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# Bi-objective optimization for a two-depot automated storage/retrieval system in container terminals

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#### 1 Introduction

Maritime trade has grown rapidly in the last decade. Especially in 2015, the estimated world seaborne trade volumes surpassed 10 billion tons. Furthermore, the containerized trade keeps growing all the time (c.f., UNCTAD 2016). Therefore, the increasing demand of the maritime transportation and seaborne trade raises a higher demand for the container terminal operational efficiency. With the development of automation technology, automated storage and retrieval system (AS/RS) is widely applied. In recent trends of container terminal development, the automation technology is being gradually applied in container terminals. The construction of automated container terminals is mainly focused on the automated quay crane, automated guided vehicle and automated transfer platform. It rarely focuses on the automated storage and retrieval system. The main reason may be the high cost of implementing such a system. However, in recent years, both the shortage of yard space and the advancements of technology have facilitated the application of AS/RS in container terminals (Gharehgozli et al. 2017).

This paper studies an AS/RS and two depots in the container yard. Each depot can be used to store and retrieve containers (i.e., requests) by the S/R machine. The S/R machine moves along the track to store and retrieve requests in the aisle. In this paper, the current depots and dedicated storage locations of storage requests are given. The current locations of retrieval requests in the AS/RS are also known. However, the depot location of retrieval requests is to be determined. The determination of the depot must consider the travel distance between two depots or the immediate next request. Furthermore, the energy consumption of the S/R machine in X (horizontal) direction is far greater than that in Y (vertical) or Z (pick up or drop off a request) direction. Therefore, terminal operators not only try to decrease the travel distance of S/R machine in X direction, but also attempt to complete all requests before their due dates. This paper aims to balance these two objectives via bi-objective optimization approaches. These two aims are measured in dimensions of times. That's to say, the distance is equal to the unit time multiplied by the unit speed. Thus, the distance is expressed in time.

To the best of our knowledge, the contributions of the paper are: (1) to establish a mixed integer programming model for the AS/RS in the container terminal with two depots, (2) to balance total travel time of the S/R machine in horizontal direction and total delay time of requests with considering the release time, (3) to develop a promising heuristic algorithm to solve the considered problem. Furthermore, the computation results show the effectiveness of the proposed algorithm.

### 2 Mathematical Model

This paper studies a single S/R machine and two depots (k1, k2) in the container yard. The parameters are expressed as follows.  $N, N_s$  and  $N_r$  represent the set of all requests, storage requests and retrieval requests, respectively, and  $N = N_s \bigcup N_r$ .  $l_i = (x_i, y_i, z_i)$  indicates the aisle location of request *i*. The depot location of request *i* is defined as  $k_i$ .  $k_i = (x_i^k, y_i^k, z_i^k)$ refers to the depot location of storage request i.  $[x_{min}^1, x_{max}^1]$  and  $[x_{min}^2, x_{max}^2]$  represent the horizontal ranges of depote k1 and k2, respectively.  $r_i$  and  $d_i$  mean the release time and due date of i. h represents the time to pick up or drop off a request. The starting location of the S/R machine is defined as  $(x_0, y_0, z_0)$ .  $a_{l_i,k_i}^x$  and  $a_{l_i,k_i}^y$  refer to the distance between aisle location of i and depot location of j in X and Y directions.  $b_{k_i,l_i}^x$  and  $b_{k_i,l_i}^y$  mean the distance between the depot location of i and its aisle location in X and Y directions.  $c_{l_i,l_j}^x$  and  $c_{l_i,l_j}^y$  represent the distance between the aisle location of i and that of j in X and Y directions.  $e_{k_i,k_j}^y$  indicates the vertical distance between the depot location of i and that of j.  $f_{k_i,l_j}^y$  means the vertical distance between the depot location of i and aisle location of j.  $g_{l_i,k_i}^y$  refers to the vertical distance between the depot location of i and its aisle location. The binary variables  $p_i$  and  $q_i$ represent which depot of the storage request i is located in.  $u_{i,j}$  and  $v_{i,j}$  mean which depot of retrieval requests i will be delivered to if the immediate next request is retrieval request j.  $s_i$ and  $w_i$  show which depot of request i is close to. M is a sufficiently large enough number.

