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Mateus A. O. Leite, Benoît Delinchant, Jean-Michel Guichon, João A. Vasconcelos, Jean-michel Guichon. Simplex-based adaptive parametric model order reduction for applications in optimization. International Journal of Numerical Modelling: Electronic Networks, Devices and Fields, 2019, 32 (4), 10.1002/jnm.2264 . hal-01692439

HAL Id: hal-01692439

<https://hal.science/hal-01692439>

Submitted on 21 Apr 2020

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1 SIMPLEX BASED ADAPTIVE PARAMETRIC MODEL ORDER 2 REDUCTION FOR APPLICATIONS IN OPTIMIZATION

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9
10 **Abstract:** A new methodology for optimization using parametric reduced order models is
11 introduced. An adaptive scheme to place the expansion points in the reduction step is combined
12 with a smart parametric space sampling in the optimization step to produce very reliable
13 reduced order models. The methodology is illustrated in a demonstrative electromagnetic
14 problem with appreciable gain in computational time. This technique is of general application
15 and can be used to speed up optimization processes in many areas of engineering.

16 **Keywords:** moment matching, parametric model order reduction, dynamic systems

17 **1. Introduction**

18 In the last decades, very successful numerical modeling techniques have been developed and
19 studied. The Finite Elements Method (FEM) or the Partial Element Equivalent Circuit (PEEC),
20 for example, are capable of building models that have a high degree of precision. It is, however,
21 hard to directly use these models for simulation and optimization. If the required accuracy is
22 very high or if the physical system is too complex, high dimensional models may be produced.
23 It is in this context that Model Order Reduction (MOR) may be applied. Given a High-Fidelity
24 Model (HFM), the objective of this technique is to build a Reduced Order Model (ROM) that
25 approximates the input/output behavior of the HFM but that has a much lower dimension,
26 allowing faster computations [1]. There are well-established methods for this purpose like
27 Balanced Truncation [2], Proper Orthogonal Decomposition [3] and Moment Matching [4]. For
28 the reduction of equivalent circuits obtained by the PEEC method, algorithms like PRIMA [5]
29 and SPRIM [6] have been shown to be particularly efficient.

30 If MOR techniques were to be used to speed up simulation processes, one would need the ROM
31 to conserve the parametric characteristics of the HFM for the design process [7]. This would
32 allow an optimization to be launched without the high cost of using the HFM for each objective
33 function evaluation. One approach to perform this task is to build several ROMs for different
34 parameter values and perform model interpolation for parameter values different from the
35 sampled points. However, this approach will become very expensive for a large number of
36 parameters [8].

37 In this article, we present an adaptive methodology for producing very reliable reduced order
38 models and a smart strategy for sampling the parameter space to minimize the number of model
39 order reductions during optimization processes. Both of these tasks are crucial for an effective
40 optimization methodology based in MOR. One cannot expect to obtain correct interpolated
41 models if the models used as basis for the interpolation are not precise. At the same time, it is

1 important to keep the number of sampled models as low as possible to reduce the computational
2 burden of the whole optimization process.

3 This article deals with systems like the one in (1): linear systems in descriptor form. In these
4 equations A and $E \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times p}$, $C \in \mathbb{R}^{q \times n}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$ and $y \in \mathbb{R}^q$. The number n
5 is the order of the model and p and q are, respectively, the number of inputs and outputs of the
6 system. The vector x is the state vector of the dynamical system and u and y are its input and
7 output vectors, respectively.

8 This set of equations is the natural form of writing many physical systems like circuits. At the
9 same time, the developed methodology is very general and may be adapted to different fields
10 of applications. This generality comes from the fact that no hypothesis is made about the
11 structures of the matrices of the state space. Any system that can be written in this form can be
12 treated.

$$\begin{cases} E \frac{dx}{dt} = Ax + Bu \\ y = Cx \end{cases} \quad (1)$$

13 This article is divided in four main sections. The first one introduces the main subject. Section
14 2 presents a review of reduced order models based in Krylov Subspaces and introduces the
15 technique to place the expansion points adaptively. The smart sampling of the parametric space
16 for parametric model order reduction is described in Section 3. Section 4 ends the article with
17 a conclusion.

