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Predicted Device-Degradation Failure-Rate

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Abstract

There is a concern that the continuing trend on miniaturization (Moore's law) in IC design and fabrication might have a negative impact on the device reliability. To understand and to possibly quantify the physics underlying this concern and phenomenon, it is natural to proceed from the experimental bathtub curve (BTC) - reliability "passport" of the device. This curve reflects the combined effect of two major irreversible governing processes: statistics-related mass-production process that results in a decreasing failure rate with time, and reliability-physics-related degradation (aging) process that leads to an increasing failure rate. It is the latter process that is of major concern of a device designer and manufacturer.

The statistical process can be evaluated theoretically, using a rather simple predictive model. Owing to that and assuming that the two processes of interest are statistically independent one can assess the failure rates associated with the aging process from the BTC data by simply subtracting the predicted ordinates of the statistical failure rates (SFR) from the BTC ordinates. The objective of this analysis is to show how this could be done.

The suggested methodology proceeds from the concepts that the actual ("instantaneous") SFR is a random variable with a known (assumed, established) probability distribution, that the experimental BTC can be represented by its infant mortality and the wear-out portions only (the steady-state portion in this case is simply the boundary between the infant mortality and wear-out portions) and that the two BTC portions considered can be approximated analytically. The cases, when the "instantaneous" SFR is distributed normally and in accordance with the Rayleigh law are used as suitable illustrations of the general concept.

The developed methodology can be employed when there is a need to better understand the relative roles of the statistics-related and physics-of-failure-related processes in reliability evaluations of electronic products. The methodology can be used also beyond the field of IC engineering, when there is a need to understand and, hence, to separate the roles of the two irreversible processes in question.

One of the major challenges of the future work is to determine the probability distributions of the actual ("instantaneous") SFRs for particular products and applications.

Introduction

There is an indication [1] that the continuing trend on miniaturization (Moor's law) might have a negative impact on the device reliability. This trend is of particular concern when it comes to deep submicron (DSM) technologies characterized by etching thicknesses below 90nm and when device is operated at the wear out portion of the BTC, when the degradation (aging) process plays the major role.

To better understand and possibly quantify the reliability physics underlying DSM technologies, it is natural to proceed from the BTC - an experimental reliability "passport" of a population of mass produced devices. The BTC considers and reflects, however, the combined effect of two governing irreversible processes: the statistics-related mass-production process and the reliability-physics-related degradation (aging) process. The first process results in a decreasing effective failure rate with time, while the second process leads to an increased failure rate. It is the latter process that is of major concern of the device manufacturer, and therefore there is an obvious need for being able to "extract" the degradation related failure rate from the total (observed) failure rate reflected by the BTC.

The statistical process can be assessed theoretically, using a simple predictive model [2]. Then, assuming that the two processes of interest are statistically independent, one can predict the failure rates associated with the degradation process by simply subtracting the SFR ordinates from the BTC data. The developed methodology shows how this could be done. It proceeds from an assumption that the actual (“instantaneous”) SFR is a random variable with a known (assumed or established) probability distribution. Normal and Rayleigh distributions of the random SFR are considered in this analysis as suitable illustrations of the general concept.

The analytical representation of the BTC and the developed methodology can be employed in the design and analysis of the next generations of electron devices. It can be used also beyond the electron device field, when there is a need to assess, in various engineering and applied science problems, the roles of the two governing irreversible processes in question.

Analysis

Analytical Bathtub Curve (BTC)

The experimental BTC (Fig.1) for the time dependent failure rate $\lambda(t)$ can be approximated as follows:

$$\lambda(t) = \begin{cases} \lambda_0 + (\lambda_1 - \lambda_0) \left(1 - \frac{t}{t_1}\right)^{n_1} = \\ = \lambda_0 \left[1 + \left(\frac{\lambda_1}{\lambda_0} - 1\right) \left(1 - \frac{t}{t_1}\right)^{n_1}\right], & 0 \leq t \leq t_1 \\ \lambda_0 + (\lambda_2 - \lambda_0) \left(\frac{t - t_1}{t_2}\right)^{n_2} = \\ = \lambda_0 \left[1 + \left(\frac{\lambda_2}{\lambda_0} - 1\right) \left(\frac{t - t_1}{t_2}\right)^{n_2}\right], & t_1 \leq t \leq t_1 + t_2 \end{cases} \quad (1)$$

