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Research Paper/

Hydraulic Tomography of Discrete Networks of Conduits and Fractures in a Karstic Aquifer by Using a Deterministic Inversion Algorithm

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Key words: Distributed modeling, Coupled discrete-continuum model, Deterministic inversion, Heterogeneity, Aquifer characterization

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1 **Abstract**

2 In this paper we present a novel inverse modeling method called Discrete Network
3 Deterministic Inversion (DNNDI) for mapping the geometry and property of the discrete
4 network of conduits and fractures in the karstified aquifers. The DNNDI algorithm is based on a
5 coupled discrete-continuum concept to simulate numerically water flows in a model and a
6 deterministic optimization algorithm to invert a set of observed piezometric data recorded
7 during multiple pumping tests. In this method, the model is partitioned in subspaces piloted by
8 a set of parameters (matrix transmissivity, and geometry and equivalent transmissivity of the
9 conduits) that are considered as unknown. In this way, the deterministic optimization process
10 can iteratively correct the geometry of the network and the values of the properties, until it
11 converges to a global network geometry in a solution model able to reproduce the set of data.
12 An uncertainty analysis of this result can be performed from the maps of posterior
13 uncertainties on the network geometry or on the property values. This method has been
14 successfully tested for three different theoretical and simplified study cases with hydraulic
15 responses data generated from hypothetical karstic models with an increasing complexity of
16 the network geometry, and of the matrix heterogeneity.

17 **1. Introduction**

18 In hydrogeological studies, the choice of the management and protection strategies of the
19 groundwater resources is mainly based on the characterization of the hydraulic properties of
20 the aquifer, such as hydraulic conductivity and specific storage. This characterization is most
21 often carried out from pumping, slug and tracer tests by intrusively recording the aquifer
22 responses, such as hydraulic pressure and tracer concentration at a set of boreholes (Butler
23 2005). The reliability of these techniques for capturing the spatial heterogeneity of the
24 hydrodynamic properties is particularly conditioned by the amount and spatial disposition of
25 wells used during the investigation, and the procedure applied to analyze the hydraulic data
26 (Yeh and Lee 2007). In karstic and/or fractured aquifers the hydrodynamic properties (such as
27 the hydraulic conductivity) can vary significantly from 10^{-10} m/s to 10^{-1} m/s, even at small
28 scales (Wang et al. 2016). This heterogeneity mainly depends on the apertures, connectivity
29 and density of the conduits and fractures network in the medium, making the groundwater
30 flow path complex (Eisenlohr et al. 1997 ; Kovacs et al. 2005 ; Borghi et al. 2016 ; Ronayne
31 2013). In this complex context, the hydraulic flow pattern is spatially disconnected and
32 principally focused in the transmissive fissures and fractured zones, wherein the geometrical
33 features and hydraulic flow regime (turbulent or laminar) are usually difficult to identify,
34 especially with a limited number of wells, or with the use of oversimplified assumptions for
35 interpreting the piezometric data to infer the hydrodynamic parameters.

36 In the hydroscience literature, several different modeling approaches based on the physical
37 theories have already been tested in order to simulate the dynamics of karstic flows for the
38 prediction of hydraulic properties (Hartmann et al. 2014). Among them, the equivalent porous
39 media model, also called the single continuum model, in which the discrete features of
40 fractures and karstic conduits are conceptualized as a porous media with continuous hydraulic
41 properties (Larocque et al. 1999 ; Illman 2014 ; Wang et al. 2016). This simplifies the

42 description of heterogeneity of karstic aquifers because it does not require an accurate
43 knowledge on the architecture of fractures and conduits networks for simulating the
44 groundwater flows. In such concept, it is sufficient to assign high hydraulic conductivity
45 values to fractured zones and very low conductivity for intact rock. Otherwise, the coupled
46 discrete-continuum distributed approach is of great interest thanks to its ability to imitate the
47 dual hydrodynamic behaviors in the fractured aquifers by using Discrete Channel or Fracture
48 Networks (DCN/DFN) for the conduits and fractures, and equivalent porous media for
49 representing the matrix blocks (Teutsch 1993 ; Liedl et al. 2003 ; De Rooij et al. 2013). In
50 contrast to the equivalent porous media model, the discrete-continuum approach requires a
51 good knowledge on the geometry of the karstic and fracture networks. The influence of the
52 discrete network geometry on the hydraulic simulations and the benefits of a coupled discrete-
53 continuum approach compared to the equivalent porous media have been widely discussed in
54 the literature (Kovacs 2003 ; Painter and Cvetkovic 2005 ; Ghasemizadeh et al. 2012 ;
55 Hartmann et al. 2014). One of these advantages is its efficiency to reproduce numerically the
56 hydraulic fluctuations of karst spring discharge, while an equivalent porous media
57 systematically generated lower values than the ones measured (Kovacs 2003).

58 The hydraulic tomography is a useful tool to predict rigorously the spatial distribution of the
59 hydraulic properties, or the structural architecture of the fractures and conduits and their
60 properties. It involves the use of inverse algorithms to analyze jointly a set of hydraulic data
61 collected during multiple pumping tests (Carrera et al. 2005 ; Cliffe et al. 2011 ; Zhou et al.
62 2014). In this framework, various inversion algorithms were successfully applied for
63 characterizing the hydraulic properties of fractured and heterogeneous aquifers using both
64 concepts of parametrization discussed in the previous paragraph: the equivalent porous media
65 and coupled discrete-continuum approach.

66 (1) Inversions in an equivalent porous media were led by using geostatistical approaches in
67 which the statistical characteristics of hydraulic properties are used as a priori information to
68 constraint the inversion. Among these tools we cite: sequential successive linear estimator
69 (Yeh and Liu 2000 ; Ni and Yeh 2008 ; Hao et al. 2008 ; Illman et al. 2009 ; Sharmeen et al.
70 2012), pilot-point (Lavenue and de Marsily 2001), transitional-probability (Wang et al. 2017),
71 anisotropy directions (Meier et al. 2001), multi-scale resolution (Ackerer and Delay 2010), or
72 structural approaches: probability perturbation method (Caers and Hoffman 2006), image-
73 guided (Soueid Ahmed et al. 2015), and cellular automata-based (Fischer et al. 2017).

74 (2) On the other hand, the parameterization of hydraulic tomography using a distributed
75 discrete-continuum approach is less flexible than the concept of the equivalent porous
76 medium because the discrete-continuum model relies on the establishment of the architecture
77 of the conduits and fractures, and their hydraulic properties. Several works have already
78 brought some solutions to these difficulties. One solution would be to generate stochastically
79 patterns of networks with various constraints: statistical constraints (Li et al. 2014 ; Le Coz et
80 al. 2017), mechanical constraints (Bonneau et al. 2013 ; Jaquet et al. 2004), geological and
81 speleogeological metrics information (Collon et al. 2017 ; Pardo-Iguzquiza et al. 2012), or
82 flows hierarchical identification (Le Goc et al. 2010). More recently, Borghi et al. (2016) have
83 combined the use of a generator of karstic networks, based on sets of fractures stochastically
84 generated, with a gradient-based parameters optimization in order to reconstruct a discrete
85 network able to reproduce a set of tracer test hydraulic data.

86 In this present article we propose a novel strategy for dealing with hydraulic tomography of
87 fractured and karstic aquifers, which we will shorten as the Discrete Network Deterministic
88 Inversion (DNDI). The DNDI algorithm permits to map the architecture of fractures and
89 conduits networks, their hydraulic properties, and the distribution of the transmissivity in the
90 hard rock (matrix).

91 The DNDI approach relies on the use of a coupled discrete-continuum concept to simulate
 92 water flows through a karstic and fractured aquifer and a deterministic optimization algorithm
 93 to invert a set of observed piezometric data recorded during multiple pumping tests. The
 94 model is partitioned in several subspaces, each one being piloted locally by a set of
 95 parameters including: the orientations of the conduit/fracture, their equivalent transmissivity
 96 values, and the transmissivity of the rock matrix. This partitioning makes it possible to locally
 97 modify the directions of the fracture network and to iteratively update the geometry of the
 98 global network in order to minimize the objective function in the inverse process. The method
 99 is tested on several hypothetical and simplified karstic aquifers with simple to more complex
 100 conduit networks and with homogeneous or heterogeneous transmissivity in the matrices.