18 2. Model Order Reduction

19 Projection based model order reduction is a framework that reduces the model by a projection
20 of the state variables into a lower dimension subspace [8]. Each method will differ in the way
21 that this subspace is chosen. For practical reasons, the subspaces are represented as matrices
22 whose columns span the subspace.

23 If one projects a system represented by (1) into the subspace spanned by V and orthogonal to
24 the span of W , the result is given by (2). In many applications, W is chosen as being equal to V .
25 For different purposes, the W matrix can be chosen to guarantee a stable or passive reduced
26 order model [9].

$$\begin{cases} W^T E V \frac{dx_r}{dt} = W^T A V x_r + W^T B u \\ y_r = C V x_r \end{cases} \quad (2)$$

27 To keep notation as simple as possible, one can introduce some definitions as in (3).

$$\begin{aligned} W^T E V &= E_r & W^T A V &= A_r \\ W^T B &= B_r & C V &= C_r \end{aligned} \quad (3)$$

28 This leads to a new state space model in descriptor form (4) that can be used instead of the
29 HFM to frequency domain and time domain simulations, if it is accurate enough.

$$\begin{cases} E_r \frac{dx_r}{dt} = A_r x_r + B_r u \\ y_r = C_r x_r \end{cases} \quad (4)$$

1 The following subsections presents how to choose the matrix V to assure very good accuracy
 2 between the high-fidelity model and the reduced one.

3 2.1. Moment Matching

4 If one applies the Laplace transformation to the system of equations in (1), it is possible to
 5 isolate a matrix (H) that relates inputs and outputs as written in (5). For the special case of a
 6 single input and single output (SISO), this matrix reduces itself to the well-known scalar
 7 transfer function.

$$H = C(Es - A)^{-1}B \quad (5)$$

8 For reasons that will become evident in the next steps, one can write (5) into a slightly different
 9 form, shown in (6). In this Equation, s_0 is an arbitrary complex number.

$$H = -C(A - s_0E - (s - s_0)E)^{-1}B \quad (6)$$

10 Using the easily proven identity (7) in (6), it is possible to expand this equation to a more useful
 11 form, written in (8).

$$(I - M)^{-1} = \sum_{i=0}^{\infty} M^i \quad (7)$$

12

$$H = - \sum_{i=0}^{i=\infty} C[(A - s_0E)^{-1}E]^i (A - s_0E)^{-1}B (s - s_0)^i \quad (8)$$

13 For simplicity, it is possible to define the i -th moment of the function at the point s_0 as done in
 14 (9).

$$M_i(s_0) = -C[(A - s_0E)^{-1}E]^i (A - s_0E)^{-1}B \quad (9)$$

15 This definition allows the writing of the transfer function of the original system in terms of its
 16 moments centered at s_0 . This can be seen in (10).

$$H = \sum_{i=0}^{i=\infty} M_i(s_0) (s - s_0)^i \quad (10)$$

17 A moment matching technique aims at finding a matrix V that produce reduced order models
 18 whose moments are the same as those of the HFM for given expansion points (s_0) up to a given
 19 order. Equation (11) contains the condition for this to be true [10]. In this equation, \mathcal{K} represents
 20 the krylov subspace up to the order i . To build such a matrix V , the well-known Arnoldi process
 21 may be used [11].

$$\text{span}\{V\} \supset \mathcal{K}\{(A - s_0E)^{-1}E, (A - s_0E)^{-1}B, i\} \quad (11)$$

22 It must be noted that it is possible to match moments around many different expansion points.
 23 It is only necessary that the above condition be satisfied for each one of the expansion points
 24 individually. This will be largely used in the next section to find a set of expansion points that
 25 produce very accurate reduced order models.

2.2. Automatic placement of expansion points

During the optimization process, it will be necessary to compute the output for a set of systems having different parameters. Each one of these systems have distinct frequency responses. Consequently, one may have an unreliable frequency response if the placement of the expansion points is not chosen individually for each system.

In this section, we provide an adaptive and very reliable heuristic method for the automatic placement of the expansion points. The objective of the method is to find points that provide a reliable frequency response and, at the same time, keeps a low order for the reduced order model. These goals are achieved by doing successive repartitions in the frequency domain and assuring that the frequency response is within an accepted error at each iteration. This idea is analogous to the one used in [12], but in another context.

