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# Integration of Timetabling, Multi-type Vehicle Scheduling and User Routing in Public Transit Network considering Fuel Consumption

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**Abstract**—This paper focuses on the integration of timetabling, multi-type vehicle scheduling and user routing in transit network considering fuel consumption. For the integrated problem, we consider the users' preferences for departure and arrival times and the capacity limits of vehicles. Additionally, the multi-type vehicle and fuel consumption factors are also the key contributions, which are firstly added into the integrated problem. The objective is to minimize the inconvenience of the users (i.e. in-vehicle times, line-transfer penalties and deviation between desired departure and arrival times), penalties of users not served and the cost of line runs and fuel consumption for operators. Finally, we establish a mixed-integer programming model for the integrated problem.

## I. INTRODUCTION

In recent decades, the transit network has become huger and huger due to the development of road network and the increasing of vehicles. The complex transit network forms a bigger challenge for the road traffic planning and the vehicle scheduling. Especially for the public transit network, more convenient service for users is an quite important problem, meanwhile, the factors of energy saving and environmental protect are also an unneglected problem.

The transit network timetabling and scheduling problem (TNTSP) includes two subproblems: the transit network timetabling problem (TNTP), vehicle scheduling problem (VSP). The transit network timetabling problem and vehicle scheduling problem in transit network have attached more attention. In this paper, we also consider the user routing problem. For the user routing, we assume that users are interested in minimizing their scheduled inconveniences related to in-vehicle times, line-transfer penalties and deviation between desired departure and arrival time. For the timetabling problem, it needs to consider the potential itinerary and departure and arrival times, then a convenient timetable can be obtained. On the other hand, the convenient timetable can also reduce the waste of public transport resources. For the vehicle scheduling

problem, most existing studies assumed that all the vehicles are identical. Laporte et al. (2017) studied the multi-objective integration of timetables, vehicle schedules and user routings, however, they also assumed the identical vehicle factor. In fact, many public transports, such as train, subway or bus, have various configurations or types. Therefore, the speed and fuel consumption are also different, then the various speeds of vehicles will also impact on the users' optimal option for itineraries and the departure times.

In this paper, we focus on the integration of timetabling, multi-type vehicle scheduling and user routing in transit network considering fuel consumption. Our main contributions are as follows. 1) For the integrated problem, we not only pursues transfer coordination but also users' preferences for departure and arrival times. 2) Each user need to be treated individually with hard time windows constraints for trip duration, departure and arrival times. 3) We consider multi-type vehicle, vehicle capacity limits and vehicle fuel consumption. 4) We establish a mixed-integer programming model for the integrated problem.

The rest of this paper is organized as follows. In Section 2, a brief literature review of previous work is given. Section 3 describes the problem under consideration, and Section 4 formulates the mathematical model. Finally, some conclusions are given in Section 5.

## II. LITERATURE REVIEW

Recent research in transit network has mainly focused on the transit network timetabling problem, vehicle scheduling problem and the integration of the two problems.

The timetabling problem has caught a lot of attention in the field of bus and train scheduling. For the train timetabling problem, Huang (2006) developed ant colony optimization (ACO), considering the trains scheduling in the transit period

between peak period and off-peak period, the conflict resolving and the balance of in and out trains for each depot. Kang et al. (2016) studied the optimization of first train timetables for an urban railway network, which focuses on designing convenient and smooth timetables for morning passengers, then proposed a mixed integer programming (MIP) model for minimizing the number of missed trains. Hassannayebi and Zegordi (2015) focused on the rail rapid transit timetabling problem, aiming at minimizing the total and maximum waiting time of the passengers through optimization of the train timetables for urban rail transit systems. To solve the problem, they developed mixed-integer linear and non-linear programming models, then proposed adaptive and variable neighbourhood search algorithms to tackle large instances. Canca et al. (2016) studied the design and optimization of non-periodic train timetables when passengers demand follows a dynamic behavior along certain planning horizon, then present four different formulations for this problem. Ibarra-Rojas et al. (2015) proposed the multiperiod synchronization bus timetabling (MSBT) problem, which specifies the departure times of the trips of all lines where each line has its own planning periods along the day, with the objective of optimizing synchronization events: maximize passenger transfers and minimize bus bunching along the network.

