Enumeration: counting difficulties are not always related to numbers

Claire Margolinas, Floriane Wozniak, Olivier Rivière

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CHAPTER FOUR

Enumeration: counting difficulties are not always related to numbers

Claire Margolinas
Université Clermont-Auvergne

Floriane Wozniak
Université de Strasbourg

Olivier Rivière
Université Clermont-Auvergne

Abstract

When parents and educators think about numeracy and how to help children's use of number, they often try to help them to memorise the numbers in order and to ask them to count how many objects are present. This is certainly useful in order to attain the level of numeracy which is important at school, but it is not sufficient. Parents and educators generally pay little attention to the way in which the objects are pointed to in order to give the right number. This chapter will focus on this aspect of numeracy that has been named "enumeration" by researchers in mathematics education (Brousseau 1984; Briand 1999).

Keywords: counting, enumeration, organisation, numbers, theory of didactical situations

What is and what is not enumeration?

In French and in English, “to enumerate something” is “to name things on a list one by one” (Oxford Advanced Learner's Dictionary). It does not thus refer to mathematics or to counting. For instance, on my table I have “pen, my glasses, an eraser” this list is an enumeration, in Brousseau’s meaning of this term, if it respects two conditions, which are not explicit in the dictionary’s’ definition:

all the objects on my table are present in the list

no object is present in the list more than once.

Another difference between the common meaning of the verb “to enumerate” and the sense we have given to enumeration is that it does not necessarily refer to an oral recitation. For instance, if I point silently to every objects on my table avoiding pointing twice the same object, we will name “enumeration” this silent procedure. Enumeration is involved in counting, as we develop in this chapter, because in order to count a collection of objects, one has to consider every objects once.

Comparing One To One Correspondence And Enumeration

Enumeration is thus a component of counting, it is, in this sense, “pre-numerical” (Briand 1999). The most well-known pre-numerical component of counting: one-to-one correspondence has been described by Piaget (Piaget and Szeminska 1941). One-to-one correspondence is linked to the acquisition of the concept of quantity. Two collections have the same quantity if they can be put in one-to-one correspondence: one egg with one egg holder, for instance, two collections have not the same quantity if this is not possible: for instance if there is no egg for some of the egg holders. It is pre-numerical in the sense that the concept of “number” is base on the construction of quantity (for a detailed description, see Margolinas and Wozniak 2012).

However, even if enumeration and one-to-one correspondence are both pre-numerical knowledge, they are different. In particular, one-to-one correspondence involves necessarily two collections which are matched by the one-to-one procedure between one element of the first collection and one element of the second collection. Enumeration involves only one collection, whose elements are pointed to exactly once.

Why is enumeration important in the context of counting? In order to understand this concept, we remain in the domain of counting and start with a basic example, which is how to count the dots in Figure 4.1.

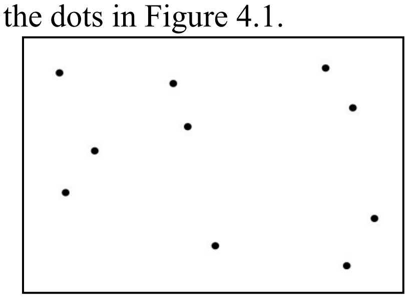

Fig. 4.1 ten dots which are not so easy to count!

In order to count the dots, you have to know the numbers in order up to ten, but it is not sufficient. You have also to be able to coordinate the recitation of the numbers and the pointing of the dots. It is in fact common to observe children who have been trained to rattle off the numbers but who are not pointing the dots at the same pace.

What this Chapter illustrates is that counting the dots requires even more competencies. We refer here to Briand (1999) who explained in detail the steps a child has to undertake in order to count a given collection.

“[the child has]:

1- to distinguish two different elements […]” (Briand, 1999, p.52, authors’ translation)

Why is this important? If you are describing a collection of animals, you can say that there are ten animals, or you can count two cats, three dogs and five birds, or you can name each animal: one ginger cat, one black cat, one terrier, etc. To count a collection requires children to distinguish each element and to consider these elements as parts of a whole. To count the dots, you have to distinguish them, even if they are identical in shape and colour, because they occupy different positions and they are elements of a whole collection of dots.