101 **2. Algorithm framework**

102 **2.1. Forward problem and model parameterization**

103 We represent a confined karstic and fractured aquifer in a two-dimensional model Γ with an
 104 equivalent porous media Γ_M (for representing the water flows where the rock is intact) and a
 105 discrete network Γ_N (for simulating the water flows in the fracture/conduit networks). The
 106 numerical simulation of groundwater flows are governed by a steady state continuity equation
 107 associated to Darcy's law, considering a laminar flow in both the matrix domain and the
 108 discrete networks:

$$109 \quad \begin{cases} \nabla \cdot (-e_M K_M \cdot \nabla h) = \frac{e_M Q_M}{V_{el.}} & \text{in the matrix } \Gamma_M \\ \nabla_T \cdot (-e_N K_N \cdot \nabla_T h) = \frac{e_N Q_N}{V_{el.}} & \text{in the network } \Gamma_N \end{cases} \quad (1)$$

110 where Q_M and Q_N are punctual water extraction or injection rates per unit of thickness
 111 ($m^3/s/m$) applied on the matrix and network respectively, K_M denotes the matrix hydraulic

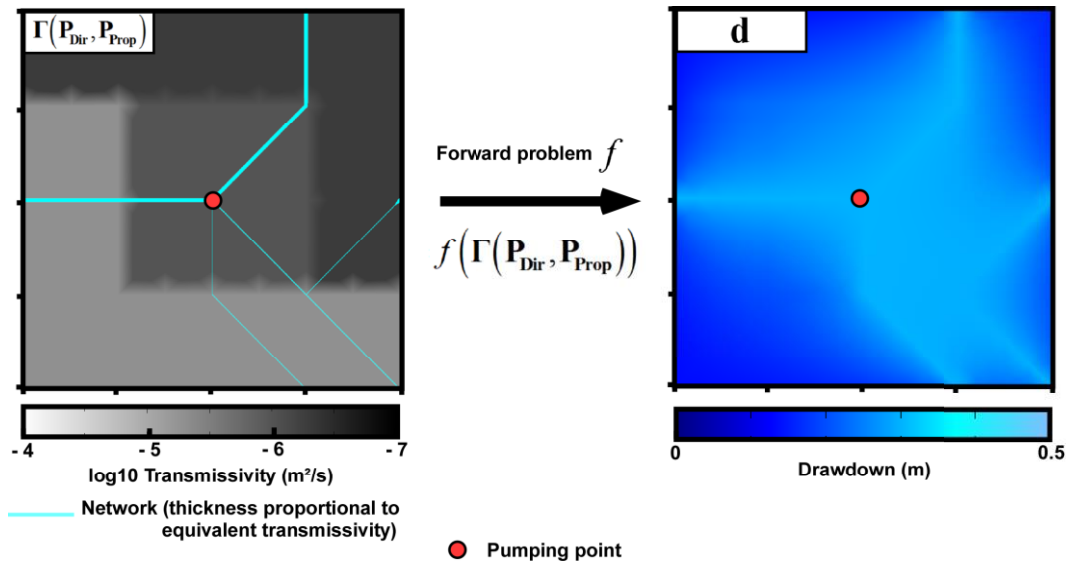
112 conductivities (m/s), K_N denotes the fractures or conduits equivalent conductivities (m/s), h
 113 is the piezometric level (m) common to both domains Γ_M and Γ_N , e_M (m) is the thickness of
 114 the matrix block, e_N (m) is the aperture of the network, and V_{el} is an elementary volume at the
 115 pumping location (m³). We mention that Darcy's law formulation in the matrix domain is
 116 described in 2D, and in 1D for fractured networks at the internal network boundaries. That's
 117 why we use the tangential gradient operator $\nabla_T = \vec{\nabla} \cdot \vec{\ell}$ (where $\vec{\ell}$ is a local directional unit
 118 vector of the network) to solve the hydraulic equation at the network. In the study cases
 119 presented later in this article, we have chosen to simulate laminar flows as presented in Eq. 1
 120 in a network of conduits. However, the property values K_N in the network can be more
 121 specifically adapted to the behavior of turbulent conduit flows or fracture flows through other
 122 empirical laws (eventually related to an aperture variable).

123 The forward problem consists in solving numerically Eq. 1 by using a finite element
 124 technique with a triangular meshing. It links the hydraulic head data simulated continuously
 125 over the coupled model to the spatial distribution in the model of the conduits or fractures
 126 with their properties in 1D, and the hydraulic transmissivities of the matrix in 2D (Figure 1).
 127 The forward problem can be formulated as:

$$128 \quad \mathbf{d} = f\left(\Gamma\left(\mathbf{P}_{Dir}, \mathbf{P}_{Prop}\right)\right) + \boldsymbol{\varepsilon}, \quad (2)$$

129 where \mathbf{d} is a vector of simulated hydraulic data ($n \times 1$), f is a forward operator that
 130 calculates the hydraulic data field from a model $\Gamma\left(\mathbf{P}_{Dir}, \mathbf{P}_{Prop}\right)$ defined by given parameters of
 131 network geometry \mathbf{P}_{Dir} and hydraulic properties \mathbf{P}_{Prop} , $\boldsymbol{\varepsilon}$ is a null mean Gaussian noise to add
 132 the uncertainties associated to the numerical discretization technique and the hydraulic
 133 experimental data. The model is enclosed in a large buffer zone associated to an equivalent
 134 porous media mean transmissivity. This zone permits to limit the influence of the boundary

135 conditions. The DNDI inversion algorithm was coded using Matlab and is linked to the
 136 COMSOL Multiphysics software which runs the forward problems.



138 Figure 1: Example of a simulated distribution of hydraulic heads (here drawdowns) by solving
 139 the forward problem f (Eq.1) for a steady state pumping in a given coupled discrete-
 140 continuum distributed model $\Gamma(\mathbf{P}_{Dir}, \mathbf{P}_{Prop})$.

141

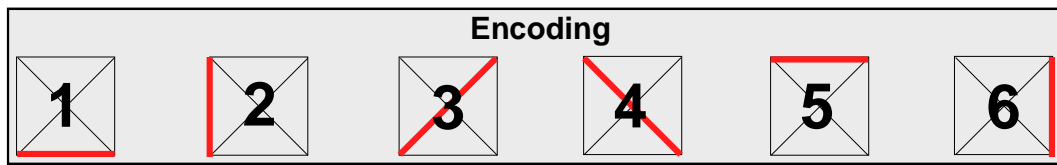
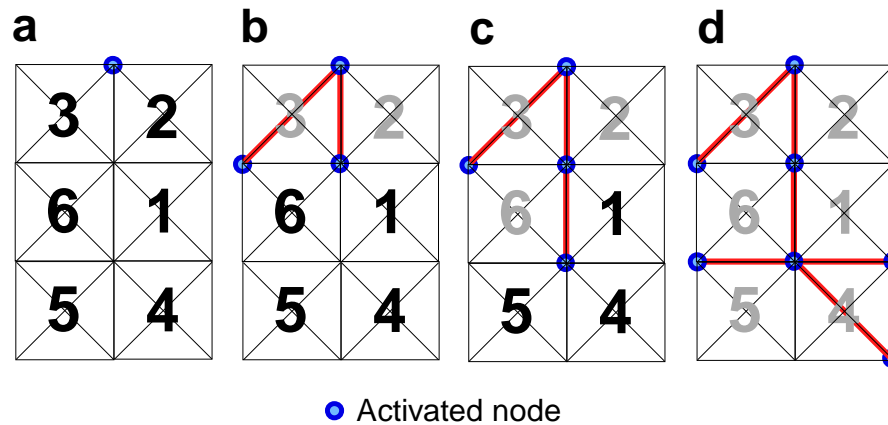
142 For the DNDI algorithm, the model domain Γ is partitioned in p_x squared subspaces along
 143 the X-axis and p_y along the Y-axis. The total of subspaces of the whole domain is then

144 $p = p_x \times p_y$. Three parameters are assigned to each subspace (Figure 3):

- 145 (1) the local direction of the conduit/fracture network,
 146 (2) the local conduit/fracture equivalent transmissivity value,
 147 (3) and the local matrix transmissivity value.

148 The geometry of the network follows the local direction in each subspace by a node-to-node
 149 principle. The network structure enters a subspace by activating one of its four nodes (corners
 150 of the square) and the subspace direction parameter will define to each other node of the

151 subspace the structure will generate. This other node will then be activated itself and permits
 152 to the structure to include new subspaces. A subspace in which the structure has already been
 153 generated becomes inhibited to another generation from the same network. The generation
 154 process is schematized in Figure 2.

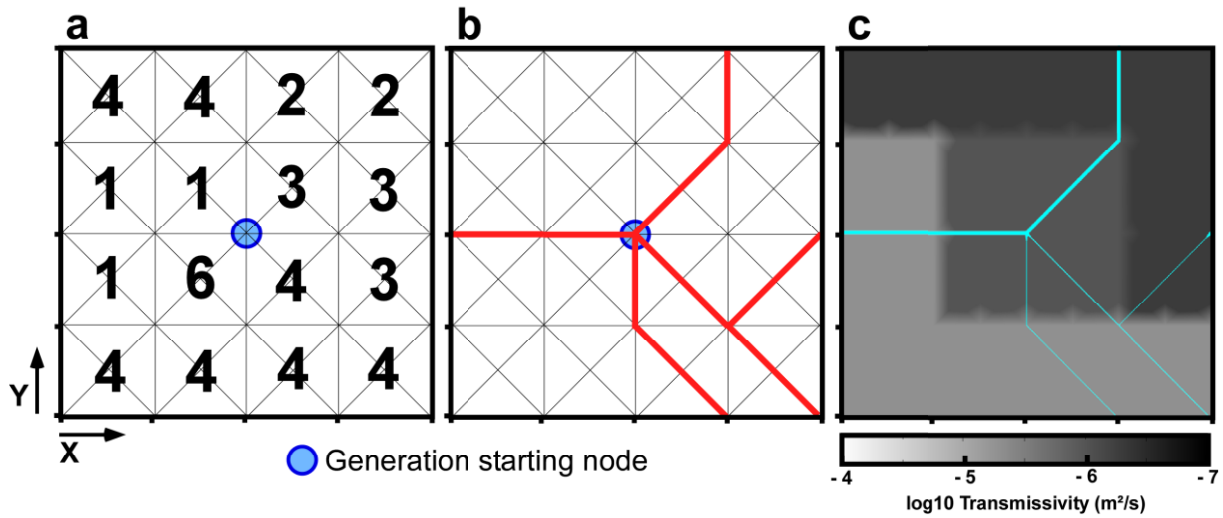


156 Figure 2: Schema of the node-to-node generation process in the DNDI method with six
 157 subspaces. An activated node in the top subspaces (a) starts the generation of the structure.
 158 The structure generates to the nodes in the bottom of these subspaces, following the local
 159 direction defined in the subspaces through the encoding rules. These reached nodes then
 160 become activated (b). The subspaces in which the structure has generated become inhibited to
 161 another generation (shown as greyed number in this figure). The structure then continues its
 162 generation from its newly activated node if the subspaces structural parameters permit it (c) –
 163 (d).

