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Abstract

The food grain supply chain problem of the Public Distribution System (PDS) of India is addressed in this paper to satisfy the demand of the deficit Indian states. The problem involves the transportation of bulk food grain by capacitated vehicles from surplus states to deficit states through silo storage. A mixed integer non-linear programming (MINLP) model is formulated which seeks to minimize the overall cost including bulk food grain shipment, storage, and operational cost. The model incorporates the novel vehicle preference constraints along with the seasonal procurement, silo storage, vehicle capacity and demand satisfaction restrictions. The management of Indian food grain supply chain network is more intricate and difficult issue due to many uncertain interventions and its chaotic nature. To tackle the aforementioned problem an effective meta-heuristic which based on the strategy of sorting elite ants and pheromone trail updating called Improved Max-Min Ant System (IMMAS) is proposed. The solutions obtained through IMMAS is validated by implementing the Max-Min Ant System (MMAS). A sensitivity analysis has been performed to visualize the effect of model parameters on the solution quality. Finally, the statistical analysis is carried out for confirming the superiority of the proposed algorithm over the other.

Keywords: Distribution system, Supply chain management, Inventory, Transportation, Mixed integer non-linear programming, Ant colony optimization

1. Introduction

India is the second largest food grains (wheat and rice) producer in the world after China. Despite this fact, India is still facing the challenge of feeding the high-quality, nutritious and safe food to more than one billion peoples in the country (Mukherjee et al. 2013). India is

ranked at 80th position out of 104 countries in the Global Hunger Index (GHI) and lagging the neighboring country such as Nepal, Sri Lanka and China (Von Grebmer et al. 2015). In India, every year around 25-30 percent of agricultural production gets wasted due to the improper handling and storage, poor logistics, inadequate storage and lack of transportation infrastructure (Sachan et al. 2005). As per the Food and Agriculture Organization (FAO) of the United Nations estimation, out of total food produced for human consumption, 32% of the food by weight was wasted across the entire food supply chain in 2009, equivalent to around 1.3 billion tons (FAO, 2011). According to World Bank Report (1999), post-harvest losses in India amount to 12 to 16 million metric tons of food grains each year, an amount that the World Bank stipulates could feed one-third of India's poor. The monetary value of these losses amounts to more than Rs 50,000 crores (7515.689 million US dollars) per year (Singh 2010; Naik and Kaushik 2011). The improper transportation planning, untimely deliveries, mismatched demand-produce scenario, inadequate infrastructure and highly inefficient supply chain are the primarily causes behind this (Maiyar & Thakkar 2017, Parwez 2014). In the past few decades, the major concern of developing countries was on the increasing the food production to feed the growing population and the advanced agricultural production technologies have helped to increase the production, but they have not given the proper attention towards the reduction of losses.

The targeted Public Distribution System (PDS) is the national food security system of India, which provides food grains to poor people of the society at a subsidized rate. The procurement, storage, movement and distribution to final consumers are the major stages of the food grain supply chain. The Food Corporation of India (FCI) is the central nodal agency which handles all these activities as shown in Fig. 1. The various states in India are categorized into producing and consuming states based on production quantity of food grains. The food grain is procured under two scheme, i.e. centralized and decentralized procurement scheme. In a centralized system, FCI and several State Government Agencies (SGAs) procure the food grain from the farmers in procurement centers, located in different parts of states, at minimum support price (MSP). Next, this procured food grain is transported to FCIs central warehouses for storage. The Government of India (GOI) makes the annual allocation at a uniform central issue price (CIP) to each consuming state and Union territories based on the demand of the state and off-take in the previous period. The consuming state takes care of the distribution of food grains from state depot to the final consumer. Primarily, the interstate movement of food grain from producing state to consuming state depot is carried out by rail mode and intrastate movement

through road. Under decentralized procurement (DCP), SGAs procures, store and distribute food grains to beneficiaries through PDS on behalf of the GOI.

Presently, FCI is facing numerous major issues and challenges related to food grain storage and transportation. FCI yearly transports around 40 to 50 million tons of food grains through rail, road and waterways across the country which incurred the average expenditure of 47.2737 billion (CAG, 2013). The key issues of food grain supply chain include a huge amount of transportation and handling cost, underutilization of existing storage facilities, leakages in PDS, manpower shortages, vague buffer stock norms, unavailability of a sufficient number of vehicles, manual loading and unloading of gunny bags and lack of modern storage facilities. FCI has to maintain the operational and buffer stock of food grains in deficit states for food security purpose. Currently, PDS is having a large network of 5.13 lakh Fair Price Shops (FPS) in throughout the country which becomes a largest retail system in the world. To handle all these real time major issues and challenges of Indian PDS, FCI needs the effective storage and movement plan of food grains with time.

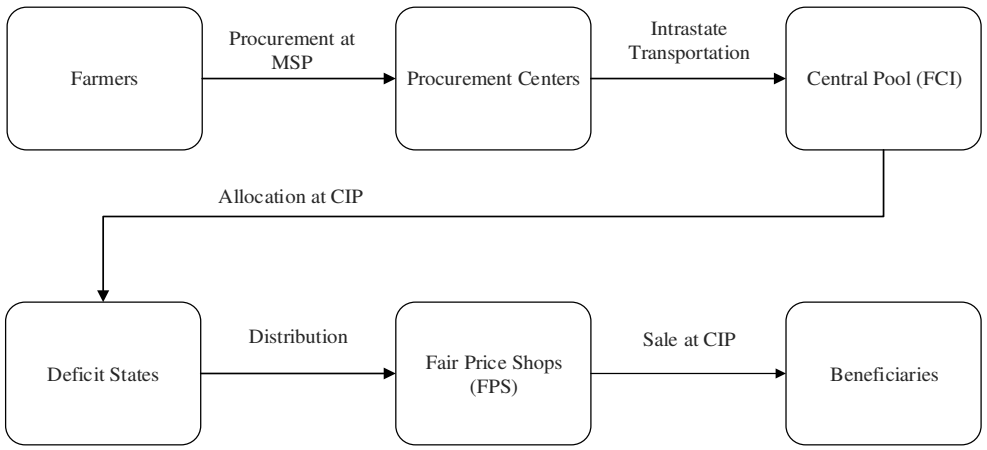

Fig. 1. PDS activities

In this paper, we have examined the issue of bulk food grain transportation between producing and consuming states along with silo storage considering operational (handling) cost inside the silos. Food grain supply chain has been divided into four stages as described follows.

1. Intrastate food grain transportation from procurement centers to silos in surplus states.

2. Interstate shipment from surplus state silos to deficit state silos.

3. The grain shipment from deficit state silos up to block level and

4. The food grain shipment from block level to fair price shops.

In this study, we consider major wheat producing and consuming states like Punjab, Haryana, Madhya Pradesh, Uttar Pradesh, Rajasthan and Maharashtra, Tamilnadu, Karnataka, West Bengal, respectively.

The food grain transportation and storage problem is complex with below mentioned numerous constraints and specificities.

The FCI has to take the efficient decisions about "from which surplus nodes to which surplus state silos and when to transport the food grains" in order to minimize the food grain supply chain cost. The operational and inventory holding cost of food grain at the surplus state silos, availability of food grain at surplus nodes, silo storage capacity, the demand of particular deficit state and availability of different capacitated vehicles are considered.

The next important goal is to minimize the total numbers of vehicles required for food grain transportation. The total time requires for movement of food grains is influenced by the different capacitated vehicles because if high capacity vehicles are utilized then it transports the food grains in fewer numbers of the trip than low capacitated vehicles. However, in real time scenario sufficient numbers of each type of capacitated vehicles may not be available during the particular time period. In general, we can say that if high capacitated vehicles are given the first priority than low capacitated vehicles, then the cost and time require for movement will be minimized. Thus, owing to all these vehicles related issues the novel vehicle preference constraints are formulated for shipping the food grain, which has not been addressed in most of the previous transportation related literature.

Two main contributions of this paper are as follows. First, a novel MINLP mathematical model is formulated to minimize the food grain supply chain cost in India. It considers simultaneously the seasonal procurement, heterogeneous vehicles and their fixed costs, inventory and operational costs of food grain, specific vehicle preference constraints, capacitated silos, intermodal transportation and a definite planning horizon. Second, we propose a variant of MMAS algorithm called IMMAS to solve the MINLP model in a reasonable computational time.

Following this introduction, in section 2 some related works are described in perspective of food supply chain transportation problems. Conventional heuristics and metaheuristics used as solution approaches are reported. Section 3 describes the problem background of the Indian food grain supply chain. Section 4 presents the MINLP mathematical formulation including the objective function and constraints. Section 5 describes the proposed

IMMAS algorithm for solving the model. Section 6 is devoted to the extensive analysis of the model results and discussion. Finally, conclusions are given in section 7 which includes some recommendation for future research.

2. Related Work

The food supply chain transportation problem is not new and many studies have been carried out before. For better understanding, the below presentation of the existing results is divided into two sub-sections. The first sub-section deals with the food supply chains and other inventory transportation-related problems addressed in the literature. The second sub-section examines several solution methodologies employed to handle these models.

2.1 Food supply chains and their models

A linear integer programming (LIP) model of wheat storage and transportation problem in Iran has been solved using the LINGO software and genetic algorithm by Asgari et al. (2013). The rail-road flexibility, operational cost, vehicle capacity and availability constraints have not been introduced in that study.

Analytic and simulation models of Canadian wheat supply chain were developed by Ge et al. (2015) for identification of effective wheat quality testing strategies to minimize the handling cost of wheat supply chain.

The mathematical model for the crop movement planning from the farm to processing plant was developed by Lamsal et al. (2016) by considering the multiple independent farmers and no storage at farms. Three types of crops were considered in this paper, i.e. sugar canes, sugar beets, and vegetables.

The strategic vehicle routing problem of dairy industries of the Canada has been solved by the two-stage technique which depends on the adaptive large neighborhood search (ALNS) (Masson et al. 2015).

In the perspective of food grain supply chain, Mogale et al. (2017) examined the two stage food grain transportation problem and solved the formulated model using Hybrid chemical reaction optimization with tabu search algorithm. The focus of the paper was restricted to minimization of transportation, storage and handling costs. Furthermore, Maiyar and Thakkar (2017) developed the cost-effective model in the context of Indian food grain supply chain by considering rail-road flexibility option. However, they have not focused on the

inventory holding cost, allocation decision, and vehicle preference constraints. In this paper, we additionally incorporate intermodal transportation, vehicle capacity, and novel vehicle preference constraints to minimize the total numbers of vehicles required for food grain transportation. In order to determine the travel time, variable cost and rail network capacity, Hyland, Mahmassani and Mjahed (2016) formulated there models of domestic grain transportation including trucking, elevator storage, and rail shipment, respectively. A mathematical model with the objective function of minimization of infrastructure investment and economic cost was developed for reduction of post-harvest losses (Nourbakhsh et al. 2016).

