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Bow-Free Tri-Component Mechanically Pre-Stressed Failure-Oriented-Accelerated-Test (FOAT) Specimen

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Abstract

In some today's and future electronic and optoelectronic packaging systems (assemblies), including those intended for aerospace applications, the package (system's component containing active and passive devices and interconnects) is placed (sandwiched) between two substrates. In an approximate stress analysis these substrates could be considered, from the mechanical (physical) standpoint, identical. Such assemblies are certainly bow-free, provided that all the stresses are within the elastic range and remain elastic during testing and operation. Ability to remain bow-free is an important merit for many applications. This is particularly true in optical engineering, where there is always a need to maintain high coupling efficiency.

The level of thermal stresses in bow-free assemblies of the type in question could be, however, rather high. High thermal stresses are caused by the thermal contraction mismatch of the dissimilar materials of the assembly components and occur at low temperature conditions. These stresses include normal stresses acting in the component cross-sections and interfacial shearing and peeling stresses. The normal stresses in the component cross-sections determine the reliability of the component materials and the devices embedded into the inner component (package). The interfacial stresses affect the adhesive and cohesive strength of the assembly, i.e. its integrity.

It should be pointed out that although the assembly as a whole is bow-free, the peeling stresses in it, whether thermal or mechanical, are not necessarily low: the two outer components (substrates) might exhibit appreciable warpage with respect to the bow-free inner component (package).

While there is an incentive for using bow-free assemblies, there is also an incentive for narrowing the temperature range of the accelerated reliability testing: elevated temperature excursions might

produce an undesirable shift in the modes and mechanisms of failure, i.e. lead to failures that will hardly occur in actual operation conditions. Failure oriented accelerated test (FOAT) specimens are particularly vulnerable, since the temperature range in these tests should be broad enough to lead to a failure, and, if a shift in the modes and mechanisms of failures takes place during significant temperature excursions, the physics of such failures might be quite different of those in actual operation conditions. Mechanical pre-stressing can be an effective means for narrowing the range of temperature excursions during accelerated testing and, owing to that, - for obtaining consistent and trustworthy information. If pre-stressing is considered, the ability to predict the thermo-mechanical stresses in the test specimen is certainly a must.

Accordingly, the objective of this analysis is to obtain simple, easy-to-use, physically meaningful and practically useful closed form solutions for the evaluation of stresses in a bow-free test specimen of the type in question. The emphasis is on the role of compliant attachments, if any, between the inner and the two outer components.

The developed model can be used at the design and accelerated test stages of the development of bow-free electronic and optoelectronic products. The compliant attachments, if any, could be particularly comprised of beamlike solder joint interconnections that, if properly designed, have a potential to relieve the thermal stresses to an extent that the low-cycle-fatigue state-of-stress is avoided.

Introduction

Excessive warpage (bow) is a significant concern of microelectronics and photonics industries at all the stages of the product design, fabrication, testing and operation. Ability to predict and minimize warpage is critical for the product fabrication (soldering requirements and assembly) and operation [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31,

[32, 33, 34, 35]. Warpage is affected by package geometries, properties of the molding compound, mechanical characteristics of the employed materials and, certainly, the direction and level of temperature excursions. Package warpage during board assembly can cause the package terminals to have opens and/or shorts during and after the reflow soldering operation. Widely used today ball grid arrays (BGA) packages have been found to be particularly susceptible to warpage. The most significant warpage usually occurs at low (room or testing) temperatures. Package warpage could also be affected also by absorbed moisture. There are many ways to fight warpage. E.g., additional (surrogate) materials could be brought in to reduce bow. To prevent the ceramic substrate in an overmolded package from excessive bow and possible cracking, a high-CTE surrogate polymeric film co-cured with the molding compound was placed on the outer side of the substrate [36]. Another way to minimize substrate warpage was to place (also co-cured) a low CTE ceramic (even a ceramic with a negative CTE) on the opposite outer surface of the package. In other cases a metal frame stiffener could be added to the package structure to keep it flat during reflow soldering, as well as during its actual operations.