164

165 In order to perform this node-to-node generation, an initially activated node has to be
 166 specified in the model (starting node in Figure 3). The model geometry in COMSOL is built
 167 as a discrete grid including all network possibilities (a grid of squares and diagonals as
 168 presented in Figure 3). This whole geometry is initially disabled in the COMSOL physical
 169 part. When different network geometries are tested in the inversion process, only the
 170 associated parts in the model grid are enabled for the solver computation. This avoids the

171 creation of a new model geometry for each modification of the network and permits to reduce
 172 the computing time in the inversion.



173

174 **Figure 3:** Parameterization of a model in the DNDI method. For each subspace of the model
 175 there are six local direction possibilities (see encoding in Figure 2) that are used to
 176 parameterize a network structure in the model (a). The structure (in red) is then generated,
 177 following a node-to-node rule, from the set of structural parameters in (a) and a chosen
 178 starting point at a node between subspaces (b). Finally a set of property values
 179 (transmissivities), also defined for each subspace, is assigned to the structural model (c).

180

181 The parameterization of the whole model is contained in two vectors piloting the subspaces.

182 (1) The local direction in a subspace is selected among six possibilities (see Figure 3) as a
 183 structural parameter $Dir \in \{1, 2, 3, 4, 5, 6\}$. The set of structural parameters for all subspaces in
 184 the model is contained in a $(p \times 1)$ vector \mathbf{P}_{Dir} . It is also possible to generate more than one
 185 network, but this would add more unknown structural parameters. For example if one would
 186 want to generate 3 independent networks in the model, each subspace would need to define 3
 187 local directions instead of one. Thus, the structural vector of direction parameters \mathbf{P}_{Dir} would
 188 become a $(3p \times 1)$ vector.

189 (2) The local equivalent transmissivity of the structure in a subspace is defined as a property
 190 parameter T_N and the matrix transmissivity as a property parameter T_M . The set of property
 191 parameters for all subspaces in the model is contained in a $(2p \times 1)$ vector \mathbf{P}_{Prop} with the p
 192 parameters T_N followed by the p parameters T_M . The entire model Γ is thus piloted only
 193 by two parameter vectors: \mathbf{P}_{Dir} and \mathbf{P}_{Prop} , and can be noted $\Gamma(\mathbf{P}_{Dir}, \mathbf{P}_{Prop})$.

194 2.2. Inverse problem

195 The inversion process in the DNDI algorithm consists in retrieving a model of network of
 196 conduit/fracture and of spatial distribution of the transmissivities of the network and matrix
 197 which permits to maximize two probability density functions $\rho_{network}$ and $\rho_{properties}$. Following
 198 the theory described by Tarantola and Valette (1982) for a least square criterion resolution of
 199 the inverse problem, we calculate $\rho_{network}$ and $\rho_{properties}$ with the Bayes theorem, by considering
 200 Gaussian laws for the probability density functions ρ , and $\rho(\mathbf{d}_{obs})$ as certain:

$$\rho_{network}(\mathbf{P}_{Dir} | \mathbf{d}_{obs}, \mathbf{P}_{Prop}) = \rho(\mathbf{d}_{obs} | \mathbf{P}_{Dir}, \mathbf{P}_{Prop}) \cdot \rho(\mathbf{P}_{Dir}) / \rho(\mathbf{d}_{obs})$$

$$\begin{aligned} 201 \quad \rho_{network}(\mathbf{P}_{Dir} | \mathbf{d}_{obs}, \mathbf{P}_{Prop}) &\propto \exp\left(-\frac{1}{2}(\mathbf{d}_{obs} - f(\Gamma(\mathbf{P}_{Dir}, \mathbf{P}_{Prop})))^T \mathbf{C}_d^{-1}(\mathbf{d}_{obs} - f(\Gamma(\mathbf{P}_{Dir}, \mathbf{P}_{Prop})))\right) \\ &\quad \times \exp\left(-\frac{1}{2}(\mathbf{P}_{Dir,prior} - \mathbf{P}_{Dir})^T \mathbf{C}_{P_{Dir}}^{-1}(\mathbf{P}_{Dir,prior} - \mathbf{P}_{Dir})\right) \end{aligned} \quad (3)$$

$$\rho_{properties}(\mathbf{P}_{Prop} | \mathbf{d}_{obs}, \mathbf{P}_{Dir}) = \rho(\mathbf{d}_{obs} | \mathbf{P}_{Prop}, \mathbf{P}_{Dir}) \cdot \rho(\mathbf{P}_{Prop}) / \rho(\mathbf{d}_{obs})$$

$$\begin{aligned} 203 \quad \rho_{properties}(\mathbf{P}_{Prop} | \mathbf{d}_{obs}, \mathbf{P}_{Dir}) &\propto \exp\left(-\frac{1}{2}(\mathbf{d}_{obs} - f(\Gamma(\mathbf{P}_{Dir}, \mathbf{P}_{Prop})))^T \mathbf{C}_d^{-1}(\mathbf{d}_{obs} - f(\Gamma(\mathbf{P}_{Dir}, \mathbf{P}_{Prop})))\right) \\ &\quad \times \exp\left(-\frac{1}{2}(\mathbf{P}_{Prop,prior} - \mathbf{P}_{Prop})^T \mathbf{C}_{P_{Prop}}^{-1}(\mathbf{P}_{Prop,prior} - \mathbf{P}_{Prop})\right) \end{aligned} \quad (4)$$

204

205 where \propto represents a proportionality relation, $\rho_{\text{network}}(\mathbf{P}_{\text{Dir}}|\mathbf{d}_{\text{obs}},\mathbf{P}_{\text{Prop}})$ represents the a
 206 posteriori probability density function of the discrete fracture network model for a given
 207 hydraulic observed data \mathbf{d}_{obs} and the transmissivity model of the network and matrix \mathbf{P}_{Prop} .
 208 $\rho_{\text{properties}}(\mathbf{P}_{\text{Prop}}|\mathbf{d}_{\text{obs}},\mathbf{P}_{\text{Dir}})$ is the a posteriori probability density function of the spatial
 209 distribution of the transmissivity parameters for a given hydraulic observed data \mathbf{d}_{obs} and
 210 network model \mathbf{P}_{Dir} . $\rho(\mathbf{d}_{\text{obs}}|\mathbf{P}_{\text{Dir}},\mathbf{P}_{\text{Prop}})$ and $\rho(\mathbf{d}_{\text{obs}}|\mathbf{P}_{\text{Prop}},\mathbf{P}_{\text{Dir}})$ represent the probability
 211 density functions of the network structure and property models, which permit to evaluate the
 212 ability of the network structure and property models to reproduce the observed data via the
 213 use of the forward operator. $\rho(\mathbf{P}_{\text{Dir}})$ and $\rho(\mathbf{P}_{\text{Prop}})$ represent prior distributions for the
 214 unknown parameters. It is well known that, on one hand, the piezometric data are insufficient
 215 to cope with the non-uniqueness of the solution of an inverse process, and on another hand,
 216 that a deterministic inversion process leads to a single local solution dependent to the initial
 217 model. For these reasons, and in order to additionally constrain the inversion to a more
 218 realistic solution in relation to the field knowledges, it can be interesting to incorporate prior
 219 distributions for the unknown parameters in $\rho(\mathbf{P}_{\text{Dir}})$ and $\rho(\mathbf{P}_{\text{Prop}})$.

220 The maximization of the a posteriori probability density functions ρ_{network} and $\rho_{\text{properties}}$ is
 221 equivalent to a minimization of the arguments of the exponentials in Eq.3 and 4. This is what
 222 we aim to minimize during the inversion process in the following objective functions Ψ :

$$223 \quad \Psi_{\text{network}}(\mathbf{P}_{\text{Dir}}) = \frac{1}{2}(\mathbf{d}_{\text{obs}} - f(\Gamma(\mathbf{P}_{\text{Dir}}, \mathbf{P}_{\text{Prop}})))^T \mathbf{C}_{\text{d}}^{-1}(\mathbf{d}_{\text{obs}} - f(\Gamma(\mathbf{P}_{\text{Dir}}, \mathbf{P}_{\text{Prop}}))) \\ + \frac{1}{2}(\mathbf{P}_{\text{Dir,prior}} - \mathbf{P}_{\text{Dir}})^T \mathbf{C}_{\mathbf{P}_{\text{Dir}}}^{-1}(\mathbf{P}_{\text{Dir,prior}} - \mathbf{P}_{\text{Dir}}) \quad (5)$$

$$\begin{aligned}
224 \quad \Psi_{\text{properties}}(\mathbf{P}_{\text{Prop}}) &= \frac{1}{2} \left(\mathbf{d}_{\text{obs}} - f(\Gamma(\mathbf{P}_{\text{Dir}}, \mathbf{P}_{\text{Prop}})) \right)^T \mathbf{C}_d^{-1} \left(\mathbf{d}_{\text{obs}} - f(\Gamma(\mathbf{P}_{\text{Dir}}, \mathbf{P}_{\text{Prop}})) \right) \\
&+ \frac{1}{2} (\mathbf{P}_{\text{Prop,prior}} - \mathbf{P}_{\text{Prop}})^T \mathbf{C}_{\text{PProp}}^{-1} (\mathbf{P}_{\text{Prop,prior}} - \mathbf{P}_{\text{Prop}})
\end{aligned} \quad (6)$$

225 where \mathbf{d}_{obs} is a vector of observed data ($n \times 1$), \mathbf{P}_{Dir} and \mathbf{P}_{Prop} are the unknown parameters to
226 estimate for imaging the geometry of the network (in \mathbf{P}_{Dir}) and the hydraulic properties
227 (defined here by the equivalent transmissivity of the conduits/fractures and the transmissivity
228 of the matrix in \mathbf{P}_{Prop}). $\mathbf{P}_{\text{Dir,prior}}$ ($p \times 1$) and $\mathbf{P}_{\text{Prop,prior}}$ ($2p \times 1$) are the prior information on the
229 geometry and on the property parameters employed to constrain the inverse problem for
230 overcoming the unrealistic solutions, \mathbf{C}_d ($n \times n$) is a covariance matrix on the observed data
231 that permits to include the uncertainties of the hydraulic data in the inversion process. \mathbf{C}_{PDir}
232 ($p \times p$) and $\mathbf{C}_{\text{PProp}}$ ($2p \times 2p$) are the covariance matrices on the structural and property
233 parameters respectively.