In order to deliver the food just in time and with least delivery cost inclusive of holding and transportation cost, Agustina et al. (2014) studied the cross docking operations of food grain supply chain. The operational planning and integrated tactical model for the production and distribution of perishable agricultural products (Bell peppers and tomatoes) have been investigated by Ahumada and Villalobos (2011a) and (2011b) respectively.

Finally, Soto-Silva et al. (2015) revised the current state of the art of literature in detail for operation research models used in the fresh fruit supply chain. Their main focus was on planning models for fruit supply chain and classification of literature by different criteria. Ahumada and Villalobos (2009) have done the comprehensive review of successfully implemented planning models in production and distribution of agri-food supply chain.

2.2 Solution methods

The use of metaheuristics for solving the computationally complex NP-hard optimization problems is significantly growing because of its effectiveness of getting the near optimal solutions in a reasonable time (Hauser and Chung 2006; Bachlaus et al. 2009; Borisovsky et al. 2009; Pal et al. 2011; Essafi et al. 2012; Lee 2017).

Stützle, and Hoos (2000) presented the Max-Min Ant System (MMAS) algorithm which was a variant of ant system and successfully applied to travelling salesmen as well as quadratic assignment problems. Recently, Tang et al. (2014) have developed a novel beam search with Max-Min ant system algorithm called BEAM MMAS for solving the weighted vehicle routing problem (WVRP). The novel WVRP model with variable weight into the routing has been considered for determining the total cost. Furthermore, a Beam-ACO algorithm was developed by hybridizing the solution construction mechanism of ACO with beam search and validated using the well-known open shop scheduling (OSS) benchmark instances (Blum 2005).

A two-stage supply chain distribution network problem with a fixed charge for a transportation route has been examined by the Panicker et al. (2013) and proposed an ACO based heuristic for solving the problem. The Frito lay's outbound supply chain was studied by Çetinkaya et al. (2009) and developed an integrated model considering inventory and transportation decisions. The suggested mixed integer programming (MIP) model was solved by an iterative solution approach by decomposing the problem into inventory and routing components.

Genetic algorithms or genetic algorithm based heuristics are widely used for solving two-stage fixed charge transportation and multi-period fixed charge transportation problems (Jawahar and Balaji 2009; Jawahar and Balaji 2012; Hajiaghaei-Keshteli, Molla-Alizadeh-Zavardehi, and Tavakkoli-Moghaddam 2010; Antony Arokia Durai Raj and Rajendran 2012). A non-linear fixed charge transportation problem has been extensively addressed by the Xie and Jia (2012). The proposed MIP model was solved with a minimum cost flow based genetic algorithm. Bilgen and Ozkarahan (2007) addressed a bulk grain bending and shipping problem of wheat supply chain and formulated a MIP model which has been solved with the ILOG CPLEX software. A new discrete event simulation tool has been proposed by Van der Vorst et al. (2009) for a food supply chain redesign problem to integrate the logistics, sustainability, and food quality analysis decisions.

Pitakaso et al. (2007) addressed an unconstrained multi-level lot-sizing problem and presented an ant-based algorithm for solving the proposed MIP model. A three-tier multi-objective model with a cost function, time function and delay punishment function of a supply chain scheduling problem with networked manufacturing has been developed by Tang, Jing and He (2013) and solved using an Improved ant colony optimization (IM-ACO) algorithm. To avoid the premature convergence and increase the search speed, Ding et al. (2012) proposed a hybrid ant colony optimization (HACO) algorithm and applied it to solve a vehicle routing problem with time windows. Moreover, Yu, Yang, and Yao (2009) proposed an improved ant colony optimization (IACO) with ant weight strategy and mutation operation for solving a vehicle routing problem. Dorigo and Stützle (2009) gave an overview of ACO and its application to NP-hard problems along with recent developments in ACO. Liu et al. (2012) addressed a product disassembly sequence planning problem which is a NP-hard combinatorial problem and used an improved max-min ant system based algorithm to solve it.

Recently, Wari and Zhu (2016) have reviewed and described the extensive applications of metaheuristic methods in the food industry comprising of modelling approaches, parameters tuning and determination of near optimal solutions. The transportation cost minimization problem of cross-docking network has been addressed by Musa, Arnaout and Jung (2010) using novel ACO algorithm. They have not considered the heterogeneous vehicles and their fixed costs, multi-period, inventory and operational cost into the integer programming model. In addition, many authors have employed the different variants of ACO for solving the various combinatorial optimization problems including integrated scheduling of production and distribution, shortest loop design and multi-floor discrete layout (Cheng, Leung and Li 2015; Eshghi and Kazemi 2006; Izadinia and Eshghi 2016).

However, research in the area of agricultural supply chain management is mainly targeted on perishable foods like fruits and vegetables, sugarcanes, sustainability aspects and milk transportation problems. There is scarce literature available on food grains supply chain optimization problems. The most of the authors have not extensively addressed all the practical aspects of the problems like vehicle capacity constraints, intermodal transportation, fixed cost of vehicles, and multi-period scenario. In our problem, steel silos are used for storage of bulk food grains which can minimize the gunny bags cost, loading and unloading time, manpower shortage problem and increases the life of food grains. Moreover, in this research work, the operational cost of food grains inside the surplus state silos is considered because of the automation of silo operations. The crucial practical aspects of the food grain supply chain problem are captured with the novel vehicle (trucks and rakes) preference constraints.

3. Problem description

The considered multi-period bulk food grains shipment and storage problem of the Indian food grain supply chain is discussed in this section. As explained earlier, procurement of food grains from the farmers at MSP is done at procurement centers located at different parts of the producing states during Rabi market season (April to June) by FCI and state government agencies. The transportation of food grains in the bulk form from the procurement centers to nearby silos is carried out by trucks and tractors. At the silo level, bulk food grains are moved through the belt conveyors and before storing it goes through sampling, cleaning and automatic weighing machine, etc. The distribution conveyor shipped the food grains to the receiving silo bins, long storage bins, and shipping bins according to the requirement. Here, operational cost is incurred because of the movement of food grains from the beginning point to silo bins and

from silo bins to loading into the rakes and trucks. The next stage is the interstate shipment of food grains from surplus states to deficit states on the basis of the allocation made by GOI to each particular states and union territories. Due to the long distances between the two states, a large volume of food grains and less transportation cost, rail mode is generally preferred for interstate shipments. Additionally, maintaining an optimal inventory into the surplus state silos for reducing the inventory holding cost is also a crucial aspect of the problem.

There are various restrictions associated with bulk food grain shipment and storage problems such as seasonal procurement, limited silo storage capacity and a limited number of different capacitated vehicles in planning time horizon, demand, fixed cost of vehicles and available mode of transportations, etc.

In this paper, we have focused on only two stages of the food grain supply chain which include the shipment between procurement centers to surplus states silos and from surplus states silos to deficit states silos. The overall depiction of this problem is as shown in Fig. 2. The main objective of this paper is to minimize the total cost of bulk food grain shipment from surplus states to deficit states along with the operational and inventory holding cost in surplus state silos in a multi-period environment.

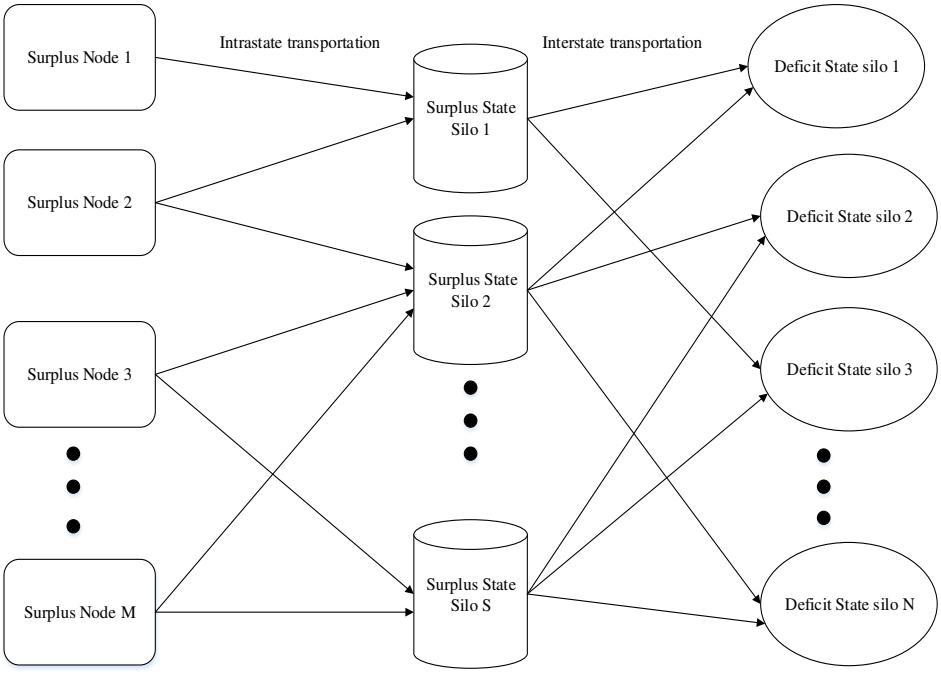

Fig. 2. Two stage transportation network

4. Mathematical model

In this section, we present a Mixed Integer Nonlinear Program (MINLP) model for the bulk food grain shipment and storage. The objective is to obtain a time dependent shipment and storage plan with minimized objective function value.

First of all, the assumptions, parameters, and decision variables of the model are defined. Then, the objective function and constraints used to solve the problem along with explanation are given.

Assumptions:

The following assumptions are considered while developing the model.

1) The clusters of procurement centers are represented by the various surplus state nodes.
2) Procurement at surplus nodes and demand of deficit states silos are deterministic in nature and well known with little variation. In this paper, we have not considered the stochastic environment. The GOI keep the records of all the people or different families in the particular states and their monthly allocation is also known and fixed, therefore demand is taken as a deterministic parameter.
3) At each surplus node and silos, finite numbers of capacitated vehicles are available in each time period. Generally, FCI uses the three different capacitated trucks and rakes for intra-state transportation and interstate transportation, respectively, and their availability is also limited in each time period.
4) The variable shipment cost is considered with the travelled distances among locations.
5) The procured food grain quantity is adequate to fulfill the demand of each deficit state silos in each time period therefore, backlog and shortages are not considered here.