There is an obvious incentive to design and use bow-free (temperature change insensitive) package assemblies without resorting to surrogate stiffeners. It has been shown [37, 38] that this is indeed possible, if statically indeterminate tri- or multi-material assemblies are used. A bi-material assembly is statically determined and therefore cannot be made bow-free: the two thermally induced forces, one in tension, another one - in compression, acting in the components of a bi-material assembly always form a non-zero bending moment. In a tri- and a multi-material assembly the resulting bending moment can be made just zero, and this could be done by the proper selection of the substrate and/or bonding materials and their thicknesses. The bow-free condition for a tri-material assembly is [37, 38]:

$$\begin{aligned} & E_0^* E_1^* h_0 h_1 (h_0 + h_1) (\alpha_1 - \alpha_0) - \\ & - E_0^* E_2^* h_0 h_2 (h_0 + h_2) (\alpha_2 - \alpha_0) + \\ & + E_1^* E_2^* h_1 h_2 (h_1 + 2h_0 + h_2) (\alpha_1 - \alpha_2) = 0. \end{aligned} \quad (1)$$

In this condition, the zero component is the inner one, and the components #1 and #2 are the outer ones. The following notation is used in (1): h_i , $i = 0, 1, 2$, are the components' thicknesses, α_i , $i = 0, 1, 2$, are the CTE of the materials, $E_i^* = \frac{E_i}{1 - \nu_i}$, $i = 0, 1, 2$, are the effective Young's moduli of the materials, E_i , $i = 0, 1, 2$, are their actual Young's moduli, and ν_i , $i = 0, 1, 2$, are their Poisson's ratios. If, e.g., the inner material is absent ($h_0 = 0$), then the condition (1) can be fulfilled only provided that $\alpha_1 = \alpha_2$. It is imperative that the assembly components' materials are elastic and remain elastic during the entire operation of the package, otherwise the condition (1) will be compromised.

If the two outer components are identical, the condition (1) is fulfilled for any inner component. Such structures were addressed, with an emphasis on the behavior of the bonding material, in connection with the design and use of holographic memory devices [39, 40, 41, 42]. The "adhesive" in these structures was, in effect, an optically sensitive holographic material. Important findings were the attributes

of the behavior of circular "wafer-like" assemblies. Note that this finding was obtained using theory-of-elasticity approach, rather than structural analysis method employed in this analysis.

In some packaging systems the package is placed (sandwiched) between two substrates, which, in an approximate analysis, could be considered identical. Such systems (assemblies) are certainly bow-free. This important merit could be helpful in maintaining high coupling efficiency in optoelectronic warpage-sensitive devices. The highest thermal stresses in such a tri-component (one inner and two outer components) bi-material (the composite material of the inner component - package and of the material of the outer components - substrates) assemblies occur at low temperature conditions, are caused by the thermal contraction mismatch of the dissimilar materials of the components, and include normal stresses acting in the cross-sections of the components and the interfacial shearing and peeling stresses. The normal stresses in the component cross-sections determine the reliability of the component materials and the devices embedded into the body of the inner component. The interfacial stresses affect the adhesive and cohesive strength of the assembly.

While there is an incentive for using bow-free assemblies, there is also an incentive for narrowing the temperature range of the reliability testing: elevated temperature excursions during testing might produce an undesirable shift in the modes and mechanisms of failure. Failure oriented accelerated testing (FOAT) [43, 44] is especially vulnerable, since the temperature range in it should be broad enough to cause failures, and if a shift in the modes of failures takes place during temperature excursions, physics of such failures might be quite different of the one in actual operation conditions. In such a situation an appropriate mechanical pre-stressing can be an effective means for narrowing the temperature excursion width during accelerated testing and obtaining consistent and trustworthy test data. It is clear that if a pre-stressing is considered, the ability to predict the thermo-mechanical stresses in the test specimen is a must. The corresponding stress models were developed in [45] for an arbitrary elongated tri-material assembly and in [46] for an elongated assembly with identical outer components. The analysis that follows is, in a way, an extension and a modification of the analysis in Ref. [46]. The emphasis of this extension is on the role and significance of the compliant bonds. Particularly, the bonds could be comprised of an array of solder joints configured as short beams [47]. The objective of the analysis is to obtain simple, easy-to-use and physically meaningful and practically useful closed form solutions for the evaluation of stresses in a bow-free test specimen comprised of an electronic or optoelectronic package sandwiched between two identical substrates. The emphasis is on the role of the compliant attachments (bonds).