Here λ_0 is the minimum (“steady-state”) value of the BTC, λ_1 is the initial value of the failure rate (at the beginning of the infant mortality portion), t_1 is the duration of this portion, λ_2 is the final value of the failure rate (at the end of the wear-out portion), t_2 is the duration of this portion, and the exponents n_1 and n_2 are expressed through the fullnesses β_1 and β_2 of the infant -mortality and the wear-out BTC portions as $n_{1,2} = \frac{\beta_{1,2}}{1 - \beta_{1,2}}$. These fullnesses are defined as the

ratios of the areas above the BTC to the areas $(\lambda_1 - \lambda_0)t_1$ and $(\lambda_2 - \lambda_0)t_2$ of the corresponding rectangulars. The exponents n_1 and n_2 change from 1 to infinity, when the fullnesses β_1 and β_2 change from 0.5 (triangle) to 1 (rectangular). The lowest $\lambda(t)$ values can be achieved in the case of the largest β_1 and β_2 (or n_1 and n_2) values.

The first formula in (1) describes the infant mortality portion of the BTC. The second formula is related to its wear-out portion. Both portions change slowly in the vicinity of their boundary. These slow changing regions of the BTC could be perceived and interpreted as the steady-state segment of the BTC. It is noteworthy that the analytical BTC in Fig.1 could be used for different applications, and should not be necessarily viewed as an 100% experimental information.

An experimental BTC [3] obtained during failure oriented accelerated testing (FOAT) of flip-chip solder joint interconnections in Bell Labs Si-on-Si technology is shown in Fig.2, where the number of cycles is used instead of time. The data for the failure rates in the second line of Table 1 are obtained with the input information of $\beta_1 = 0.8$ ($n_1 = 4$), $\beta_2 = 0.75$ ($n_2 = 3$), $t_1 = N_1 = 75$, $t_2 = N_2 = 300$, $\lambda_0 = 0.5 \times 10^{-4}$, $\lambda_1 = 7.5 \times 10^{-4}$, $\lambda_2 = 17.5 \times 10^{-4}$. The probabilities P_{NF} of non-failure are shown in the third line of Table 1 assuming that the exponential law $P_{NF} = e^{-\lambda N}$ is applicable for the entire duration of testing. The calculated data indicate that the time-dependence of the failure rate has a strong effect on the probability of non-failure.

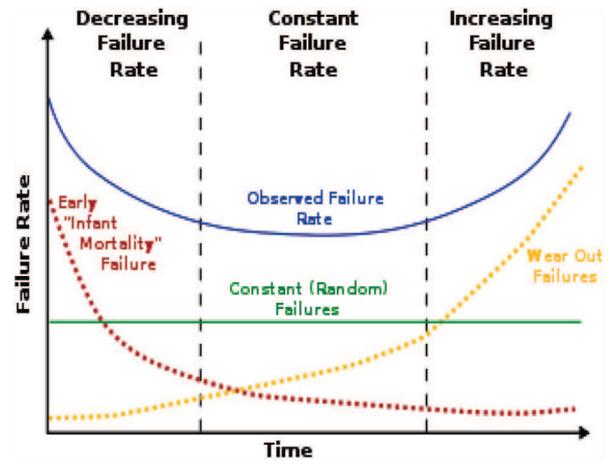


Figure 1. Bathtub curve

Table 1. Failure rate and the probability of non-failure vs. number of cycles (see Fig.2)

N	0	25	50	75	100
$\lambda \times 10^4$	7.5	1.8827	0.5864	0.5000	0.5098
P_{NF}	1.0	0.9953	0.9971	0.9963	0.9949
N	150	200	250	300	350
$\lambda \times 10^4$	0.7656	1.7297	3.8744	7.6719	13.5943
P_{NF}	0.9886	0.9660	0.9077	0.7944	0.6214