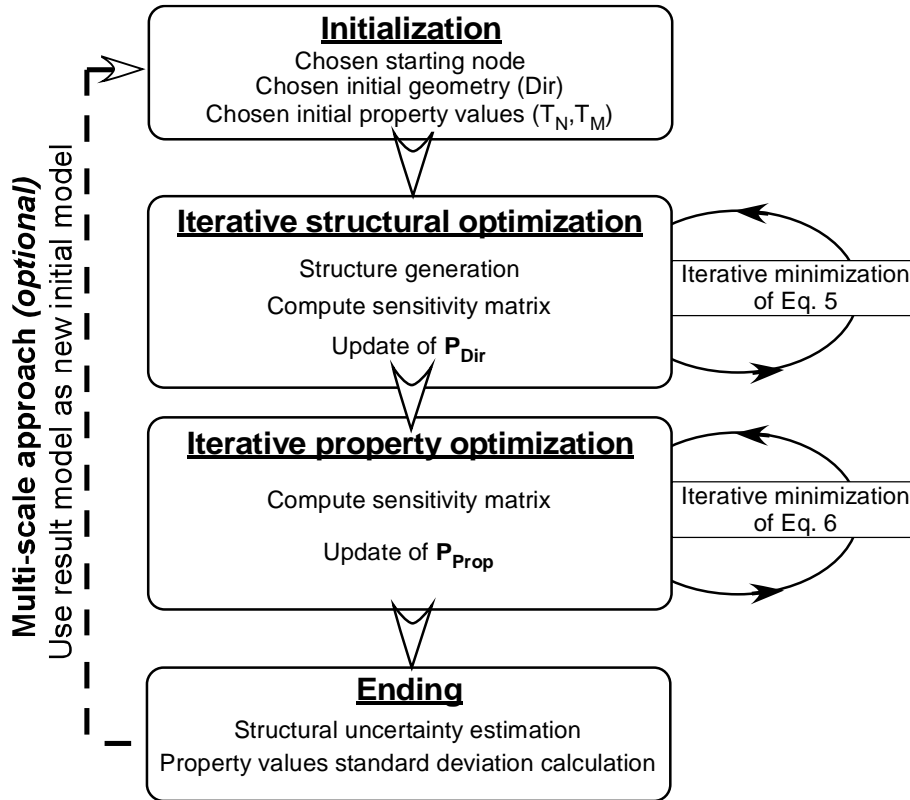
234 This separated formulation of the probability density functions between network and
235 properties permits to sequentially estimate the two dependent unknown models \mathbf{P}_{Dir} and
236 \mathbf{P}_{Prop} . In a first step, we focus on the characterization of the network with the piezometric data
237 by fixing the model of the transmissivity distributions in the conduits/fractures and matrices.
238 The model of network resulting from the first step will then be used in a second step as known
239 parameter to infer the transmissivity pattern.

240 **2.3. Optimization and uncertainty analysis**

241 The minimization of Eq. 5 and Eq. 6 can be done by optimizing the network geometry and the
242 property values during two sequential iterative processes. These optimizations consist in
243 successively modifying the structural and property parameters \mathbf{P}_{Dir} and \mathbf{P}_{Prop} .

244 The inversion process is led as a sequential optimization (Figure 4) of

- 245 (1) the structural geometry (considering as fixed the initially chosen property values),
 246 (2) and the property values T_N, T_M (considering as fixed the previously inverted structural
 247 geometry).



248

249 **Figure 4:** A flowchart of the inversion steps used in the DNDI algorithm. After the
 250 initialization of the parameters, a sequential iterative optimization is led on the structure
 251 geometry and on the property values in order to minimize both objective functions (Eq. 5 and
 252 Eq. 6). An eventual re-run of the inversion process (multi-scale option) using the result as new
 253 initial model can be performed in order to improve this result.

254

255 The structural optimization is performed iteratively by modifying the structural parameter \mathbf{P}_{Dir}
 256 through a structural sensitivity analysis and by considering the hydraulic properties \mathbf{P}_{Prop} as
 257 fixed. At a given iteration step k , the sensitivity analysis of the network geometry toward the
 258 observed data is recorded into a $(6 \times p)$ sensitivity matrix \mathbf{J}_n^k . For a local direction $i \in [1, 6]$
 259 and for a subspace $j \in [1, p]$ the element of the matrix \mathbf{J}_n^k is calculated as:

$$\begin{aligned}
260 \quad \mathbf{J}_n^k(i, j) = & \frac{1}{2} \left(\mathbf{d}_{\text{obs}} - f \left(\Gamma \left(\mathbf{P}_{\text{Dir}}^k \Big|_{\mathbf{P}_{\text{Dir}}(j)=i}, \mathbf{P}_{\text{Prop}} \right) \right) \right)^T \mathbf{C}_d^{-1} \left(\mathbf{d}_{\text{obs}} - f \left(\Gamma \left(\mathbf{P}_{\text{Dir}}^k \Big|_{\mathbf{P}_{\text{Dir}}(j)=i}, \mathbf{P}_{\text{Prop}} \right) \right) \right) \\
& + \frac{1}{2} \left(\mathbf{P}_{\text{Dir}, \text{prior}}(j) - i \right)^T \mathbf{C}_{\mathbf{P}_{\text{Dir}}}^{-1} \left(\mathbf{P}_{\text{Dir}, \text{prior}}(j) - i \right)
\end{aligned} \quad (7)$$

261 with $\mathbf{P}_{\text{Dir}}^k \Big|_{\mathbf{P}_{\text{Dir}}(j)=i}$ the structural geometry parameter at the iteration k considering a modified
262 local direction in the subspace j , and $\left(\mathbf{P}_{\text{Dir}, \text{prior}}(j) - i \right)$ the gap between the prior local direction
263 of subspace j and the modified local direction.

264 Thus, the sensitivity matrix guides the evolution of the objective function in Eq. 5 by testing
265 successively the modification of the network with all possible local direction in each
266 subspace. The minimal value in the matrix $\mathbf{J}_n^k(i_{\min}, j_{\min})$ designates the local direction i_{\min} in
267 the subspace j_{\min} which would produce the best decrease in the objective function.

268 Then the parameters set $\mathbf{P}_{\text{Dir}}^k$ is updated from the previous set $\mathbf{P}_{\text{Dir}}^k = \mathbf{P}_{\text{Dir}}^{k-1}$ by taking into
269 account the sensitivity analysis minimum $\mathbf{P}_{\text{Dir}}^k(j_{\min}) = i_{\min}$ in order to minimize the objective
270 function at each step of the optimization.

271 Once the sensitivity analysis cannot find any more \mathbf{P}_{Dir} configuration decreasing the objective
272 function, the iterative structural optimization is stopped. The last structural iteration represents
273 the local solution, dependent to the initial model. The uncertainty analysis of the inverted
274 network geometry can be inferred from the computation of the posterior covariance matrix as:

$$275 \quad \mathbf{C}_{\mathbf{P}_{\text{Dir}}}^{\text{post}}(j) = \left(\frac{1}{6} \sum_{i=1}^6 \mathbf{J}_n^{\text{post}}(i, j) - \Psi_{\text{network}}^{\text{post}} + \mathbf{C}_{\mathbf{P}_{\text{Dir}}}^{-1}(j, j) \right)^{-1}, \quad (8)$$

276 where $\mathbf{C}_{\mathbf{P}_{\text{Dir}}}^{\text{post}}(j)$ is the posterior structural uncertainty value for the local direction in the
277 subspace j , $\mathbf{J}_n^{\text{post}}$ is the last iteration structural sensitivity matrix and $\Psi_{\text{network}}^{\text{post}}$ is the value of
278 the minimized structural objective function. If the structural uncertainty value is low, then

279 another direction in the subspace would lead to a deterioration of the reproduction of the data.
 280 On the other hand if the structural uncertainty value is high, then the structure in the subspace
 281 could have another local direction without significantly degrading the reproduction of the
 282 observed data.

