Pricing and Risk Analysis of a Long-Term Care Insurance Contract in a non-Markov Multi-State Model

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Introduction ●○			
Context	t		

- Multi-state models are the most natural tools for pricing and reserving Long Term Care (LTC) insurance guarantees.
- ▶ In literature, a Markov model is generally used \rightarrow may be too strong.
- Lack of (detailed) national data. It's hard to have precise and accurate insurance data.
- Classical inference methodologies:
 - are based on crude intensities,
 - use GLM Poisson models that depend on age or duration time (e.g. Haberman and Pitacco, 1998; Pritchard, 2006; Levantesi and Menzietti, 2012; Fong *et al.*, 2015).

Introduction ○●			
Aims			

- Helms et al. (2005) use a direct regression model for annual transition probabilities with a Markov model based on pseudo-values (Andersen et al., 2003).
- This allows to measure the effect of observed covariates. However, this framework requires that the Markov assumption to be satisfied.
- ► We propose dynamic regression methods using directly the transition probabilities of a non-Markov illness-death model.
- Interests for pricing:
 - Flexible and takes into account the effect of covariates over time.
 - Offers a way to avoid or reduce the bias induced by the Markov assumption.

Results

Long-Term Care Insurance Model

- ► Consider an illness-death model X with only one heavy dependency state.
- ▶ Transition probabilities $h \rightarrow j$ between s and t with individual covariates Z

 $p_{hj}(s,t;Z(s)) = \mathbb{P}\left(X(t) = j \mid X(s) = h, Z(s)\right).$

A very simple annuity guarantee paid in dependency.



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Long-Term Care Insurance Model

- Actuarial calculation in discrete time at the valuation time 0 for a s-year old insured.
- Actuarial value of premium payments

$$a_{1}(s; Z(s)) = \sum_{\tau=0}^{\omega-s-1} v^{\tau} p_{11}(s, s+\tau; Z(s)) \pi.$$

• Actuarial values of annuity payments in state h = 1, 2

$$A_{h}\left(s;Z\left(s\right)\right) = \sum_{\tau=0}^{\omega-s-1} v^{\tau} p_{h2}\left(s,s+\tau;Z\left(s\right)\right) B.$$

- ► Under the Markov assumption, we can estimate $p_{hj}(t) = p_{hj}(t-1,t)$, and then use the Chapman-Kolmogorov equations.
- Calculating these probabilities is not simple without the Markov assumption. For state 1, we suppose that it's verified (no previous state).

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Non-Parametric Transition Probabilities

- Based on de Uña-Álvarez and Meira-Machado (2015) and Guibert and Planchet (2016)
- ▶ Introduce *S*, the lifetime in healthy state, and *T*, the overall lifetime.
- ► Let *C* and *L* be a right-censoring and a left-truncation variables. Truncation only occurs in state 1.
- ▶ Suppose $(C,L) \perp (S,T,Z)$ and $C \perp L$. If $L \leq S$, we observe

$$\begin{cases} \widetilde{S} = \min(S, C) \text{ and } \gamma = \mathbb{1}_{\{S \le C\}}, \\ \widetilde{T} = \min(T, C) \text{ and } \delta = \mathbb{1}_{\{T \le C\}}. \end{cases}$$

Consistent estimators of transition probabilities

By selecting individuals in $\mathcal{L}_{s}^{1} = \left\{i: \widetilde{S}_{i} > s\right\}$, we have

$$\widehat{p}_{11}\left(s,t\right) = 1 - \widehat{H}_{s}^{1}\left(t\right), \ \widehat{p}_{12}\left(s,t\right) = \widehat{H}_{s}^{1}\left(t\right) - \widehat{F}_{s}^{1}\left(t\right),$$

with $\hat{H}_{s}^{1}(t)$ and $\hat{F}_{s}^{1}(t)$ be the Kaplan-Meier c.d.f of *S* and *T* on \mathcal{L}_{s}^{1} (Tsai *et al.*, 1987).

How Calculate Jackknife Pseudo-Values?

Let n_s^1 , the number of individuals alive and not truncated at time *s*.