“[the child has]:

2- to choose an element of the collection

3- to pronounce a number word (one or the succeeding number word in the list of number words). ” (Briand, 1999 p. 52, authors’ translation)

If you start to count with point A (Figure. 4.2), it is possible to find a simple path to count the dots, following a pattern which forms some kind of horizontal parallels, path 1 and path 2 are examples of this procedure.

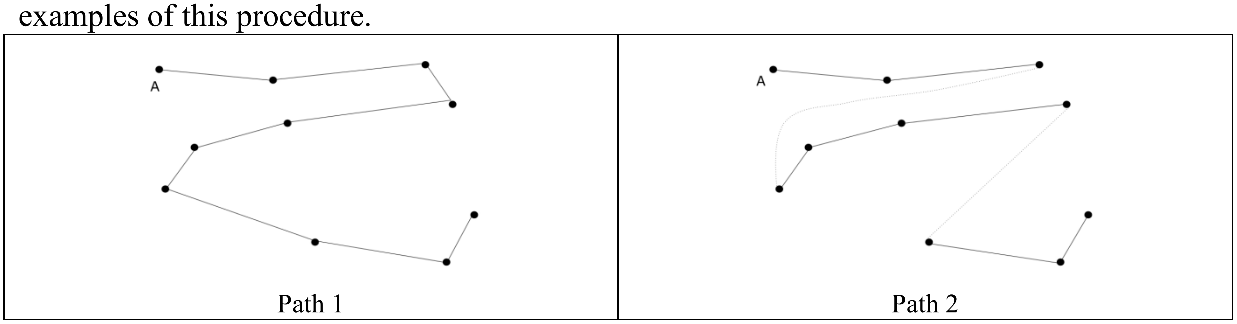

Fig. 4.2 Successful “horizontal” paths.

But if you start to count with point B (Figure 4.3), it is more difficult to find a path which allows remembering all the dots which have already been counted. For instance you can go up, turn right and count all the dots going around the borders and forget that you have already counted the point of departure. Or you can go around and forget that you have not counted the point in the middle, etc. Of course you can also succeed, but it is more difficult than with the horizontal paths shown in Figure 4.2.

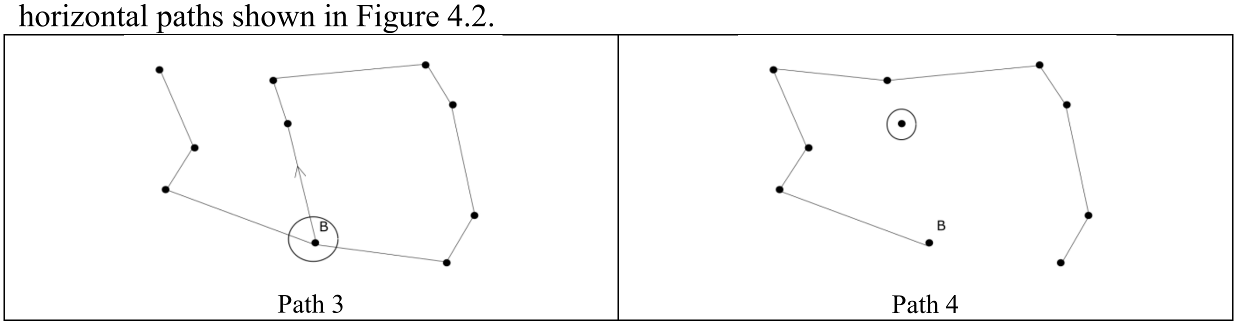

Fig. 4.3 Two ways to fail to count the dots, starting from point B.

Starting with any point, you have thus:

4- to memorise which elements of the collection have already been chosen

5- to conceive the collection of the elements which has not already been chosen

6- to repeat (for the collection of the elements which has not already been chosen) steps 2-3-4-5 until the collection of elements to be chosen is empty

7- to know that the last element has been chosen

8- to pronounce the last number word. " (Briand 1999 pp. 52-53, authors' translation).