283 Following the network geometry optimization, the property parameters optimization will
 284 iteratively modify the transmissivities with the previously inverted geometry in order to
 285 minimize the objective function in Eq. 6. The network equivalent transmissivities and the
 286 matrix transmissivities are optimized simultaneously. At a given iteration step k , the
 287 parameters set \mathbf{P}_{Prop} , which contains the transmissivities for both the network and the matrix,
 288 is updated by linearizing Eq. 6, which can be formulated as:

$$289 \quad \mathbf{P}_{\text{Prop}}^{k+1} = \mathbf{P}_{\text{Prop}}^k + \left((\mathbf{J}_p^k)^T \cdot \mathbf{C}_d^{-1} \cdot \mathbf{J}_p^k + \mathbf{C}_{\mathbf{P}_{\text{Prop}}}^{-1} \right)^{-1} \cdot (\mathbf{J}_p^k)^T \cdot \mathbf{C}_d^{-1} \cdot \left(\mathbf{d}_{\text{obs}} - f(\Gamma(\mathbf{P}_{\text{Dir}}, \mathbf{P}_{\text{Prop}}^k)) \right) \quad (9)$$

$$+ \mathbf{C}_{\mathbf{P}_{\text{Prop}}}^{-1} \cdot (\mathbf{P}_{\text{Prop}, \text{prior}} - \mathbf{P}_{\text{Prop}}^k)$$

290 where \mathbf{J}_p^k is the Jacobian matrix ($n \times 2p$) that holds the sensitivity for each modeled data f_i
 291 (at the positions of the observed data) toward the property values in the matrix and the
 292 network. This Jacobian matrix can be calculated by using a finite difference approach, with a
 293 finite difference step $\Delta \mathbf{P}_{\text{Prop}}$:

$$294 \quad \mathbf{J}_p^k(i, j) = \left. \frac{\partial f_i}{\partial \mathbf{P}_{\text{Prop}}^k} \right|_{\mathbf{P}_{\text{Prop}}^k(j) = \mathbf{P}_{\text{Prop}}^k(j) + \Delta \mathbf{P}_{\text{Prop}}} \quad (10)$$

295 Finally, once the objective function has iteratively converged to a minimum, the property
 296 optimization is stopped. The posterior covariance matrix on the inversion of the property
 297 values can be calculated as:

$$298 \quad \mathbf{C}_{\mathbf{P}_{\text{Prop}}}^{\text{post}} = \left((\mathbf{J}_p^{\text{post}})^T \cdot \mathbf{C}_d^{-1} \cdot \mathbf{J}_p^{\text{post}} + \mathbf{C}_{\mathbf{P}_{\text{Prop}}}^{-1} \right)^{-1}, \quad (11)$$

299 where $\mathbf{C}_{\text{Prop}}^{\text{post}}$ is the posterior covariance matrix and $\mathbf{J}_{\text{p}}^{\text{post}}$ is the Jacobian matrix of the last
 300 iteration step. The diagonal entries of the posterior matrix represent the variances on the
 301 property values of each subspace.

302 **3. Validation of the DNDI algorithm on hypothetical study cases**

303 The DNDI inversion algorithm has been tested on three hypothetical and simplified confined
 304 karstic fields with network of conduits:

- 305 • in a first case, we treat a simple network case with heterogeneity in the equivalent
 306 transmissivity of the conduits and a homogenous transmissivity assigned to the matrix,
- 307 • a second case is similar to the first one but adding a transmissivity variability also in
 308 the matrix,
- 309 • in a third case, we seek to image a complex network geometry with the use of two
 310 different initial models to start the inverse problem.

311 We considered in the forward problem (Eq. 1) a unit thickness for the matrix (2D modeling)
 312 and a unit aperture for the network (but with a variable equivalent transmissivity). The buffer
 313 zone boundaries were associated to a $h_{\text{bound.}} = 0$ m Dirichlet condition and the hydraulic heads
 314 were set to $h_0 = 0$ m initially over the model. These theoretical study cases were used to
 315 produce 2,401 hydraulic drawdown data from 49 pumping/measurement boreholes (a
 316 pumping test is performed alternatively in each borehole) distributed homogeneously over the
 317 100×100 m² models. The pumping rates were set to 0.6 L/min for a borehole in the matrix
 318 and 5 L/s for one in the conduit network.

319 In these different cases, for the inversion of the geometry of the network, no a priori
 320 information has been added. On the other hand, we have constrained the inversion of the
 321 hydraulic properties with a priori values. The a priori models on the properties are used also

322 as initial model to launch the inversion process. For the property optimization, for the matrix
 323 we took the $-\log_{10}(T)$ as transmissivity parameter T_M (for example a transmissivity parameter
 324 equal to 6 represents in the model a 10^{-6} m²/s transmissivity value) and for the network we
 325 took directly the T value as transmissivity parameter T_N . The covariance matrices C_d and
 326 $C_{P_{Prop}}$ are built as diagonal matrices with a constant variance value σ^2 (in the case of the
 327 matrix transmissivity the variance value $\sigma_{T_M}^2$ applies to the exponent of the transmissivity, in
 328 the case of the network transmissivity the variance value $\sigma_{T_N}^2$ applies to the transmissivity).
 329 The partitioning of the models and the chosen inversion parameters values for each study case
 330 are given in Table 1. The different study cases inversions were led on a 64Go RAM PC on 2
 331 processors of 16 cores.

332 **Table 1:** Parameters used in the inversion study cases

	Study case 1	Study case 2	Study case 3
Partitioning	4x4	4x4	8x8
A priori T_N	0.06 m ² /s	0.04 m ² /s	0.1 m ² /s
A priori T_M	10^{-6} m ² /s	10^{-6} m ² /s	10^{-6} m ² /s
Data cov. matrix $C_d = \sigma_{data}^2 \cdot Id(n)$	$\sigma_{data} = 10^{-2}$ m	$\sigma_{data} = 10^{-2}$ m	$\sigma_{data} = 10^{-2}$ m
Property cov. matrix $C_{P_{Prop}} = \sigma_{T_N/T_M}^2 \cdot Id(2p)$	$\sigma_{T_N} = 10^{-6}$ m ² /s $\sigma_{T_M} = 10^{-6}$	$\sigma_{T_N} = 10^{-6}$ m ² /s $\sigma_{T_M} = 10^{-1}$	$\sigma_{T_N} = 10^{-6}$ m ² /s $\sigma_{T_M} = 10^{-6}$

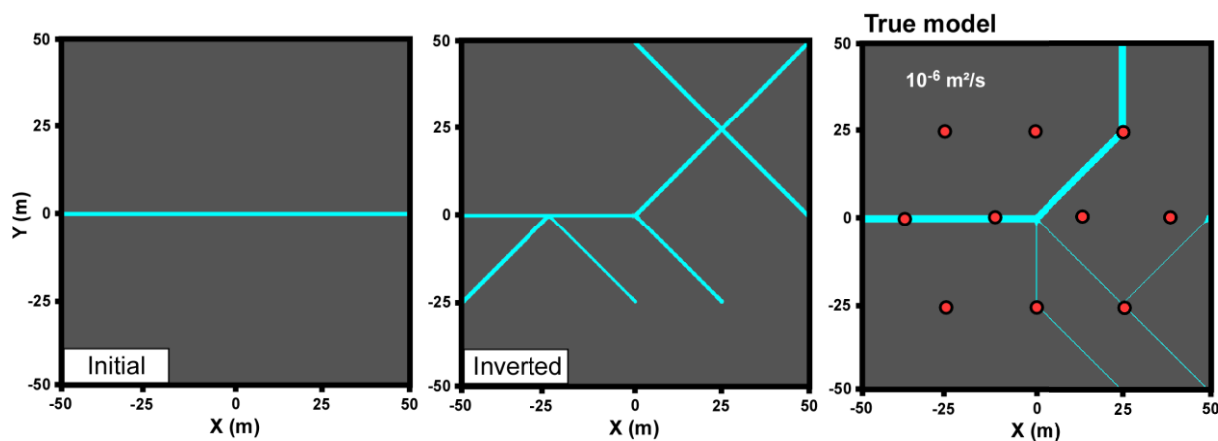
333

334 **3.1. Study case 1**

335 In a first study case, we have tested the ability of the inversion method to reproduce a network
 336 geometry with variable conduit equivalent transmissivities in a homogeneous matrix. We
 337 generated drawdown data from a theoretical model with a 10^{-6} m²/s matrix transmissivity and
 338 a principal conduit associated to a 0.1 m²/s transmissivity and secondary conduits associated

339 to a $0.01 \text{ m}^2/\text{s}$ transmissivity. Firstly we tested an inversion with a small set of data (100
 340 drawdown data from 10 boreholes, see the ‘True model’ in Figure 5).

341 We started the inversion from a simple initial model with a single horizontal $0.06 \text{ m}^2/\text{s}$ conduit
 342 and a homogeneous $10^{-6} \text{ m}^2/\text{s}$ matrix transmissivity. The structural optimization converged in
 343 10 iterations and the properties optimization in 1 iteration.