Compared with the transit network timetabling problem, the research of vehicle scheduling problem has formed rather mature methods and contents. Foster et al. (1976) described an integer programming formulation of the vehicle scheduling problem and illustrated how such a formulation can be extended to incorporate restrictions on work load, coverage and service that occur in real world vehicle scheduling problems. Huisman et al. (2004) focused on dynamic scheduling and considered three measurements throughout the paper, namely the number of vehicles used, the percentage of trips starting late, and a virtual measure for delay costs. Hadjar et al. (2006) considered the multiple depot vehicle scheduling problem (MDVSP) and proposed a branch-and-bound algorithm for solving it that combines column generation, variable fixing, and cutting planes. Haghani and Banihashemi (2012) presented new models for multiple depot vehicle scheduling problem (MDVS) and multiple depot vehicle scheduling problem with route time constraints, considering the route time constraints.

Based on the two aforementioned problems, some literature gradually turns to the integration problem. Ibarra-Rojas et al. (2014) focused the trade-off between the level of service and operating costs in transit system, then presented two integer linear programming models for the timetabling and vehicle scheduling problems, and combined them in a bi-objective integrated model. Ceder (2011) addressed the timetable development and vehicle-scheduling with different vehicles types, and proposed a method to construct timetables with the combination of both even-headway and even-load concepts, then the vehicle-scheduling problem are based on given sets of trips and vehicle types arranged in decreasing order of vehicle cost. Liu and Shen (2007) established a bi-level programming to study the regional bus operating problem. In the upper

model, the regional bus vehicle scheduling, which is designed to minimize the total number of the required vehicles and the total time of the deadheading trips, is formulated as a class of the scheduling problem complying strictly with the vehicle chain-running time, the maximum capacity, and the required retaining vehicles in each depot. In the lower model, the objective is to minimize the total transfer time of passengers in every connection stop, and the synchronization coefficient describing the cases of lines-crossing in one connection stop and the satisfaction criteria. Guihaire and Hao (2010) focused on combining important features of these two steps and proposed a simultaneous solution approach to redefine timetables with the objective of bringing improvements to both quality of service and vehicle costs incurred, then introduced an optimization procedure based on Iterated Local Search to solve the problem. Fedorko and Weiszer (2012) presented a selection of crossover operator and determination of maximum number of generations in algorithm for integrated timetable and vehicle scheduling optimization in public transport. Michaelis and Schobel (2009) suggested to reorder the classic sequence of the planning steps: first designing the vehicle routes, then splitting them to lines and finally calculating a (periodic) timetable. Laporte et al. (2017) studied the transit network timetabling and scheduling problem, aiming at determining an optimal timetable for each line of a transit network by establishing departure and arrival times at each station and allocating a vehicle to each timetable.

Some of the above studies only focused on a subproblem, such as timetabling problem or vehicle scheduling problem. In addition, most of the studies about the integration problem ignored the user factor, even assumed that all the vehicles in transit network are identical. In this paper, we develop the integration of timetabling, multi-type vehicle scheduling and user routings, based on Laporte et al. (2017). Specifically, an user can choose different types of vehicles to complete his itinerary. Meanwhile, each user has the desired departure and arrival times.