Analysis

1. Thermal Stresses

1.1. Shearing Stress

The longitudinal interfacial displacements in the assembly (specimen) components can be sought, in an approximate analysis based on the concept of the interfacial compliance [1, 2, 3, 4], in the form:

$$\begin{aligned}
u_0(x) &= -\alpha_0 \Delta t x - \lambda_0 \int_0^x T(\xi) d\xi + \kappa_0 \tau(x), \\
u_1(x) &= -\alpha_1 \Delta t x + \lambda_1 \int_0^x T(\xi) d\xi - \kappa_1 \tau(x).
\end{aligned}
\tag{2}$$

Here $u_0(x)$ and $u_1(x)$ are the interfacial displacements of the inner and the outer components, respectively, α_0 and α_1 are the effective coefficients of thermal expansion (CTE) of the composite materials, Δt is the change in temperature from an elevated to a low temperature,

$$\lambda_0 = \frac{1-\nu_0}{E_0 h_0}, \quad \lambda_1 = \frac{1-\nu_1}{2E_1 h_1}
\tag{3}$$

are the axial compliances of the inner and the outer components, respectively, E_0 and ν_0 are the effective elastic constants of the inner component material, E_1 and ν_1 are the elastic constants of the outer component materials, h_0 and h_1 are the component thicknesses,

$$T(x) = \int_{-l}^x \tau(\xi) d\xi
\tag{4}$$

is the axial force acting in the cross-sections of the inner component (the forces acting in the same cross-section of the outer components are obviously half this value), $\tau(x)$ are the interfacial shearing stresses, l is half the assembly length,

$$\kappa_0 = \frac{h_0}{6G_0}, \quad \kappa_1 = \frac{h_1}{3G_1}
\tag{5}$$

are the longitudinal interfacial compliances of the inner and each of the two outer assembly components, and

$$G_0 = \frac{E_0}{2(1+\nu_0)}, \quad G_1 = \frac{E_1}{2(1+\nu_1)}
\tag{6}$$

are the effective shear moduli of the component materials. The origin of the coordinate x is at the mid-cross-section of the assembly. The first formula in (5) was obtained for bonded assemblies with identical adherends [40], and the formula for the compliance κ_1 was obtained for a bi-metal thermostat or, more generally, for a bi-material adhesively bonded or soldered assembly [1].

The first terms in the equations (2) are unrestricted (stress free) displacements. The second terms are the displacements caused by the thermally induced forces (4) and are based on Hooke's law. This law assumes that all the longitudinal displacements in the given cross-section are the same. The third terms are, in effect, corrections to this assumption. They consider, in an approximate fashion, that the interfacial displacements are somewhat larger than the displacements of the inner points of the cross-section. The structure of these corrections is based on an assumption that these corrective terms

could be sought as a product of the thus far unknown interfacial shearing stress acting in the given cross-section and the interfacial compliance of the given assembly component. The condition of the displacements (2) compatibility can be written as

$$u_0(x) = u_1(x) - \kappa_a \tau(x).
\tag{7}$$

Here

$$\kappa_a = \frac{h_a}{G_a}
\tag{8}$$

is the compliance of one of the bonding layers, h_a is its thickness, and G_a is shear modulus of the attachment material. Introducing the equations (2) into the condition (7), the following equation for the shearing stress function can be obtained:

$$\tau(x) - k^2 \int_0^x T(\xi) d\xi = \frac{\Delta \alpha \Delta t}{\kappa} x.
\tag{9}$$

Here $\Delta \alpha = \alpha_0 - \alpha_1$ is the difference in the effective coefficients of expansion of the inner and the outer components,

$$k = \sqrt{\frac{\lambda}{\kappa}}
\tag{10}$$

is the parameter of the interfacial shearing stress,

$$\lambda = \lambda_0 + 2\lambda_1
\tag{11}$$

is the total axial compliance of the assembly, and

$$\kappa = \kappa_0 + 2\kappa_1 + 2\kappa_a
\tag{12}$$

is its total longitudinal interfacial compliance.