As seen in the last subsection, the reduced order model obtained by moment matching will have the same moments as those of the high-fidelity model. The 0-th moment for a given expansion point, is the frequency response of the system for the frequency s_0 . This can be directly verified by plugging 0 in the value of i in (9). This means that the frequency response of both reduced and high fidelity model are the same at that point. This is the main idea behind the following heuristic, shown as a pseudo algorithm and detailed in the following paragraphs.

Input parameters: sys (system to be reduced), m_0 (number of moments to be matched at the first iteration), m (number of moments to be matched at subsequent iterations), f_l (lower value of the frequency range), f_u (upper value of the frequency range), α (repartition proportion), δ (Error tolerance)

Output values: red (reduced order system)

```
1   START
2
3   // Put the first interval into a stack
4   Stack.push([fl, fu])
5   // Obtains the reduced order model with m0 moments at each expansion point
6   Red ← reduce(sys, [fl fu], m0)
7
8   // Iterate until the stack is empty
9   WHILE stack not empty DO
10  // Receive the interval in the top of the stack
11  interval ← stack.pop();
12
13  // Compute where to place the next expansion point
14  cutPoint ← interval.lower ^ (1 - α) * interval.upper ^ α
15  // Reduced Order Model with addition of new expansion point
16  redTry ← reduce(sys, [red.freqs cutPoint], m)
17
18  // Determine the two new intervals
19  int1 ← [interval.lower, cutPoint]
20  int2 ← [cutPoint, interval.upper]
21
22  // Verify if there is significant change in the frequency response of the model
23  for the subspace at the left
```

```

14      IF max(|simulation(redTry) – simulation(red)| > δ) in int1 THEN
15          Stack.push(int1)
16          Keep ← true
17      END IF
18
19      // Verify if there is significant change in the frequency response of the model
for the subspace at the right
19      IF max(|simulation(redTry) – simulation(red)| > δ) in int2 THEN
20          Stac.push(int2)
21          Keep ← true
22      END IF
23
24      // If there is significant change, keep the current Reduced Order Model
24      IF Keep is true THEN
25          red ← redTry
26      END IF
27  END WHILE
28  END

```

1

2 The user must supply lower and upper bounds for the frequency range of interest and an
3 acceptable error tolerance. These bounds are used to build the first frequency interval. Each
4 extreme receives one expansion point with m_0 moments matched ($m_0=2$ by default). This will
5 produce a first reduced order model of very low order but that is exact for the chosen point and
6 matches the first derivative of the full model. This model and the whole interval delimited by
7 the lower and upper bounds supplied by the user are placed in a stack (FIFO data structure). At
8 each iteration on interval of the stack is analyzed and process is repeated until the structure is
9 empty.

10 At each iteration, the interval in the top of the stack is split in two new intervals. A new
11 expansion point is chosen using Equation (12), where f_e , f_l and f_u are, respectively, the
12 frequencies of the expansion points, lower and upper frequencies values of the interval.

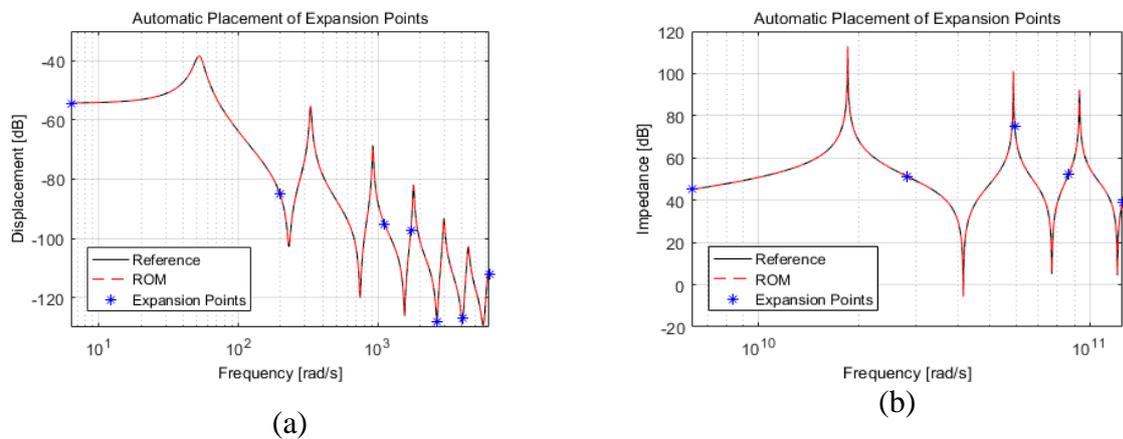
$$f_e = f_l^{1-\alpha} f_u^\alpha \quad (12)$$