### III. PROBLEM STATEMENT AND MATHEMATICAL MODEL

#### A. Problem Statement

We define a public transportation network (PTN) as a graph  $G = (S, A)$  with a set of nodes  $S$  representing stations and a set of arcs  $A$ . Each arc denotes a direct connection between two stations of  $S$ . A directed graph  $G_l = (S_l, A_l)$  stands for a public transportation line (PTL), and  $l$  belongs to the set of lines  $\mathcal{L}$ ,  $S_l \subseteq S$  is the set of stations and  $A_l \subseteq A$  contains the direct connections between two stations using line  $l$ , the set of direct path lines  $\overline{\mathcal{L}} \subseteq \mathcal{L}$  and the set of direct cycle lines  $\hat{\mathcal{L}} \subseteq \mathcal{L}$ . Additionally, the set of lines  $\mathcal{L}$  can also be divide into two categories: going forward lines set  $\overline{\mathcal{L}}$  and going backward lines set  $\hat{\mathcal{L}}$ . Figure 1 shows a PTN with 4 nodes  $S = \{1, 2, 3, 4\}$  and 8 arcs  $A = \{(1, 2), (2, 1), (2, 4), (2, 3), (3, 2), (3, 4), (4, 2), (4, 3)\}$ . We define the set of lines  $\mathcal{L} = \{1, 2, 3, 4\}$ , and  $A_1 = \{(1, 2), (2, 3)\}$ ,  $A_2 = \{(2, 3), (3, 4), (4, 2)\}$ ,  $A_3 = \{(3, 2), (2, 1)\}$ ,  $A_4 = \{(2, 4), (4, 3), (3, 2)\}$ . Meanwhile,  $\overline{\mathcal{L}} = \{1, 3\}$ ,  $\hat{\mathcal{L}} = \{2, 4\}$ ,

$\vec{\mathcal{L}} = \{1, 2\}$ ,  $\overleftarrow{\mathcal{L}} = \{3, 4\}$ . Additionally, users can transfer lines on the nodes (stations) included by no less than two lines, for example, a user departing station 1 along line 1 can transfer to line 4 at their common station 2.

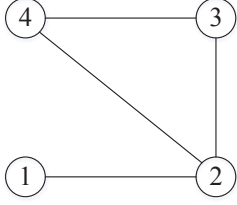


Fig. 1. A PTN with four nodes and eight arcs

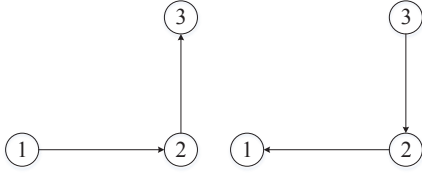


Fig. 2. The going forward line 1 and its opposite direct line 3

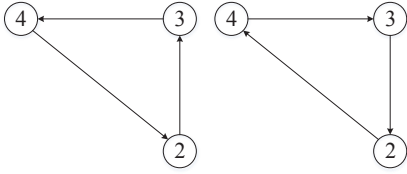


Fig. 3. The going forward line 2 and its opposite direct line 4

For timetabling problem, we first define a line run as the journey of a vehicle making stops for boardings and alightings at every station along the line. Each line is provided two types of vehicle: express-vehicle and general-vehicle, users can choose any one along this line. The travel time and capacity of two types of vehicles are different, and their cost of implementing a line run in a same line are also different. In addition, when a vehicle allocated to  $l \in \vec{\mathcal{L}}$  can start a line run at any time slot. Once the line run is completed, the associated vehicle becomes part of the fleet size of the line  $l + |\vec{\mathcal{L}}|$  ( $l - |\vec{\mathcal{L}}|$  if  $l \in \overleftarrow{\mathcal{L}}$ ). Circular lines, are similar to path lines except that they have only one terminal station. Therefore, any itinerary that involves traversing the terminal station will require a transfer at that station. Each user has fixed upper and lower bounds associated to the departure and arrival times. Additionally other inconveniences related to in-vehicle times, line-change penalties and deviation between desired departure and arrival times will be taken into account.

Each itinerary offers different travel options according to each combination of the potential timetables from the different lines that can be used for completing a trip. Let  $\Pi$  be the set