If you can draw the path or strike the dots which have been already counted, it may prove easy, but if you cannot do that (which is often the case when you have to count something), you have to find an organised path that you can remember. This is why path 1 and 2 shall be easier to remember, because these paths begin with a straight line from left to right which is similar to the writing. Path 2 is consistent with a "write-like" disposition, whereas path 1 is similar to writing only because it is composed by horizontal lines but different because it is not always oriented from left to right. These paths reflect what Goody (1977) calls the "domestication of the mind" which is the result of the familiarity of writing, even for children who are not yet able to read or write.

In Briand's citation, only lines 3 and 8 refer to numbers which are necessary steps for counting a given set of objects, the other lines do not refer to any number. In fact, if what you want to do is not to count but to point silently exactly one time at every object, you have to follow steps 1, 2, 4, 5, 6, 7. These steps are a way to characterise "enumeration" as knowledge. In order to understand why this is important, consider what happens when you fail to achieve some of those steps.

Step 1: if you do not distinguish elements you might point to the entire collection and give the recitation of the "number song" and stop at some indeterminate number.

Step 2: to choose an element of the collection is not difficult in itself but, as is seen above, it is very important to have an efficient strategy.

Step 4: If you do not remember the elements of the collection you have already chosen, you might count again some object which has already been counted and fail to give the correct final number.

Step 5: If you do not conceive the collection of the elements which have not already been chosen, you might forget to count some object and fail to give the correct final number.

Step 6 and Step 7 emphasise the importance of steps 4 and 5: you have to know that all the elements have been counted when the collection of the elements which has not already been chosen is empty. If you are not aware of this, you will continue to count, not knowing that you have to stop exactly at the last element.

Didactical Engineering and Task Design

Didactical Engineering has been developed in France within the context of Brousseau's Theory of Didactical Situation in Mathematics (Brousseau 1997). Artigue (2009; 2015) describes didactical engineering as similar to the work of the engineer, who is acquainted with the major scientific knowledge and accepts the scientific methods but at the same time is obliged to work with very complex objects, far from the simplified objects which are studied by science. On the other hand, the theoretical framework gains from the results of didactical engineering.
Sometimes it is comforted by the result of the experiments but most of the time, the process involved during the research and experiment of didactical situations lead to important discoveries in the core of theories. The specificity of the French paradigm of research in mathematics education might be in the fact that the design in itself is not viewed as the final goal of the research and that the theoretical developments are most of the time more important than the design in itself (Margolinas and Drijvers 2015).

Enumeration as a knowledge in situation

In particular for early knowledge, it is quite impossible to describe the knowledge at stake without referring to real situations. In fact, a formal mathematical definition of enumeration can involve high level mathematics (Briand 1993; Margolinas, Wozniak and Rivière 2015). This is one of the reasons that we now describe some situations which can be considered as characteristics of enumeration.

These situations have been observed in clinical teaching experiments (Wittman 1995 pp. 367-368):

> "[…] 'clinical teaching experiments' in which teaching units can be used not only as research tools, but also as objects of study.
> The data collected in these experiments have multiple uses: they tell us something about the teaching/ learning processes, individual and social outcomes of learning, children's productive thinking, and children's difficulties. They also help us to evaluate the unit and to revise it in order to make teaching and learning more efficient."

Our research reported here is not strictly based on clinical teaching, since the experiment was conducted in individual interviews. However, to understand the concept of enumeration exactly, we describe what the children produce when they carry out a certain kind of task in a situation where enumeration was required to be used. Thus, in this chapter we give examples of social situations and games designed to enhance children's abilities to enumerate. This illustrates that it is possible to educate children in order to enhance their abilities to enumerate. In the first part of the following text, we describe some situations which are similar to "counting the dots", that is situations where it is not possible to move the object you want to enumerate. In the second part, we consider the enumeration of objects you can move and we show that these situations are very different.

Enumeration of fixed objects

When a child is counting, she has to pay a lot of attention to the number words, their order, etc. Moreover, a child cannot count a collection without knowing the number word sequence matches up to the number of elements. If we want to improve enumeration, we have to think about designed games and social situations where enumeration is present without counting. The following is an example of such a game.