Effective SFR of Mass-Produced Devices

A customer that receives products from n vendors can evaluate the probability of non-failure for the received products, assuming that the exponential law of reliability is applicable, as

$$P(t) = \sum_{k=1}^n p_k \exp(-\lambda_k t). \quad (2)$$

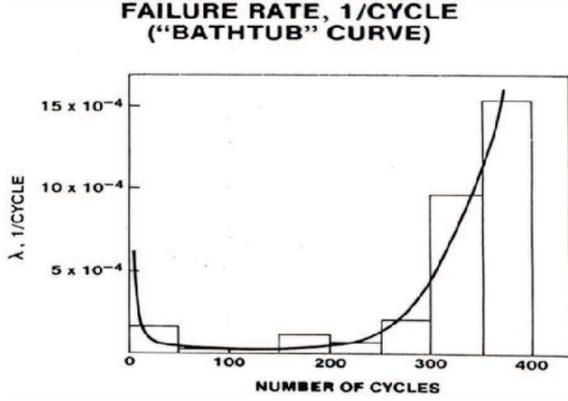


Figure 2. Experimental BTC obtained for solder joint interconnections in Si-on-Si flip-chip multi-chip module Bell-Labs design

Here p_k is the fraction of the products received from the k -th vendor, λ_k is the (random) failure rate of these products, and t is time. Assuming that the actual ("instantaneous") random failure rate can be, in effect, any number between zero and infinity, the sum in the formula (2) can be substituted, for a large n number, by the integral:

$$P(t) = \int_0^{\infty} \exp(-\lambda t) dF(\lambda) = \int_0^{\infty} \exp(-\lambda t) \frac{dF(\lambda)}{d\lambda} d\lambda = \int_0^{\infty} \exp(-\lambda t) f(\lambda) d\lambda. \quad (3)$$

Here $F(\lambda)$ is the probability distribution function and $f(\lambda)$ is the probability density distribution function of the random failure rate λ . The rationale behind the formulas (2) and (3) is as follows. The probability of non-failure of a large population of devices is determined by the failure rate of each particular device. The failure rate

$$\lambda(t) = \frac{dN_f(t)}{N_s(t)} \quad (4)$$

changes in a random fashion from one device to another and is defined as the ratio of the current rate $\frac{dN_f(t)}{dt}$ of the number $N_f(t)$

of devices that failed by the time t to the number $N_s(t)$ of devices that remained sound by that time. Substituting, in accordance with the ergodic theorem, the number of the sound items with the probability

$P(t)$ of their non-failure and the number of the failed items - with the probability $Q(t) = 1 - P(t)$ of failure one could write the formula (4) as

$$\lambda(t) = \frac{d[1 - P(t)]}{P(t)} = -\frac{1}{P(t)} \frac{dP(t)}{dt}. \quad (5)$$

Considering (3), this formula can be written in the form:

$$\lambda(t) = \frac{\int_0^{\infty} \lambda \exp(-\lambda t) f(\lambda) d\lambda}{\int_0^{\infty} \exp(-\lambda t) f(\lambda) d\lambda}. \quad (6)$$

Computations based on this formula confirm that the effective SFR decreases with time. In an extreme case, when the failure rate $\lambda(t)$ is distributed uniformly, the formula (6) yields: $\lambda_{ST}(t) = \frac{1}{t}$. This

result leads to a very sharp decrease in the effective SFR with time and is viewed as a non-realistic.

Normally Distributed "Instantaneous" SFR

To assess the decrease in this rate with time for a more realistic distribution, let us assume that the SFR is normally distributed:

$$f(\lambda) = \frac{1}{\sqrt{2\pi D}} \exp\left(-\frac{(\lambda - \bar{\lambda})^2}{2D}\right). \quad (7)$$

Here $\bar{\lambda}$ is the mean value of the random failure rate λ and D is its variance. The effective SFR $\lambda_{ST}(t)$ can be found as a function of time from the formula (6) and the distribution (7):

$$\lambda_{ST}(t) = \frac{\int_0^{\infty} \lambda \exp\left(-\frac{(\lambda - \bar{\lambda})^2}{2D} - t\lambda\right) d\lambda}{\int_0^{\infty} \exp\left(-\frac{(\lambda - \bar{\lambda})^2}{2D} - t\lambda\right) d\lambda} = \sqrt{2D} \varphi_N(\xi). \quad (8)$$