of itineraries and  $\Pi_i \subseteq \Pi$  the subset the itinerary that can be available for user  $i$ . Note that  $\mathcal{L}_\pi$  is the set of lines related with itinerary  $\pi \in \Pi$ . Once his path is defined, a user can consider different options of departure times, depending on the combinations of timetables that can be implemented on each line of the path. By  $R_{i\pi}^e$  ( $R_{i\pi}^g$ ) we denote the set of options that can be used to serve request  $i$  under the itinerary  $\pi$  by a express-vehicle (general-vehicle). We define a strategy as a combination of an itinerary and a potential timetable that a user can choose to travel in a PTN. For the Figure 1, if a user want to travel from station 2 to station 4, he can choose three itineraries: 1) itinerary 1, starting from the station 2 to station 4 along the line 4; 2) itinerary 2, starting from station 2 to station 4 along the line 2; 3) itinerary 3, starting from the station 2 to station 3 along line 1 and transferring to the line 2 at the station 3, then starting the second trip from station 3 to station 4 along line 2. For the itinerary 1 of the user, we will introduce the available options about the departure times. If the traveling time of the express-vehicle and general-vehicle between adjacent stations are 1 and 2 respectively. In addition,  $t_i^-$  denotes the earliest time at which user  $i$  can start the trip and  $t_{i+|J|}^+$  denotes the latest time at which  $i$  can reach his destination. If the two parameter are 3 and 8 respectively. Therefore, if user  $i$  chooses itinerary 3 by express-vehicle, then the options for departure time and transferring are as follows: 1) starting his trip at time slot 3 and transferring at any of the time slots  $\{4, 5, 6, 7\}$ ; 2) starting his trip at time slot 4 and transferring at any of the time slots  $\{5, 6, 7\}$ ; 3) starting his trip at time slot 5 and transferring at any of the time slots  $\{6, 7\}$ ; 4) starting his trip at time slot 6 and transferring at the time slots 7; so he has 10 options under itinerary 3 by express-vehicle. On the other hand, if he chooses itinerary 3 by general-vehicle, then his options are as follows: 1) starting his trip at time slot 3 and transferring at any of the time slots  $\{5, 6\}$ ; 2) starting his trip at time slot 4 and transferring at the time slots 6; so there are only three options by general-vehicle to carry out his trip in itinerary 3.

The remaining of this paper is based on the following preliminary assumptions.

- (1) All express-vehicle (general-vehicle) have the same speed and stopping time at every (non-terminal) station along a line.
- (2) Once a line run is completed, the associated vehicle becomes part of the fleet size of the line  $l + |\vec{\mathcal{L}}|$  ( $l - |\vec{\mathcal{L}}|$  if  $l \in \overleftarrow{\mathcal{L}}$ ).
- (3) Each user can choose express-vehicle or general-vehicle to carry out his trip, and the express-vehicle has a higher speed than the general-vehicle.
- (4) All the users only consider the strategies involving at most one transfer.

## B. Model formulation

In this section, we adopt some notions in Laporte et al. (2017). Additionally, our models are also an extension of the models in this paper.

### Indices

- $s$ : indices of nodes (stations);
- $a$ : indices of arcs;
- $l$ : indices of lines;
- $r$ : indices of options of departure times for users;
- $i$ : indices of user requests;
- $\pi$ : indices of itineraries;
- $t$ : indices of time slots;

