

$$-R_1 = \sum_{i\in\partial S_1}\sum_k d_{ik}u_k = \sum_k\left(\sum_{i\in\partial S_1} d_{ik}\right)u_k = \sum_k \tilde{d}_{S_1k}u_k \tag{10}$$

where $\tilde{d}_{S_1k} = \sum_{i\in\partial S_1} d_{ik}$. Thus, equation 10 defines a new row of DSM which is the sum of all rows corresponding to the nodal force f_i with $i \in \partial S_1$.

In a similar way, we have

$$f_k = \sum_{i\notin\partial S_1} d_{ki}u_i + \sum_{i\in\partial S_1} d_{ki}u_i \quad \forall k \tag{11}$$

By substituting equation 8 into the aforementioned equation, we obtain

$$f_k = \sum_{i\notin\partial S_1} d_{ki}u_i + \left(\sum_{i\in\partial S_1} d_{ki}\right)w_1 = \sum_{i\notin\partial S_1} d_{ki}u_i + \tilde{d}_{kS_1}w_1 \tag{12}$$

where $\tilde{d}_{kS_1} = \sum_{i\in\partial S_1} d_{ki}$. Hence, equation 12 defines a new column in DSM which is the sum of all column corresponding to DOF u_i with $i \in \partial S_1$.

Therefore, we can replace rows and columns of DSM which correspond to DOF and nodal force at the contact surface with each sleeper by theirs sums to obtain a new row and a new column which justify equation 7 with DOF and nodal forces replaced by the sleeper displacement and contact force. In the other way, if we denote $\mathbf{w}_S = (w_1 \cdots w_N)^T$ and $\mathbf{R}_s = (R_1 \cdots R_N)^T$ the vectors of the displacements and reaction forces of all sleepers, we can obtain the following result from equation 7

$$\begin{pmatrix} \mathbf{D}^*_{SS} & \tilde{\mathbf{D}}^*_{SL} & \mathbf{D}^*_{SI} \\ \tilde{\mathbf{D}}^*_{LS} & \tilde{\mathbf{D}}_{LL} & \tilde{\mathbf{D}}_{LI} \\ \mathbf{D}^*_{IS} & \tilde{\mathbf{D}}_{IL} & \mathbf{D}_{II} \end{pmatrix}\begin{pmatrix} \mathbf{w}_S \\ \mathbf{u}_L \\ \mathbf{u}_I \end{pmatrix} = \begin{pmatrix} -\mathbf{R}_S \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{13}$$

By substituting the second and third rows of the aforementioned equation into the first one, we can deduce

$$\mathbf{R}_S = -\tilde{\mathbf{D}}_S\mathbf{w}_S \tag{14}$$

where

$$\tilde{\mathbf{D}}_S = \tilde{\mathbf{D}}_{SS} - \left(\tilde{\mathbf{D}}^*_{SL}\ \mathbf{D}^*_{SI}\right)\begin{pmatrix} \tilde{\mathbf{D}}_{LL} & \tilde{\mathbf{D}}_{LI} \\ \tilde{\mathbf{D}}_{IL} & \mathbf{D}_{II} \end{pmatrix}^{-1}\begin{pmatrix} \tilde{\mathbf{D}}^*_{LS} \\ \mathbf{D}^*_{IS} \end{pmatrix} \tag{15}$$

On the other side, the beam and its supports is modelled as a periodically supported beam. This analytical model permits to obtain a relation between the supports displacements and reaction forces by using the beam dynamical equation and the steady state condition as follows (see [1])

$$\mathbf{C}_e\tilde{\mathbf{R}}_S = \tilde{\mathbf{w}}_S + \mathbf{w}_e \tag{16}$$

where $\tilde{\mathbf{R}}_S$ and $\tilde{\mathbf{w}}_S$ are the sleeper responses in the reference of the moving forces. That means $\tilde{\mathbf{R}}_S = \mathbf{T}\,\mathbf{R}_S$, $\tilde{\mathbf{w}}_S = \mathbf{T}\,\mathbf{w}_S$, with $\mathbf{T}$ is transformation matrix of the two references which is given by

$$\mathbf{T} = \mathrm{diag}\left(\mathrm{e}^{\mathrm{i}\omega\frac{a_1}{v}}\ \mathrm{e}^{\mathrm{i}\omega\frac{a_2}{v}} \cdots \mathrm{e}^{\mathrm{i}\omega\frac{a_N}{v}}\right) \tag{17}$$

where a_p is the coordinate of the sleeper p. The functions $\mathbf{C}_e, \mathbf{w}_e$ are calculated by [1]

$$\mathbf{C}_e = \begin{pmatrix} \eta_0 & \eta_1 & \cdots & \eta_{m-1} \\ \eta_{m-1} & \eta_0 & \cdots & \eta_{m-2} \\ \vdots & & & \\ \eta_1 & \eta_2 & \cdots & \eta_0 \end{pmatrix}, \quad \mathbf{w}_e = \frac{1}{vEI}\sum_{j=1}^{K}\frac{Q_j\mathrm{e}^{-\mathrm{i}\frac{\omega}{v}D_j}}{\left(\frac{\omega}{v}\right)^4 - \lambda^4}\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \tag{18}$$

