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Bi-objective optimization of a reentrant flow shop scheduling with exact time lag considering energy cost

Ming Liu¹, Xin Liu², FeiFeng Zheng³ and Feng Chu⁴

Abstract—In this paper, we study a bi-objective optimization of the two-machine reentrant flow shop scheduling problem with an exact time lag considering energy consumption cost. This problem is proposed by Amrouche et al. (2016), in which each job must be operated from the first machine $M_1$ to the second machine $M_2$ and then back to the first machine $M_1$ with an exact time lag $l_j$ between the two operations on the first machine. The two objectives are: (1) minimization of the total energy consumption cost and (2) makespan. To describe the problem precisely, we propose a bi-objective mixed integer programming formulation. An $\epsilon$-constraint method and NSGA-II approach are proposed to obtain the optimal Pareto front and approximate Pareto solutions for the problem. The frameworks of the proposed methods are presented in this paper.

I. INTRODUCTION

With the development of technology and industry, new research issues keep emerging in the field of flow shop scheduling. The usual flow shop scheduling problem assumes that jobs are operated on all machines following a preset sequence, and a job visit each machine only once (c.f. Yang and Chern (2000), Yang et al. (2008), Wu and Lee (2009), Wang et al. (2015)). However, this assumption may be inconsistent with some production conditions in reality. Reentrant flow shop scheduling problem has been attracting more and more attention with the assumption that a job can visit a certain machine more than once since it was first proposed by Graves (1983).

Generally, the objective of reentrant flow shop scheduling problem is to minimize customer dissatisfaction and related cost, such as the number of tardy jobs, makespan and total tardiness etc. Environmental effects and energy consumption cost are seldomly considered in the previous relevant studies. As people become more and more concerned about the environmental problems and energy wast nowadays, it is necessary to take energy consumption cost into consideration.

In this paper, we study a bi-objective optimization of the two-machine reentrant flow shop scheduling problem with an exact time lag. The problem is proposed by Amrouche et al. (2016), where each job must be operated from the first machine $M_1$ to the second machine $M_2$ and then back to the first machine $M_1$ with an exact time lag $l_j$ between the two operations on the first machine. Amrouche et al. (2016) prove the NP-hardness of the problem and propose heuristics for the problem. We extended their work by establishing a bi-objective mathematical formulation and considering energy consumption cost. The main contributions of this paper are presented as follows.

1) We consider two objectives optimization of minimizing the total energy consumption cost and the makespan simultaneously;

2) A mixed integer programming formulation is established;

3) An $\epsilon$-constraint method and NSGA-II approach are proposed to obtain the optimal and approximate Pareto solutions for the problem.

The rest of this paper is structured as follows. Section 2 reviews the literature on reentrant flow shop scheduling problem and shop scheduling problem with an exact time lag and energy consumption. Section 3 describes the problem, and a mixed integer programming formulation is proposed. In Section 4, we propose solution methods to obtain the Pareto solutions for the problem. Section 6 concludes the work and indicate further research directions.

II. LITERATURE REVIEW

There has been a limited number of researches on reentrant flow shop scheduling problem (c.f. Hwang and Sun (1998), Jing et al. (2009), Chu et al. (2010), Huang et al. (2014), Sangsawang et al. (2015), Zhou et al. (2016), Amrouche et al. (2016)). Moreover, the number of previous relevant researches focusing on reentrant flow shop with exact time lags is relatively few.

A. Reentrant flow shop

The reentrant flow shop scheduling problem is first proposed by Graves (1983), and the author proposes a heuristic algorithm for the problem. Lev and Adiri (1984) investigate the V-shop problem, in which jobs are operated on $m$ machines in a preset sequence $M_1, M_2, \ldots, M_{m-1}, M_m, M_{m-1}, \ldots, M_2, M_1$. They prove that this problem is NP-hard, and also propose polynomial algorithms for some special cases. Wang et al. (1997) study a chain-reentrant shop scheduling problem, where each job is operated in the preset sequence $M_1, M_2, M_3, \ldots, M_m$, and then back to the primary machine $M_1$. The authors prove some properties of optimal solutions for the two-machine case.
Chen (2006) study a reentrant permutation flow shop scheduling problem, in which each job is planned to be operated on all machines in the following sequence, \( M_1, M_2, ..., M_m, M_1, M_2, ..., M_m \). The objective is to minimize the makespan. They propose a branch and bound algorithm for the problem. Based on that, Chen et al. (2008) develop a tabu search algorithm and a hybrid genetic algorithm for this problem.