From (9) one can obtain by differentiation:

$$\tau'(x) - k^2 T(x) = \frac{\Delta \alpha \Delta t}{\kappa}.
\tag{13}$$

The zero boundary condition $T(l) = 0$ for the induced force can be translated, using the relationships (4) and (13), into the boundary condition

$$\tau'(l) = \frac{\Delta \alpha \Delta t}{\kappa}
\tag{14}$$

for the sought interfacial shearing stress $\tau(x)$.

The solution

$$\tau(x) = k \frac{\Delta\alpha\Delta t}{\lambda} \frac{\sinh kx}{\cosh kl} \quad (15)$$

to the [equation \(13\)](#) satisfies the boundary condition [\(14\)](#) and considers that the thermal interfacial shearing stress is antisymmetric with respect to the origin. The interfacial shearing stress [\(15\)](#) reaches its maximum value

$$\tau_{\max} = \tau(l) = k \frac{\Delta\alpha\Delta t}{\lambda} \tanh kl \quad (16)$$

at the assembly ends. For sufficiently long assemblies with stiff interfaces ($kl \geq 2.5$) this stress becomes assembly-length-independent:

$$\tau_{\max} = \tau(l) = k \frac{\Delta\alpha\Delta t}{\lambda} \quad (17)$$

The force acting in the cross-sections of the inner component can be determined, in accordance with the formula [\(4\)](#), by integrating the solution [\(15\)](#):

$$T(x) = -\frac{\Delta\alpha\Delta t}{\lambda} \left(1 - \frac{\cosh kx}{\cosh kl} \right) \quad (18)$$

The forces in each of the outer components are positive (tensile) and are half this value. The expression $\frac{\Delta\alpha\Delta t}{\lambda}$ in front of the parentheses in the formula [\(18\)](#) determines the force acting in the mid-portion of a long assembly with stiff interfaces ($kl \geq 2.5$). The corresponding normal stresses in the cross-sections of the assembly component can be evaluated by simply dividing the forces [\(18\)](#) by the component thicknesses.

1.2. Peeling Stress

When seeking the interfacial peeling stress $p(x)$, the normal stress acting in the through-thickness direction of the assembly, one could assume that this stress is proportional to the deflection function $w(x)$ of the outer component:

$$p(x) = Kw(x) \quad (19)$$

This relationship is based on the natural assumption that if the component experiences no deflections in the given cross-sections, the peeling stress is zero in this cross-section. It is assumed also, that the interfacial through-thickness spring constant K can be assessed by the approximate formula

$$K = \left[\frac{(1-\nu_0)h_0}{E_0} + \frac{2(1-\nu_1)h_1}{3E_1} + \frac{2(1-\nu_a)h_a}{E_a} \right]^{-1} \quad (20)$$

This formula is based on more or less elementary considerations using Hooke's law: the terms in the brackets represent, in an approximate way, the anticipated through-thickness displacements of the constituent materials.

Treating an outer component of the assembly as a thin elongated plate, the following equation of its equilibrium can be applied:

$$D_1 w''(x) = \frac{h_1}{2} T(x) - \int_0^x \int_0^x p(\xi) d\xi d\xi \quad (21)$$

The left part of this equation is the elastic bending moment, where

$$D_1 = \frac{E_1 h_1^3}{12(1-\nu_1^2)} \quad (22)$$

is the flexural rigidity of the component. The first term in the right part of [\(21\)](#) is the external bending moment caused by the thermally induced force $T(x)$ and the second term is the bending moment due to the peeling stress. Considering the relationship [\(19\)](#) and the equilibrium equation [\(21\)](#), the following equation for the peeling stress can be obtained:

$$p''(x) + 4\beta^4 \int_0^x \int_0^x p(\xi) d\xi d\xi = 4\beta^4 \frac{h_1}{2} T(x) \quad (23)$$