Similarly, the decision variables are presented below.  $\lambda_i, \theta_i, \pi_i$  and  $\rho_i$  mean the start time, completion time, delay time and process sequence of request *i*.  $(X_i^k, Y_i^k, Z_i^k)$  indicates the depot location of retrieval request *i*.  $e_{k_i,k_j}^x, f_{k_i,l_j}^x$  and  $g_{l_i,k_i}^x$  are correspond to the meaning of  $e_{k_i,k_j}^y, f_{k_i,l_j}^y, g_{l_i,k_i}^y$  in X direction.  $l^e = (x'_0, y'_0, z'_0)$  means the ending location of the S/R machine.  $t_{i,j}$  is defined as the travel time between *i* and *j*.  $t_{i,0'}^x$  is the horizontal travel time of the last request.  $t_{i,0'}, t_{i,j}^e$  and  $t_{i,j}^f$  are assistant variables to linearize non-linear constraints. For the binary variables below.  $\alpha_{i,j}$  indicates *i* is processed before *j*,  $\beta_{i,j}$  represents *j* is immediately processed after *i*,  $\phi_{i,j}$  shows retrieval request *i* is delivered to *k*1 based on any *j* when  $\xi_{i,j}$  refers to k2,  $\delta_i, \tau_i$  represent the first and the last request, respectively.

In the minimization item of  $F_1$ , we calculate the total travel time of the S/R machine in X direction. In the minimization item of  $F_2$ , we calculate the total delay time of all requests.

$$F_{1} = \min\left(\sum_{i \in N_{s}} (x_{i}^{k} - x_{0})\delta_{i} + \sum_{i \in N_{r}} (x_{i} - x_{0})\delta_{i} + \sum_{i \in N_{s} \setminus \{j\}} \sum_{j \in N_{s} \setminus \{i\}} a_{l_{i},k_{j}}^{x}\beta_{i,j} + \sum_{i \in N_{s}} b_{k_{i},l_{i}}^{x}\right)$$
$$+ \sum_{i \in N_{s}} \sum_{j \in N_{r}} c_{l_{i},l_{j}}^{x}\beta_{i,j} + \sum_{i \in N_{r}} \sum_{j \in N_{s}} t_{i,j}^{e} + \sum_{i \in N_{r} \setminus \{j\}} \sum_{j \in N_{r} \setminus \{i\}} t_{i,j}^{f} + \sum_{i \in N_{r}} g_{l_{i},k_{i}}^{x} + \sum_{i \in N_{s}} t_{i,0'}\right)$$
$$F_{2} = \min\sum_{i \in N} \pi_{i}$$

Because of the limitation of the paper, the complete model can not be shown. Here, we mainly present some important constraints.

Constraint (1) states the calculation of the delay time.

$$\pi_i \ge \theta_i - d_i, \quad i \in N \tag{1}$$

The calculation of the completion time of each request is expressed as Constraints (2)-(7).

$$\theta_{i} \geq \lambda_{i} + h + z_{i}^{k} + b_{k_{i},l_{i}}^{x} + z_{i} + h - 1, \quad i \in N_{s} \quad (2)$$
  
$$\theta_{i} \geq \lambda_{i} + h + z_{i} + g_{l_{i},k_{i}}^{x} + Z_{i}^{k} + h - 1, \quad i \in N_{r} \quad (3)$$
  
... 
$$(4) - (7)$$

Constraints (8)-(13) give the calculation of the start time.

$$\lambda_i \ge r_i, \quad i \in N \tag{8}$$

$$\lambda_i \ge x_i^k - x_0 + z_i^k + 1, \quad i \in N_s$$
 (9)  
... (10) - (13)

The process sequence of each request and the relationship between  $\alpha_{i,j}$  and  $\beta_{i,j}$  are expressed as Constraints (14)-(25).

$$\rho_{j} = \sum_{i \in N} \alpha_{i,j} + 1, \quad j \in N, i \neq j$$

$$\rho_{j} - \rho_{i} \ge 1 - (1 - \alpha_{i,j})M, \quad i, j \in N, i \neq j$$

$$\dots$$
(14)
(15)
(16) - (25)

The travel time between any two requests can be calculated based on  $a_{l_i,k_j}^x, a_{l_i,k_j}^y, c_{l_i,l_j}^x, c_{l_i,l_j}^y, c_{l_i,l_j}^x, c_{l_i,l_j}^y, c_{l_i,l_j}^x, c_{l_i,l_j}^y, c_{l_i,l_j}^x, c_{l_i,l_j}^y, c_{l_i,l_j}$ 

$$t_{i,j} \ge z_i + a_{l_i,k_j}^x + z_j^k - (1 - \beta_{i,j})M, \quad i, j \in N_s, i \ne j$$

$$t_{i,j} \ge z_i + c_{l_i,l_j}^x + z_j - (1 - \beta_{i,j})M, \quad i \in N_s, j \in N_r$$

$$\dots$$

$$(26)$$

$$(27)$$

$$(28) - (37)$$

According to the value of  $p_i, q_i, u_{i,j}$  and  $v_{i,j}$ , Constraints (38)-(47) show the definition of  $\phi_{i,j}$ and  $\xi_{i,j}$ .

$$\phi_{i,j} \ge 1 - (2 - \beta_{i,j} - p_j)M, \quad i \in N_r, j \in N_s$$
(38)
...
(39) - (47)