Description of the game of hidden objects

The first game we describe has been designed and observed by a team of researchers (Margolinas, Wozniak and Rivière 2015), based on ideas by Briand (1993) and Berthelot and Salin (1992).

Fig. 4.4 Game of hidden objects.

The rationale for the game is this: if you have only dots on a sheet of paper and you ask children to point to every dot one at a time, you cannot verify the work of the child and, what is more important, the child herself is unaware of the result. Thus, if you want to design a game for enumerating the dots, you have to find a way for the validation to be apparent to both adult and child. It is possible now to do so with a digital device, because the device can memorise the dots which have been touched or not and give feedback about this information to the child. However, it seems also important that children understand that this kind of procedure is not due to any sophisticated device, but is related to common activities in real life. What we present now is one solution. We first explain the game and then we analyse the reason for the different materials and phases.

In order to play the game of hidden objects, you need a large sheet of paper, some little objects (e.g., counters) and little cups (e.g., cups for baking little cupcakes), the cups should be larger than the objects in order to hide the object totally when covered by the cup.

On the sheet of paper, you draw some dots. The number of dots is important: if you increase the number of dots, the game is more difficult. For children aged 2-4 year-old, ten dots might be sufficient to be a real challenge, for older children, you can draw up to twenty. For some adults, even thirty might be a challenge. We discuss later other variables of the game, apart from the number of dots. You need the same number of objects and cups plus one cup, and an open box near the sheet of paper. You can set the game on a table or on the floor, so that the child can reach easily all the materials. In order to introduce the game, the adult says something like this:

You will play a special game with all the materials. I will explain how it is played. First we have to set the game, can you help me? We have to put one counter on each dot.

When this is done, you explain the goal of the game:

To win the game you have to take all the counters and put them in the box. But this is really too easy! We will hide together the counters with the cups, this way, the game will be really challenging.

The result of the setting is shown in Figure 4.4. It is important for the child to be associated with the setting of the game: she thus knows exactly what is hidden by the cups.

The adults thus explain the rules of the game, demonstrating these rules using the extra set of counter and cup in order to show how to play. It is very important not to disturb the game which has already been set, because if the adult himself takes one cup, this cup might be considered by the child as the one to be taken first. The adult explains the rules, setting the cup over the counter and he manipulates the cup during the explanation:

When you play, you take the cup; you thus pick up the counter; you put the counter in the box and immediately after you put back the cup on the dot. You will do that with all the counters. When you think you have gathered all the counters, you say: "I have finished". We will then remove together all the cups. If you have put all the counters in the box, you win the game. If you have forgotten to pick some counters, you lose the game. You have to be careful with something else: during the game, if you take a cup and there is no counter, you also loose. Look: if I take this cup, there is no counter there! It has already been taken, it's here, in the box! In order to win the game, you have to be careful not to forget any counter and not to take off a cup more than one time.

In this game there are: dots, counters and cups, we now explain the role of these elements of the game.

The game's cognitive role is to induce children to enumerate without any other interfering difficulties, in particular there is no counting involved here. However, as stated above, enumerating the dots does not immediately produce facts which can be validated. In order to obtain a validation, there are two main possibilities: to mark the enumerated dots (e.g., with a pencil) or to find a way to determine which dot has been enumerated without any mark. To mark the elements is an interesting choice, which has been explored with older children by Briand (1999).

In the hidden objects game, the function of the dots is to provide a set of fixed objects. Since pointing at the dots does not result in anything tangible, the counters permit the child and the adult to know that a dot/counter has been enumerated. If the counters were always visible, the enumeration would have been too simple and not similar to the procedure required in order to count a collection. The function of the cups is to hide the presence/absence of the counters. To understand the importance of hiding the counters, if we refer to Figure 4.3 path 3, if there were no cups, the child would have seen that there was already no counter on dot B. The decision not to enumerate this dot again would not derive from the memorization of the path but from the simple absence of the counter. The cups, which are always at the same place (they are only momentarily displaced when the counter is picked up), hide the fact that the counters have been already taken or not.

