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# Second Order Sliding Mode Controllers for Altitude Control of a Quadrotor UAV: Real-Time Implementation in Outdoor Environments

Filiberto Muñoz, Iván González-Hernández, Sergio Salazar, Eduardo S. Espinoza and Rogelio Lozano

**Abstract**—This article deals with the design and real-time implementation of different second order sliding mode controllers for the altitude tracking of an aerial vehicle known as Quadrotor aircraft, a comparative study based in the error analysis is realized in order to know which controller has the best performance in a real-time application in outdoors environments. The strategies to compare are Classical First Order Sliding Mode Controller, Super Twisting Sliding Mode Controller, Modified Super Twisting Sliding Mode Controller and Nonsingular Terminal Super Twisting Sliding Mode Controller. The last three controllers mentioned previously are based on the second order sliding mode technique by the fact that ensure robustness with respect to modelling errors even under external disturbances while reducing the chattering phenomenon in comparison with first order sliding mode controllers. The Lyapunov stability theory is used to guarantee the convergence in finite time of the altitude tracking error in the different control laws proposed. To demonstrate the performance of the proposed solutions, an extensive set of simulation and real-time experiments results outdoor are presented. From the theoretical and experimental results it can prove the advantages and disadvantages of each of the techniques onto the aerial vehicle in external conditions.

## I. INTRODUCTION

In recent years, Unmanned Aircraft Systems (UASs) have attracted significant attention because of their wide range of applications. For example, sports media is making use of different kind of multi-rotorcrafts equipped with imaging systems to obtain and record video feed that can not be obtained by conventional means. In the domain of agriculture, UASs are being used for monitoring crop fields in a more efficient way. Highway monitoring is another area where UASs are being implemented to collect important data about the situation in crowded streets to improve the flow of vehicles. Disaster relief is another area that gets a lot of support from the UAS community. Having a UAS providing a first status of the disaster situation reduces risks for humans and increases the efficiency of rescue crews. Therefore, both rescuers and victims can benefit from this technology.

It is expected that Unmanned Aircraft Systems (UASs) will displace manned aircraft in diverse activities, before that happens, UASs must be safely integrated into the airspace system along with other manned and unmanned aerial vehicles. At present, the required level of autonomy for safe UASs operations is yet to be fully developed. Whether the quadrotor

is in Hover Flight Mode or in Trajectory Tracking Flight Mode, it is absolutely crucial maintain a desired altitude in order to have a satisfactory performance in autonomous mode for any of the applications previously mentioned. For this reason and because of their importance, the robust altitude control problem for an quadrotor UAS is studied in this research work.

In order to accomplish the autonomy necessary, robust flight control systems are required to track desired trajectories in the presence of wind or other disturbances. Robust Control strategies based on Sliding Mode Control (SMC) have shown to be an effective method for the control of dynamic systems with uncertainties. SMC strategies are appealing due to their ability to reject both disturbances and parameter variations, which is very useful for practical applications. Unfortunately, SMC induces signal chattering due to the discontinuous terms in the control law. These type of signals are undesirable because they can cause damage and accelerate tear and wear in electromechanical systems. Recently, SMC has been extended to incorporate novel techniques known as higher-order sliding mode control (HOSMC) [1],[2]. HOSMC preserves the main advantages of SMC, while at the same time yields improved performance with respect to (w.r.t) chattering effects as well as higher accuracy. To date, diverse systems have been controlled in real-time by means of HOSMC, see for instance[3],[4].

One of the most powerful HOSMC is the Second Order Super Twisting Sliding Mode Controller and their variations (Modified Super Twisting-SMC, Non-Singular Terminal-SMC) [5],[6],[7]. This algorithm generates a continuous control function that drives the sliding variable and its derivative to zero in finite time. The Super Twisting algorithm ensure robustness with respect to modelling errors and external disturbance, also this controller contains a discontinuous function under the integral term, attenuating the chattering effect [8]. In practice the controller gains are not easy to adjust due the bounds of perturbations, non-modeling dynamics and their time derivatives are need to known [9]. In order to overcome this disadvantage adaptive gains versions of the Super Twisting SMC have been developed. In [10], [4] in these papers an adaptive sliding mode super-twisting algorithm is proposed in order to reduce at minimum the chattering phenomena by using the adaptation law for the control gains.

## Related Works

In the literature, a wide class of controllers based on SMC have been proposed for the quadrotor position and orientation control problem. Most of them controllers are validated only with simulation results, a set of these works are presented to continuation. For example, the authors in [11] developed a SMC for a small quadrotor UAV, in this approach a First Order SMC using a Super twisting sliding mode disturbance observer for the estimation of perturbations. In [12], [13] the

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authors proposed a Robust Block Second Order SMC for a quadrotor, in this work an altitude control law based on Block Linearization Control technique is used and the Super Twisting control is employed for the longitudinal, latitudinal and heading motions control loops independently.

An Integral Backstepping control combined with adaptive terminal sliding mode to control the attitude of the quadrotor, and an adaptive robust PID controller is designed for the position control was developed in [14]. In [15] a backstepping method combined with sliding mode control for the position and orientation is applied for the subsystems of altitude, latitudinal and longitudinal, and heading control. A super twisting second order sliding mode control considering motor dynamics for position and orientation was designed in [16]. In [17] a robust backstepping sliding mode controller with adaptive radial basis function neural network is proposed to control the attitude of a eight-rotor unmanned aerial vehicle. Furthermore, in [18] a Second Order SMC is used in the controllers design for the actuated and underactuated systems of a small quadrotor UAV.