13 This equation reduces to a geometric mean if α is one half, the default choice. A new reduced
14 order model and its frequency response are obtained. It must be pointed out that the projection
15 subspaces obtained for previous iterations are not recomputed. The algorithm only proceeds
16 with the computation of the vectors associated to the new point. After the addition of this new
17 expansion point with m moments matched ($m=1$ by default), two new frequency intervals will
18 have been delimited, one between the lower bound and the expansion point and a second one
19 from the expansion point to the upper bound. The order of the moments to be matched are
20 somehow arbitrary. If m is chosen to be one, this means that the model will be exact for the
21 selected expansion point and no derivatives are matched. There is indeed a trade-off between
22 accuracy and computational cost.

23 The difference of the frequency response is computed for each one of these intervals. If any of
24 the values is bigger than the accepted precision, this interval is added to the stack. Otherwise,
25 it is discarded and one considers that the algorithm has converged for that region. It is Important
26 to note that if the interval is discarded, the expansion point is not kept in the final reduced order

1 model. If there is no change, this means that the information added by the new point was already
2 present in the model. Therefore, keeping it would increase the order of the model without a
3 correspondent increase in accuracy. This guarantees that the order of the ROM is as small as
4 possible.

5 This method was applied to two different systems of different domains: a cantilever beam
6 obtained by finite elements methods [13] and a loop antenna obtained by the PEEC method
7 [14]. The beam receives the force at the extreme of the beam and outputs the total displacement.
8 It consists of a set of 1200 equations. The loop antenna is excited by a unitary current source
9 and outputs the impedance of the system for different frequencies. The total number of
10 equations for this system is 1035. For both systems, the accepted error tolerance is of 0.1 dB
11 and the frequency range where 1 Hz – 1 kHz and 1GHz – 20 GHz, respectively. Figure 1 presents
12 these results.



13 *Figure 1 (a) Comparison of the ROM ant the HFM for the cantilever beam. (b) Comparison of the ROM*
14 *ant the HFM for loop antenna.*

15 It must be noted that all the computations are done only using successive reduced order models.
16 The High-Fidelity Model is never used for a comparison. The simulation of the ROM can be
17 done in a very efficient way using the Hessenberg algorithm in [15]. Thus, this process is very
18 fast. The orders of the obtained system are 14 and 18, respectively, compared to the originals
19 of 1200 and 1035, respectively. Even though the final reduced order models are much smaller
20 than the high-fidelity ones, the adaptive algorithm has successfully chosen points that leads to
21 very good approximations, as can be seen by the comparison between the reference and the
22 obtained Reduced Order Models.

23 **3. Parametric Model Order Reduction for Optimization**

24 The process of model order reduction is capable of producing a very good approximation of
25 one given model with a certain parameter set. However, for the optimization process it is
26 necessary to evaluate the behavior of these models for different values of the parameters at each
27 iteration. Reducing a new model for each of these points may be very time consuming. To
28 overcome this difficulty, one may use techniques of parametric model order reduction [8].
29 Many methods like [16] and [17] need a kind of parameter space sampling as a starting point
30 to produce interpolated models. For the optimization process, equally sampling the whole
31 parameter space is not useful. During the execution of the algorithm only regions with

1 promising values of the objective function will be consulted. This particularity can be used to
2 guide the sampling strategy to be used. The next subsection presents a way of doing this.

3 3.1. Choosing the sampling points with a simplex based strategy

4 In polynomial interpolation, one provides measures for a set of independent variables and
5 intermediate values are obtained by an interpolating scheme. For the case of model
6 interpolation, the process is analogous. The user must provide a set of known models for
7 different parameter sets and an interpolating strategy is used to produce intermediate models.