Table 1. Parameters of a railway track

Parameters	Unit	Value
Rail mass (ρS)	kg/m	60
Rail stiffness (EI)	MNm^2	6.3
Train speed (v)	km/h	160
Charge per wheel (Q)	kN	100
Distance between sleepers (l)	m	0.6
Sleeper width	m	0.3
Mass density of foundation	kg/m^3	2 710
Young's modulus of foundation	GPa	50
Young's modulus of defect zone	GPa	1.0
Poisson's coefficient of foundation		0.25

where $\lambda = \sqrt[4]{\frac{\rho S \omega^2}{EI}}$ with ρ, S, E and I are the mass density, beam section, Young's modulus and the inertia of the rail, Q_j, D_j are the loads and their relative distances as shown in Figure 1. The functions η_p $(0 \leq p \leq m-1)$ depend only on the parameter of the beam and the moving forces as follows

$$\eta_p = \frac{1}{LEI} \sum_{n \in \mathbb{Z}} \frac{e^{i2\pi n \frac{p}{m}}}{\left(\frac{\omega}{v} + \frac{2\pi n}{L}\right)^4 - \lambda^4} \tag{19}$$

By substituting equation 16 into equation 14, we obtain

$$\mathbf{R}_S = \left(\mathbf{C}_e \mathbf{T} + \mathbf{T}\, \tilde{\mathbf{D}}_S^{-1}\right)^{-1} \mathbf{w}_e \tag{20}$$

Equation 20 permits to compute the sleeper response. Then, the foundation response can be obtained by using this result and equation 13. We note that this model has the same number of DOF as the foundation. Therefore, we can calculate the dynamic response of the railway track without involving DOF of the rail and its supports in this model.

3. Example

Let us consider a railway track based on a 2D elastic foundation of depth $h = 1.2$m. This foundation contain a defect zone of width $a = 1.8$m where the Young's modulus is lower. We compute the response of the track interval of length $L = 10.2$m which contains 17 sleeper spacing and the defect zone is at the center subjected a moving load $Q = 100$kN as shown in Figure 1 and the railway track parameters are given in Table 1.

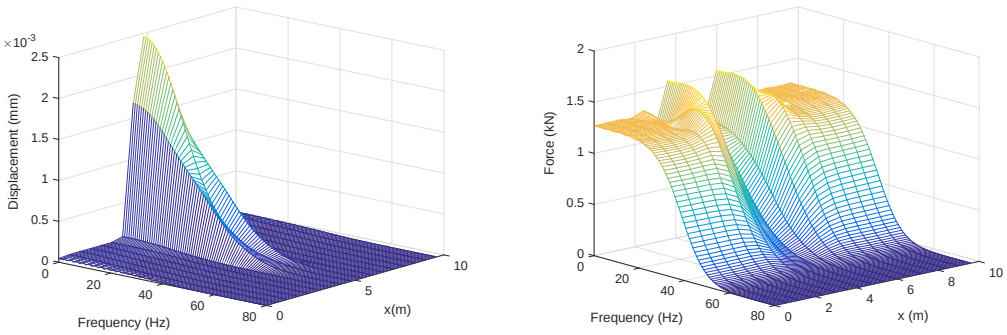

Fig. 3. Response of sleepers

In this example, we suppose that the nodes at the bottom boundary of the foundation are fixed. The mesh is created with size of 0.2m with a bilinear plane strain quadrilateral element (type 'CPE4' in ABAQUS) with thickness equal

to a half of the sleeper length. Each sleeper of width 0.3m covers 2 nodes of the contact surface among 4 nodes corresponding to the sleeper spacing l = 0.6m. Figures 3 shows the response of the sleepers in the frequency domain. The calculation is performed for the frequency [0 80Hz]. The sleeper response in the time domain is calculated by the inverse Fourier transform and the results are shown in Figure 4. We see that while the sleeper displacements increase in the defect zone, the reaction force decreases. Moreover, the reaction force of the sleeper next to the defect zone increases.

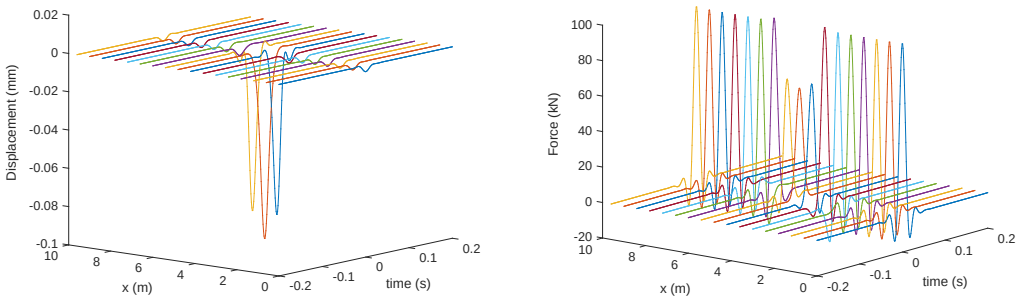

Fig. 4. Response of sleepers

4. Conclusion

A new method to calculate the response of a railway track has been developed by coupling analytical and numerical methods. From the finite element model of the foundation, the dynamical stiffness matrix is transformed to obtain a reduced DSM but this is a global matrix of the foundation together with the track. Therefore, this method reduces the number of DOF in a significant way. In perspective, we can include the finite elements of sleepers in the numerical model in order to analyse the dynamics of the sleepers together with the foundation.

Acknowledgements

This work has been developed in the context of a partnership between Eurotunnel Group and Ecole des Ponts ParisTech.

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