Here

$$\varphi_N(\xi_N) = -\xi_N + \frac{1}{\Phi(\xi_N)} \quad (9)$$

is a function of the dimensionless time

$$\xi_N = \frac{Dt - \bar{\lambda}}{\sqrt{2D}} \quad (10)$$

and so are the auxiliary function

$$\begin{aligned} \bar{\Phi}(\xi_N) &= \sqrt{\pi} \exp(\xi_N^2) [1 - \Phi(\xi_N)] \approx \\ &\approx \frac{1}{\xi_N} \left[1 + \sum_{k=1}^{\infty} (-1)^k \frac{1 \times 3 \times \dots \times (2k-1)}{2^k \xi_N^{2k}} \right] \approx \\ &\approx \frac{1}{\xi_N} \left(1 - \frac{1}{2\xi_N^2} + \frac{3}{4\xi_N^4} - \frac{15}{8\xi_N^6} + \frac{105}{16\xi_N^8} \dots \right) \end{aligned} \quad (11)$$

and the probability integral (Laplace function)

$$\Phi(\xi_N) = \frac{2}{\sqrt{\pi}} \int_0^{\xi_N} \exp(-\eta^2) d\eta. \quad (12)$$

The function $\text{Erfc}(x) = 1 - \Phi(x)$ is sometimes referred to as Weierstrass' zeta-function. The asymptotic expansion of the function $\bar{\Phi}(\xi_N)$ can be used for large ξ_N values, exceeding, say, 2.5. This expansion has been used when calculating the [Table 2](#) data. The [formula \(10\)](#) indicates that the “physical” (effective) time depends not only on the actual time t , but also on the mean and variance of the accepted (established) distribution of the statistical failure rate. The function $\varphi_N(\xi_N)$ is tabulated in [Table 2](#). This function changes from infinity to zero, when the dimensionless time ξ_N changes from $-\infty$ to ∞ . For dimensionless times below -2.5 the function $\bar{\Phi}(\xi_N)$ is significant, so that the second term in [\(9\)](#) becomes small compared with the first term, and the function $\varphi_N(\xi_N)$ can be put equal to the dimensionless time ξ_N itself, with an opposite sign though.

At the initial moment of time ($t = 0$) the [formulas \(10\)](#), [\(11\)](#) and [\(8\)](#) yield:

$$\begin{aligned} \xi_N &= -\frac{\bar{\lambda}}{\sqrt{2D}}, \quad \bar{\Phi}(\xi_N) = \sqrt{\pi} \exp\left(\frac{\bar{\lambda}^2}{2D}\right) \left[1 + \Phi\left(\frac{\bar{\lambda}}{\sqrt{2D}}\right) \right], \\ \lambda_{ST} &= \bar{\lambda} + \sqrt{\frac{2D}{\pi}} \frac{\exp\left(-\frac{\bar{\lambda}^2}{2D}\right)}{1 + \Phi\left(\frac{\bar{\lambda}}{\sqrt{2D}}\right)} \end{aligned} \quad (13)$$

With the initial value $\lambda_{ST} = \lambda_1$ (the degradation failure rate λ_{DG} is obviously zero at the initial moment of time), the third formula in [\(13\)](#) yields:

$$\frac{\lambda_1}{\sqrt{2D}} = \frac{\bar{\lambda}}{\sqrt{2D}} + \frac{1}{\sqrt{\pi}} \frac{\exp\left(-\frac{\bar{\lambda}^2}{2D}\right)}{1 + \Phi\left(\frac{\bar{\lambda}}{\sqrt{2D}}\right)} \quad (14)$$

Table 2. The governing function $\varphi_N(\xi_N)$ of the effective dimensionless time ξ_N

ξ_N	-3.0	-2.5	-2.0	-1.5
$\varphi_N(\xi)$	3.0000	2.5005	2.0052	1.5302
ξ_N	-1.0	-0.5	-0.25	0
$\varphi_N(\xi)$	1.1126	0.7890	0.6652	0.5642
ξ_N	0.25	0.5	1.0	1.5
$\varphi_N(\xi)$	0.4824	0.4163	0.3194	0.2541
ξ_N	2.0	2.5	3.0	3.5
$\varphi_N(\xi)$	0.2080	0.1618	0.1456	0.1300

ξ_N	4.0	4.5	5.0	6.0
$\varphi_N(\xi)$	0.1166	0.1053	0.0958	0.0809

ξ_N	7.0	8.0	9.0	10.0
$\varphi_N(\xi)$	0.0699	0.0615	0.0549	0.0495
ξ_N	11.0	12.0	13.0	15.0
$\varphi_N(\xi)$	0.0451	0.0414	0.0391	0.0332
ξ_N	20.0	30.0	50.0	100.0
$\varphi_N(\xi)$	0.0249	0.0166	0.0100	0.0050

ξ_N	200.0	500		
$\varphi_N(\xi)$	0.0025	0.001		

When the ratio $\frac{\bar{\lambda}}{\sqrt{2D}}$ changes from zero to infinity, the ratio

$\frac{\lambda_1}{\sqrt{2D}}$ changes from $\frac{1}{\sqrt{\pi}} = 0.5642$ to infinity. The relationship

(14) is tabulated in Table 3.