### Input parameters

- $G$ : Graph corresponding to the public bus transit network;
- $S$ : the set of nodes, i.e., public bus stations,  $s \in S$ ;
- $A$ : the set of arcs,  $a \in A$ ;
- $\mathcal{L}$ : the set of lines,  $l \in \mathcal{L}$ ;
- $S_l$ : the subset of stations used by line  $l$ ,  $S_l \subseteq S$ ;
- $A_l$ : the subset of arcs that used by line  $l$ ,  $A_l \subseteq A$ ;
- $\overline{\mathcal{L}}$ : the set of directed path lines,  $\overline{\mathcal{L}} \subseteq \mathcal{L}$ ;
- $\hat{\mathcal{L}}$ : the set of directed cycle lines,  $\hat{\mathcal{L}} \subseteq \mathcal{L}$ ;
- $\overrightarrow{\mathcal{L}}$ : the set of path lines going forward,  $\overrightarrow{\mathcal{L}} \subseteq \mathcal{L}$ ;
- $\overleftarrow{\mathcal{L}}$ : the set of path lines going backwards,  $\overleftarrow{\mathcal{L}} \subseteq \mathcal{L}$ ;
- $\mathcal{N}$ : the set of stations of all lines,  $\mathcal{N} = \{(l, i) : l \in \mathcal{L}, i \in S_l\}$ ;
- $\mathcal{A}$ : the set of arcs of all lines,  $\mathcal{A} = \{(l, i, j) : l \in \mathcal{L}, (i, j) \in A_l\}$ ;
- $\tilde{\mathcal{N}}$ : the set of transfer stations;
- $\mathcal{A}^{(tra)}$ : the set of transfer edges,  $\mathcal{A}^{(tra)} = \{(l, i, n) : l \in \mathcal{L}, i \in S_l, n \in \tilde{\mathcal{N}}\}$ ;
- $T$ : the set of time slot,  $T = \{1, \dots, |T|\}$ ,  $|T|$ : the planning horizon,  $t \in T$ ;
- $\tau_i^e$ : the fixed travel time required by a express-vehicle to complete a line run in line  $l$ ;
- $\tau_i^g$ : the fixed travel time required by a general-vehicle to complete a line run in line  $l$ ;
- $Q^e$ : the capacity of a express-vehicle;
- $Q^g$ : the capacity of a general-vehicle;
- $c_l^e$ : the cost to locate a line run by a express-vehicle in line  $l$ , including fuel cost;
- $c_l^g$ : the cost to locate a line run by a general-vehicle in line  $l$ , including fuel cost;
- $\lambda^e$ : the cost of a express-vehicle;
- $\lambda^g$ : the cost of a general-vehicle;
- $\rho$ : the total available budget to locate line runs by the express-vehicle and general-vehicle;
- $\mathcal{K}^e$ : the fleet size of express-vehicle;
- $\mathcal{K}^g$ : the fleet size of general-vehicle;
- $I$ : the set of users' transportation requests;
- $t_i$ : the preferred departure times for user request  $i$ ;
- $t_{i+|I|}$ : the preferred arrival times for user request  $i$ ;
- $t_i^+$ : the earliest times that can serve the user request  $i$ ;
- $t_{i+|I|}^+$ : the latest times that can serve the user request  $i$ ;
- $\Pi$ : the set of possible itineraries in the public bus transit network,  $\pi \in \Pi$ ;
- $\Pi_i$ : the subset of itineraries that user  $i$  can choose from,  $\Pi_i \subseteq \Pi$ ;
- $\mathcal{L}_\pi$ : the set of lines related the itinerary  $\pi \in \Pi$ ;

- $|\mathcal{L}_\pi|$ : the number of lines contained by the itinerary  $\pi \in \Pi$ ;
- $\mathcal{L}_{i\pi}$ : the set of lines chosen by user  $i$  when itinerary  $\pi \in \Pi$  is selected;
- $\mathcal{R}_{i\pi}^e$ : the set of options available for user  $i$  by express-vehicle when itinerary  $\pi \in \Pi$  is selected,  $r \in \mathcal{R}_{i\pi}^e$ ;
- $\mathcal{R}_{i\pi}^g$ : the set of options available for user  $i$  by general-vehicle when itinerary  $\pi \in \Pi$  is selected,  $r \in \mathcal{R}_{i\pi}^g$ ;
- $m_{\pi a}$ : a binary parameter equals to 1 if the itinerary  $\pi \in \Pi$  contains arc  $a \in A$ , 0 otherwise;
- $\varphi_{i\pi r}^e$ : the cost that user  $i$  chooses express-vehicle under the itinerary  $\pi$  and option  $r$ ;
- $\varphi_{i\pi r}^g$ : the cost that user  $i$  chooses general-vehicle under the itinerary  $\pi$  and option  $r$ ;
- $t_{i\pi r}^e$ : the departure time slot of a express-vehicle in line  $l$  when user  $i$  chooses the itinerary  $\pi$  and the option  $r$ ;
- $t_{i\pi r}^g$ : the departure time slot of a general-vehicle in line  $l$  when user  $i$  chooses the itinerary  $\pi$  and the option  $r$ ;

### Decision Variables

- $\rho_l^e$ : the number of line runs to be deployed in line  $l$  by express-vehicle;
- $\rho_l^g$ : the number of line runs to be deployed in line  $l$  by general-vehicle;
- $\mathcal{K}_l^e$ : the number of express-vehicle initially allocated to line  $l$ ;
- $\mathcal{K}_l^g$ : the number of general-vehicle initially allocated to line  $l$ ;
- $x_{lt}^e$ : a binary variable equals to 1 if a express-vehicle starts a line run in line  $l$  at time slot  $t$ , 0 otherwise;
- $x_{lt}^g$ : a binary variable equals to 1 if a general-vehicle starts a line run in line  $l$  at time slot  $t$ , 0 otherwise;
- $y_{i\pi r}^e$ : a binary variable equals to 1 if user  $i$  chooses express-vehicle under the itinerary  $\pi$  and option  $r$ , 0 otherwise;
- $y_{i\pi r}^g$ : a binary variable equals to 1 if user  $i$  chooses general-vehicle under the itinerary  $\pi$  and option  $r$ , 0 otherwise;