In this situation, it is possible to try different procedures: we consider, like Tsamir et al. (2010), that pupils aged 5–6 years are capable of solving problems using several methods. We also consider that it is important for a game not to be too easy and in particular to offer the possibility to win… and to fail! To be able to fail is in fact important for learning mathematics. It is the fact that reality does not always match our anticipation which triggers the will to improve our procedure (Brousseau 1997). Of course, as it is the case with any game, adults have to be supportive, but the child has to reach a new ability to overcome the difficulty. Our observations show that children (from 3 to 10 years old): are not all able to win the first time, even the older ones; already adapt their procedure the second time, either winning or finding a better path, without any intervention of the adult.

Results of experimentation

Table 4.1 shows the results of the experiment with 44 children. The experiment was conducted by a researcher in a room situated near the children's classroom. The game was explained by the researcher to each child. The children had the opportunity to try a second time if they did not win the game the first time. The number of dots was 11 for children aged 3-5, 15 for children 6-8 and 20 for children 7-11, in order to take into account the ability of older children to recall a greater number of facts. The whole process was video recorded.

Age range	Number of pupils	Total of winners after the 1st game	Total of winners after the 2nd game
3-5	17	29%	53%
6-8	9	11%	67%
7-11	18	22%	56%
Total	44	23%	57%

Table 4.1
Results of an experiment of the hidden objects' game.

The number of children involved in the experiment at each level does not permit a valid statistical analysis, thus we examine only the major trends. The first result is the improvement between the first try and the second try (the winners of the 1st game were not allowed to play another time). The second result, consistent with Briand's findings (1993) is the stability of the proportion of winners across the groups.

We now examine some of the procedures employed by the children in this situation. The problem is to remember the spots already dealt with. Since there is no possibility of distinguishing the cups, you have to remember the positions of the cups taken or not. The number of the cups involved render almost impossible the memorization of the spots one by one. For instance, if you choose a spot in the middle and you take a spot left and now right, etc. it will be extremely difficult to win. Figure 4.5 shows two trials of a child who failed the first time after 9 cups and the second time with the 11th cup.

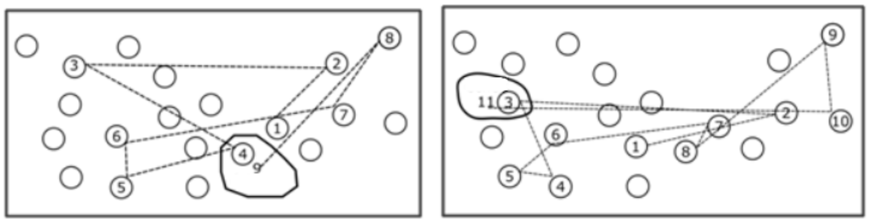

Fig.4.5 First and second trial of a 9 year-old with 20 hidden objects.

In order to win, children have to organise the path. One of the easier ways to do so is to draw mentally horizontal or vertical paths. Learning to write includes too the ability to recognise and to imagine these paths (Goody, 1977). Another way to win is to mentally separate the collection of objects in subsets, as seems to be the case in Figure 4.6.

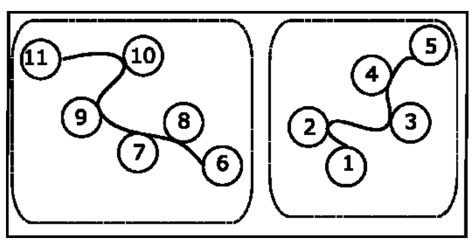

Fig. 4.6 First winning game of a 5 year-old with 20 hidden objects.

Each subset is easier to enumerate than the whole set because you can memorise a rather linear path.

Role of teachers or educators

What can thus be the role of an adult who is trying to help the child to build a successful procedure in the game of hidden objects? First of all the adult has to engage the child in the activity to ensure that it is possible to win the game, even if it is a challenging one. We have seen, in fact, that the mere repetition of the game was in itself sufficient to improve their procedure for some children.