Kim (2005) investigates the reentrant hybrid flow shop scheduling problem with the aim of minimizing the total tardiness. The author develops a heuristic algorithm for the problem and evaluates it by conducting numerical experiments. Based on that, Choi and Kim (2009) study the two-machine reentrant flow shop scheduling problem with the objective of minimizing the total tardiness. They propose a branch and cut algorithm for the problem.

Desprez et al. (2009) address an industrial production scheduling problem in reality, which is a hybrid reentrant flow shop scheduling problem with the objective of minimizing the total weighted number of tardy jobs. The authors develop a genetic algorithm according to the characteristics of the studied problem.

Chu et al. (2010) study a reentrant flow shop scheduling problem, which can be regarded as a special case of the problem investigated by Wang et al. (1997). The authors propose an optimal schedule for the problem with the aim of minimizing the makespan. For the problem that minimize the total flow time, they divide the problem into many subproblems according to the relationship between the processing time in different procedure, and propose algorithms and some dominant properties for the different subproblems.

Huang et al. (2014) propose a particle swarm optimization algorithm for the reentrant shop problem, which is a hybrid flow shop scheduling problem considering due dates and time windows. The objective of the studied problem is minimizing the total earliness and tardiness. Xu et al. (2014) develop a memetic algorithm for the reentrant permutation flow shop scheduling problem with the objective of minimizing the makespan.

Sangsawang et al. (2015) address a reentrant two-stage hybrid flow shop scheduling problem considering the blocking constraint. The objective of the problem is minimizing the makespan. They propose a hybrid particle swarm optimization and an adaptive auto-tuning hybrid genetic algorithm, which is based on the idea of relaxation and modification.

Zhou et al. (2016) investigate the reentrant hybrid flow shop scheduling problem with the objective of minimizing the total weighted completion time. They take inspection and repair operation into consideration, and these operations are conducted at the end of each cycle of processing. The authors propose a mathematical formulation for the problem and a differential evolution algorithm for the problem.

Other studies on reentrant shop scheduling may include Pan and Chen (2005), Dugardin et al. (2010), Jing et al. (2011), Ying et al. (2014), Shen et al. (2016), Rifai et al. (2016), etc.

B. Shop scheduling with time lags

The idea of time lags has been studied in various scheduling problems, in particular in the two-machine flow shop and the single machine case with the name of coupled-task problem. In the two-machine flow shop problem, the notation of time lag of a job represents that the time difference between the start time of the job on the second machine and the completion time of it on the first machine. Yu et al. (2004) prove the NP-hardness in the strong sense of the two-machine flow shop scheduling problem. For the single machine case with the name of couple-task problem, the process of each job contains two individual operations with required delay time between them. Ahr et al. (2004) present an exact algorithm based on dynamic programming for this problem.

Amrouche and Boudhar (2013) introduce the notion of time lags to the two-machine reentrant shop scheduling problem with the objective of minimizing the total completion time. In the problem, each job must be processed following the sequence \( M_1, M_2 \) and then back to \( M_1 \) with an identical exact time lag between the completion time of the first operation and the start time of the third operation. Based on that, Amrouche and Boudhar (2016) prove the NP-hardness of a special case and given some special subproblems that can be solved in polynomial time.

Amrouche et al. (2016) thoroughly study a chain-reentrant shop scheduling problem with an exact time lag in order to minimize the makespan. They propose heuristic algorithms for the problem with identical time lags, and establish a new NP-hardness result and some polynomial cases. However, the authors have not given the mathematical formulation for the problem. Based on this work, we extend the problem by considering energy consumption cost as one of the objectives.