**Mathematical Model:** The model for the problem can be expressed as follows. the objective function:

$$\begin{aligned} \min \{ & w_1 \sum_{i \in I} \sum_{\pi \in \Pi_i} \left( \sum_{r \in \mathcal{R}_{i\pi}^e} \varphi_{i\pi r}^e y_{i\pi r}^e + \sum_{r \in \mathcal{R}_{i\pi}^g} \varphi_{i\pi r}^g y_{i\pi r}^g \right) \quad (1) \\ & + w_2 \sum_{i \in I} \left[ 1 - \sum_{\pi \in \Pi_i} \left( \sum_{r \in \mathcal{R}_{i\pi}^e} y_{i\pi r}^e + \sum_{r \in \mathcal{R}_{i\pi}^g} y_{i\pi r}^g \right) \right] \\ & + w_3 \sum_{l \in \mathcal{L}} \sum_{t \in T} (c_l^e x_{lt}^e + c_l^g x_{lt}^g) + w_4 \sum_{l \in \mathcal{L}} (\lambda^e \mathcal{K}_l^e + \lambda^g \mathcal{K}_l^g) \} \end{aligned}$$

The objective function is to minimize the total user inconvenience. The first term denotes the inconvenience related to in-vehicle times (express-vehicle or general-vehicle), the penalties of changing lines and deviation between desired departure and arrival time, which are all about the users who have been served in transit network. The second term computes

the penalty cost of users not assigned to any service in transit network. The third term computes the cost of line runs and the last term computes the cost of fleet size. The first and second terms are user-oriented, while the the third and last terms are operator-oriented.

$$\sum_{\pi \in \Pi_i} \left( \sum_{r \in \mathcal{R}_{i\pi}^e} y_{i\pi r}^e + \sum_{r \in \mathcal{R}_{i\pi}^g} y_{i\pi r}^g \right) \leq 1, \quad i \in I \quad (2)$$

Constraint (2) ensures each user  $i$  can choose no more than one strategy, specifically, a user can only choose more than one combination of an itinerary and a potential timetable of express-vehicle or general-vehicle vehicle.

$$\sum_{t \in T} x_{it}^e \leq \rho_l^e, \quad l \in \mathcal{L} \quad (3)$$

$$\sum_{t \in T} x_{it}^g \leq \rho_l^g, \quad l \in \mathcal{L} \quad (4)$$

Constraint (3) guarantees no more than  $\rho_l^e$  line runs by express-vehicle can be located on each line  $l$ . Similarly, Constraint (4) guarantees no more than  $\rho_l^g$  line runs by general-vehicle can be located on each line  $l$ .

$$|\mathcal{L}_\pi| y_{i\pi r}^e \leq \sum_{l \in \mathcal{L}_\pi} x_{l t_{i\pi r l}^e}^e, \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi}^e \quad (5)$$

$$|\mathcal{L}_\pi| y_{i\pi r}^g \leq \sum_{l \in \mathcal{L}_\pi} x_{l t_{i\pi r l}^g}^g, \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi}^g \quad (6)$$

Constraint (5)-(6) ensure no request will be allocated to a strategy that cannot be carried out with the available line runs. i.e., if the right term equals to 0, then the left term must be 0;

$$\sum_{l \in \mathcal{L}} (c_l^e \rho_l^e + c_l^g \rho_l^g) \leq \rho \quad (7)$$

Constraint (7) guarantees the total cost of incurred by all the line runs by express-vehicle and general-vehicle cannot exceed the total available budget.