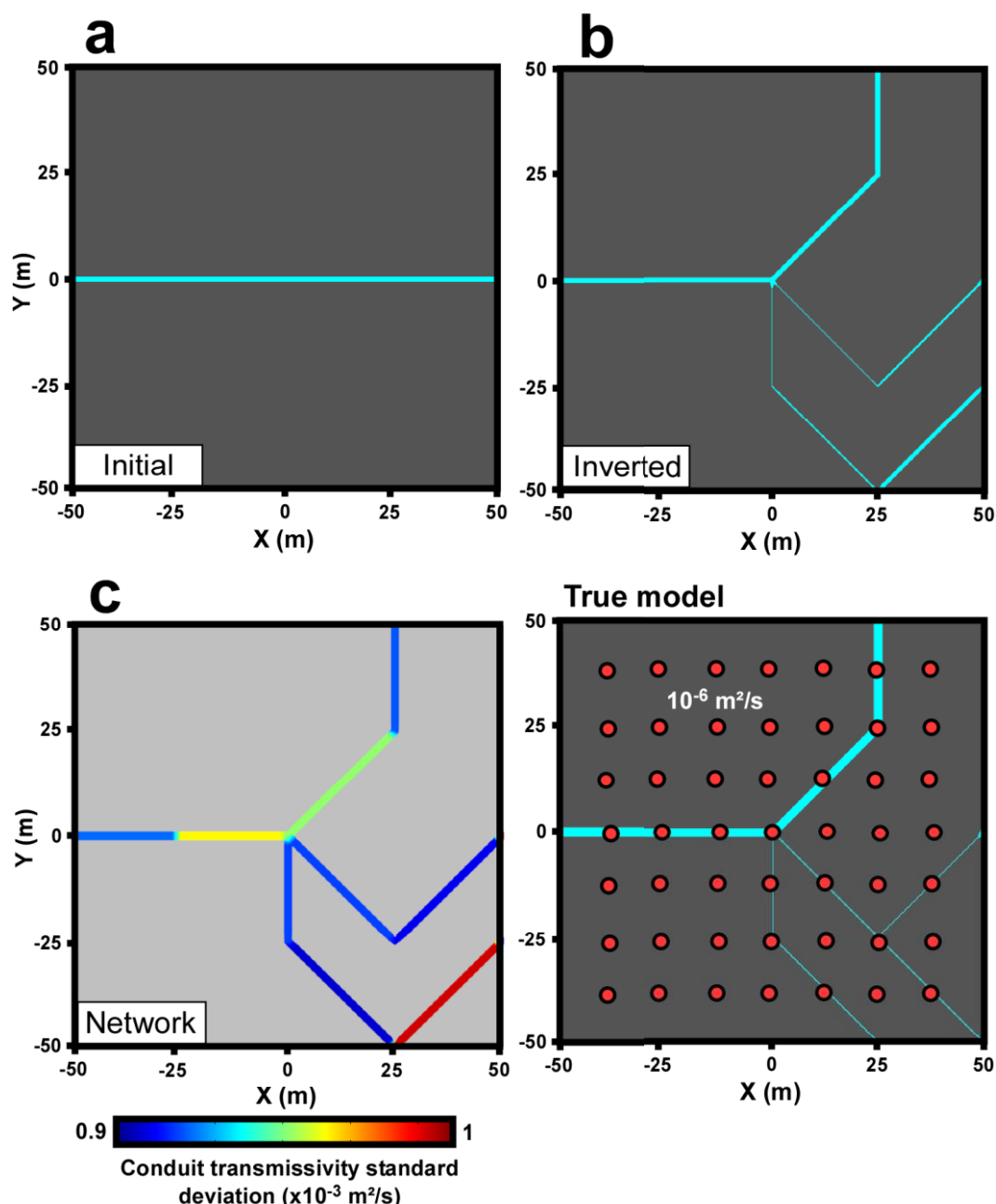


344

345 Figure 5: Initial and inverted models for an inversion using drawdown data produced from a
 346 true model (on the right) with a homogeneous matrix. The red dots on the true model
 347 symbolize the pumping/measurement boreholes for the hydraulic data. The inverted model
 348 permits to localize approximatively the karstic network connections but in this case the
 349 amount of data is insufficient to have a proper imagery.

350

351 The inverted model reproduces the data set ($R^2 = 0.97$) and approximately the connectivity
 352 between the points in the network, however this reconstruction remains distant from the true
 353 geometry. This is due to a lack of data to correctly identify the shape of the conduit network.
 354 Therefore, the efficiency of the inversion for mapping the heterogeneity of the hydraulic
 355 parameters and retrieving the principal karstic conduits is highly dependent to the number of
 356 wells and their locations. In the next test, we used a denser distribution of wells (49 wells) for
 357 providing a better spatial resolution in order to image the heterogeneity of the aquifer
 358 presented for the same ‘True model’ in Figure 6.



359

360 Figure 6: Initial (a) and inverted (b) models for an inversion using drawdown data produced
 361 from a true model with a homogeneous matrix, and associated map of the conduit properties
 362 posterior standard deviations (c). The inverted model in (b) permits a good localization the
 363 true karstic network. It also reduced locally the initial transmissivity ($0.06 \text{ m}^2/\text{s}$ to $0.01 \text{ m}^2/\text{s}$)
 364 of the conduits connected to the primary drain in the bottom right part of the model (the
 365 conduit thickness is proportional to its transmissivity value). The red dots on the true model
 366 symbolize the pumping/measurement boreholes for the hydraulic data.

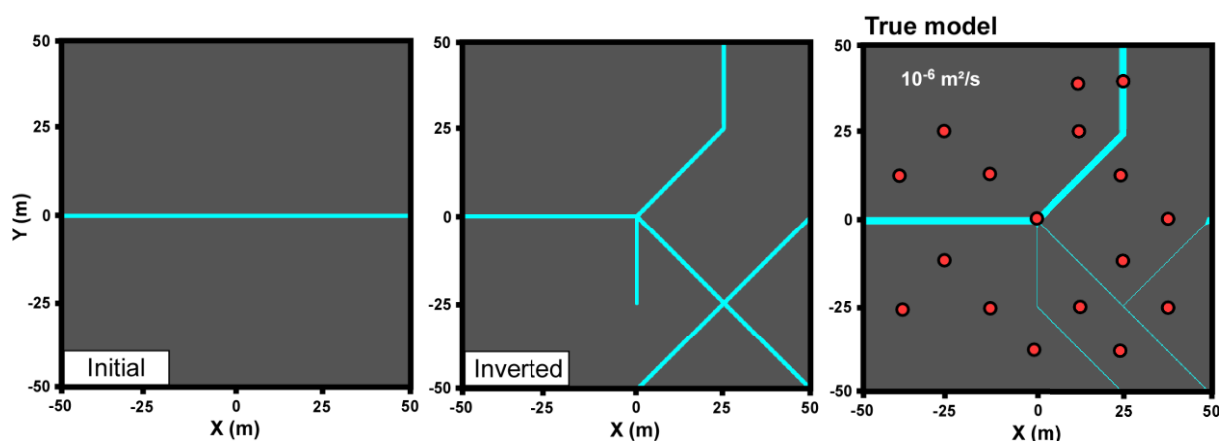
367

368 The structural optimization converged in 11 iterations and the properties optimization in 1

369 iteration. The inverted model reproduces now the data set ($R^2 = 0.95$) and also a very good

370 representation of the true geometry (Figure 6). The property optimization permitted to correct
 371 the initial equivalent transmissivity of $0.06 \text{ m}^2/\text{s}$ to $0.01 \text{ m}^2/\text{s}$ for the conduits connected in the
 372 bottom right area of the network. It permits to reduce the flow rates coming to this zone and
 373 enhance the reproduction of the true cones of depression. The flows in this zone are mainly
 374 conditioned by the properties of the conduits connected directly to the primary drain. This
 375 affirmation can be supported by the conduit transmissivity standard deviation map produced
 376 from Eq. 11 (Figure 6), that shows that the properties of the conduits directly connected to the
 377 primary drain have lower uncertainties than the primary drain itself in the center of the
 378 inverted model. The conduit in the bottom right periphery of the inverted model does not
 379 image correctly the true model. But as the data reproduction is perfect, this periphery zone
 380 might not be sufficiently described by the data to permit a very good reproduction. The
 381 uncertainty map confirms that this part of the network has a more uncertain transmissivity
 382 value than the rest of the network.

383 We have also tested another configuration with more available boreholes than in the case in
 384 Figure 5, but in which only two boreholes intersect the true karstic network. The true model
 385 and the inversion result are presented in Figure 7.



386

387 Figure 7: Initial and inverted models for an inversion using drawdown data produced from a
 388 true model (on the right) with a homogeneous matrix. The red dots on the true model
 389 symbolize the pumping/measurement boreholes for the hydraulic data, primarily located in

390 the matrix. The inverted model permits to almost reproduce the karstic network even if only
391 two measurement points are located in the true network.

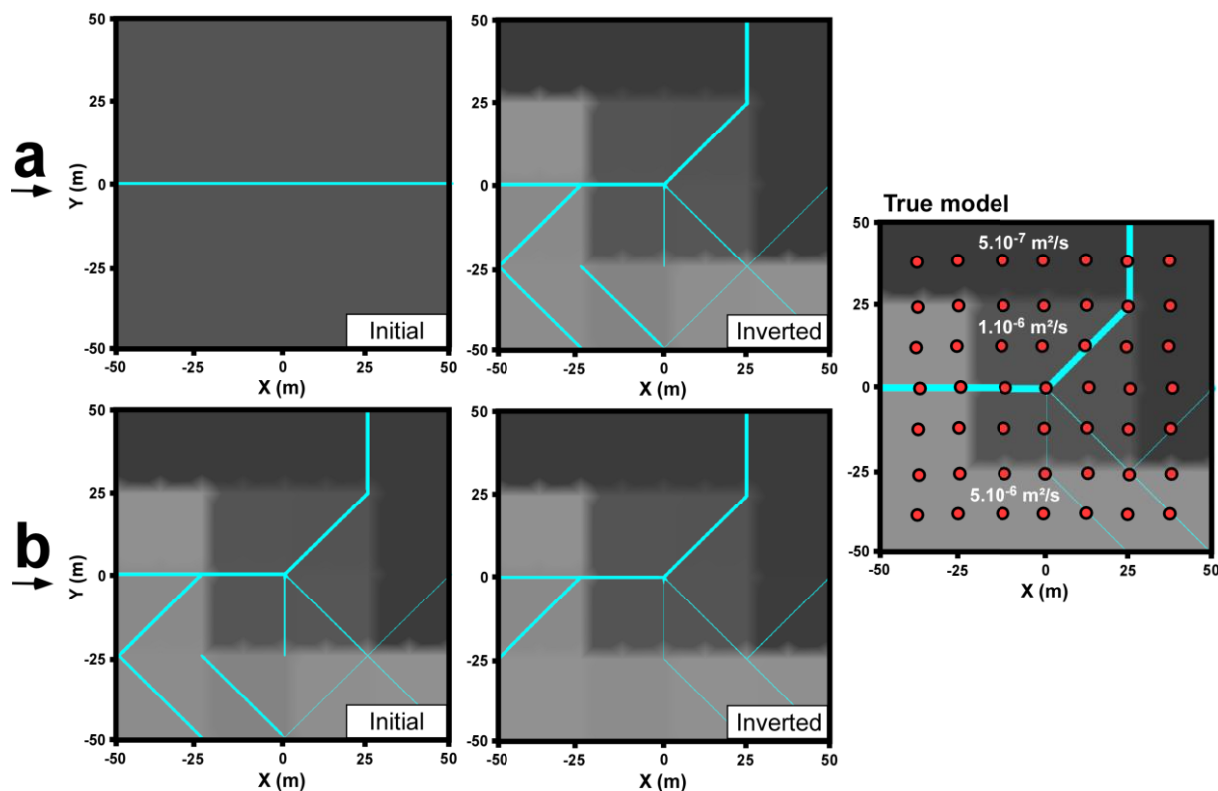
392

393 The inverted model can almost reproduce the true network geometry, which shows that
394 boreholes, even in the matrix, can provide information about the localization of nearby
395 conduits. This is especially true for the thin conduits which appear in the inverted model
396 although no boreholes are intersecting them. Therefore the DNDI method can be used with
397 dataset with only a few boreholes intersecting the conduits as long as there are a sufficient
398 number of other boreholes, in the matrix, in suitable locations for characterizing the nearby
399 conduit network.