Here $\beta = \sqrt[4]{\frac{K}{4D_1}}$ is the parameter of the peeling stress, which is similar to the parameter [\(10\)](#) of the interfacial shearing stress. Differentiating the [equation \(23\)](#) twice with respect to the coordinate x , the following equation for the peeling stress function $p(x)$ can be obtained:

$$p^{IV}(x) + 4\beta^4 p(x) = 4\beta^4 p_0 \frac{\cosh kx}{\cosh kl} \quad (24)$$

where the notation

$$p_0 = h_1 \frac{\Delta\alpha\Delta t}{\kappa} \quad (25)$$

is used. The [equation \(24\)](#) has the form of the equation of bending of a beam supported by a continuous elastic foundation (which, however, determines the deflections, not the stress) and therefore its solution can be sought in the form [\[48\]](#):

$$p(x) = C_0 V_0(\beta x) + C_2 V_2(\beta x) + \frac{\eta^4}{1+\eta^4} p_0 \frac{\cosh kx}{\cosh kl} \quad (26)$$

Here $\eta = \frac{\beta\sqrt{2}}{\kappa}$ is the characteristic of the level of the peeling stress in comparison with the shearing stress, the functions $V_i(\beta x)$, $i = 0, 1, 2, 3$, are expressed as

$$V_0(\beta x) = \cosh \beta x \cos \beta x, \quad V_2(\beta x) = \sinh \beta x \sin \beta x,$$

$$V_{1,3}(\beta x) = \frac{1}{\sqrt{2}} (\cosh \beta x \sin \beta x \pm \sinh \beta x \cos \beta x),$$
(27)

and obey the following simple and convenient rules of differentiation:

$$V_0'(\beta x) = -\beta\sqrt{2}V_3'(\beta x); \quad V_1'(\beta x) = \beta\sqrt{2}V_0'(\beta x);$$

$$V_2'(\beta x) = \beta\sqrt{2}V_1'(\beta x); \quad V_3'(\beta x) = \beta\sqrt{2}V_2'(\beta x);$$
(28)

The first two terms in (26) is the general solution to the homogeneous equation that can be obtained from (26) by putting its right part equal to zero, and the last term is the particular solution to the inhomogeneous equation (26). The constants

$$C_0 = \frac{2\eta p_0}{1+\eta^4} \frac{\sqrt{2} \cosh u \cos u \tanh kl + \eta (\cosh u \sin u - \sinh u \cos u)}{\sinh 2u + \sin 2u},$$

$$C_2 = \frac{2\eta p_0}{1+\eta^4} \frac{\sqrt{2} \sinh u \sin u \tanh kl - \eta (\cosh u \sin u + \sinh u \cos u)}{\sinh 2u + \sin 2u}$$
(29)

of integration can be found from the boundary conditions

$$p''(l) = 0, \quad p'''(l) = 0.$$
(30)

The notation $u = \beta l$ is used in the formulas (29). The conditions (30) follow from (19) (indeed, the bending moments and the lateral forces are zero at the ends of the assembly, and therefore the second and the third derivatives of the deflection function are zero as well) and are equivalent to the conditions

$$\int_0^l \int_0^x p(\xi) d\xi d\xi = 0, \quad \int_0^l p(x) dx = 0$$
(31)

of self-equilibrium of the peeling stress. While the interfacial shearing stresses are anti-symmetric with respect to the assembly mid-cross-section and, hence, act in the opposite directions at the assembly ends and are therefore always in equilibrium, the peeling stresses are symmetric with respect to the assembly's mid-cross-section and have to satisfy the imposed self-equilibrium conditions (31) at each of the assembly end portions.

In the practically important case of an elongated assembly with stiff interfaces (large $u = \beta l$ values) the formulas (29) can be simplified:

$$C_0 = \frac{2\eta}{1+\eta^4} p_0 e^{-u} \left[\sqrt{2} \cos u + \eta (\sin u - \cos u) \right],$$

$$C_2 = \frac{2\eta}{1+\eta^4} p_0 e^{-u} \left[\sqrt{2} \sin u - \eta (\sin u + \cos u) \right]$$
(32)

Then the solution (26) results in the following expression for the distributed peeling stress:

$$p(x) = \frac{\eta}{1+\eta^4} p_0 \left[\eta^3 e^{-k(l-x)} + 2e^{-\beta(l-x)} \left((\sqrt{2} - \eta) \cos \beta(l-x) + \eta \sin \beta(l-x) \right) \right]$$
(33)

The first term in the brackets considers the direct impact of the interfacial shearing stress, and the second term is the response of the assembly to the longitudinal gradient of the interfacial shearing load. At the assembly ends ($x = l$)

$$p(l) = \frac{\eta}{1+\eta^4} p_0 \left(\eta^3 + 2(\sqrt{2} - \eta) \right).$$
(34)

When the parameter η (reflecting the relative roles of the interfacial peeling and shearing interfacial stresses) is significant, the formula (34) yields: $p(l) = p_0$. This result explains the physical meaning of the p_0 value (25): it is the peeling stress at the ends of a long assembly with a stiff enough through-thickness interface.