Another step might be taken in order to help the children understand that the crucial point in the game is to memorise the path. Questions like, "which path have you taken?" might lead the child to understand that when you pick up the different cups, you might follow a path, and that some paths are better than others.

The number of objects might be adjusted to the possibilities of the child, with her acknowledgment: “do you want to try with fewer objects” or, in the case the child has been successful: “do you want to try with more objects?”

Of course the number of objects is not the only variable: in all our examples, the dots were purposefully set without any obvious order on the page, thus rendering the task quite difficult. For instance, if there were two visible subsets of dots, the procedure shown in Figure 4.7 would be easier to achieve. Or, if there were some visible horizontal directions for the dots, the procedure shown in Figure 4.2 would be favoured.

It is important for the adult to observe the child without any initial prompting from his part, in order to better understand which scaffolding might be useful if the child fails to win the game and build a successful procedure. For instance, it is different trying to help a child who has not yet understood that you have to build a path in order to win (Figure 4.5) than helping a child who has almost successfully built an horizontal path as in Figure 4.7.

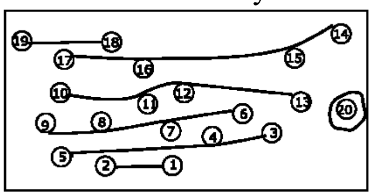

Fig. 4.7 11 year-old horizontal procedure, one cup has been forgotten.

Social and school situations similar to the hidden objects game in terms of enumeration. All the situations where fixed objects have to be all considered one by one require some procedure in order to enumerate all objects.

It is for instance the case in the experimental situation devised by Cornell and Heth (1983), who were investigating “spatial cognition” in the context of “hidden objects”. The common features of the situations they studied were: “the unconstrained search for objects in open environments” (p. 94). The second experiment (pp. 99-108) is interesting to reconsider using the concept of enumeration. The experiment involved “32 children in each of the two age-groups [3 year-olds and 5 year-olds]” (p. 101). The children were tested individually in interaction with a tester. Each child was seated on a short rotating chair at the centre of the test area and was shown a puzzle.

The tester started by saying:

> *This is like an Easter egg hunt. You sit here and watch me hide these puzzle pieces. When I have hidden 12 pieces, I will sit down and you get to find them. I'll bet you can find every piece, but you'll have to watch very carefully.*
> *The tester then rotated the chair so that the child faced a predetermined hiding place. There were 24 hiding places equidistant from the chair. (p. 101)*

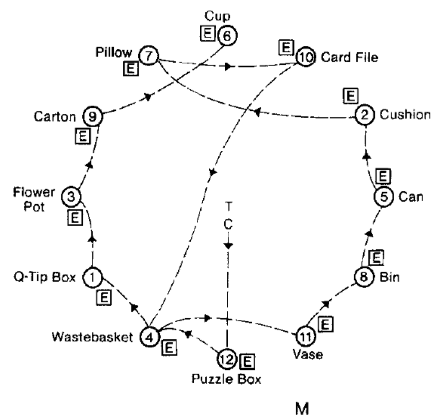

Fig. 4.8 An overhead schematic of the layout of events in the children's laboratory from Cornell and Heth (1983, p. 102).

T is the tester, sitting in the swivel chair, C is the child, standing at start, M is the mother, seated within view outside the testing area. Twelve puzzle pieces have been hidden in different containers, represented by labeled circles. Identical foils are represented as E, envelope. Numbers indicate the order of hiding. The dashed lines depict the path of a 5-year-old boy allowed to gather at will. He committed one error, a repetition at 7 o'clock." (Cornell and Heth 1983 p. 102).

Different conditions were included in the experiment. We focus on the following conditions: in the first, the child had to bring immediately the gathered piece in order to complete the puzzle at the centre of the area. In the second, the child was allowed to collect pieces and complete the puzzle afterwards (see figure above for an example).