400 **3.2. Study case 2**

401 A second study case was led to test the ability of this inversion method to reproduce the data
402 in a case of a karstic network with various conduit properties developed in a heterogeneous
403 matrix. We simulated the piezometric data from a theoretical model with the same karstic
404 network than in study case 1, but in a matrix with a transmissivity varying from $5 \cdot 10^{-6}$ m²/s to
405 $5 \cdot 10^{-7}$ m²/s (Figure 8).

406 We started the inversion from a simple initial model with a single horizontal 0.04 m²/s conduit
407 and a homogeneous 10^{-6} m²/s matrix transmissivity. The structural optimization converged in
408 10 iterations and the properties optimization in 3 iterations (Figure 8a).



409

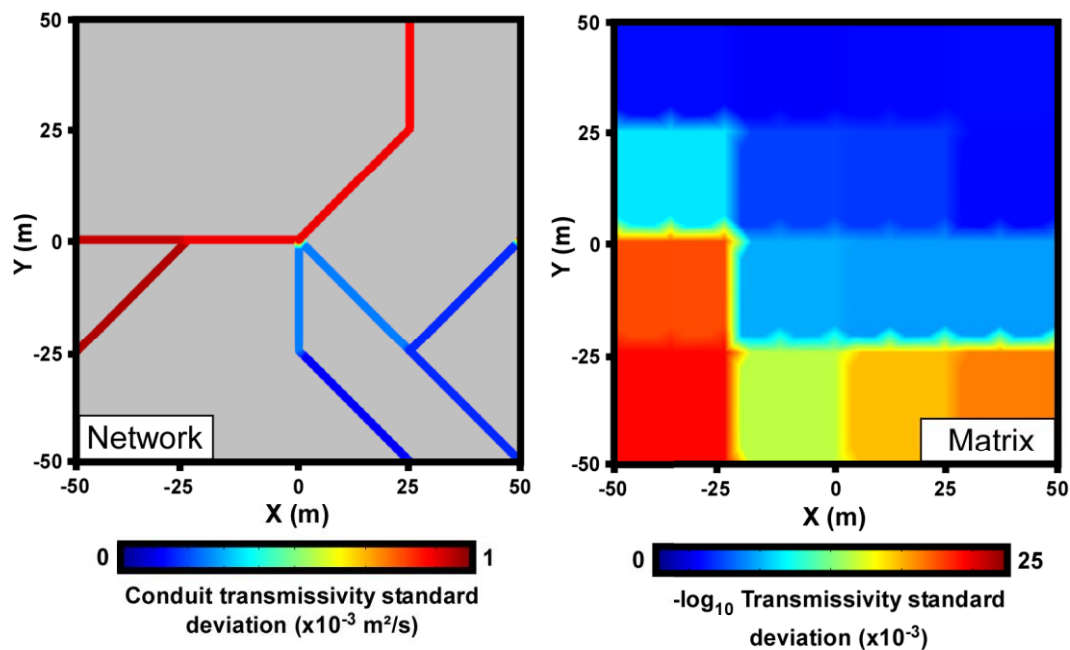
410 Figure 8: Initial and inverted models for an inversion using drawdown data produced from a
 411 true model (on the right) with a heterogeneous matrix. The red dots on the true model
 412 symbolize the pumping/measurement boreholes for the hydraulic data. A first inverted model
 413 (a) permits to localize the true karstic network but also generates conduits to simulate the
 414 more transmissive part of the true model. A second inversion (b) starting from the previous
 415 inverted model permits to correct the geometry and produces an inverted model matching
 416 more accurately the true model.

417

418 The structural optimization permitted to retrieve the true geometry of the conduits network,
 419 but it also added conduits in the bottom left part of the model to reproduce the drawdown data
 420 of the more transmissive area of the matrix. Then the property optimization could reproduce
 421 the true transmissivity values distribution in the matrix. In the end the inverted model can
 422 reproduce the true drawdowns data, but its network geometry incorporates parts, inexistent in
 423 the true model, that has been generated in order to simulate a more transmissive area of the
 424 matrix before the matrix transmissivity values could be optimized.

425 We started a second inversion using the previously inverted model (indicated in Figure 4 as
 426 the ‘multi-scale option’). The structural optimization converged in 2 iterations and the

427 parameter property optimization in 1 iteration (Figure 8b). The only changes were made
 428 during the structural optimization step, with an important improvement in the identification of
 429 the shape of the conduits. In this case the inverted model reproduces the drawdowns data (R^2
 430 $= 0.99$) but is also a good representation of the true network geometry.



431

432 Figure 9: Maps of the conduit and matrix transmissivities posterior standard deviations. The
 433 matrix higher transmissivity zones in the inverted model (bottom left) have a higher
 434 uncertainty value than the lower transmissivity zones (top right). On the contrary, the
 435 uncertainty on the transmissivities of the conduits of the primary drain is higher than the
 436 secondary conduits.

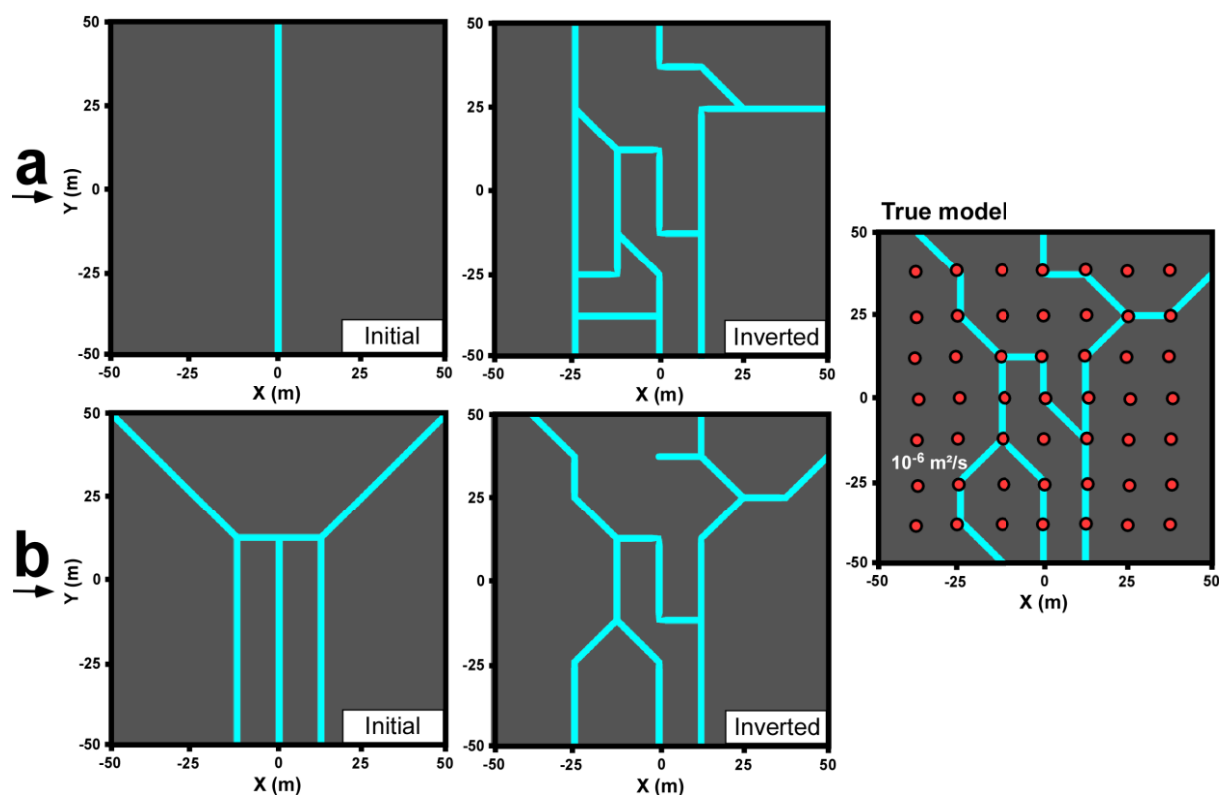
437

438 The posterior standard deviation maps produced from Eq. 11 (Figure 9) show, for the conduit
 439 property values, a smaller uncertainty for the secondary conduits and a higher uncertainty for
 440 the primary drain, especially for the part of the network on the left of the model. Concerning
 441 the matrix transmissivity property values, the highest uncertainty are located mostly in the
 442 most transmissive areas.