Table 3. The initial SFR vs. its mean value

$\bar{\lambda} / \sqrt{2D}$	0	0.5000	1.0000	1.5000
$\lambda_1 / \sqrt{2D}$	0.5642	0.7890	1.1126	1.5302
$\bar{\lambda} / \sqrt{2D}$	2.0000	2.5000	2.7500	2.8000
$\lambda_1 / \sqrt{2D}$	2.0052	2.5005	2.7501	2.8001
$\bar{\lambda} / \sqrt{2D}$	2.9000	3.0000		
$\lambda_1 / \sqrt{2D}$	2.9001	3.0000		

As evident from the computed data, the initial failure rate can be put

equal to its mean value, if the ratio $\frac{\bar{\lambda}}{\sqrt{2D}}$ exceeds 2.5. This is

usually the case indeed, since the accepted normal distribution, when applied to a random variable that cannot be negative, should be characterized by a significant ratio of its mean value to the standard deviation, so that the negative values of the assumed distribution, although exist, are meaningless, i.e., do not contribute appreciably to the predicted information.

The statistics-related and physics-of-failure-related modes and mechanisms of failure take place concurrently. Assuming that the two irreversible processes in question are statistically independent, the total probability of non-failure at the given moment of time can be determined as a product of the statistics-related and reliability-physics-related probabilities of non-failure:

$$P(t) = P_{ST} P_{DG} = \exp\left(-\frac{t}{\tau_{ST}}\right) \frac{1 - \Phi\left(\frac{t - \tau_{DG}}{\sqrt{2D_\sigma}}\right)}{1 + \Phi\left(\frac{\tau_{DG}}{\sqrt{2D_\sigma}}\right)} . \quad (15)$$

Table 4. Calculated probabilities of non-failure

1	t, hr	0	48	100
2	$\xi_N = t\sqrt{D/2} - \bar{\lambda} / \sqrt{2D}$	3.000	2.993	2.986
3	$\varphi_N(\xi_N) = -\xi_N + \frac{1}{\Phi(\xi_N)}$	3.000	2.993	2.986
4	$\lambda_{ST}(t) = \sqrt{2D}\varphi_N(\xi_N) \times 10^5 / hr$	84.85	84.67	84.45
5	$\lambda_{BTC} \times 10^5, 1/hrs$	84.85	84.67	96.00
6	$\lambda_{DG} \times 10^5, 1/hrs$	0	0	11.55
7	$P_{ST} = \exp(-\lambda_{ST}t)$	1.000	0.960	0.919
8	$P_{ST}^* = \exp(-\lambda_{ST}^*t)$	1.000	0.960	0.919
9	P_{DG}	1.000	0.999	0.999
10	$P = P_{ST} P_{DG}$	1.000	0.960	0.919
1	t, hr	250	1000	5000
2	$\xi_N = t\sqrt{D/2} - \bar{\lambda} / \sqrt{2D}$	-2.965	-2.859	-2.293
3	$\varphi_N(\xi_N) = -\xi_N + \frac{1}{\Phi(\xi_N)}$	2.965	2.859	2.347

4	$\lambda_{ST}(t) = \sqrt{2D}\varphi_N(\xi_N) \times 10^5 / hr$	83.86 2	80.86 4	64.937
5	$\lambda_{BTC} \times 10^5, 1/hr$	96.00 0	96.00 0	96.193
6	$\lambda_{DG} \times 10^5, 1/hr$	11.54 8	15.13 6	31.256
7	$P_{ST} = \exp(-\lambda_{ST}t)$	0.810 9	0.445 5	0.0389
8	$P_{ST}^* = \exp(-\lambda_{ST}^*t)$	0.808 9	0.428 0	0.0144
9	P_{DG}	0.999 3	0.994 2	0.0767
10	$P = P_{ST}P_{DG}$	0.810 3	0.442 9	0.0030