Notations:

Index sets:

$\boldsymbol{T}$ Set of time periods $(t = 1, 2, ..., T)$, where $\boldsymbol{T}$ being the upper limit on number of time periods

$\boldsymbol{M}$ Set of surplus nodes $(m = 1, 2, ..., M)$, where $\boldsymbol{M}$ being the upper limit on number of surplus nodes

$\boldsymbol{S}$ Set of surplus state silos $(s = 1, 2, ..., S)$, where S being the upper limit on number of surplus state silos

N Set of deficit state silos $(n = 1, 2, ..., N)$, where N being the upper limit on number of deficit state silos

P Set of trucks $(p = 1, 2, ..., P)$, where P being the upper limit on types of trucks

R Set of rakes $(r = 1, 2, ..., R)$, where R being the upper limit on types of rakes

Vehicle-related parameters:

ψ_p Capacity of trucks of type p where $p \in P$

δ_r Capacity of rakes of type r where $r \in R$

Q_{pm}^t Number of p type trucks available at node m in time period t where $m \in M$, $p \in P, t \in T$

A_{rs}^t Number of r type rakes available at surplus state silo s in time period t where $r \in R, s \in S, t \in T$

Cost and distance parameters:

c_{ms} Unit shipment cost (road transportation) per km from surplus node m to surplus state silo s where $m \in M$, $s \in S$

c_{sn} Unit shipment cost (rail transportation) per km from surplus state silo s to deficit state silo n where $s \in S, n \in N$

d_{ms} Distance from surplus node m to surplus state silo s where $m \in M$, $s \in S$

d_{sn} Distance from surplus state silo s to deficit state silo n where $s \in S, n \in N$

f_{ms}^p Fixed transportation cost on route (m, s) for truck type p where $m \in M$, $s \in S$, $p \in P$

f_{sn}^r Fixed transportation cost on route (s, n) for rake type r where $s \in S, n \in N, q \in Q$

b_s Inventory carrying cost per Metric Tonne (MT) per unit time in surplus state silo s where $s \in S$

ω_s Operational cost per MT at surplus state silo s where $s \in S$

Procurement, capacity and demand parameters:

V_m^t Food grain quantity available at surplus node m in period t where $s \in S$ and $t \in T$

H_s Capacity of surplus state silo s where $s \in S$

D_n^t Demand of deficit state silo n in period t where $n \in N$ and $t \in T$

Decision variables:

Binary variables

J_{ms}^t $\begin{cases} 1 & \text{if surplus node } m \text{ is assigned to surplus state silo } s \text{ in period } t; \\ 0 & \text{otherwise} \end{cases}$

L_{sn}^t $\begin{cases} 1 & \text{if surplus state silo } s \text{ is assigned to deficit state silo } n \text{ in period } t; \\ 0 & \text{otherwise} \end{cases}$

g_{pm}^t $\begin{cases} 1 & \text{if all } p \text{ type trucks at surplus node } m \text{ are loaded/full in period } t; \\ 0 & \text{otherwise.} \end{cases}$

h_{rs}^t $\begin{cases} 1 & \text{if all } r \text{ type rakes of surplus state silo } s \text{ are loaded/full in period } t; \\ 0 & \text{otherwise} \end{cases}$

Continuous variables:

w_{ms}^t The food grain quantity in MT shipped through road from surplus node m to surplus state silo s in time period t where $m \in M, s \in S, t \in T$

v_{sn}^t The food grain quantity in MT shipped through rail from surplus state silo s to deficit state silo n in time period t where $s \in S, n \in N, t \in T$

E_s^t Food grain quantity available in surplus state silo s at time t where $s \in S, t \in T$

Integer variables

k_{ms}^{pt} Number of p type trucks used for food grain transportation from surplus node m to surplus silo s in time period t where $m \in M, s \in S, p \in P$ and $t \in T$

q_{sn}^{rt} Number of r type rakes used for food grain transportation from surplus state silo s to deficit state silo n in time period t where $s \in S, n \in N, q \in Q$ and $t \in T$

Objective function:

Minimize

$$\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{t=1}^{T}\left[\left(f_{ms}^{p}k_{ms}^{pt}\right)+\left(c_{ms}d_{ms}w_{ms}^{t}\right)\right]J_{ms}^{t}+\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{r=1}^{R}\sum_{t=1}^{T}\left[\left(f_{sn}^{r}q_{sn}^{rt}\right)+\left(c_{sn}d_{sn}v_{sn}^{t}\right)\right]L_{sn}^{t}$$
$$+\sum_{t=1}^{T}\left[\sum_{m=1}^{M}\sum_{s=1}^{S}w_{ms}^{t}+\sum_{s=1}^{S}\sum_{n=1}^{N}\left(v_{sn}^{t}\right)\right]\omega_{s}+\sum_{s=1}^{S}\sum_{t=1}^{T}b_{s}E_{s}^{t}$$

The objective function minimizes the overall total cost required for transporting the bulk food grain from a set of surplus nodes to a set of surplus state silos through road, from a set of surplus state silos to a set of deficit state silos through rail, the operational cost inside the surplus state silos and inventory holding cost at surplus state silos.

Subject to

$$\sum_{s=1}^{S}w_{ms}^{t}J_{ms}^{t}\le V_{m}^{t}\qquad\forall m,\forall t \tag{1}$$

$$\sum_{n=1}^{N}v_{sn}^{t}L_{sn}^{t}\le E_{s}^{t}\qquad\forall s,\forall t \tag{2}$$

Constraint (1) limits the quantity of food grain shipped from surplus node to surplus state silo, to maximum quantity of food grain available at the surplus node during each time period. Constraint (2) restricts the quantity of food grain shipped from surplus state silo to deficit state silo, to maximum available inventory at given surplus state silo in given time period.

$$\sum_{s=1}^{S}v_{sn}^{t}L_{sn}^{t}=D_{n}^{t}\qquad\forall n,\forall t \tag{3}$$

Constraint (3) ensures that total quantities shipped from surplus state silo to deficit state silo through rail must be equal to the demand of the given deficit state silo.

$$E_{s}^{t-1}+\sum_{m=1}^{M}w_{ms}^{t}J_{ms}^{t}\le H_{s}\qquad\forall s,\forall t \tag{4}$$

Constraint (4) is a capacity constraint on surplus state silo which states that sum of inventory available and quantity of food grains arrived cannot increase the capacity of the silo in any period for all the silos.

$$E_s^{t-1} + \sum_{m=1}^{M} w_{ms}^t J_{ms}^t - \sum_{n=1}^{N} v_{sn}^t L_{sn}^t = E_s^t \qquad \forall s, \forall t \tag{5}$$

Constraint (5) is an inventory balancing constraint represents that total inventory at the end of this period is the sum of quantities received in the current period, leftover inventory from the previous period, minus quantities transferred to deficit state silos.

$$E_s^{t=0} = 0 \qquad \forall s, \forall t \tag{6}$$

Constraint (6) specifies inventory in time period $t = 0$ is set to zero

$$w_{ms}^t J_{ms}^t \le \sum_{p=1}^{P} k_{ms}^{pt} \psi_p \qquad \forall m, \forall s, \forall t \tag{7}$$

$$\sum_{s=1}^{S} k_{ms}^{pt} \le Q_{pm}^t \qquad \forall m, \forall p, \forall t \tag{8}$$

Constraint (7) describes the trucks capacity constraint. Constraint (8) limits the number of trucks used on the route (m,s), to maximum trucks available at the surplus node during each time period.

$$v_{sn}^t L_{sn}^t \le \sum_{r=1}^{R} q_{sn}^{rt} \delta_r \qquad \forall s, \forall n, \forall t \tag{9}$$

$$\sum_{n=1}^{N} q_{sn}^{rt} \le A_{rs}^t \qquad \forall s, \forall r, \forall t \tag{10}$$

Constraint (9) limits the maximum quantity that is being transferred from surplus state silo *s* to deficit state silo *n* to maximum capacity of all the rakes being used in that period from *s* to *n*. Constraint (10) restricts the number of rakes used on the route (s,n), to maximum rakes available at surplus state silo during each time period.

Trucks preference constraints:

$$g_{p_3 m}^t \le g_{p_2 m}^t \le g_{p_1 m}^t \qquad \forall m, \forall p, \forall t \tag{11}$$

$$g_{p_1 m}^t Q_{p_1 m}^t \le \sum_{s=1}^{S} k_{ms}^{p_1 t} \le Q_{p_1 m}^t \qquad \forall m, \forall p, \forall t$$

$$(12)\ g_{p_2 m}^t Q_{p_2 m}^t \le \sum_{s=1}^{S} k_{ms}^{p_2 t} \le Q_{p_2 m}^t g_{p_1 m}^t \qquad \forall m, \forall p, \forall t$$

$$g^{t}_{p_3 m} Q^{t}_{p_3 m} \le \sum_{s=1}^{S} k^{p_3 t}_{ms} \le Q^{t}_{p_3 m} g^{t}_{p_2 m} \qquad \forall m, \forall p, \forall t \tag{13}$$

(14)

Constraints (11)-(14) defines the trucks preference constraints. In this paper, we considered three types of different capacitated trucks. These constraints ensure that high-capacity trucks should fill first in order to minimize the cost and time for movement of food grains.

Rakes preference constraints:

$$h^{t}_{r_3 s} \le h^{t}_{r_2 s} \le h^{t}_{r_1 s} \qquad \forall s, \forall r, \forall t \tag{15}$$

$$h^{t}_{r_1 s} A^{t}_{r_1 s} \le \sum_{n=1}^{N} q^{r_1 t}_{sn} \le A^{t}_{r_1 s} \qquad \forall s, \forall r, \forall t \tag{16}$$

$$h^{t}_{r_2 s} A^{t}_{r_2 s} \le \sum_{n=1}^{N} q^{r_2 t}_{sn} \le A^{t}_{r_2 s} h^{t}_{r_1 s} \qquad \forall s, \forall r, \forall t$$

$$h^{t}_{r_3 s} A^{t}_{r_3 s} \le \sum_{n=1}^{N} q^{r_3 t}_{sn} \le A^{t}_{r_3 s} h^{t}_{r_2 s} \qquad \forall s, \forall r, \forall t \tag{17}$$

(18)

Similarly, the priority for filling of the three different capacitated rakes is depicted by the constraints (15)-(18).

$$J^{t}_{ms}, L^{t}_{sn}, g^{t}_{pm}, h^{t}_{rs} = \{0,1\} \qquad \forall m, \forall s, \forall n, \forall p, \forall r, \forall t \tag{19}$$

$$w^{t}_{ms}, v^{t}_{sn}, E^{t}_{s} \ge 0 \qquad \forall m, \forall s, \forall n, \forall t \tag{20}$$

$$k^{pt}_{ms}, q^{rt}_{sn} \in Z \qquad \forall m, \forall s, \forall n, \forall p, \forall r, \forall t \tag{21}$$

Constraint (19), (20) and (21) are domain constraints.