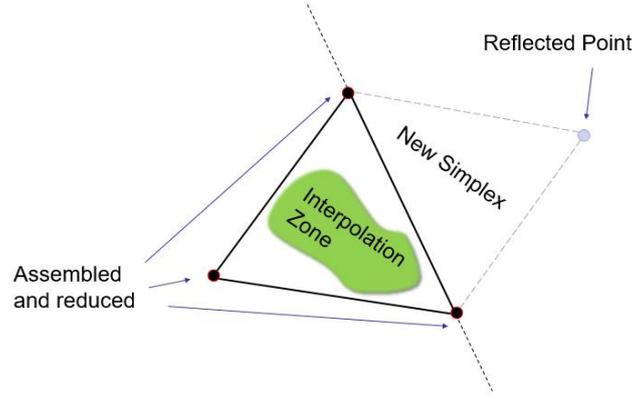
8 One may choose the sampling points in very different manners. Naive strategies like building
9 a hypercube in the parameter space will only work for very low dimensional parameter spaces
10 due to the curse of dimensionality. To overcome this problem, strategies based on sparse grids
11 may be applied [18]. However, it is still inefficient if an optimization process is the target
12 application. One does not need to sample the whole parameter space, only the regions that are
13 requested by the optimization algorithm.

14 To do this, we propose a new strategy based on a simplex, the generalization of a triangle to an
15 arbitrary number of dimensions. When the optimization process starts, one must provide a
16 starting point. A simplex having this point as its center is built using a process like the one
17 presented in [19]. For the sake of completeness, we briefly describe this procedure. In this
18 discussion, n is the dimension of the space and, consequently, the simplex is composed of $n +$
19 1 vectors. One starts by building a symmetric matrix $M \in R^{n \times n}$ as shown in (13).

$$M_{ij} = \begin{cases} 1 & \text{if } i = j \\ -\frac{1}{n} & \text{if } i \neq j \end{cases} \quad (13)$$

20 The columns of the upper triangular Cholesky factor of this matrix contains the first n vectors
21 that compose the simplex. The last one can be computed by taking the negative of the sum of
22 the first n vectors. A simplex that is not centered at zero can be obtained by a simple translation
23 of the obtained points. In a similar fashion, its points can be rotated to obtain different
24 simplexes. The ideal size of the simplex is a problem that is left as an open issue. For the current
25 discussion, physical knowledge of the problem can be used to infer an appropriate size.

26 At each one of the simplex vertices, a High-Fidelity Model is reduced using the adaptive
27 strategy presented in Section 2b. As long as the optimization algorithm keeps analyzing points
28 inside this simplex, interpolated models are used to evaluate the objective function. When the
29 algorithm steps out of the simplex, the farthest point from the current solution is reflected. The
30 Figure 2 shows an illustration of this procedure and identifies the interpolation zone and a
31 possible new reflected point.



1

2

Figure 2 - Example of interpolation zone inside a simplex

3 To successfully implementing this method, one needs a way to detect if a point is contained on
 4 the simplex. This can be easily done by using the convex hull definition of a simplex. If all the
 5 coefficients of this linear combination in (14) are positive, the point is inside the simplex, and
 6 outside otherwise. In this equation, the vectors v_i are the vectors from the origin to the i -th
 7 vertex of the simplex and p is the point to be analyzed.

$$p = \sum_{i=1}^{n+1} \alpha_i v_i \quad (14)$$

8 Equation (15) imposes the constant that the weights of this linear combination all sum up to
 9 one.

$$\sum_{i=1}^{n+1} \alpha_i = 1 \quad (15)$$

10 These two equations can be arranged as a linear system to allow direct resolution. Notice that
 11 there is a slight abuse of notation in (16) to write the matrix composed of vectors and scalars.

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_{n+1} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n+1} \end{bmatrix} = \begin{bmatrix} p \\ 1 \end{bmatrix} \quad (16)$$

12 The simplex will follow the current solution in a way that only very few points are sampled in
 13 the parameter space, resulting in a very few reductions processes.