$$\sum_{i \in I} \sum_{\pi \in \Pi_i} \sum_{l \in \mathcal{L}_\pi} \sum_{r \in \mathcal{R}_{i\pi}^e} m_{\pi a} y_{i\pi r}^e \leq Q^e x_{lt}^e, \quad (8)$$

$$l \in \mathcal{L}, a \in A_l, t \in T$$

$$\sum_{i \in I} \sum_{\pi \in \Pi_i} \sum_{l \in \mathcal{L}_\pi} \sum_{r \in \mathcal{R}_{i\pi}^g} m_{\pi a} y_{i\pi r}^g \leq Q^g x_{lt}^g, \quad (9)$$

$$l \in \mathcal{L}, a \in A_l, t \in T$$

Constraint (8)-(9) consider the capacity limit of the express-vehicle and general-vehicle, ensuring the total number of users use an same arc by the same type vehicle that cannot exceed

the vehicle capacity of the express-vehicle or general-vehicle.

$$\sum_{l \in \mathcal{L}_\pi} (x_{l t_{i\pi r_1 l}^e}^e + x_{l t_{i\pi r_2 l}^g}^g) \leq (|\mathcal{L}_\pi| - 1) \quad (10)$$

$$+ \sum_{\pi' \in \Pi_i} \sum_{r' \in \mathcal{R}_{i\pi}^e} y_{i\pi' r'}^e, \quad i \in I, \pi \in \Pi_i, r_1 \in \mathcal{R}_{i\pi}^e, r_2 \in \mathcal{R}_{i\pi}^g$$

$$\sum_{l \in \mathcal{L}_\pi} (x_{l t_{i\pi r_1 l}^e}^e + x_{l t_{i\pi r_2 l}^g}^g) \leq (|\mathcal{L}_\pi| - 1) \quad (11)$$

$$+ \sum_{\pi' \in \Pi_i} \sum_{r' \in \mathcal{R}_{i\pi}^g} y_{i\pi' r'}^g, \quad i \in I, \pi \in \Pi_i, r_1 \in \mathcal{R}_{i\pi}^e, r_2 \in \mathcal{R}_{i\pi}^g$$

Constraint (10)-(11) ensure that if there exists a given itinerary and a vehicle type for a given user, then the user must be served.

$$\sum_{l \in \mathcal{L}_\pi} (x_{l t_{i\pi r_1 l}^e}^e + x_{l t_{i\pi r_2 l}^g}^g) + \sum_{\pi' \in \Pi_i} \sum_{r' \in \mathcal{R}_{i\pi}^e} y_{i\pi' r'}^e > \varphi_{i\pi}^e, \quad (12)$$

$$+ \sum_{\pi' \in \Pi_i} \sum_{r' \in \mathcal{R}_{i\pi}^g} y_{i\pi' r'}^g > \varphi_{i\pi}^g, \quad y_{i\pi' r'}^g \leq |\mathcal{L}_\pi|,$$

$$i \in I, \pi \in \Pi_i, r_1 \in \mathcal{R}_{i\pi}^e, r_2 \in \mathcal{R}_{i\pi}^g$$

Constraint (12) guarantees that if there exists a given itinerary and a vehicle type for a given user, then the user will not choose another combination of itinerary and vehicle type with higher cost.

$$\sum_{t'=1}^{\tau_l^e} x_{l t'}^e \leq \mathcal{K}_l^e, \quad l \in \mathcal{L} \quad (13)$$

$$\sum_{t'=1}^{\tau_l^g} x_{l t'}^g \leq \mathcal{K}_l^g, \quad l \in \mathcal{L} \quad (14)$$

Constraint (13) ensures that the total number of line runs by express-vehicle cannot exceed the fleet size of express-vehicle in line  $l$  during  $\tau_l^e$ , which is time required to complete a line run along line  $l$ . Specifically, during the time interval  $[1, \tau_l^e]$ , there are no more than  $\mathcal{K}_l^e$  express-vehicle. Similarly, no more than  $\mathcal{K}_l^g$  general-vehicle can be located on a same arc simultaneously.