The fixed costs presents in the objective function create discontinuities (nonlinearity) therefore the problem becomes much more difficult to solve. Our problem is a variant of FCTP and FCTPs are known to be NP-hard (Jawahar and Balaji 2009; Jawahar and Balaji 2012; Panicker et al. 2013; Antony Arokia Durai Raj and Rajendran 2012). The complexity of our problem is even increasing because of inclusion of time-dependent inventories, capacitated vehicles and operational cost. These types of problems are very difficult and hard to solve using the exact algorithms because as the problem size increases the computational effort needed to find the

best solutions grows exponentially. Any optimization algorithm cannot solve them in reasonable computational time or obtained solution may not be very high quality. The conventional technique-based commercial software are incapable to solve the models with non-linear and discrete decision variables (Yu et al. 2017). Thus, heuristic algorithms which can provide the near optimal solutions in a relatively short computation time are adopted by many authors to tackle the NP hard problems (Dorigo and Stützle 2009; Ding et al. 2012; Pratap et al. 2017).

The ant colony optimisation (ACO) has been successfully used to solve combinatorial problems such as the Traveling Salesman Problem, Quadratic Assembly and Vehicle Routing (Dorigo and Stützle 2009), FCTP (Xie and Jia 2012), job-shop scheduling (Chang et al. 2008) and the sequencing problems (Zhu and Zhang 2011). Therefore, the Improved Max-Min Ant System (IMMAS) algorithm is used to solve the model and its detail description is given in the next section.

5. Improved Max-Min Ant System (IMMAS)

ACO is a population-based metaheuristic which is inspired from the natural behavior of ants for finding the shortest route between the nests (colony) to the food source. While travelling from the nest to the food source ants deposits the chemical substance called pheromone. They use the stigmergic communication through the pheromones trails. With the time progress, the deposited pheromones on the chosen path evaporate. The evaporation rate is high on a longer path than the shorter path, therefore more pheromones will be condensed on shorter routes than a longer route. Therefore, more number of ants will follow the shorter path compared to the longer path. Initially, we deposit the small amount of pheromones on each edge of the graph. Next, ants travel from current nodes to another node on the basis of the rule that specified the preference of possible nodes. The probability of choosing the next node mainly depends on the pheromone intensity (τ) and problem dependent heuristic information (η). Furthermore, after all ants completed their tour from the nest to food source pheromones will be updated locally and globally on the basis of the solution obtained through the tour (cost value) and evaporation rate (ρ). The performance of an ACO is depended on the different parameters such as a number of ants, greedy heuristic to determine the visibility, evaporation rate and the importance of pheromones versus visibility.

In the present situation, different variants of the basic ant system are developed by several researchers for improving the performance of the ant system such as Elitist strategy, Ant Colony System (ACS), rank-based Ant System (AS rank), MAX–MIN Ant System (MMAS) and Population based ACO (Stützle and Hoos 2000; Guntsch and Middendorf 2002; Dorigo and Stützle 2009;). Along with these variants, many other authors have hybridized or improved the original ACO algorithm by hybridizing the solution construction mechanism and using the various strategies or new operators (Tang et al. 2014; Cheng, Leung and Li 2015; Izadinia and Eshghi 2016). For example, a stronger exploitation of the best solution found in the search space can improve the performance of the algorithm.

In this paper, we have modified the MMAS variant of ACO based on the strategy of sorting elite ants and pheromone trail updating to improve the performance of the algorithm. MMAS avoids the premature convergence of the search by restricting the pheromone trail between the upper and lower bounds. The MMAS differs from the ant system in three key features:

i) The strong elitist strategy is used for updating the pheromones. Therein, only the best solution obtained after each iteration or global best solution is allowed to update the pheromones trails.

ii) The pheromones trails will be updated on each solution component in the interval of $\left[\tau_{Min}, \tau_{Max}\right]$ to avoid the stagnation of the search space.

iii) In order to achieve the higher exploration by the algorithm, the pheromones trails will be initialised to τ_{Max} at the start of the algorithm (Stützle, and Hoos 2000). Initially, we will give all the problem specific information i.e. input parameters value required for solving the developed MINLP model.

Figure 3 shows the steps used along with an algorithm to find out the optimal solution. In next section, each step represented in the Fig. 3 is explained in detail along with the formulations used.

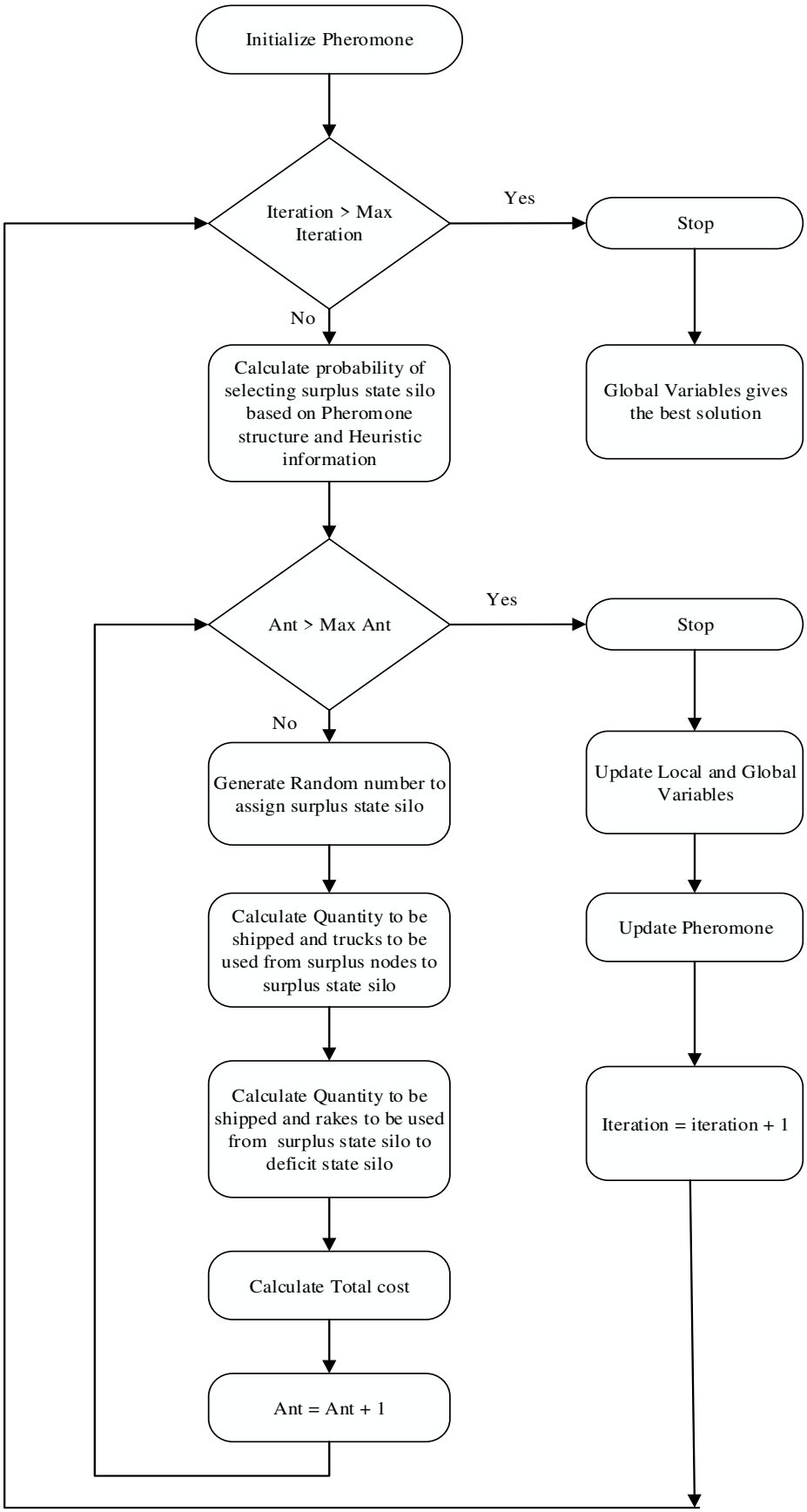

Fig. 3. Algorithm procedure

5.1 Initialize Pheromone

Initially, pheromones are set to some arbitrarily high value so that after first iteration pheromone trails are forced to take a value in limits set by us. This type of initialization leads to a greater exploration of the solution space.

5.2 Determination of Probability of Selecting a Route

The probability for selecting a surplus state silo, which further determines the route of the solution is calculated using the given formula:

$$p_{msnt} = \frac{\left[\tau_{msnt}\right]^{\alpha} \cdot \left[\eta_{msnt}\right]^{\beta}}{\sum_{s \in S} \left[\tau_{msnt}\right]^{\alpha} \cdot \left[\eta_{msnt}\right]^{\beta}}$$

This is the probability for transporting the food grains The aim of this study was to determine *m* to deficit state silo *n* through surplus state silo *s* by an ant and it updated after every iteration. In numerator the first term represents the pheromone accumulated, it is raised to the power of α which signifies the importance given to pheromone trail.

Where α is a parameter of the algorithm which has to be tuned such that the algorithm performs efficiently. The second term indicates heuristic information which is available locally.

In our problem, we used a combination of shipment costs and distances. Given below is the formulae used to determine η on route *m, s* and *n* during iteration *t.*

$$\eta_{msnt} = [1/(C_{ms} \times d_{ms})] + [1/(C_{sn} \times d_{sn})]$$

Here, β is a parameter similar to α which signifies the importance given to local heuristic information.

5.3 Solution Generation

Here, an ant represents a complete solution which is found out by first selecting a surplus state silo for each surplus node and deficit state silo selected randomly during each time interval. A matrix *H*: $m \times n \times t$ is generated using the probability function explained in above section for each iteration. The generated matrix is to be populated up to the maximum number of surplus state silos *(s=1, 2, 3….S).*

Let us assume that for simple sample case we have three surplus nodes (*m*=3), three deficit state silos (*n*=3), two surplus state silos (*s*=2) and one time period (*t*=1). In order to fill

the cell entry in the first column and first row in the decision matrix *H*, the probabilities for shipping the load from *m*=1 to *n*=1 through the *s*=1 and 2 in time period *t*=1 is to be evaluated using the probability function. After calculating probabilities of selecting *s*=1 or *s*=2, the cumulative density function (CDF) is generated out from these two probability values. Next, a value is selected as follows. Suppose 0.3 and 0.7 are the probabilities for *s*=1 and 2 respectively. This indicates that surplus state silo two (*s*=2) is more likely to be selected for shipping the food grains. The corresponding CDF values are 0.3 and 1.0 (0.3+0.7). Thus, we generate a random number between 0 and 1. If the generated random number is 0.6, for example, then we will choose the surplus state silo 2 (*s*=2) because that generated random number (0.6) value lies in the range between 0.3 and 1. To fill the value in the other cell of the matrix, we simply repeat the same procedure. The matrix H is described as follows for this example.

$$H = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

This matrix shows that the food grains from *m*=1 to *n*=1, *m*=1 to *n*=2, *m*=2 to *n*=2, *m*=3 to *n*=1 and *m*=3 to *n*=3 are to be transported to deficit state silos through the surplus state silo one (*s*=1), whereas the remaining nodes use the surplus state silo 2 for shipping the food grains.