3	$\varphi_N(\xi_N) = -\xi_N + \frac{1}{\Phi(\xi_N)}$	0.4073	0.2822	0.1567
4	$\lambda_{ST}(t) = \sqrt{2D}\varphi_N(\xi_N) \times 10^5 / hr$	11.521	7.982	4.4327
5	$\lambda_{BTC} \times 10^5, 1/hr$	120.848	138.980	198.000
6	$\lambda_{DG} \times 10^5, 1/hr$	109.327	130.998	193.567
7	$P_{ST} = \exp(-\lambda_{ST}t)$	0.0561	0.0912	0.1698
8	$P_{ST}^* = \exp(-\lambda_{ST}^*t)$	6.13E-10	8.80E-12	1.82E-15
9	P_{DG}	0	0	0
10	$P = P_{ST}P_{DG}$	0	0	0

1	t, hr	7500	10000	20000
2	$\xi_N = t\sqrt{D/2} - \bar{\lambda} / \sqrt{2D}$	-1.9394	-1.5858	-0.1716
3	$\varphi_N(\xi_N) = -\xi_N + \frac{1}{\Phi(\xi_N)}$	1.9460	1.6089	0.6319
4	$\lambda_{ST}(t) = \sqrt{2D}\varphi_N(\xi_N) \times 10^5 / hr$	55.041	45.506	17.874
5	$\lambda_{BTC} \times 10^5, 1/hr$	96.662	97.577	108.704
6	$\lambda_{DG} \times 10^5, 1/hr$	41.621	52.071	90.830
7	$P_{ST} = \exp(-\lambda_{ST}t)$	0.0161	0.0106	0.0280
8	$P_{ST}^* = \exp(-\lambda_{ST}^*t)$	1.72E-3	2.10E-4	4.26E-8
9	P_{DG}	0	0	0

1	t, hr	25000	30000	40000
2	$\xi_N = t\sqrt{D/2} - \bar{\lambda} / \sqrt{2D}$	0.5355	1.2426	2.6568

The calculated probabilities $P(t)$ are shown, for the carried out numerical example, in the tenth line of Table 4. At the wear-out portion of the BTC they are obviously dominated by the low non-failure probabilities of the degradation process. The numerical example is carried out for the following input data: mean value (initial value) factor of the random failure rate:

$$\frac{\bar{\lambda}}{\sqrt{2D}} = \frac{\lambda_1}{\sqrt{2D}} = 3.0; \text{ standard deviation of the failure rate:}$$

$\sqrt{D} = 2 \times 10^{-4} / hr$; the initial failure rate: $\lambda_1 = 8.4853 \times 10^{-4} / hr$; the lowest failure rate: $\lambda_0 = 9.6000 \times 10^{-4} / hr$; the highest (allowable) failure rate: $\lambda_2 = 19.8 \times 10^{-4} / hr$; duration of the infant mortality portion: $t_1 = 48 hr$ (burn-in time); duration of the wear out portion $t_2 = 39,952 hr$ (obtained as the difference between the total time of operation of 40,000 hrs and the duration of the infant mortality portion); “fullnesses” of the infant mortality portion: $\beta_1 = 0.8$ ($n_1 = 4$); and the wear out portion: $\beta_2 = 0.75$ ($n_2 = 3$). Calculations are performed in Table 4 and the probabilities of non-failure associated with the degradation process are shown in Table 5. The degradation related failure rates are computed, for each particular moment of time, as the difference between the ordinates of the experimentally obtained BTC (line 5 in Table 4) and the calculated (predicted) SFR (line 4 in Table 4). It is assumed that the infant mortality portion is short, so that no degradation takes place during this time, regardless of whether a burn-in effort is applied or not.

Table 5. Calculated probabilities-of-non-failure caused by the degradation process

1	$tx10^{-3}, hr$	0	0.048	0.1	0.25
2	$\lambda_{DG}.x10^5, 1/hrs$	0	11.345	11.548	11.548
3	$\tau_{DG} = 1/\lambda_{DG}, hrs$	∞	8814.46	8659.51	8659.51
4	$\sqrt{2D_t} = \tau_{DG} / 2$	∞	4407.23	4329.75	4329.75
5	$t/\sqrt{2D_t}$	0	0.0109	0.0231	0.0577
6	P_{DG}	1.000	0.9999	0.9997	0.9993