Based on the value of  $\phi_{i,j}$  and  $\xi_{i,j}$ , we can obtain the depot location of retrieval requests, which is expressed as Constraints (48)-(51).

$$X_{i}^{k} \leq x_{min}^{1} + (1 - \phi_{i,j})M, \quad i \in N_{r}, j \in N, i \neq j$$

$$X_{i}^{k} \geq x_{min}^{1}, \quad i \in N_{r}$$

$$\dots$$
(48)
$$(49)$$

$$\dots$$
(50) - (51)

Constraints (52)-(57) present the ending location of the S/R machine.

$$x'_{0} \le x^{1}_{min} + (2 - \tau_{i} - s_{i})M, \quad i \in N$$
(52)

$$x'_{0} \ge x^{2}_{max} - (2 - \tau_{i} - w_{i})M, \quad i \in N$$
(53)

$$(56) - (57)$$

If the last request is storage request i, then the travel time between i and the ending location of the S/R machine in X direction is decided by the distance between the two depots, which is expressed as Constraints (58)-(59)

$$t_{i,0'}^x \ge x_0' - x_i - (1 - \tau_i)M, \quad i \in N$$
(58)

$$t_{i,0'}^x \ge x_i - x_0' - (1 - \tau_i)M, \quad i \in N$$
(59)

Constraints (60)-(68) are used to linearize non-linear constraints.

...

$$t_{i,0'} \ge t_{i,0'}^x - (1 - \tau_i)M, \quad i \in N_s$$
(60)

$$t_{i,j}^e \le \beta_{i,j} M, \quad i \in N_r, j \in N_s \tag{61}$$

$$t_{i,j}^e \le e_{k_i,k_j}^x, \quad i \in N_r, j \in N_s \tag{62}$$

... 
$$(63) - (68)$$

Constraints (69)-(70) give the range of decision variables.

$$\lambda_{i}, \theta_{i}, \pi_{i}, \rho_{i}, X_{i}^{k}, e_{k_{i}, k_{j}}^{x}, f_{k_{i}, l_{j}}^{x}, g_{l_{i}, k_{i}}^{x}, l^{e}, t_{i, j}, t_{i, 0'}^{x}, t_{i, 0'}, t_{i, j}^{e}, t_{i, j}^{f} \in Z^{+}, \quad i, j \in N, i \neq j$$

$$(69)$$

$$\alpha_{i,j}, \beta_{i,j}, \phi_{i,j}, \xi_{i,j}, \delta_i, \tau_i \in \{0, 1\}, \quad i, j \in N, i \neq j$$
(70)

#### 3 Solution approach

In this paper, we first adopt  $\epsilon$ -constraint method to obtain the optimal Pareto front. The main idea of it is to transform an objective into a constraint with a gradual reduction value  $\epsilon$  to connect the two objectives. We transform  $F_2$  into a constraint and set  $F_1$  as a single objective. Meanwhile, the value of  $\epsilon$  can be chosen from an appropriate set, i.e., the range of  $F_2$  values. For ten instances that each of them includes ten requests, the average running time is 528.22s. However, for instances with twelve requests, CPLEX takes more than 7200s for one iteration because of its NP-hard nature. Therefore, more efficient heuristic algorithms to generate an approximate Pareto front for medium- and large-scale instances are necessary.

(1) NSGA-II To solve the problem for medium- or large-scale instances efficiently, we devise a variant of non-dominated sorting genetic algorithm II (NSGA-II for short). In NSGA-II, we first generate the initial individuals. Then all the initial individuals are sorted with considering the non-domination rule. For each generation, the offsprings are generated by genetic algorithm operators including crossover and mutation operations. Following closely, the recombination process is necessary. The details of NSGA-II are omitted here.

(2) **R-OLRD** According to the characteristic of the problem, we propose a new heuristic algorithm to solve it. The difference from other approaches is that this method can obtain more solutions as much as possible. We denote this approach as R - OLRD (i.e., Random sequence, Original sequence, Location sequence, Release time sequence and Due date sequence). The core idea of R - OLRD algorithm is to obtain the solution through the comparison of the results with different rules (such as FCFS, EDD) for each iteration. Then put all the solution into a data pool which contains the optimal solutions through entire iteration. Finally, the Pareto front is obtained with different iteration scales. The details are omitted here.

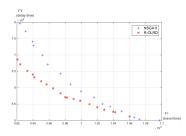


FIG. 1: Non-dominated solutions produced by NSGA II and R-OLRD with 300 requests

Figure 1 plots the non-dominated solutions produced by NSGA II and R - OLRD in an instance with 300 requests. In this figure, with the increase of the value of F1, F2 decreases. The results demonstrate that R - OLRD is always better than NSGA II for the sake of the solutions and running time. Due to the limitation of the article, other details are omitted here.

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