Once nodes are assigned, quantities of food grain to be transferred is calculated. A greedy optimization technique is used to find out the quantities to be shipped. A lowest cost route is selected in *H* matrix as the probability of such a route being selected would be high due to large pheromone concentration. To minimize the cost, maximum quantity should be shipped from the route with minimum cost. Quantities to be shipped is further divided. First, we consider quantities to be shipped from *m* to *s,* then we consider quantities to be shipped from *s* to *n* for time periods *t=1* to T. Maximum quantity to be shipped from *m* to *s* is calculated based on three limiting constraints:

i) quantity available at node *m* in time period *t*

ii) maximum capacity of different trucks available in time period *t* at node *m*

iii) difference between maximum possible inventory and current inventory at selected surplus state silo at time period *t*

Minimum value obtained from the above three values is the maximum quantity that can be shipped without violating any constraints. Now, we calculate the quantities to be shipped from *s* to *n*. The maximum quantity that can be shipped is decided taking into account three variables:

i) Inventory available at *s* in time period *t*

ii) Maximum capacity of different rakes available in time period *t* at surplus state silo *s*

iii) Remaining demand to be fulfilled at deficit state silo *n*

Here quantity to be shipped and updated values of the variables are decided in exactly same way as done in the case of *m* to *s*. Once the route of ant and quantity of food grains to be carried through each route is decided, we have the values of all the decision variables. So, the total cost for each ant is found out using the objective function.

5.4 Pheromone Trail Updating

After all ants generate a solution and one iteration is completed, pheromones on each path are updated. MMAS follows an elitist approach, and solutions are updated only when a better solution is generated, i.e. a solution having a lower cost than that of the global best solution. Flowchart of the algorithm used to update pheromone structure in case of MMAS is shown in Fig. 4.

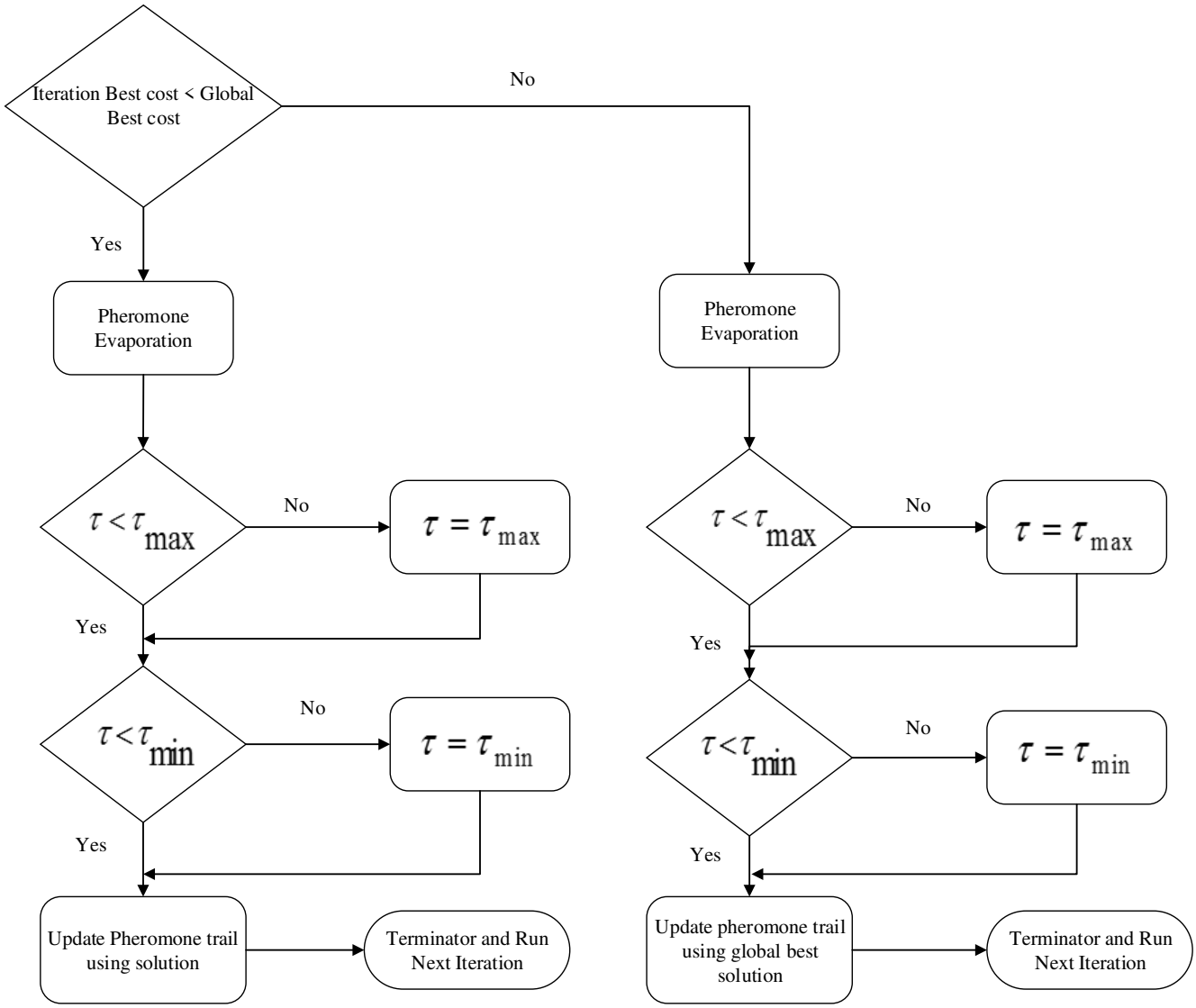

Fig. 4. Pheromone Updating Flowchart using MMAS

To improve the solution quality and search speed, MMAS has been modified using the strategy of sorting elite ants (σ) in ascending order and updating the top best solutions. The main difference between MMAS and IMMAS is the solution update methodology. In MMAS if a current best solution is better than the global best solution then only the best solution (best ant) updates the pheromone structure whereas in IMMAS first all current solutions are sorted in ascending order then instead of only best solution updating pheromone structure, top *n* solutions (elite ants) update pheromone structure where $1 < n(\sigma) <$ number of ants . The number of elite ants in the algorithm is denoted by σ . The pheromone update rule used here is as shown below:

$$L^1(t) \le L^2(t) \le .... \le L^\sigma(t)$$

$$\tau_{msnt}(t+1) = (1-\rho)\tau_{msnt}(t) + \sum_{r=1}^{\sigma} \Delta\tau^r_{msnt}(t),$$

Where, $\Delta\tau^r_{msnt}(t) = \begin{cases} 1/L^r(t) & \text{if route } (m,s,n) \text{ is used by} \\ & \text{ant } r \text{ in iteration } t \\ 0 & \text{otherwise} \end{cases}$

Therein, $L^r(t)$ is the solution cost of *r*th ant in *t*th iteration.

The flowchart and procedure of IMMAS are depicted in Figs. 5 and 6 respectively.