C. Sustainability considerations in shop scheduling

Sustainability considerations in shop scheduling have been attracting more and more attention. Moreover, electricity is the main form of energy consumption in manufacturing industry (Sun and Li, 2013). Mouzon et al. (2007) develop operational methods for the minimization of the energy consumption of manufacturing equipment by turning machines off at idle times. Mouzon and Yildirim (2008) propose a metaheuristic to decide the timing and length of operation and turn on operation to minimize total tardiness and total energy consumption simultaneously. Che et al. (2016) address an energy-conscious single machine scheduling problem under time-of-use (TOU) electricity tariffs, where the prices of electricity are various in different moment of a day. They propose a MILP formulation and a greedy insertion heuristic algorithm for the problem.

Demir et al. (2014) study a bi-objective Pollution-Routing problem, and the two objectives are minimizing the fuel consumption and the driving time of vehicles. Ding et al. (2015) consider a permutation flow shop scheduling problem with the objective of minimizing the carbon emissions and the makespan simultaneously. Mansouri et al. (2016)
investigate a two-machine flowshop scheduling problem by analyzing the trade-off between minimizing the makespan and total energy consumption. They propose a mixed integer bi-objective optimization formulation and a heuristic. Wang et al. (2016) investi-gate a single machine batch scheduling problem considering energy cost with the objective of minimizing the makespan and total energy cost simultaneously. A bi-objective discrete-time mathematical formulation and two heuristic algorithms are proposed.

Other studies on shop scheduling problem considering energy consumption may include Luo et al. (2013), Shrouf et al. (2014), Sharma et al. (2015), etc.

The framework presented in this paper extends the literature on two-machine reentrant flow shop scheduling by considering two objectives of minimizing the total energy consumption and the makespan.

III. PROBLEM DESCRIPTION AND FORMULATION

In the problem we discussed here, there are \( n \) jobs planned to be processed on two machines. Every job is operated on the machines following a preset sequence. A job \( j \) should be processed in the order \( M_1, M_2 \) and then back to machine \( M_1 \), and there is an exact time lag \( l_j \) between the two operations on the first machine.

The aim of this problem is to find a set of sequences of the jobs and determine the starting time for each job in different procedure. The two objectives are minimizing the total energy consumption and minimizing the makespan. We study the problem by considering non-identical job sizes and the time-of-use (TOU) electricity prices. Besides, we consider the reentrance by transforming the second procedure on the primary machine into a virtual machine, as shown in Fig. 1.

![Fig. 1. Framework of initial solution generation](image)

To illustrate the problem, a mixed integer programming formulation is established below.

A. Notation

Indices:
- \( i, j \): indices of jobs, \( i \neq j \).
- \( h \): index of procedures.

Parameters:
- \( N \): set of jobs, i.e. \( N = \{1, ..., n\} \).
- \( M \): set of machines, i.e. \( M = \{1, ..., m\} \).
- \( H \): set of procedures.
- \( P_k \): set of procedures on machine \( k \).
- \( l_i \): the exact time lag of job \( i \).
- \( p_{hi} \): the processing time of job \( i \) in procedure \( h \).
- \( c_p \): the unit price of energy consumption at time \( t \).
- \( L \): a large enough number.

Variables:
- \( x_{ij}^h \): equal to 1 if job \( i \) is processed before job \( j \) in procedure \( h \), 0 otherwise.
- \( z_{hi}^t \): equal to 1 if job \( i \) is being processed in procedure \( h \) at time \( t \), 0 otherwise.
- \( S_{hi} \): positive integer, the start time of job \( i \) in procedure \( h \).
- \( C_{hi} \): positive integer, the completion time of job \( i \) in procedure \( h \).
- \( C_{max} \): positive integer, the makespan.

B. A mathematical formulation

We describe the two objective functions and the main constraints below. The first objective \( f_1 \) is to minimize the makespan, and the second one \( f_2 \) represents the total energy cost.

\[
\min f_1 = C_{max} \quad (1)
\]

In the minimization of item \( f_2 \), as the total energy cost is the function of the time of energy consumption and the price of electricity, we calculate it based on cumulative energy cost.

\[
\min f_2 = \sum_{i \in N} \sum_{t \in T} \sum_{h \in H} z_{hi}^t c_p \quad (2)
\]

In the problem of reentrant flow shop scheduling with exact time lag, some constraints must be satisfied and we describe them below. For two jobs \( i \) and \( j \) processed in a certain procedure \( h \), job \( i \) is either processed before job \( j \) or after it.