After the peeling stress is determined, the deflections can be found, in accordance with the formula (19), by simply dividing the peeling stress in this cross-section by the through-thickness spring constant

$$w(x) = \frac{p(x)}{K}.$$
(35)

1.3. Numerical Example

Input data:

Elastic constants:

$$E_0 = 12000 \text{ kg / mm}^2; \quad E_1 = 8000 \text{ kg / mm}^2;$$

$$E_a = 2000 \text{ kg / mm}^2; \quad \nu_0 = 0.25; \quad \nu_1 = 0.30; \quad \nu_a = 0.33;$$

Thicknesses:

$$h_0 = 2.0 \text{ mm}, \quad h_1 = 1.0 \text{ mm}; \quad h_a = 0.1 \text{ mm};$$

CTE's:

$$\alpha_0 = 10 \times 10^{-6} 1/^\circ\text{C}; \quad \alpha_1 = 25 \times 10^{-6} 1/^\circ\text{C};$$

Temperature change:

$$\Delta t = 200^\circ\text{C};$$

Half assembly length:

$$l = 100 \text{ mm}.$$

Computed data:

Thermal strain:

$$\Delta\alpha\Delta t = 0.0030;$$

Longitudinal interfacial compliances:

$$\kappa_0 = 6.9444 \times 10^{-5} \text{mm}^3/\text{kg}; \kappa_1 = 10.8331 \times 10^{-5} \text{mm}^3/\text{kg};$$

$$\kappa_a = 13.2979 \text{mm}^3/\text{kg}; \kappa = 62.1500 \times 10^{-5} \text{mm}^3/\text{kg};$$

Parameter of the interfacial shearing stress:

$$k = 0.5761 \text{mm}^{-1};$$

Maximum thermal shearing stress:

$$\tau_{\max} = 8.3800 \text{kg} / \text{mm}^2;$$

Through-thickness spring constant:

$$K = 3994.7270 \text{kg}/\text{mm}^3;$$

Flexural rigidity of the outer component treated as an elongated thin plate:

$$D_1 = 732.6007 \text{kgmm};$$

Parameter of the peeling stress:

$$\beta = 1.0805 \text{mm}^{-1};$$

Peeling-to-shearing stress parameters ratio:

$$\eta = 2.6524;$$

Maximum peeling stress in an assembly with an infinitely high through thickness spring-constant:

$$p_0 = 4.8270 \text{kg}/\text{mm}^2;$$

Maximum peeling stress in the actual assembly:

$$p(l) = 4.1035 \text{kg}/\text{mm}^2;$$

Thermally induced force in the inner component:

$$T = 14.5454 \text{kg} / \text{mm};$$

Normal thermal stresses in the cross-sections of the assembly components:

$$\sigma_0 = \sigma_1 = 7.2727 \text{kg}/\text{mm}^2;$$

Maximum bow:

$$w(l) = 1.027 \mu\text{m}.$$

Thus, the magnitudes of the maximum interfacial shearing stress, the maximum peeling stress and the maximum thermal normal stresses acting in the cross-sections of the assembly components are

comparable. If the thickness of the attachment were increased to $h_a = 0.4 \text{mm}$, the normal stresses in the cross-section of the assembly components will not change, the maximum interfacial shearing stress will reduce to $\tau_{\max} = 4.1920 \text{kg}/\text{mm}^2$; the maximum peeling stress will become $p(l) = 1.1258 \text{kg}/\text{mm}^2$; and the bow of the outer components will only $w(l) = 0.5081 \mu\text{m}$. In the absence of the compliant attachment, the predicted interfacial stresses and the maximum bow are $\tau_{\max} = 11.0783 \text{kg}/\text{mm}^2$; $p(l) = 6.8785 \text{kg}/\text{mm}^2$; and $w(l) = 1.2610 \mu\text{m}$. Thus, the application of a $100 \mu\text{m}$ thick strain buffer resulted in 24% reduction in the maximum shearing stress, in 40% reduction in the maximum peeling stress, and in 18.5% reduction in the maximum bow. The application of a $400 \mu\text{m}$ thick strain buffer resulted in 62% reduction in the maximum shearing stress, in 84% reduction in the maximum peeling stress, and in 59.7% reduction in the maximum bow.