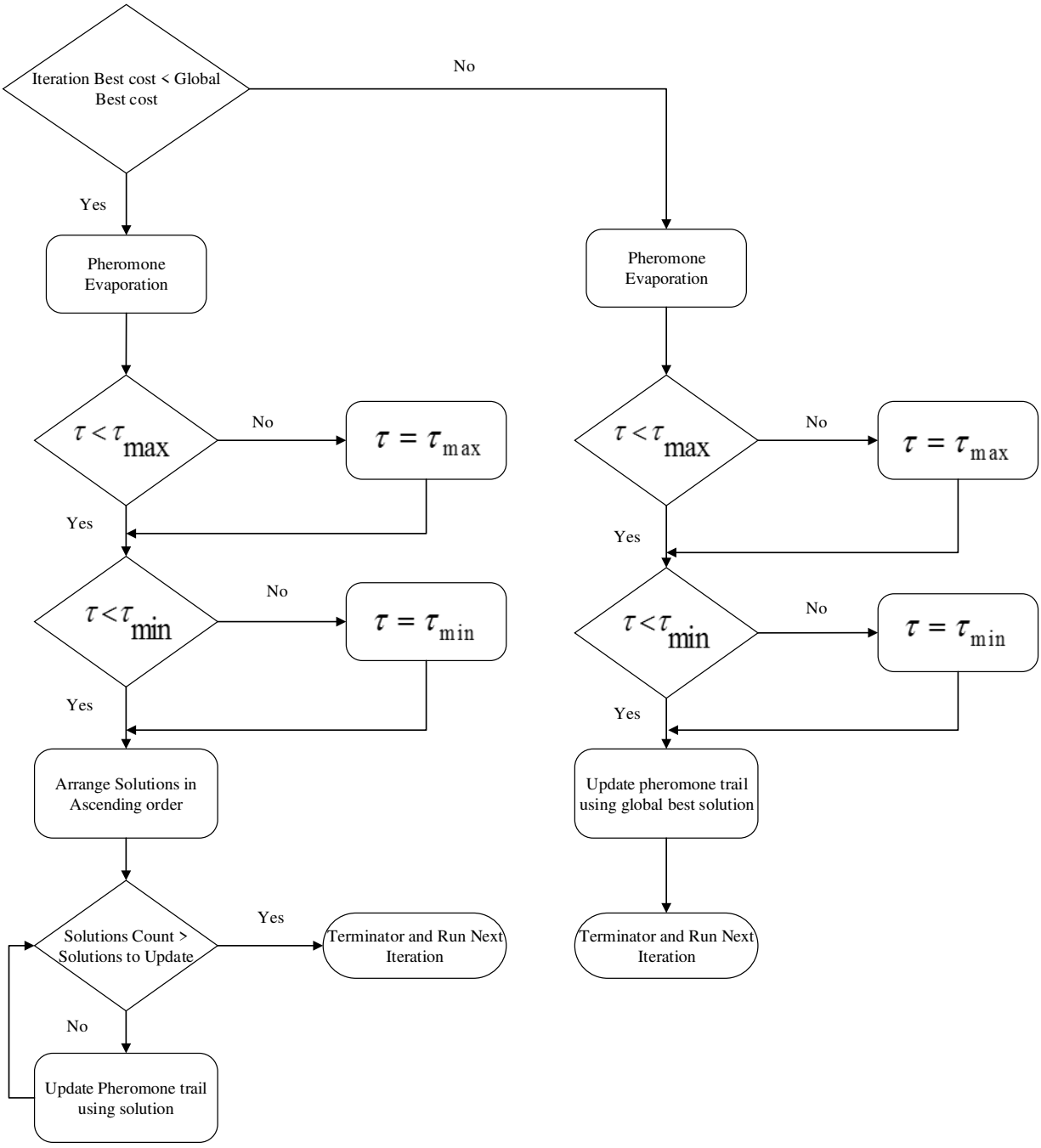

Fig. 5. Pheromone Updating Flowchart using IMMAS

Input : Problem specific information
Initialize the pheromone structure τ_{msnt}
For i = 1 to I **do**
Calculate probability of selecting a surplus state silo p_{msnt}
For a = 1 to P **do**
Get a random number to select surplus state Silo s
Assign route for the ant J_{ms}^{t} and L_{sn}^{t}
Compute the maximum quantity that can be shipped from m to s (w_{ms}^{t})
Assign trucks required k_{ms}^{pt}
Assign silo capacity used E_{s}^{t}
Deplete quantity available at surplus node V_{m}^{t}
Compute the maximum quantity that can be shipped from s to n (v_{sn}^{t})
Assign rakes required q_{sn}^{rt}
Fulfil demand for selected deficit state silo D_{n}^{t}
Deplete quantity available at selected surplus state Silo E_{s}^{t}
Compute and store total cost for ant a
store route of ant, quantity shipped and vehicles assigned
End for
compute iteration best cost
Sort all the solutions
update pheromone values of elite ants
using the local best or global best route
End for
compute the global best cost
Output : an optimal solution for porblem

Fig. 6. Procedure of the IMMAS

6. Computational Results and Discussions

In this section, various problem instances, parameter tuning of the algorithm and results of the several computational experiments are described. The three problem categories i.e., small, medium and large-scale, each one with 10 instances (total 30 instances) with increasing complexity are considered to extensively investigate the efficiency of the developed mathematical model. This categorization has been done considering the number of surplus

nodes, surplus state silos, deficit state silos and time periods. The small scale problem category comprises of 3-10 surplus nodes, 1-5 surplus state silos, 1-10 deficit state silos and 2-4 time periods. The description of the other two problem categories along with small scale category are given in Table 1. The data require for solving mathematical model has been taken from several reliable secondary sources such as PDS Portal of India, Ministry of Consumer Affairs, Food and Public Distribution (http://pdsportal.nic.in/main.aspx), CAG 2013 report, Report of the high-level committee on reorienting the role and restructuring of FCI and PDS annual reports (Functioning of the PDS, an Analytical report, 2013).

Table 1 Description of problem instances

Category	Surplus nodes (I)	Surplus state silos (J)	Deficit state silos (K)	Time periods (T)
Small- scale	[3-10]	[1-5]	[1-10]	[2-4]
Medium-scale	[11-20]	[6-10]	[11-20]	[5-7]
Large-scale	[21-30]	[11-15]	[21-30]	[8-10]

Additionally, for improving the performance of the algorithm different control parameters i.e. m, α, β, ρ and σ are set up after carrying out the parameter tuning, and the tuned parameters values are shown in Table 2. The proposed algorithms are coded in MATLAB R2014a. Furthermore, all the experiments are executed on the workstation of 8 GB RAM and Intel Core i5 with 2.90 GH processor. The termination criteria of IMMAS and MMAS algorithms are set as the maximum iteration of 100 in each run.

Table 2 Table depicting the tuned parameters values for all the problem instances

Parameter	Values
Number of ants (m)	50
Pheromone Intensity (α)	1.5
Heuristic Information (β)	3
Evaporation Rate (ρ)	0.7
Initial deposited pheromones	0.5
Maximum number of iterations	100
Number of elite ants (σ)	15

All the problem instances are solved using IMMAS and results are validated by comparing with MMAS algorithm. The results of the 10 each problem instances of three categories along

with the number of variables and constraints, near optimal solutions (total cost) and computational time are summarized in Table 3, 4 and 5, respectively. Moreover, the problem complexity can be observed from the total number of variables and constraints presents in each one of the instances. The total costs obtained in each instance of three categories are the average of 30 independent runs. It was observed from these tables that for each instance of each category the total costs obtained through IMMAS are substantially lower than the MMAS. In addition, the IMMAS provide superior performance for all the considered instances in a reasonable computational time.

Table 3 Total cost obtained through IMMAS and MMAS for small-scale problem category

Problem No	Surplus node	Surplus state silo	Deficit state silo	Time period	Number of variables	Number of constraints	IMMAS		MMAS	
							TC	CT (s)	TC	CT(s)
1	3	2	3	2	154	886	8,093,629	6.35	8,538,508	9.401
2	3	3	3	2	222	1278	11,268,950	7.131	12,558,759	11.165
3	4	3	4	2	288	2122	15,786,703	10.407	17,158,594	14.636
4	5	3	4	3	486	3924	23,783,618	18.00	25,747,743	24.886
5	5	3	5	3	531	4791	30,659,295	21.213	34,596,515	27.234
6	6	4	5	3	762	7503	28,787,960	28.625	31,432,306	36.859
7	7	4	6	3	891	10314	33,699,774	35.255	37,717,138	47.669
8	8	5	6	4	1576	19436	47,224,178	49.59	52,154,574	65.043
9	9	5	10	4	2088	35576	78,767,428	82.463	84,461,425	98.677
10	10	5	10	4	2200	39460	76,877,001	87.85	85,634,557	105.402
Average							35,494,854		39,000,012	
SD							25,072,003		27,431,902	

Table 4 Total cost obtained through IMMAS and MMAS for medium-scale problem category

Problem No	Surplus node	Surplus state silo	Deficit state silo	Time period	Number of variables	Number of constraints	IMMAS		MMAS	
							TC	CT (s)	TC	CT (s)
1	11	6	12	5	3140	64690	115,043,231	137.585	125,320,550	161.777
2	13	6	11	5	3915	83895	106,296,143	152.273	113,803,320	175.784
3	14	7	11	5	4725	105125	84,931,458	167.671	97,606,317	188.498
4	14	6	13	5	4380	106305	123,661,179	184.814	134,650,684	202.772
5	15	6	16	6	5994	167496	184,571,318	279.808	201,617,484	310.843
6	15	7	17	6	7158	207174	195,987,801	304.688	213,768,406	338.828
7	16	7	19	6	7806	246510	227,066,439	354.227	239,761,885	386.99
8	18	9	17	6	9990	318702	194,732,254	377.82	215,422,137	428.302
9	19	9	17	7	11991	392343	256,029,007	470.336	271,088,331	535.251
10	20	8	18	7	11284	388598	272,413,231	493.625	277,443,931	557.771
Average							176,073,206		189,048,305	
SD							65,607,371		66,353,129	

Table 5 Total cost obtained through IMMAS and MMAS for large-scale problem category

Problem No	Surplus node	Surplus state silo	Deficit state silo	Time period	Number of variables	Number of constraints	IMMAS		MMAS	
							Total cost	CT (s)	Total cost	CT (s)
1	21	11	21	8	19336	646048	355,995,505	686.774	387,849,363	747.369
2	22	11	23	8	20680	854664	387,896,097	816.803	406,898,988	867.41
3	23	12	22	8	22536	932208	378,080,649	858.274	405,475,898	925.819
4	24	12	25	8	24480	1104200	428,935,917	974.709	444,358,741	1038.577
5	25	13	24	9	29808	1345572	477,070,541	1121.838	514,436,098	1191.685
6	25	12	26	9	28647	1345194	495,477,749	1193.821	548,621,934	1287.444
7	26	13	25	9	31005	1457109	481,449,129	1203.832	523,332,212	1317.534
8	27	13	25	9	31617	1512945	483,618,441	1256.351	519,539,592	1372.605
9	28	14	26	10	39200	1951440	560,920,054	1534.845	577,123,670	1656.621
10	30	15	28	10	45000	2410630	573,492,957	1733.733	587,042,857	1895.934
Average							462,293,704		491,467,935	
SD							73,991,391		74,215,571	

The convergence graph of the problem instances 1, 5 and 7 from each category are shown in Figs. (7-9) respectively. It can be seen that the rate of convergence of IMMAS algorithm is faster than MMAS in each instance.