14 3.2. Subspace Interpolation

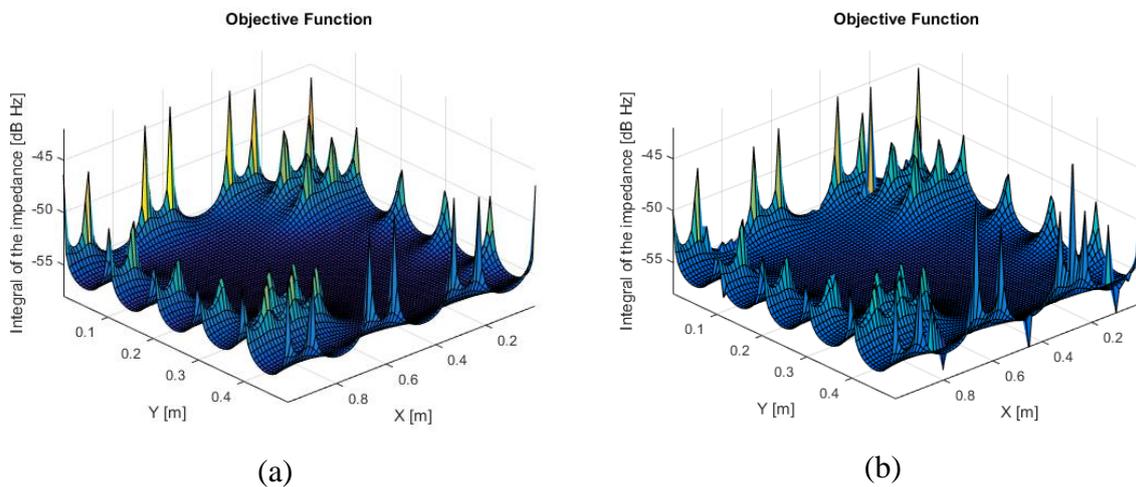
15 The main goal of projection based model order reductions is to find a suitable subspace in which
 16 the state space vector should be projected in order to reduce the number of unknowns and at the
 17 same time produce accurate input/output behavior. It is then natural to use the subspaces
 18 obtained for different parameter values in order to produce a reduced order models in a quicker
 19 manner [17].

20 To achieve this in a systematic way, one can use a very successful technique that is the
 21 interpolation of these subspaces in a tangent space to the Grassmann manifold.

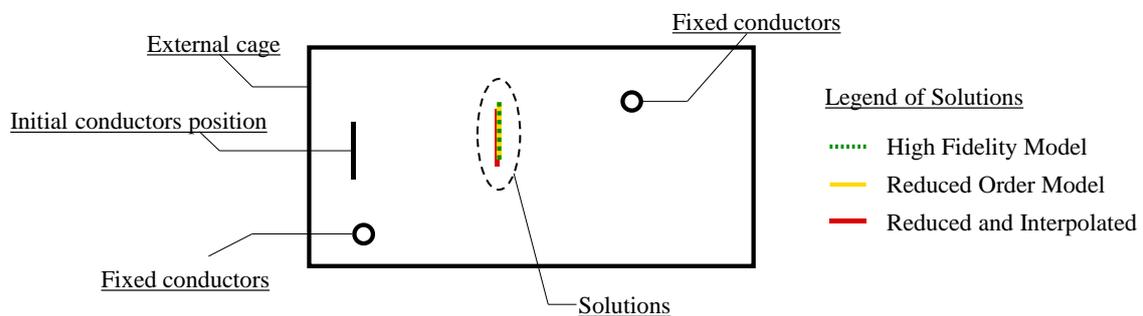
1 For the case of only one variables, one can interpret subspace interpolation as being a process
 2 that transforms the principal vectors of one subspace into those of a second one by means of
 3 rotation. This idea was used to deduce suitable formulas for this particular case in [20] and [21].
 4 Later, this idea was generalized by [17] to deal with an arbitrary number of dimensions. For the
 5 optimization process, the generalized process described in [17] and resumed in [8] is used.