2. Mechanical Stresses

Let a compressive external mechanical pre-stressing force \hat{T} be applied to the inner component of the specimen. This component experiences thermal compression that is intended to be enhanced by mechanical pre-stressing. If the load were applied to the outer components, one should simply reverse the subscripts “zero” and “one” in the final formulas. When the applied forces are tensile, the signs in the obtained solutions should be reversed.

The longitudinal interfacial displacements in the assembly components can be sought in the form:

$$\begin{aligned} u_0(x) &= -\lambda_0 \int_0^x T_0(\xi) d\xi - \kappa_0 \tau(x), \\ u_1(x) &= -\lambda_1 \int_0^x T_1(\xi) d\xi + \kappa_1 \tau(x). \end{aligned} \quad (36)$$

Since the sum of the forces $T_0(x)$ and $T_1(x)$ should be equal to the external force \hat{T} in all the cross-sections of the specimen, then

$$T_0(x) = \hat{T} - T_1(x), \quad (37)$$

and the displacement compatibility condition (7) results in the following equation for the mechanical interfacial shearing stress function $\tau(x)$:

$$\tau(x) - k^2 \int_0^x T_1(\xi) d\xi = -\frac{\lambda_0 \hat{T} x}{\kappa}. \quad (38)$$

Comparing the right parts of the equations (9) and (38) we conclude that the product $\lambda_0 \hat{T}$ plays in the case of mechanical loading the same role as the “external” thermal strain $\Delta\alpha\Delta t$ plays in the case of thermal loading. The equation (38) indicates also that the level of the parameter

$$k_0 = \sqrt{\frac{\lambda_0}{\kappa}} \quad (39)$$

of the interfacial stress is crucial: when this parameter is low (stiff inner component and/or high longitudinal interfacial compliance), the load \hat{T} will not be transmitted to the assembly interfaces and, hence, to its outer components.

The [equation \(38\)](#) has the following solution:

$$\tau(x) = -k \frac{\lambda_0}{\lambda} \hat{T} \frac{\cosh kx}{\sinh kl}. \quad (40)$$

Unlike the thermal stress, the mechanical stress is symmetric with respect to the mid-cross-section of the assembly and changes from its minimum (but not zero!) value

$$\tau(0) = -k \frac{\lambda_0}{\lambda} \hat{T} \frac{1}{\sinh kl} \quad (41)$$

at the origin to its maximum value

$$\tau_{\max} = \tau(l) = -k \frac{\lambda_0}{\lambda} \hat{T} \coth kl \quad (42)$$

at the specimen ends.

The mechanically induced forces acting in the cross-sections of the outer and the inner components of the assembly can be obtained by integration the [expression \(40\)](#) for the interfacial shearing stress and are as follows:

$$T_1(x) = -\frac{\lambda_0}{\lambda} \hat{T} \left(1 - \frac{\sinh kx}{\sinh kl} \right),$$

$$T_0(x) = -\hat{T} - \frac{\lambda_0}{\lambda} \hat{T} \left(1 - \frac{\sinh kx}{\sinh kl} \right) = -\frac{\hat{T}}{\lambda} \left(\lambda_1 + \lambda_0 \frac{\sinh kx}{\sinh kl} \right) \quad (43)$$

The compressive force $T_0(x)$ acting in the cross-sections of the inner component is equal to $-\hat{T}$ at the specimen ends and to $T_0(0) = -\frac{\lambda_1}{\lambda} \hat{T}$ at the mid-cross-section of a long enough specimen.