\[
s.t. \quad x_{ij}^h + x_{ji}^h = 1, \quad h \in H, i, j \in N, i \neq j \quad (3)
\]

Each job must be processed in a procedure without interrup-tion, which means that there is no preemption allowed in the problem.

\[
C_{hi} = S_{hi} + p_{hi}, \quad h \in H, i \in N \quad (4)
\]

The start time of a job \( i \) in procedure \( h \) must be after the its completion time in the previous procedure \( h - 1 \).

\[
S_{hi} \geq C_{h-1,i}, \quad h \in H, i \in N \quad (5)
\]

Each job is planned to be processed in the order \( M_1, M_2 \) and then return back to \( M_1 \) after an exact time lag \( l_i \).
\[ S_{3i} = C_{1i} + l_i, \quad i \in N \]  

The start time of a job \( i \) must be ensured to be after the completion time of jobs prior to it in procedure \( h \).

\[ S_{hj} + (1 - x^h_{ij})L \geq C_{hi}, \quad h \in H, i, j \in N, i \neq j \]  

Item \( z^t_{hi} \) represents whether job \( i \) is being processed in procedure \( h \) at time \( t \).

\[ C_{hi} \geq (1 - z^t_{hi})L + t, \quad h \in H, i \in N, t \in T \]  

\[ S_{hi} \leq (1 - z^t_{hi})L + t - 1, \quad h \in H, i \in N, t \in T \]  

\[ \sum_{t \in T} z^t_{hi} = p_{hi}, \quad h \in H, i \in N \]  

Since that procedure 1 and procedure 3 are operated on the machine \( M_1 \), constraint below ensures that there cannot be more than one job being processed on the machine \( M_1 \) at a certain time \( t \).

\[ \sum_{i \in N} z^t_{i1} + \sum_{i \in N} z^t_{3i} \leq 1, \quad t \in T \]  

The ranges of decision variables are given below.

\[ x^{h}_{ij} \in \{0, 1\}, \quad h \in H, i, j \in N, i \neq j \]  

\[ z^t_{hi} \in \{0, 1\}, \quad h \in P, i \in N, t \in T \]  

\[ S_{hi}, C_{hi} \geq 0, \quad i \in N, h \in H \]  

IV. SOLUTION APPROACH

For bi-objective optimization problems, the relationship between the two objectives is usually negative correlation. There is no single optimal solution, and the desired output is a solution set (i.e. Pareto front) instead. In this section, to solve the two-machine reentrant flow shop scheduling problem with an exact time lag, we attempt to obtain the Pareto solutions with \( \epsilon \)-constraint method and NSGA-II approach. In the following, we present the frameworks of the proposed methods.

A. \( \epsilon \)-constraint method

The \( \epsilon \)-constraint method is widely used to obtain the optimal Pareto solutions. It is based on the idea of focusing on a single objective with restricting remaining objectives. We solve a multi-objective solution with \( \epsilon \)-constraint method by setting one of the objectives, i.e. \( f_1 \), as a single objective and transforming the other objective into a constraint with a gradual reduction value \( \epsilon \) in each iteration. The upper bound and lower bound of the two objectives need to be obtained and considered as a constraint. For the bi-objective two-machine reentrant flow shop problem with an exact time lag, the following concepts are needed (Bérubé et al. (2009)).

- Ideal point: let \( f^I = (f^I_1, f^I_2) \) with \( f^I_1 = \min \{ f_1(X) \} \) and \( f^I_2 = \min \{ f_2(X) \}, X \in \mathbb{Z}^N \);
- Nadir point: let \( f^N = (f^N_1, f^N_2) \) with \( f^N_1 = \min \{ f_1(X) : f_2(X) = f^I_2 \} \) and \( f^N_2 = \min \{ f_2(X) : f_1(X) = f^I_1 \} \), \( X \in \mathbb{Z}^N \);
- Extreme point: \( f^E = (f^E_1, f^E_2) \) are two extreme points on the Pareto front.