- **1** For each individual $i \in \{1, ..., n_s^1\}$, remove *i* from the sample
- **2** Compute probabilities of interest on this subsample of cardinal $(n_s^1 1)$

 $\widehat{p}_{11}^{(-i)}\left(s,t
ight)$ and $\widehat{p}_{12}^{(-i)}\left(s,t
ight)$

3 Compute their jackknife pseudo-values

$$n_{s}^{1}\widehat{p}_{11}(s,t) - \left(n_{s}^{1}-1\right)\widehat{p}_{11}^{(-i)}(s,t)$$

and

$$n_{s}^{1}\widehat{p}_{12}(s,t) - \left(n_{s}^{1}-1\right)\widehat{p}_{12}^{(-i)}(s,t)$$

Dynamic Pseudo-Values

- ▶ Define a set of landmark time points $S = \{s_1, ..., s_K\}$ and, for each s_k , a grid of points T_{s_k} .
- ► Let be a (multivariate) parameter $\theta = \mathbb{E}[f(S,T)]$ for some function *f*. For varying landmark time points *s* and for several prediction points *t*, our aim is to construct a regression GLM model of the form

$$\theta_i = \mathbb{E}\left[f\left(S_i, T_i\right) \mid Z_i\right] = g^{-1}\left(\beta^{\top} Z_i\right),$$

with g, an invertible link function.

For each landmark *s*, define the jackknife pseudo-values for individual *i* at each time $t \in T_s \rightarrow$ responses in the regression model.

$h \rightarrow j$	f_{st}^{hj}	Pseudo-values	Selection
$p_{11}\left(s,t\right)$	$1_{\{S>t\}}$	$\widehat{\theta}_{ist}^{11} = n_s^1 \widehat{p}_{11}\left(s,t\right) - \left(n_s^1 - 1\right) \widehat{p}_{11}^{(-i)}\left(s,t\right)$	\mathcal{L}^1_s
$p_{12}\left(s,t\right)$	$1_{\{S \le t\}} - 1_{\{T \le t\}}$	$\widehat{\theta}_{ist}^{12} = n_s^1 \widehat{p}_{12} \left(s, t \right) - \left(n_s^1 - 1 \right) \widehat{p}_{12}^{(-i)} \left(s, t \right)$	\mathcal{L}^1_s

Stacked models for each transition h → j: hj index is suppressed to simplify.

		Estimation Method ○○○●○○		
F	Regression Approach			

regression Approach

- Idea: If landmark times are close enough, the effect of covariates should vary smoothly over s. Our approach is inspired by the dynamic regression model developed by Nicolaie et al. (2013).
- Consider $f = (f_{st}, s \in S, t \in T_s), \theta_i = (\theta_{ist}, s \in S, t \in T_s)$ and a link

$$\theta_{ist} = g^{-1} \left(\beta \left(s \right)^{\top} Z_{it} \left(s \right) \right).$$

- Each component l of the vector $\beta(s)$ can be written as a polynomial function $\beta_l(s) = \beta_l^{\top} q_l(s)$.
- We consider simultaneously all landmarks, which can now interact with covariates. Polynomial function can depend on time s and time t.

	Estimation Method ○○○○●○		

Regression Approach

The vector β, such that β (s) = Q (s) β, is estimated using the GEE approach (Liang and Zeger, 1986), i.e. by resolving the score equation

$$\mathcal{U}\left(eta\left(s
ight)
ight)=\sum_{i}\left(rac{\partial}{\partialeta\left(s
ight)} heta_{is}
ight)M_{i}^{-1}\left(\widehat{ heta}_{is}- heta_{is}
ight)=0$$

- ► A working covariance matrix M_i should be fixed (e.g. independance, AR1). To have consistent results, we follow Nicolaie *et al.* (2013) and choose an independence working correlation matrix.
- ► Model selection can be done using the QIC criteria (Pan, 2001).
- ► Variance of $\hat{\beta}(s)$ is obtained by using a sandwich estimator.
- Convergence and asymptotic normality of β (s) is not formally demonstrated here, but is undoubtedly satisfy (Graw *et al.*, 2009). However, performances should be investigated.