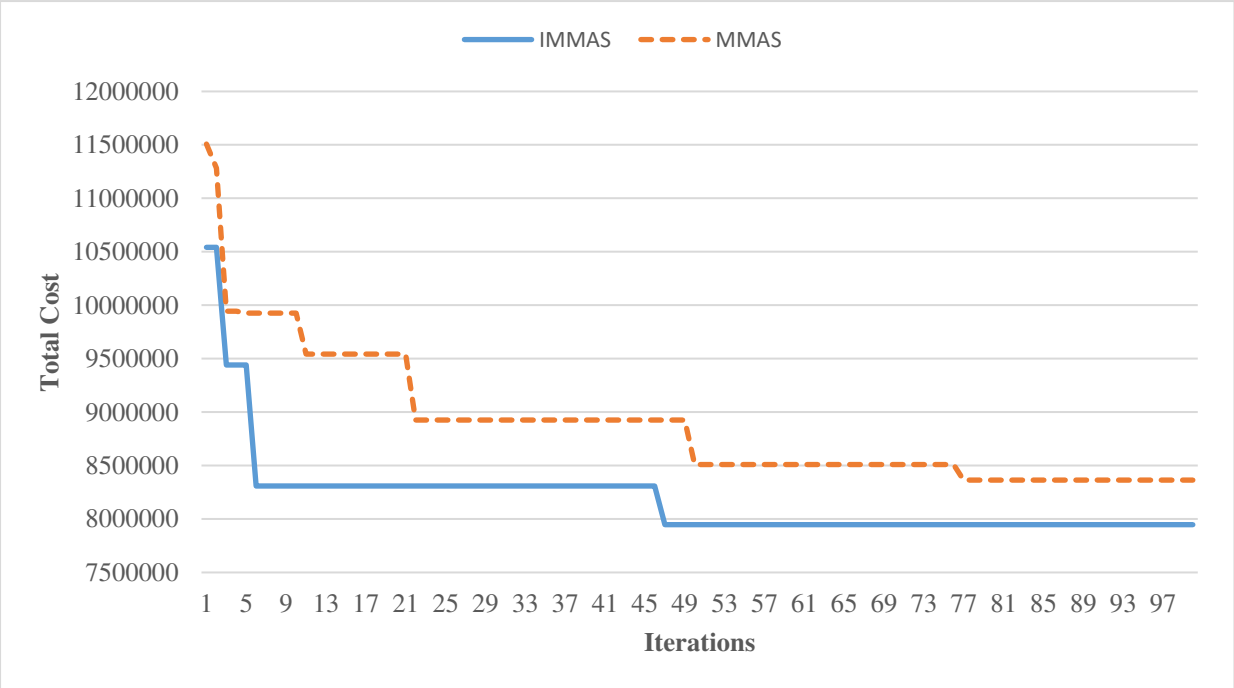

Fig. 7. Convergence graph of instance 1 of small category

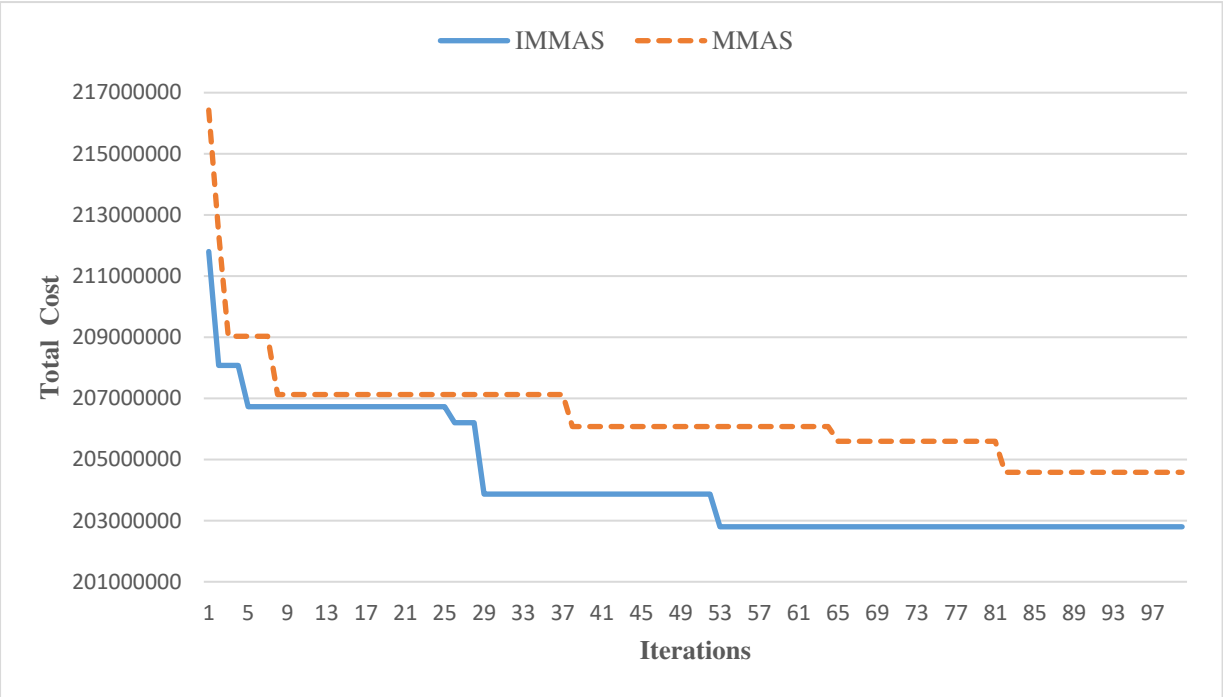

Fig. 8. Convergence graph of instance 5 of medium category

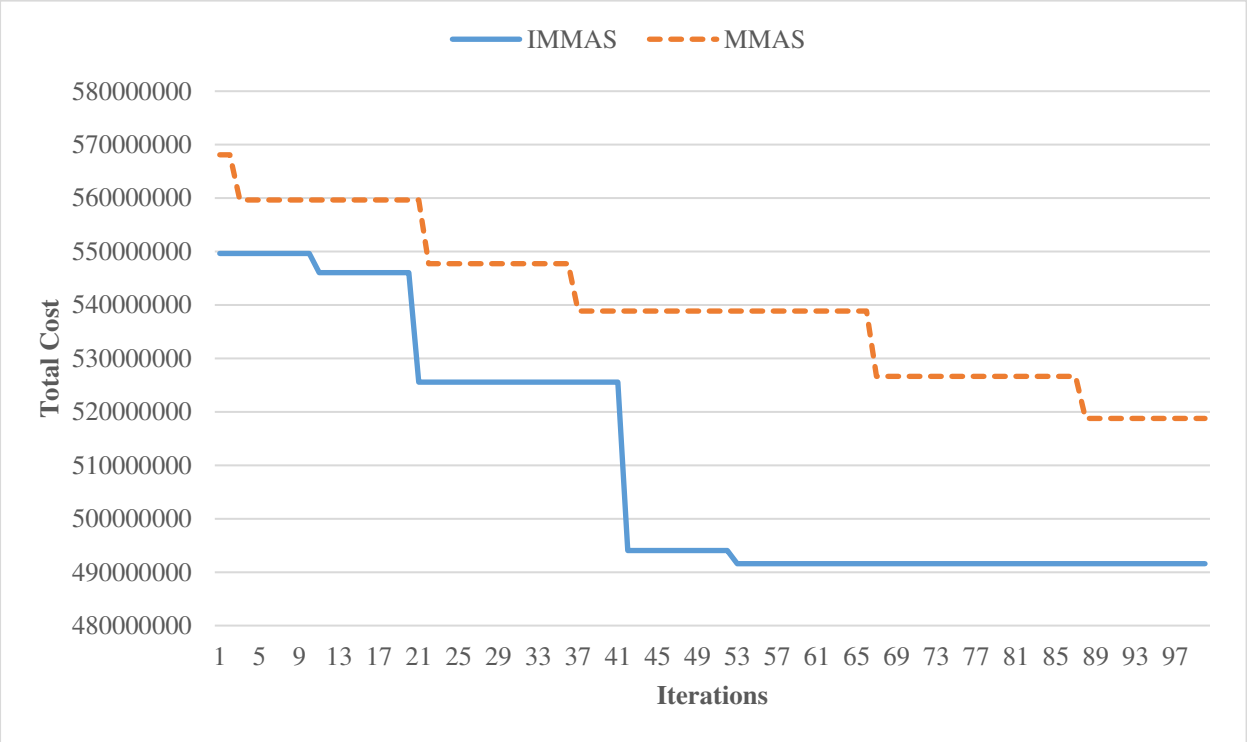

Fig. 9. Convergence graph for instance 7 of large category

The sample values of all the decision variables including the food grain quantity transferred from all the surplus nodes to all the surplus state silos, from all the surplus state silos to all deficit state silos, total inventory available in all the surplus state silos and total number of each types of trucks and rakes used in all the time periods for selected three instances from each category are reported in Table 6. It can be observed from this table that the total food grain quantity transported from all surplus nodes to all surplus state silos is more than the quantity moved from surplus state silos to deficit state silos in a definite planning horizon.

Table 6 sample values of all the decision variables

Problem category	Problem no (Description)	$\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{t=1}^{T} w_{ms}^{t}$	$\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t=1}^{T} v_{sn}^{t}$	$\sum_{s=1}^{S}\sum_{t=1}^{T} E_{s}^{t}$	$\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{t=1}^{T} k_{ms}^{p_1 t}$	$\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{t=1}^{T} k_{ms}^{p_2 t}$	$\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{t=1}^{T} k_{ms}^{p_3 t}$	$\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t=1}^{T} q_{sn}^{r_1 t}$	$\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t=1}^{T} q_{sn}^{r_2 t}$	$\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t=1}^{T} q_{sn}^{r_3 t}$
Small scale	2 (3 3 3 2)	56995.25	37800.65	19195.33	811	689	772	8	6	2
	5 (5 3 5 3)	143425.27	105300.40	38125.15	1841	1959	1961	23	11	11
	10 (10 5 10 4)	375790.83	326400.09	49390.24	5099	4884	5036	60	48	40
Medium scale	4 (14 6 13 5)	533350.64	475200.81	58150.65	8900	7858	3495	95	64	50
	7 (16 7 19 6)	914990.17	841500.95	73490.30	12130	12230	12267	159	120	99
	9 (19 9 17 7)	1217645.79	1085100.14	132545.57	16375	16183	16091	200	157	135
Large scale	4 (24 12 25 8)	1749045.26	1471500.69	277545.73	23610	23137	23116	263	205	209
	7 (26 13 25 9)	2178080.84	1927500.48	250580.07	28795	29478	28864	369	260	235
	10 (30 15 28 10)	2746450.08	2475600.87	270850.20	36535	36708	36635	460	367	290

The statistical confirmation of the evolutionary algorithms outcomes has been carried out using various *t*-tests such as independent t-test and pairwise comparison test (Lin, Gen & Wang 2009; Mousavi et al. 2014; Panicker et al. 2013; Antony Arokia Durai Raj and Rajendran 2012). Here, in order to verify that whether there is a significant pairwise difference between costs obtained through both algorithms, the paired comparison *t*-test has been conducted at 5% level of significance with respect to solutions obtained from 10 instances of each category. Initially, the difference between the total costs obtained through MMAS and IMMAS is computed and represented by the term δ .

$$\delta = Total\ \cos t_{MMAS} - Total\ \cos t_{IMMAS}$$

The null hypothesis states that there is no significant paired deviation between the average total cost obtained through MMAS and IMMAS. The alternative hypothesis indicates that average total cost found using the IMMAS is less than the average total cost obtained through MMAS.

$$H_0 : \delta \leq 0 \text{ and } H_1 : \delta > 0$$

The results of this paired comparison *t*-test are mentioned in Table 7. All the P values are less than 0.05, therefore the null hypothesis is rejected in each category. Hence, it statically proves that there is a significant difference between the total cost obtained using IMMAS and MMAS algorithm.

Table 7 The paired comparison *t* test results for all problem categories

Problem category	N	Mean of differences between pairs	SD of differences between pairs	SE-Mean	T value	P value
Small scale	10	3,505,158	2,512,164	794,416	4.42	0.000845
Medium scale	10	12,975,098	4,806,581	1,519,974	8.54	0.00001
Large scale	10	29,174,231	13,182,275	4,168,602	7.00	0.000032

6.1 Sensitivity analysis

In this section, the sensitivity analysis has been carried out by changing the problem environment on the first instance of small scale category problems. We have mainly focused on the three perspectives to visualize the effects on the performance of the model and algorithm.