Comparing the formulas [\(17\)](#) and [\(43\)](#) for the maximum thermal and the maximum mechanical interfacial shearing stresses we conclude that the mechanical compressive force of the magnitude

$$\hat{T} = -\frac{\Delta\alpha\Delta t}{\lambda_0} \tanh^2 kl \quad (44)$$

results in the same maximum interfacial shearing stress at the assembly ends as the thermal mismatch strain $\Delta\alpha\Delta t$ does. For sufficiently long and/or stiff assemblies ($kl \geq 2.5$) this force becomes assembly length independent:

$$\hat{T} = -\frac{\Delta\alpha\Delta t}{\lambda_0}. \quad (45)$$

Thus, if one attributes the anticipated structural (“physical”) failures of the assembly of interest to the maximum value of the interfacial shearing stress, and intends to substitute thermal loading with an equivalent mechanical loading, he/she should apply a mechanical pre-stressing that is by a factor of $\frac{\lambda}{\lambda_0}$ higher than the maximum thermal force $\frac{\Delta\alpha\Delta t}{\lambda}$ in the mid-cross-section of a long specimen. The ratio $\frac{\lambda}{\lambda_0}$ changes from one, in the case of ideally stiff outer components (when $\lambda_1 = 0$), to infinity, when these components are ideally compliant ($\lambda_1 \rightarrow \infty$), and, in the above numerical example, is as high as 2.4. This means that the external compression should be by a factor 2.4 greater than the thermally induced force in the mid-portion of the thermally loaded specimen in order to result in the same maximum interfacial shearing stress. In practice, however, such a high axial compliance ratio might not be necessary. The practically important consideration is that the application of mechanical pre-stressing enables one to reduce considerably the temperature change to a “safe” level and nonetheless to achieve a high enough level of the induced thermo-mechanical stress. Based on the above analysis, the maximum interfacial shearing stress caused by the combined action of the thermal loading and mechanical pre-stressing is

$$\tau_{\max} = \tau(l) = \frac{k}{\lambda} \left(\Delta\alpha\Delta t + \lambda_0 \hat{T} \right). \quad (46)$$

Hence, the required change in temperature is

$$\Delta t = \frac{1}{\Delta\alpha} \left(\frac{\lambda \tau_{\max}}{k} - \lambda_0 \hat{T} \right). \quad (47)$$

If, e.g., using the data from the above example, an external compressive force $\hat{T} = 40.0000 \text{ kg/mm}$ is applied to the inner component, and if the same maximum interfacial shearing stress of $\tau_{\max} = 25.9800 \text{ kg/mm}^2$ is intended to be achieved, then the above formula yields: $\Delta t = 116.7^\circ\text{C}$. This temperature change is considerably, by about 42%, lower than change in temperature of $\Delta t = 200^\circ\text{C}$ for the non-pre-stressed specimen.

It should be pointed out, however, that because the thermal and the mechanical loadings are of different physical nature, it is impossible to “kill two birds with one stone”, i.e., to reproduce satisfactorily both the interfacial stresses and the stresses acting in the assembly components by applying a single level compressive force. The obtained results indicate that the action of the thermal strain of the magnitude

$$\Delta\alpha\Delta t = -\lambda_0\hat{T}\coth^2 kl \quad (48)$$

leads to the same maximum thermal interfacial shearing stress as the mechanical external \hat{T} force does.

Concluding Remarks

- Simple, easy-to-use and physically meaningful formulas are obtained for the evaluation of the thermo-mechanical stresses in a tri-component bi-material assembly (test specimen), when the inner component (package) is bonded (sandwiched) between two identical outer components (substrates).
- The carried out numerical examples indicate that the expected maximum thermal interfacial shearing stress and the maximum peeling stress are comparable, and that the predicted maximum peeling stress is as high as about 85% of the maximum peeling stress in an assembly with an infinitely high through-thickness stiffness.
- The computed interfacial stresses are also comparable with the normal stresses acting in the cross-sections of the assembly components (about half its magnitude).
- The application of mechanical pre-stressing enables one to reduce considerably the temperature range in accelerated testing.
- Compliant bonds enable one to reduce considerably the induced stresses.
- The developed models can be used at the early stages of the physical-design-for-reliability and accelerated testing of systems of the type in question. They can be used also beyond the electronic or photonic engineering field.

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