1	$tx10^{-3}, hr$	1.0	2.5	5.0
2	$\lambda_{DG}.x10^5, 1/hrs$	15.136	21.168	31.256
3	$\tau_{DG} = 1/\lambda_{DG}, hrs$	6606.77	4724.11	3199.4
4	$\sqrt{2D_t} = \tau_{DG} / 2$	3303.38	2362.06	1599.7
5	$t/\sqrt{2D_t}$	0.3027	1.0584	3.1256
6	P_{DG}	0.9942	0.9121	0.0767

1	$tx10^{-3}, hr$	7.5	10.0	15.0	20.0
2	$\lambda_{DG}.x10^5, 1/hrs$	41.621	52.071	72.395	90.830
3	$\tau_{DG} = 1/\lambda_{DG}, hrs$	2402.63	1892.6	1381.31	1100.96
4	$\sqrt{2D_t} = \tau_{DG} / 2$	1201.32	946.29	690.65	550.48

5	$t/\sqrt{2D_t}$	6.2432	10.5676	21.7185	36.3319
6	P_{DG}	0	0	0	0

1	$tx10^{-3}, hr$	25.0	30.0	40.0
2	$\lambda_{DG}.x10^5, 1/hrs$	109.327	130.998	193.567
3	$\tau_{DG} = 1/\lambda_{DG}, hrs$	914.69	763.37	516.62
4	$\sqrt{2D_t} = \tau_{DG} / 2$	457.34	381.68	258.31
5	$t/\sqrt{2D_t}$	54.6635	78.5988	154.8536
6	P_{DG}	0	0	0

For short times at the beginning of the infant mortality (burn-in) process, the function $\bar{\Phi}(\xi)$ is significant, and the second term in the formula (9) is small compared to the first term, and the linear formula $\lambda_{ST} = \lambda_1 - Dt$ can be used to evaluate the SFR. Indeed, this simplified formula predicts the SFR at the end of the infant mortality time as $\lambda_{ST} = 84.834x10^{-5}hr^{-1}$. The exact number $\lambda_{ST} = 84.668x10^{-5}hr^{-1}$ is only 0.2% lower.

The obtained data indicate that the statistical probability of non-failure decreases with time at the rather significant portion of time despite the decrease in the SFR. At some moment of time (beginning with about 10,000 hours in the Table 4 example), the effect of the decreasing SFR starts to prevail, and the statistical probability of non-failure begins to increase with time. This circumstance does not play, however, an important role, because the degradation failure rates become significant and suppress the slight increase in the probability of non-failure associated with the SFR. In the line 8 of Table 4 the decrease in the probabilities of non-failure are shown assuming that the SFR remained at the initial level. The difference is large, especially for long times of operation, so that the change in the SFR with time should always be accounted for.

SFR Distributed According to Rayleigh Law

Assume now that the “instantaneous” failure rate is distributed in accordance with Rayleigh law, so that its probability distribution density function is

$$f(\lambda) = \frac{\lambda}{D} \exp\left(-\frac{\lambda^2}{2D}\right).$$

The standard deviation in this distribution is also its maximum value (mode). The formula (6) yields:

$$\lambda_{ST}(t) = \frac{\int_0^{\infty} \lambda^2 \exp\left(-\frac{\lambda^2}{2D} - t\lambda\right) d\lambda}{\int_0^{\infty} \lambda \exp\left(-\frac{\lambda^2}{2D} - t\lambda\right) d\lambda} = \sqrt{\frac{\pi D}{2}} \varphi_R(\xi_R). \quad (17)$$

Here

$$\varphi_R(\xi_R) = \frac{2}{\sqrt{\pi}} \frac{\left(\frac{1}{2} + \xi_R^2\right) \overline{\Phi}(\xi_R) - \xi_R}{1 - \xi_R \overline{\Phi}(\xi_R)} \quad (18)$$

is a function of the dimensionless time

$$\xi_R = \sqrt{\frac{D}{2}} t, \quad (19)$$

and so are the auxiliary function

$$\begin{aligned} \overline{\Phi}(\xi_R) &= \sqrt{\pi} \exp(\xi_R^2) [1 - \Phi(\xi_R)] \approx \\ &\approx \frac{1}{\xi_R} \left[1 + \sum_{k=1}^{\infty} (-1)^k \frac{1 \times 3 \times \dots \times (2k-1)}{2^k \xi_R^{2k}} \right] \approx \\ &\approx \frac{1}{\xi_R} \left(1 - \frac{1}{2\xi_R^2} + \frac{3}{4\xi_R^4} - \frac{15}{8\xi_R^6} + \frac{105}{16\xi_R^8} \dots \right) \end{aligned} \quad (20)$$

and the probability integral (Laplace function)

$$\Phi(\xi_R) = \frac{2}{\sqrt{\pi}} \int_0^{\xi_R} \exp(-\eta^2) d\eta \quad (21)$$