6.1.1 Effect of different capacitated trucks

There is a different fixed cost associated with each type of capacitated truck. Fig. 10 illustrates the effect on the total cost against the four different scenarios i.e. if all different capacitated trucks (ACT), only high capacitated trucks (HCT), only medium capacitated trucks (MCT) and finally only low capacitated trucks (LCT) used. As we can see from Fig. 10, if we are filling the trucks in descending order of their capacities, then the total cost will be reduced. Furthermore, if we change the problem size, the values may change, but the nature of the graph will remain same. Thus, this result gives better authentication to our novel truck preference constraints.

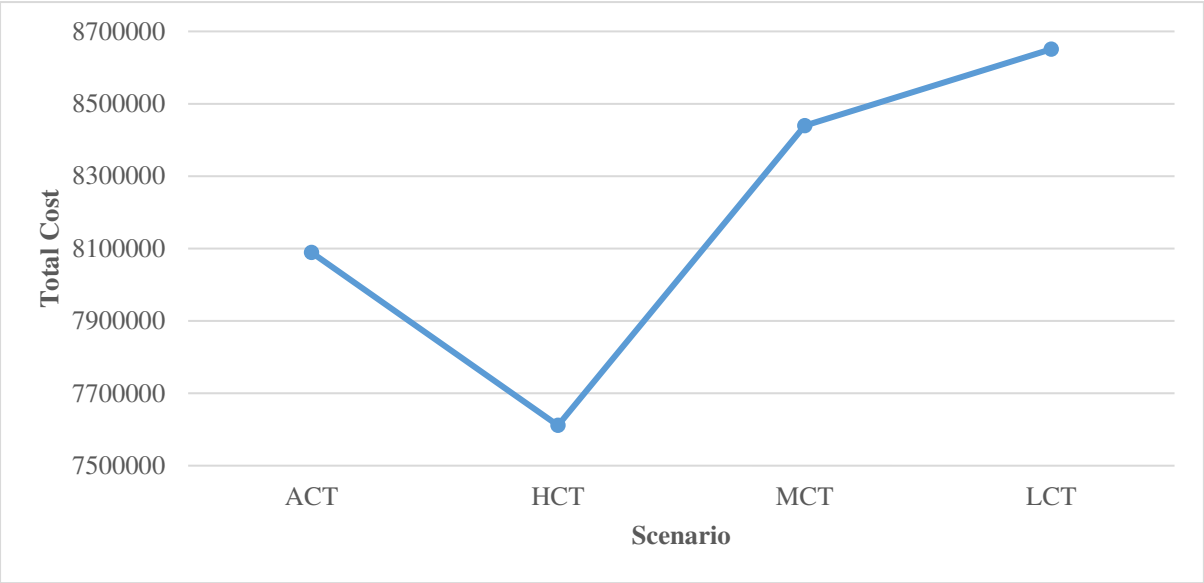

Fig. 10. The total cost against various scenarios of capacitated trucks

6.1.2 Effect of different capacitated rakes

Similar to the above section, we have conducted experiments using four scenarios with respect to rakes i.e., all three capacitated rakes (ACR), high (HCR), medium (MCR) and low capacitated rakes (LCR) used for transportation. Fig. 11 shows the graph of various scenarios versus the total cost obtained and it also depicts the similar type of nature which is obtained in Fig. 10. It can be observed that when we use small capacitated rakes instead of large capacitated ones the total cost is increased from 7,834,730 to 9,157,988 INR. This analysis proves that the vehicle preference constraints help to minimize the transportation cost.

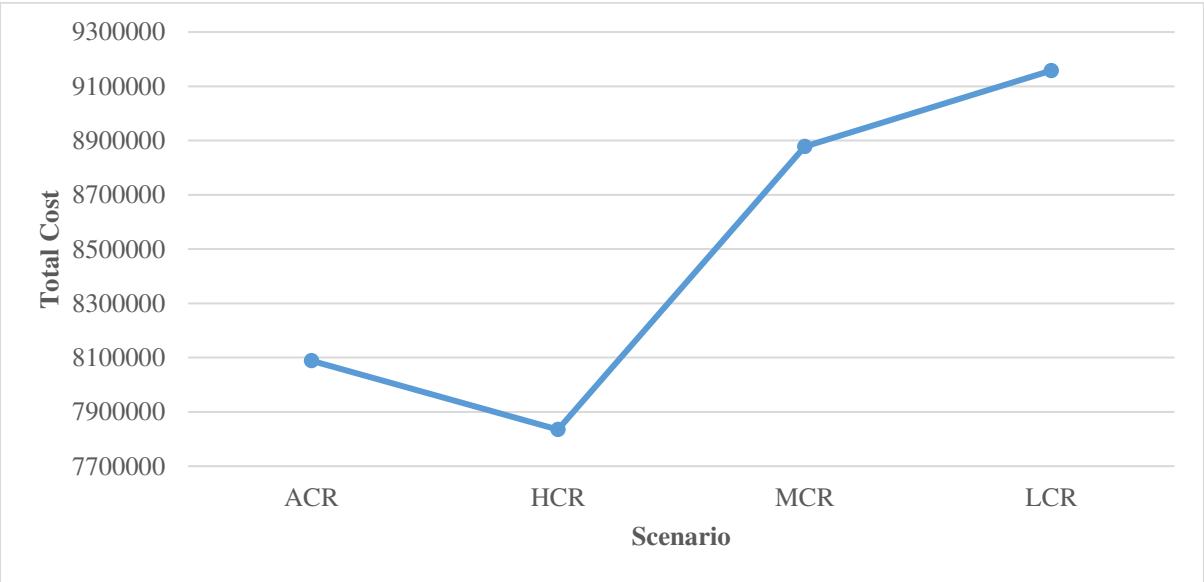

Fig. 11. The total cost vs various scenarios of capacitated rakes

6.1.3 Effect of increasing the base silo capacity on transportation cost

The effect of increasing the base silo capacity on the transportation costs is shown in Fig. 12. The downward trend of line illustrates that the transportation cost will be decreased by increasing the surplus state silo capacity, but the silo construction cost may be increased.

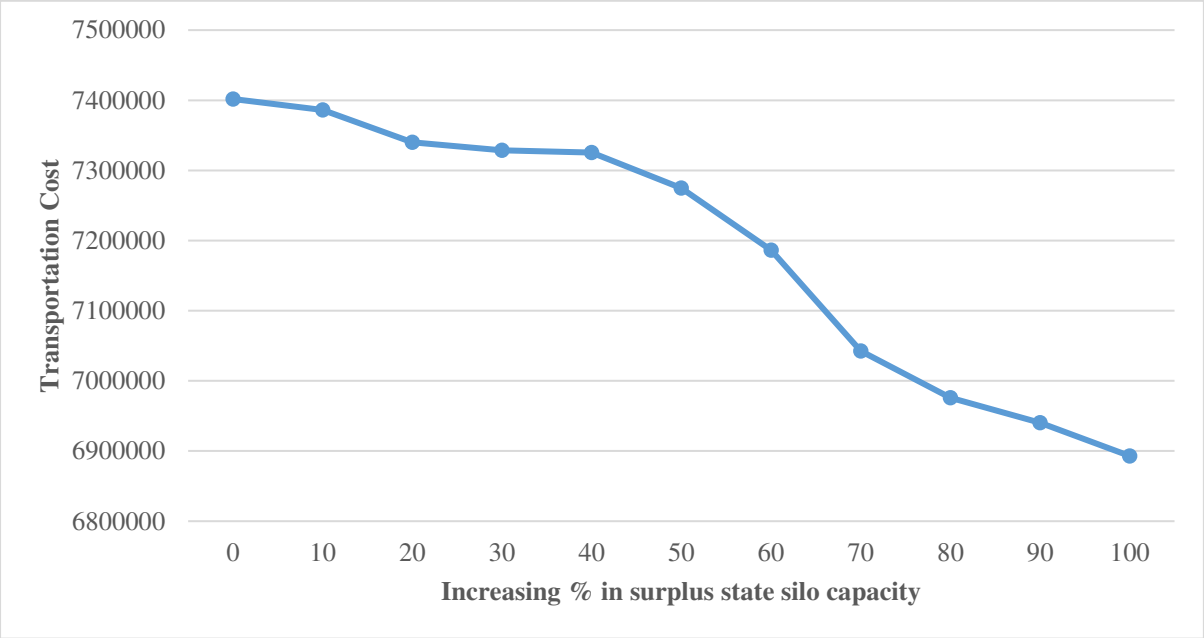

Fig. 12. The transportation cost against percent increase of surplus state silo capacity

The FCI, SGAs, Railways and other entities of the food grain supply chain can get the valuable and important insights from this study. First, FCI can get the effective monthly movement and storage plan if the previous month data like procurement quantity, capacity of silos, availability of capacitated vehicles (trucks and rakes) and demand of the deficit states is available. The results of this model will be helpful for maintaining the proper operational and buffer stock in the deficit states. In the present situation some deficit states have high buffer stocks than their requirement. FCI can resolve the concern of shortages of vehicles (trucks and Rakes) through proper planning and management among the FCI, SGAs, private contractors and Railways. This study gives the number of different capacitated vehicles (Trucks and rakes) required for transportation of food grains in the particular time period, therefore FCI in advance can inform to the contractors and railways about their requirement of trucks and rakes per month, respectively. In addition, the timely movement plan will be helpful for scheduling of vehicles at particular stages including surplus nodes and silos which can curb the demurrage charges of

vehicles. The issue of underutilization of existing storage facilities can be confronted through the storage activity plan. Since we have dealt with the bulk food grain transportation and storage, the wastages of the food grain will be reduced.

7. Conclusion and future work

In this work, a food grain supply chain problem of the public distribution system has been addressed. A mathematical model was developed to minimize the shipment, holding and handling costs while shipping the food grains from a set of surplus nodes to set of deficit state silos through storing the food grains into a set of surplus state silos. The prosposed mixed integer non-linear mathematical model incorporates the multi-period, intermodal shipment, inventory, vehicle capacity and novel vehicle preference constraints.

The IMMAS and MMAS algorithms were developed to solve the formulated mathematical model. The solutions obtained from the IMMAS algorithm for different problem instances are completely dominating the solutions of MMAS algorithm. The effect of different capacitated vehicles (trucks and rakes) and silo capacity on the solution quality have been visualized through sensitivity analysis. Furthermore, the statistical validation of the results has been carried out by using the paired comparison *t*-tests.

The present model can be extended by considering a stochastic demand and procurement. The multi food grain scenario is another future modification of this work. Multi-modal transportation can be used instead of intermodal transportation for transporting the food grains. In the future research, a multi-objective optimization model can be made by adding the transportation time minimization objective into the current model.

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