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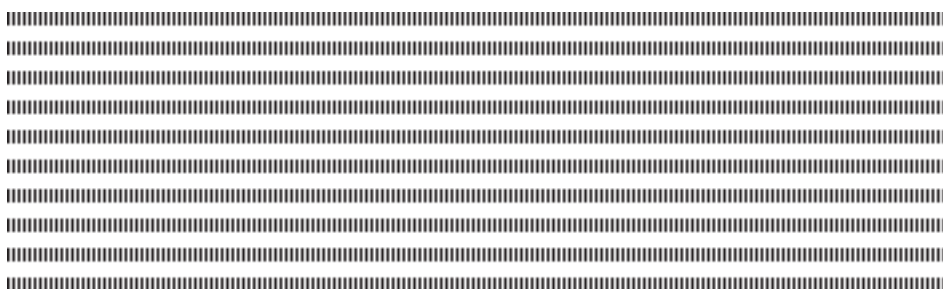
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Mathematical Modelling and analysis of microbial ecosystems and bioprocesses

Research done within the network Treasure ¹

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ABSTRACT. In this symposium, we present mathematical analyses of models related to the growth and coexistence of species of micro-organisms in the chemostat. These works were carried out within the TREASURE network. The symposium consists in 4 presentations : Anaerobic digestion (by N. Abdellatif), Syntrophic relationship in the chemostat (by Y. Daoud), Intra and interspecific competition in the chemostat (by C. Lobry) and Flocculation in the Chemostat (by R. Fekih-Salem).

RÉSUMÉ. Dans ce symposium, nous présentons une analyse mathématique de modèles de croissance et de coexistence d'espèces de micro-organismes dans le chemostat. Ces travaux ont été réalisés dans le cadre du réseau TREASURE. Ce symposium consiste en 4 présentations : La digestion anaérobie (par N. Abdellatif), la relation syntrophique dans le chemostat (par Y. Daoud), la compétition intra et interspécifique dans le chemostat (par C. Lobry) et la flocculation dans le Chemostat (par R. Fekih Salem).

KEYWORDS : Bifurcation, Chemostat, Coexistence, Competition, Density-dependence, Global stability, Bistability, Enzymatic degradation, Hydrolysis, Mortality, Flocculation Microbial ecosystems, Syntrophic relationship.

MOTS-CLÉS : Bifurcation, Chemostat, Coexistence, Compétition, Densité-dépendance, Stabilité globale, Bistabilité, Dégradation enzymatique, Hydrolyse, Mortalité, Flocculation Écosystèmes microbiens, Relation syntrophique



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1. On a three-step model of Anaerobic digestion with biomass mortality

Presented by N. Abdellatif.

In this work, we analyse a mathematical model of the chemostat with enzymatic degradation of an organic substrate under a solid form. We study the existence and the stability of equilibrium points of the model considering monotonic growth rates and positive mortality terms. The analysis of the 3-step model is derived from a smaller order sub-model. We show that the considered sub-model may exhibit bistability. The study of 3-step model shows the existence of at most four positive equilibrium and we prove that introducing decay in the model preserves the stability of the equilibria.

This work is an extension of [3]

2. The effect of a new input concentration substrate on a syntrophic relationship of two species in a chemostat

Presented by Y. Daoud (joint work with N. Abdellatif, T. Sari and J. Harmand [2])

In this work, we study a model describing a syntrophic relationship of two microbial species with two input substrates, including both decay terms and inhibition of the first species by an excess of the second substrate. This model can be seen as a reduced and simplified version of the anaerobic digestion process. We discuss the existence and stability of all equilibria and we give necessary and sufficient conditions on the control parameters of the system (the dilution rate and the input concentrations of the two substrates).

For complements and other research on this subject, done in the Network Treasure, see [9, 10, 11]

3. How intra- and interspecific competitions affect the coexistence in the chemostat

Presented by C. Lobry (joint work with R. Fekih Salem and T. Sari [5]).

This paper deals with a two-microbial species model in competition for a single-resource in the chemostat including general intra- and interspecific density-dependent growth rates with distinct removal rates for each species. In order to understand the effects of intra- and interspecific competition, this general model is first studied by determining the conditions of existence and local stability of steady states. The operating diagrams show how the model behaves by varying the operating parameters and illustrate the effect of the intra- and interspecific competition on the disappearance of coexistence region and the occurrence of bi-stability region. The bifurcations of steady states according to interspecific competition parameters are analyzed in a particular case of density-dependent growth rates which are usually used in the literature.

For complements and other research on this subject, done in the Network Treasure, see [1, 8]

4. Analysis of a model of flocculation in the chemostat

Presented by R. Fekih Salem (joint work with T. Sari, [7])

In this work, we study a flocculation model with a single resource and a single species which is present in two forms: isolated bacteria and attached bacteria. With monotonic growth rates and distinct removal rates, we show that this model presents a rich behavior with multiplicity of positive equilibria and bi-stability.

For complements and other research on this subject, done in the Network Treasure, see [4, 6]

Acknowledgments

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5. References

- [1] N. ABDELLATIF, R. FEKIH-SALEM, T. SARI, Competition for a single resource and coexistence of several species in the chemostat, *Math. Biosci. Eng.*, **13** (2016), 631–652.
- [2] Y. DAOUD, N. ABDELLATIF, T. SARI, J. HARMAND, Steady-state analysis of a syntrophic association of two species in a chemostat : The effect of a new input concentration substrate, *Fourth International Conference on Complex Dynamical Systems in Life Sciences: Modeling and Analysis* (4th ICCDS'2016) Agadir, Morocco - October 26-28, 2016.
- [3] R. FEKIH-SALEM, N. ABDELLATIF, T. SARI, J. HARMAND, Analyse mathématique d'un modèle de digestion anaérobie à trois étapes, *ARIMA Journal*, **17**, 53–71, 2014.
- [4] R. FEKIH-SALEM, J. HARMAND, C. LOBRY, A. RAPAPORT, T. SARI, Extensions of the chemostat model with flocculation, *J. Math. Anal. Appl.*, **397**, 2013, 292–306.
- [5] R. FEKIH-SALEM, C. LOBRY, T. SARI, A density-dependent model of competition for one resource in the chemostat, *To appear*.
- [6] R. FEKIH-SALEM, A. RAPAPORT, T. SARI, Emergence of coexistence and limit cycles in the chemostat model with flocculation for a general class of functional responses, *Appl. Math. Modell.*, **40** (2016), 7656–7677.
- [7] R. FEKIH-SALEM, T. SARI, The mathematical analysis of a model of flocculation in the chemostat, *To appear*
- [8] R. FEKIH-SALEM, T. SARI, N. ABDELLATIF, Sur un modèle de compétition et de coexistence dans le chémostat, *ARIMA J.*, **14** (2011), 15–30.
- [9] M. EL HAJJI, F. MAZENC AND J. HARMAND, A mathematical study of a syntrophic relationship of a model of anaerobic digestion process, *Math. Biosci.Eng.*, **7** (2010), 641–656.
- [10] T. SARI, M. EL HAJJI AND J. HARMAND, The mathematical analysis of a syntrophic relationship between two microbial species in a chemostat, *Math. Biosci. Eng.*, **9**, 627–645, 2012.
- [11] T. SARI AND J. HARMAND, A model of a syntrophic relationship between two microbial species in a chemostat including maintenance, *Math. Biosci.*, **275**, 1–9, 2016.