Calculations are carried out in Table 6. The input data for the calculated probabilities of non-failure for the degradation process are the same as in Table 5 for the case of the normal distribution of the

“instantaneous” SFR, namely, the ratio $\frac{\tau_{DG}}{\sqrt{2D\sigma}} = \frac{1}{\lambda_{DG} \sqrt{2D\sigma}}$ was assumed to be equal to 2.

Table 6. Calculated probabilities-of-non-failure caused by the degradation process

1	$\xi_R \times 10^3$	0	0.6788	1.4142
2	$\varphi_R(\xi_R)$	1.000	0.9998	0.9995
3	t, hr	0	48	100
4	$\lambda_{BC} \times 10^5 hr^{-1}$	84.85	84.67	84.45
5	$\lambda_{ST} \times 10^5 hr^{-1}$	25.07	25.06	25.05
6	$\lambda_{DG} \times 10^5 hr^{-1}$	59.78	59.61	59.40
7	$\lambda_{DG} / \lambda_{BC} \%$	70.45	70.40	70.34
8	P_{ST}	1.000	0.9880	0.9753
9	P_{DG}	1.000	0.9993	0.9984
10	$P = P_{ST} P_{DG}$	1.000	0.9873	0.9737

1	$\xi_R \times 10^3$	3.53	14.1	70.7
2	$\varphi_R(\xi_R)$	0.9988	0.9951	0.9780
3	t, hr	250	1000	5000
4	$\lambda_{BC} \times 10^5 hr^{-1}$	96.00	96.00	96.193
5	$\lambda_{ST} \times 10^5 hr^{-1}$	25.036	24.943	24.514
6	$\lambda_{DG} \times 10^5 hr^{-1}$	70.96	71.06	71.68
7	$\lambda_{DG} / \lambda_{BC} \%$	73.92	74.02	74.52
8	P_{ST}	0.9393	0.7792	0.2936

9	P_{DG}	0.9923	0.9878	0
10	$P = P_{ST}P_{DG}$	0.9321	0.7597	0

1	$\xi_R \times 10^3$	106.1	141.4	282.8
2	$\varphi_R(\xi_R)$	0.9695	0.9369	0.8763
3	t, hr	7500	10000	20000
4	$\lambda_{BC} \times 10^5 hr^{-1}$	96.662	97.577	108.704
5	$\lambda_{ST} \times 10^5 hr^{-1}$	24.301	23.484	21.965
6	$\lambda_{DG} \times 10^5 hr^{-1}$	72.36	74.09	86.74
7	$\lambda_{DG} / \lambda_{BC} \%$	74.86	75.93	79.79
8	P_{ST}	0.1616	0.0955	0.0124
9	P_{DG}	0	0	0
10	$P = P_{ST}P_{DG}$	0	0	0

1	$\xi_R \times 10^3$	353.6	424.3	567.7
2	$\varphi_R(\xi_R)$	0.8563	0.8258	0.7784
3	t, hr	25000	30000	40000
4	$\lambda_{BC} \times 10^5 hr^{-1}$	120.848	138.980	198.00
5	$\lambda_{ST} \times 10^5 hr^{-1}$	21.464	20.699	19.511
6	$\lambda_{DG} \times 10^5 hr^{-1}$	99.384	118.28	178.49
7	$\lambda_{DG} / \lambda_{BC} \%$	82.24	85.11	90.15
8	P_{ST}	0.00467	0.00201	0.000408

As one could see, the predicted data is rather different in the cases of the normal and Rayleigh distributions, and therefore the future work should include the analyses of the most suitable actual (“instantaneous”) SFR distributions.

Summary/Conclusions

- Easy-to-use and physically meaningful predictive model that can be used in application to both die and packaging technologies has been developed for the assessment of the level of the time-dependent material degradation (aging) process from the available experimental BTC.
- Normal distribution of the actual (“instantaneous”) SFR results in substantially lower resulting probabilities of non-failure than Rayleigh law.
- Future work should include the assessment of the actual distributions of the “instantaneous” random failure rates of the statistical process for various electronic products and applications.

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