The results show that:

Children were surprisingly good at finding all the pieces. […] the analysis of the total number of searches indicated that the older children used less (mean 14.6) than the younger children (mean 16.3). […] At least two kinds of errors could lead to more searches – intrusion (searches in containers where a piece had not been hidden) and repetitions (searches in containers where a piece had been previously found). […] Repetitions were more common […]. Younger children were more likely than older children to search at a place they had already searched (mean repetitions 19% and 10% respectively). Children who returned to the centre of the test area after finding each piece repeated 18% of their searches; children who were not so constrained averaged 11% repetitions. […] Children were more likely to exhibit least-distance searches in the no return condition, but this was primarily evident in the older age group. (p. 103)

Using a Monte Carlo simulation, the authors were able to conclude that children use a "spatially organised heuristic, such as searching the container immediately clockwise of the last searched" (p. 108-109).

This experiment was not designed to observe the spatial strategies of the enumeration puzzle pieces. In fact, repeating the search, which was considered as an "error" by the authors, has no impact on the reconstitution of the puzzle, which was the task given to the children. However, it shows that this kind of task can be analysed in terms of enumeration in a spatial context where no counting is needed. The results also show the possible 'spatial' abilities of children of pre-school age (3-5 year-old).

A great number of social and school situations involve enumeration abilities and procedures, like giving a treat to each seated child, etc. In this evocation, what happens if the children are not seated in a fixed position but are able to move freely or to be ordered to move in a certain way by an adult? This is basically the difference between the distribution of food in a traditional restaurant, where the guests are seated and the distribution of food in a fast-food, where the customers are moving to get their order. This is what we explore in the next section.

Enumeration of mobile objects

Let us return to counting. If you count some counters on a table, you take any counter, say one, discard this counter in another part of the table and so on. At the end of the process, all the counters have been displaced from their original place (in Figure 4.9, from left to right).

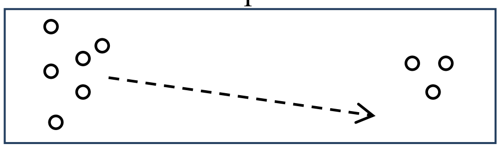

Fig. 4.9 counting counters on a table.

This gesture is so natural for adults that they often fail to understand the reason for this procedure. The target of this procedure is to enumerate the collection of counters: consider them all one by one in order to count the right amount of counters. When you can move objects, instead of following a path (which is often an alternate procedure), you can move the object from the place where the objects are initially to a place where you decide to put the objects you have already counted. In order to be successful, the places for the different status (already considered or already counted/ not considered or not counted) have to be distinct during the whole process.

Children often learn to count objects imitating adults, which is useful in order to accumulate some ready-made procedures. However, this way of learning does not permit variations in the procedures. For instance, it is easy to observe children who are able to count objects which lay on a table and unable to do so when the same objects are on their legs when seated. Since the same gesture (push to the right) is not available (the objects fall on the ground) they cannot adapt their procedure. It is thus important to devise situations where counting is not at stake but enumeration is the explicit purpose. This is for instance the case in the game of the "marked counters".

Description of the game of marked counters

For this game you only need some identical counters (15 to 25 counters) and some identical stickers (the relative sizes have to allow stickers to be placed on the counters). You put a sticker on only one face of some counters (for instance 10 counters). You have thus a collection of counters, some of them have a sticker and some have not. You also need a little box which contains all the counters you will use for the game.

In order to play (with children from 3 year-old), you first give the box to the child, tell her "to open the box and slowly overturn the box on the table". At this point it is important that the adult does not touch anything: the child has to make all the decisions to move everything (part of the box/counters). You thus ask the child to observe the counters, and tell what she observes, encouraging her to touch the counters. The child may tell you that some of the counters have a sticker and some have not. If it is not the case, you can prompt the child to take one of each sort of counters in hand and observe them. You thus ask the child to sort the counters, if the child does not understand 'to sort', you tell her to separate the counters with stickers and the counters without stickers.

Analysis of an observation

In order to understand the importance of this kind of situation, we show the procedure of 4 year-old Pauline which is characteristic of the evolution of procedures in the enumeration of this kind of collection.

Fig 4.10 the beginning of Pauline's work.

At the beginning Pauline sees that there are some visible stickers, she thus decides to take them. At this point she could have chosen to use the nearby box to place all the stickered counters (some children this age use this procedure) but it was not her choice. We can see that she has started to accumulate some stickered counters near her right hand.