In this paper, the two objectives are the total energy consumption cost and makespan. The makespan is regarded as a constraint in the method. The \( \epsilon \)-constraint method is detailed below:

**Algorithm 1 Framework of \( \epsilon \)-constraint method**

**Require:** Parameters for proposed problem

\[ f^I_1 = \min \{ f_1(X) \}; \quad f^I_2 = \min \{ f_2(X) \}; \quad f^N_1 = \min \{ f_1(X) : f_2(X) = f^I_2 \}; \quad f^N_2 = \min \{ f_2(X) : f_1(X) = f^I_1 \}; \quad F_{apo} = [f^I_1, f^I_2]; \quad \Delta = 1; \quad \epsilon = f^N_2 - \Delta; \]

**while** \( \epsilon \geq f^N_2 \) **do**

\[ f_1 = \min \{ f_1(X) : f_2(X) \leq \epsilon \}; \quad f_2 = \min \{ f_2(X) : f_1(X) = f_1; \quad F_{apo} = [f_{apo}, f_1, f_2]; \quad \epsilon = f_2 - \Delta; \]

**end while**

\[ F = \text{non-dominated sort}(F_{apo}); \]

B. Optimal solutions by CPLEX

We adopt \( \epsilon \)-constraint method to obtain optimal solutions for the proposed mathematical model. The parameters are generated randomly, as shown below.

- Number of jobs, \( N = 4 \);
- Number of machines, \( M = 2 \);
- Number of procedures, \( H = 3 \);
- Planning time horizon, \( T = 80 \);
- Processing time of jobs in different procedure:

\[
\begin{pmatrix}
3 & 4 & 5 & 2 \\
2 & 3 & 4 & 3 \\
3 & 2 & 4 & 5
\end{pmatrix}
\]

- Time lags, \( l_j = [5, 6, 7] \);
- The prices of electricity in different time of a day, which is randomly generated from 1 to 4.

We use \( \epsilon \)-constraint method to obtain the optimal Pareto solutions for the instance above, as shown in the Fig.2. The computational time is 300 seconds. Table 1 represents the solution combinations of the results obtained.

C. NSGA-II approach

In this section, we present NSGA-II method in order to solve the large scale cases efficiently. NSGA-II has been widely used to solve the multi-objective problems, which is first proposed by Deb et al. (2002). Details of NSGA-II
Fig. 2. Framework of initial solution generation

TABLE I

<table>
<thead>
<tr>
<th>Pareto frontier</th>
<th>total energy cost</th>
<th>makespan</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>69</td>
<td>59</td>
</tr>
<tr>
<td>3</td>
<td>71</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>74</td>
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<td>6</td>
<td>79</td>
<td>33</td>
</tr>
<tr>
<td>7</td>
<td>81</td>
<td>32</td>
</tr>
<tr>
<td>8</td>
<td>85</td>
<td>31</td>
</tr>
</tbody>
</table>

are presented as follows, the initialize_variables() procedure generates a set of feasible solutions for the problem, the initial solutions are sorted by non_domination_sort(). For each iteration, the offspring solutions are generated by genetic_operator() procedure. Then the solutions are sorted by non_domination_sort() procedure. K members from the next non_selected level \( F_i \) are needed to meet the requirement of \( pop \).

Algorithm 2 Framework of NSGA-II method

Data: Parameters for proposed problem.
Result: Pareto front solution set.

for \( i = 1; \text{gen do} \)

pool = gen/2 (% Matching pool size);
tour = 2 (% Tournament size);
parent\( P = \) tournament_selection(P, pool, tour);
offspring\( P = \) genetic_operator(parent\( P \));
intermediate\( P = parent\( P \cup offspring\( P \));
\( F_i = \) non_domination_sort(intermediate\( P \));
P = \emptyset;
i = 1 (%non_domination level index);
while \(|P| + |F_i| \leq pop \) do

P = P \cup F_i;
i = i + 1;
end while
K = pop - |P| (% Number of individuals to be further added);
P = P \cup selection(P, F_i, K)
end for

V. CONCLUSIONS

This paper investigates a bi-objective two-machine reentrant flow shop scheduling problem with an exact time lag, which aims to minimize the total energy consumption cost and makespan simultaneously. A bi-objective mathematical formulation is established to illustrate the problem. To obtain Pareto solutions of the problem, an exact \( \epsilon \)-constraint method and NSGA-II approach are proposed.

In future research, we should pay more attention on (i) more efficient bi-objective algorithm and (ii) some other factors that may affect the objectives, such as the speed of machines and the salary of operators.
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REFERENCES