443 3.3. Study case 3

444 Finally, a third study case was led to test the ability of this inversion method to reproduce the
 445 data in a case of a complex karstic network geometry. We generated drawdown data from a
 446 theoretical model with a karstic network with a constant equivalent transmissivity of $0.1 \text{ m}^2/\text{s}$
 447 in a homogeneous matrix with a transmissivity of $10^{-6} \text{ m}^2/\text{s}$ (Figure 10).

448 We started an inversion from a simple initial model with a single vertical $0.1 \text{ m}^2/\text{s}$ conduit and
 449 a homogeneous $10^{-6} \text{ m}^2/\text{s}$ matrix transmissivity. The structural optimization converged in 33
 450 iterations and the parameter optimization in 1 iteration (Figure 10a). The inverted model
 451 permits to fit the data set approximately ($R^2 = 0.78$) and represents the global geometry of the
 452 conduits network of the true model. Regarding the simplicity of the initial model, the result
 453 model remains satisfying.

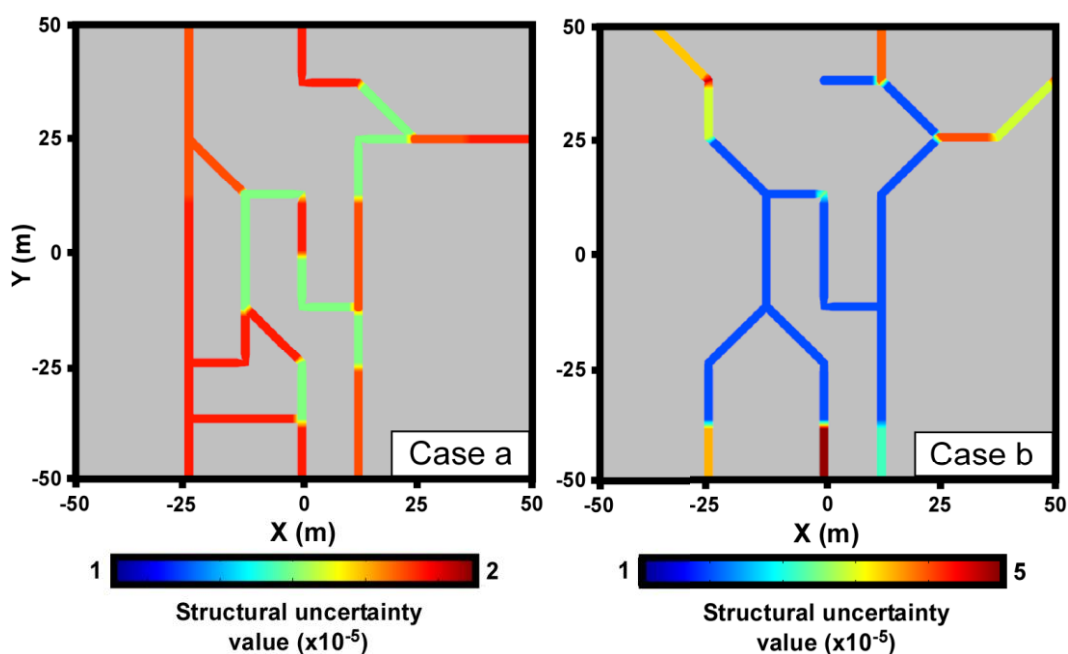


454

455 **Figure 10:** Initial and inverted models for an inversion using drawdown data generated from a
 456 true model (on the right) with a homogeneous matrix. The red dots on the true model
 457 symbolize the pumping/measurement boreholes for the hydraulic data. A first inverted model
 458 (a), starting from a simple initial model, permits to localize approximately the true network
 459 geometry. A second inversion (b), starting from a more detailed initial model, permits to
 460 produce a more precise network geometry.

461

462 We also started an inversion from a more complex initial model with two vertical 0.1 m²/s
 463 conduits diverging in the upper part of the model in a homogeneous 10⁻⁶ m²/s matrix
 464 transmissivity. This initial model geometry (representing a simple approximation of the true
 465 geometry) can be associated to a priori field knowledge information. The structural
 466 optimization converged in 17 iterations and the parameter optimization in 1 iteration (Figure
 467 10b). In this case, the inverted geometry of the discrete network permits a good reproduction
 468 of the data ($R^2 = 0.97$) and is closer to the real network than the case in Figure 10a.



469

470 Figure 11: Maps of the posterior uncertainties of the network local directions for the Cases a
 471 and b. In the Case a, started from a simple initial model, the highest uncertainties are
 472 distributed uniformly over the inverted network. In the Case b, started from a more detailed
 473 initial model, the highest uncertainties are located in the periphery of the model.

474

475 The structural posterior uncertainty maps produced from Eq. 8 are presented in Figure 11.

476 These maps show that, in the Case a, the highest uncertainties are distributed relatively
 477 uniformly among the inverted model, while in Case b, they are mostly located in the periphery

478 of the model. Here, the structural posterior uncertainties are giving important information
479 about the local validity of the different inverted networks.

480 **4. Discussion**

481 We have successfully tested the DNDI method on three theoretical and simplified study cases
482 with steady state drawdowns. However as we have seen, an inversion process is limited by the
483 non-uniqueness of its solution. Therefore using the DNDI method requires several
484 prerequisites and the modeler needs to be critical toward the result.

485 As we have seen in the first study case, the efficiency of the inversion is dependent to the
486 hydraulic data set, and in particular the number and the localization of observation wells on
487 the field. We note that even wells in the matrix can provide information on nearby conduits
488 for the inversion. Globally it appears that the most important point about a steady-state dataset
489 is to have a homogeneous and sufficiently dense distribution of wells on the site, in order to
490 characterize successfully the network.

491 Concerning the inversion process itself, we note, in the third study case, the ability of the
492 DNDI method to image complex networks. However, as the inversion is deterministic, the
493 precision of the result model is dependent to the initial model. The inversion process will
494 converge to a local solution dependent to the initial model. In fact, in Figure 10 we show that
495 a simple initial model permitted to reproduce a satisfying global representation of the true
496 model, but with local approximations, while a more complex initial model permitted a more
497 accurate reproduction of the true model and a faster convergence. Therefore an inverted
498 model using the DNDI method should be analyzed critically, like any deterministic inverse
499 methods, depending from the initial model. The study of the computed structural and property
500 uncertainty values (with Eq. 8 and Eq. 11) can supply this critical analysis on the result
501 model.

502 The second study case also illustrates some limits of the sequential optimization of the
503 method, especially when starting from a too simple initial model. Therefore the amount of a
504 priori information introduced in the initial model is important for the accuracy of the result
505 model. Otherwise, as we demonstrate in Figure 8, a simple possible operation would be to re-
506 run the inversion with a first inversion result to slightly improve the result. We would also
507 recommend the coupling of this inversion method to a multiscale method (Grimstadt et al.
508 2003) which consists in a re-run of the inversion starting from a previous result with a
509 refinement of the partitioning. It permits to lead several inversions with an initial model each
510 time more precise while saving time as we initially start with a coarsely partitioned model.

511 **5. Conclusion**

512 We present in this paper a novel deterministic inversion method that permits to characterize,
513 in a partitioned model, the karst conduits and fractures network geometry and their hydraulic
514 properties, including the transmissivity distribution of the matrices. The DNDI method let the
515 modeler choose the partitioning of the model for the inversion. This ‘cursor’ permits to define
516 either an inversion with a coarse partitioning for a quick approximation model, or with a fine
517 partitioning and a longer computation time for a better fitting model. The use of a discrete
518 network model permits to associate a specific behavior to the flows in the network and thus,
519 produces more realistic models than an equivalent porous media model. This method can be
520 easily adapted for channels or fractures network models by modifying the properties
521 associated to the discrete network (these properties can also be directly linked to an aperture
522 value, by choosing an adapted law). Therefore we believe that the DNDI method is an
523 interesting new imagery tool for the distributed modeling associated to a set of data from an
524 investigation in a karstic and/or fractured aquifer.

525 We have realized different tests in three theoretical and simplified study cases with an
526 increasing complexity, and the DNDI could always produce satisfying results, both on the
527 reproduction of the generated data and on finding the network geometry and property values
528 from the true model. As we have seen in the first study case, the result of the structural
529 inversion is dependent on the positioning and the amount of observed data. This is true for
530 any inversion, but is especially important in the case of highly heterogeneous aquifers for
531 delineating the position of the heterogeneities. Therefore, the result of the inversion has to be
532 interpreted critically regarding the set of data used for it. A first critical analysis can be
533 performed from the maps of posterior uncertainties on the structure or on the property values
534 that can be produced by using the formulas we propose in this paper. The a priori information
535 on the geometry of the network and on the property values is also a way to constrain the
536 inversion in addition to the data. This information can be inferred from general field
537 knowledges (geological and geophysical information, conduits observation in wells through
538 video camera, other studies, etc.)

539 Because this method is deterministic, the choice of the initial model should be based on a
540 relatively coherent possibility and should not be too far from the real solution in order to
541 produce a good result. Therefore, we propose to couple the DNDI method to a multi-scale
542 method. This consists in a first inversion started from an initial model which is followed by a
543 new one that starts from the first inversion solution with a refined partitioning. This strategy
544 permits to start from a simple initial model and to progressively make the model more
545 complex and improve the solution.

546 An application of this method for mapping the conduits and fractures network with real data
547 from a karstic field is planned for future works. These works will be more specifically
548 focused on the sensitivity of the method to the spatial distribution of the measurement
549 boreholes and on delineating the preferential flow paths in the network.

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