Fig. 4.11 Some stickered counters are separated.

Pauline has now progressed in her procedure and delimited a special space for the stickered counters, pushing them closer to her. She is still taking out the obvious stickered counters.

Fig. 4.12 Apparition of a place for plain counters.

During her work, she observed that some stickers were not immediately visible (remember that only one face has a sticker). At first, she simply rejected these plain stickered counters in the heap of non-treated counters. But after discovering more plain counters (in order to be sure you have to examine both faces) she began to create a new place, visible near her left hand: the place for the plain counters.

However, you can see that this may not be completely conceptualised for different reasons. The first, is that she is still not using the different parts of the box (which are both usable to store the counters). The second, is that she has not rigorously separated the three spaces: one space for the non-treated counters, one space for the treated counters with two places: one for the stickered counters, one for the plain counters.

Fig. 4.13 Pauline has not yet finished her work, some counters have been mixed up.

It is not thus surprising to find that at the end she has mixed up some plain counters with non-treated counters which do not obviously appear to be stickered.

She thus failed to sort the whole collection of counters, but during the game she encountered some useful procedures. In this experiment, the researcher did not intervene or say anything to Pauline, but if it had been in another context (for instance an educative context) some intervention of an adult would have been useful.

Role of teachers and educators

The crucial point here is for the child to understand that different spaces are needed: a space for the initial stock, a space for the stickered counters, and a space for the plain stickers that have been verified on both faces. Questions like: "what are you putting in there?" might help the child to realise that she had an implicit intention underneath her action. If needed, the teacher or educator may insist: "where are you putting the stickered counters?"

However, the most important role of the educator is certainly to help the child to build bridges between different activities having the same characteristics in terms of enumeration. For instance, "where do you put the counters you have already counted? Where do you put the paper you have already written upon?" etc. These phrases can be associated with more standard phrases like "where do you put the objects you have already dealt with?" The different activities can be linked by the adult: "remember when you mixed up the stickered counters with the plain, it is the same in this activity, you have to carefully determine the spaces".

The importance of the observation of children strategies is also high here. For instance, if you ask the child to put the stickered counters in the box, she might not understand the meaning of this action, which may be only a material action and not a procedure. On the contrary, if the child has observed other children using the box, it is interesting for the adult to see if and how she understands this procedure. Some children may put all the counters in the box, misunderstanding the procedure. Prompting from the part of an adult is interesting only if the child can understand the reasons for the suggestion. Involving children in meaningful situations gives the opportunity for the adult to give some suggestions, at the right moment when they are really useful in situation.

Social and school situations similar to the marked counters game

Many situations are similar to the marked counters game and we do not try to encompass all of them. However, an historical and social remark might be of interest here (Margolinas, René De Cotret and Giroux 2006). The games and activities which are proposed to children are different from one family to the other, and not always dependent on social conditions.

For instance, some families like to play social games: cards, board games, etc. The children involved in these social games may have many occasions to count in different situations. They may have developed also enumeration strategies in these situations. However, when they are young, children are rarely able to count collections that have more than a dozen elements, and thus to enumerate these collections might not require any sophisticated procedures.

On the other hand, in the beginning of the 20th century, children were generally involved in domestic tasks. For instance in France, they sometimes had to sort pebbles from lentils, which was a painstaking task which required them to be organised, because the number of lentils was great. Nowadays, some children may have regularly to sort a huge collection of building blocks in order to find the exact colour and shape they want to finish their construction. But you may find educated children who have never had opportunity to sort collections of objects, for instance because they like books so much that they never play with blocks!

This is why it is important for educators and teachers to be aware that enumeration is not a spontaneous behaviour. The procedures will develop only if the children have the occasion to encounter the right situations and the appropriate prompting from educators.

Conclusion

This chapter has been dedicated to the development of some competencies which are necessary during counting but are independent of counting. These competencies should be considered as aspects of numeracy, but are often considered only in relation to a number's construction. Our experiments show that children do not develop these competencies in usual social situations: there is a need for specifically designed situations.

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