6 3.3. Illustration of the method

7 For illustrating the efficacy of the developed methodology, an electromagnetic optimization
 8 problem is solved using this process. It consists of maximizing the impedance, in a certain
 9 frequency range, between a set of conductor wires inside a cage. Figure 3 shows the difference
 10 of the landscape for the full system and the reduced and interpolated one. It is possible to see
 11 that both have the same tendency. The simplex used for this computation has a side of 50 mm
 12 and the grid step is 2 mm. The maximum error obtained is of 23.7% at (0.2794, 0.4980), a point
 13 in the boundary of the domain. The mean absolute error is 0.05%. One can conclude that even
 14 though in some place the error is appreciable, in average, the interpolated model allows very
 15 accurate predictions.



16 *Figure 3 (a) Objective Function surface for the high-fidelity model. (b) Objective function surface for*
 17 *the interpolated model.*

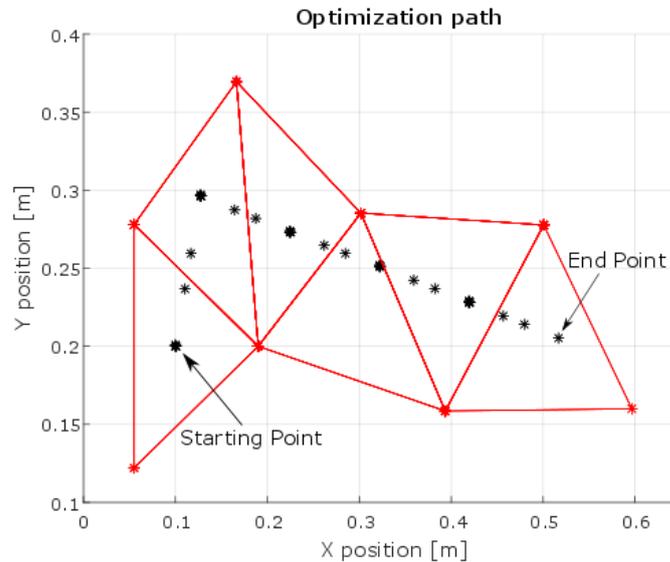


18
 19 *Figure 4 Result for the optimization problem for the high-fidelity model, reduced model and reduced*
 20 *and interpolated model.*

21 Figure 4 shows the result of the didactic optimization problem for the High-Fidelity Model, the
 22 Reduced Order one and for the Reduced and Interpolated Model. It is possible to conclude that
 23 all the processes have almost found the same solution, but with very different computational

1 times. The distance of the simplex nodes to its geometric center were chosen to be 90 mm,
2 which results in an area of approximately 2.1% of the area of the domain.

3 Table 1 shows in details the results of all the optimization processes. Table 2 contains the
4 number of times that the different optimization processes used the high-fidelity model, number
5 of reductions and interpolations. Figure 5 shows the progression of the optimization algorithm
6 and the corresponding simplexes that were constructed to perform the interpolations.



7
8 *Figure 5 - Example of the simplexes following the optimization path*

9 **4. Conclusion**

10 A complete methodology for optimization using parametric model order reduction to speed up
11 objective function evaluations was presented. It has the advantage of lowering the
12 computational cost of the process while keeping high accuracy. A very efficient heuristic to
13 adapt automatically the placement of the expansion points was developed. It was combined
14 with a smart parameter space sampling strategy to compose a very solid methodology for
15 optimization large-scale dynamical systems. Further work is needed in order to determine an
16 automatic way of choosing the size of the simplex.

17 The presented technique is applicable to any system that can be written in a linear state space
18 formulation, being applicable to many different domains of engineering.

19 **5. References**

20

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Table 1 Results of the optimization process

	<i>X [m]</i>	<i>Y [m]</i>	<i>Objective Function [dB Hz]</i>	<i>Time [s]</i>
<i>HFM</i>	0.4230	0.2431	-55.5542	624
<i>ROM only</i>	0.4232	0.2431	-55.5543	163
<i>ROM and Interpolation</i>	0,4195	0,2282	-55.5611	65

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Table 2 Number of reductions, interpolations and calls to the high-fidelity model

	<i>Model Reductions</i>	<i>Model Interpolations</i>	<i>HFM Calls</i>
<i>HFM</i>	0	0	36
<i>ROM only</i>	36	0	0
<i>ROM and Interpolation</i>	8	28	0

5