$$\sum_{t'=1}^t x_{l t'}^e - \sum_{t'=1}^{t-\tau_l^e} x_{l+|\vec{\mathcal{L}}|, t'}^e \leq \mathcal{K}_l^e, \quad (15)$$

$$l \in \vec{\mathcal{L}}, t \in [\tau_l^e, |T| - \tau_l^e]$$

$$\sum_{t'=1}^t x_{l t'}^g - \sum_{t'=1}^{t-\tau_l^g} x_{l+|\vec{\mathcal{L}}|, t'}^g \leq \mathcal{K}_l^g, \quad (16)$$

$$l \in \vec{\mathcal{L}}, t \in [\tau_l^g, |T| - \tau_l^g]$$

Constraint (15) (Constrain (16)) describes the difference between the number of express-vehicle (general-vehicle) that have departed from line  $l$  and the number of express-vehicle

(general-vehicle) that have arrival at  $l$  cannot exceed the fleet size of express-vehicle (general-vehicle) in line  $l$ .

$$\sum_{t'=1}^t x_{lt'}^e - \sum_{t'=1}^{t-\tau_l^e} x_{l-|\mathcal{L}|,t'}^e \leq \mathcal{K}_l^e, \quad (17)$$

$$l \in \overleftarrow{\mathcal{L}}, t \in [\tau_l^e, |T| - \tau_l^e]$$

$$\sum_{t'=1}^t x_{lt'}^g - \sum_{t'=1}^{t-\tau_l^g} x_{l-|\mathcal{L}|,t'}^g \leq \mathcal{K}_l^g, \quad (18)$$

$$l \in \overleftarrow{\mathcal{L}}, t \in [\tau_l^g, |T| - \tau_l^g]$$

Constraint (17) (Constrain (18)) ensures the difference between the number of express-vehicle (general-vehicle) that have arrival at line  $l$  and the number of express-vehicle (general-vehicle) have departed from line  $l$  that cannot exceed the fleet size of express-vehicle (general-vehicle) in line  $l$ .

$$\sum_{l \in \mathcal{L}} \mathcal{K}_l^e \leq \mathcal{K}^e \quad (19)$$

$$\sum_{l \in \mathcal{L}} \mathcal{K}_l^g \leq \mathcal{K}^g \quad (20)$$

Constraint (19) (Constrain (20)) states the total number of express-vehicle general-vehicle in all lines, i.e. the total fleet size of express-vehicle general-vehicle, cannot exceed the number of available express-vehicle general-vehicle.

$$\rho_l^e, \rho_l^g \in \mathcal{Z}^+, \quad l \in \mathcal{L} \quad (21)$$

$$\mathcal{K}_l^e, \mathcal{K}_l^g \in \mathcal{Z}^+, \quad l \in \mathcal{L} \quad (22)$$

$$x_{lt}^e, x_{lt}^g \in \{0, 1\}, \quad l \in \mathcal{L}, t \in T \quad (23)$$

$$y_{i\pi r}^e \in \{0, 1\}, \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi}^e \quad (24)$$

$$y_{i\pi r}^g \in \{0, 1\}, \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi}^g \quad (25)$$

#### IV. CONCLUSION

This paper focuses on the integration of timetabling, multi-type vehicle scheduling and user routing in transit network considering fuel consumption. For the integrated problem, we not only pursues transfer coordination but also users' preferences for departure and arrival times. Each user need to be treated individually with hard time windows constraints for trip duration, departure and arrival times. Due to the capacity limits of vehicles, there may be some users not assigned to any service in transit network. Additionally, the multi-type vehicle and fuel consumption factors are also the key contributions. The objective is to minimize the inconvenience of the users ( i.e. in-vehicle times, line-transfer penalties and deviation between desired departure and arrival times), penalties of users not served and the cost of line runs and fuel consumption for operators.

For the integrated problem, we also establish a mixed-integer programming model for the integrated problem and solve it via CPLEX for small-size instances. From some numerical experiments, we get some conclusion: 1) If the express-vehicle and general-vehicle are identical, the problem in this paper will degenerate into the problem proposed in Laporte et al. (2017). 2) The express-vehicle with a higher

speed have a higher efficiency, then the fuel consumption of the former is less from the long term. 3) When the difference between the ticket price of the two types of vehicle is not big enough, users will be more partial to the express-vehicle.

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