The approach of Helms *et al.* (2005)

- ► Annual transition probabilities p₁₁ (t), p₁₂ (t), and p₂₃ (t) are estimated based on the Aalen-Johansen estimator (Aalen and Johansen, 1978) for Markov models.
- Compute the jackknife pseudo-values for these probabilities.
- > Define a regression GLM model for their jackknife pseudo-values.
- Parameters are also estimated using the GEE approach.

	Results	
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Insurance Data

- Individual insurance data with left-truncation (only in healthy state) and right-censoring.
- ▶ \simeq 15,900 (63% of women) contracts observed on a 13-years period and almost 47% are censored in healthy state \rightarrow only an extract of the database to have acceptable computation times.
- Age period considered: 65-90.
- Mortality varies a lot with pathologies causing entry into dependency.

	Results	
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Estimated Transition Probabilities



Figure: $\hat{p}_{11}(s, t)$

	Results	
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Estimated Transition Probabilities



Figure: $\widehat{p}_{12}(s,t)$

Premiums Calculation - Markov vs. non-Markov estimation

Principle of equivalence for a s-year old insured

 $a_{1}\left(s;Z\left(s\right)\right) = A_{1}\left(s;Z\left(s\right)\right)$

- Annual payments.
- B = 1 Eur, $v = (1 + 0.01)^{-1}$, Age limit= 90.



Figure: Comparison of Premiums.

Results for $p_{11}(s, t)$ and $p_{12}(s, t)$ with non-Markov specification

- ▶ $S = \{65, 67, \dots, 85\}$ and, for each $s, T_s = \{s + 1, s + 2, \dots, 90\}$.
- Link tested for the GLM specification: identity, logit, clog-log.
- Set of factors tested: the arrival age t, t² and t³, the gender, the generation, and their interactions with age s and s².
- Model selection: QIC, log-Lik and the number of parameters.
- Validation: Graphical analysis of residuals.

Best model specifications

▶
$$p_{11}(s,t)$$
:
clog-log (θ_{ist}) ~ $s * (t + t^2 + Gender) + s^2 * (t + Gender) + s * t^3 * 1_{\{t-s>3\}}.$

▶
$$p_{12}(s,t)$$
: $\theta_{ist} \sim s * t + t * Gender + s^2$.

- The gender' effect is important and increases with age.
- Surprisingly, the generation has no real effect. Possible explanation: the length of our data.

	Results	
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Fitted values for $p_{12}(s, t)$



Figure: $\hat{p}_{12}(s, t)$ - Predicted transition probabilities for men and women with associated 95% prediction intervals.

Results for $p_{11}(t)$, $p_{12}(t)$ and $p_{23}(t)$ with Markov specification

▶ $t \in \{65, 66, \dots, 90\}.$

- Working covariance matrix: independence and autoregressive(AR-I)
- Same analysis and set of factors.

Best model specifications

▶
$$p_{11}(t)$$
: clog-log $(\theta_{it}) \sim t + t^2 + t^3 + Gender$.

•
$$p_{12}(t): \theta_{it} \sim t + t^2 + t^3$$
.

▶
$$p_{23}(t)$$
: clog-log $(\theta_{it}) \sim t + t^2 + t^3 + t^4 + t^5 + Gender$.

- The models estimated with independent and autoregression working covariance matrices give similar results.
- ► The gender' effect is significant, but remains very small, whereas it's seems to be higher if we consider residuals for p₁₁ (t) and p₂₃ (t).
- The generation has no real effect.

	Results	
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Actuarial Application - Premiums Comparison



Figure: Comparison of Premiums for men and women. Solid line: Markov. Dotted line: Non-Markov.

		Conclusion	

Conclusion

- New dynamic regression approach based on pseudo-values for the transition probabilities of a non-Markov illness-death model.
 - flexible methodologies,
 - \blacktriangleright the effects of covariates are directly estimated on pseudo-values \rightarrow more interpretable.
- Measure the bias induced by the use of a Markov specification.
- New researches are need to check models' performances and analyze the asymptotic properties of our estimators.

Results

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