Terminal Sliding Mode Controllers with *nonlinear* sliding mode surface have been developed for the control problem of a quadrotor also. In [19],[20] a position and attitude tracking control is proposed, in this approach a robust terminal sliding mode control is develop for the altitude control, in this approach is not included the term corresponding to the Super Twisting control. A Fixed-time Terminal Sliding Mode Trajectory Tracking Control of a quadrotor is designed in [21], the novelty of this approach is that the settling time of the system can be assigned ahead of time and does not depend on the system initial conditions. The authors in [22] developed an adaptive second order terminal backstepping sliding mode for attitude control for a quadrotor. In [23] the dynamical model of a Quadrotor UAV is divided into actuated and underactuated subsystem, and a terminal sliding mode control is used to ensure that the yaw angle and the altitude can reach their desired value.

Unlike the large number of simulation results, only a reduced number of works related with the Super Twisting implementations in quadrotors have been developed until now. The following works present results of Super Twisting Control implementation in different subsystems of the quadrotor, all these works have been implemented in indoor environments. In [24],[25] a real time implementation of the Super Twisting controller for the attitude tracking in a quadrotor where set of indoor experiments were performed. In a first set of experiments a stationary ball joint base is employed restricting the aircraft yaw movement around  $\pm 30^\circ$  and the movement in the three-dimensional space in a fixed point. Moreover, in [25] was carried an experiment in free flight where the attitude is stabilized around zero with the altitude controller at around 1 m over the floor by an operator through a remote-control system. Another experiment results for attitude stabilization of a quadrotor UAS using Classical and Second Order SMCs was realized in [26], in this work the controller implemented was based on Super Twisting algorithm.

In [27] a sliding mode controller using a nonlinear sliding surface for the rotational and traslational subsystem of a quadrotor is designed e implemented in a indoor experimental fixed test bed with displacements in three-dimensional space

around of  $\pm 0.5$  meters in axis X, Y and Z. In [28] a Super Twisting and a Modified Super Twisting are applied to the altitude control of a quadrotor system in a indoor environment using a Motion Capture System Optitrack, in this work the maximum altitude is about 1.1 meters over the floor.

### Main Contributions

Since the Second Order Sliding Mode Controllers have been implemented only in a few works related to the robust altitude control of UAS, and the reported works present results only on indoor environments, in this work the implementation problem for altitude controller based on Super Twisting Sliding Mode Controller in outdoor environments is addressed.



Fig. 1: Photo of the Quadrotor aircraft used with the embedded control system *pixhawk*.

The main contribution of this work consists in the design and implementation in real time of a set of three Second Order Sliding Mode Controllers based on the Super Twisting algorithm for the altitude control of a Quadrotor aircraft. The vehicle built to test these robust controllers is shown in Figure 1.

The controllers presented in this works are

- Classical First Order Sliding Mode Controller
- Super Twisting Sliding Mode Controller (ST-SMC)
- Modified Super Twisting Sliding Mode Controller (MST-SMC)
- Nonsingular Terminal Super Twisting Sliding Mode Controller (NSTST-SMC)

With the aim to verify in real time the higher performance of the Second Order SMC, a conventional First Order SMC is also implemented.

The rest of the paper is organized as follows. The altitude dynamic model of an underactuated UAS and the Second Order Sliding Mode Controllers: ST-SMC, MST-SMC are described in Section II. Numerical simulations results are presented in Section III. Furthermore, an extensive set of experimental

results are presented in Section IV. Finally, the manuscript closes in Section V with some conclusions and future works.

## II. SECOND ORDER SLIDING MODE CONTROLLERS DESIGN

The adopted mathematical model describing the altitude dynamics for a quadrotor UAS is given by [29]

$$\ddot{z} = -g + \frac{1}{m} (\cos \theta \cos \phi) u + \xi(t, z, \dot{z}) \quad (1)$$

where  $z$  represent the altitude,  $\dot{z}$  is the altitude velocity and  $\ddot{z}$  is the altitude acceleration,  $\phi$  and  $\theta$  are the roll and pitch angles respectively (roll and pitch angles are measured in the experiments),  $m$  is the mass of the UAS,  $g$  is the gravitational force and  $\xi(t, z, \dot{z})$  includes the perturbations and uncertainty in the process dynamics ( $\xi$  in the following). Defining a state variables  $z_1$  and  $z_2$  as  $z_1 = z$  y  $z_2 = \dot{z}$ , the system (1) is rewritten as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{\cos \theta \cos \phi}{m} u - g + \xi \end{aligned}$$

The control objective is design and implement three robust controllers (ST-SMC, MST-SMC, NSTST-SMC)  $u$  such that the system output  $z_1$  can track a given reference  $z_{1d}$ , even in the presence of uncertainty in the process dynamics, as well as external disturbances. In order to develop the Super Twisting-SMC and the Modified Super Twisting-SMC a *linear* sliding mode surface  $s$  is defined as

$$s = \dot{e} + \beta_1 e \quad (2)$$

where  $e$  and  $\dot{e}$  represent the tracking error and the derivative of the tracking error and are defined as  $e = z_1 - z_{1d}$  and  $\dot{e} = z_2 - \dot{z}_{1d}$  respectively, and  $\beta_1 > 0$ . The objective of the controllers is to enforce the sliding mode into the surface  $s = 0$ .

### A. Super Twisting-SMC

The proposed Super Twisting-SMC strategy is given by the following equation

$$\begin{aligned} u_{ST} &= \frac{m}{\cos \theta \cos \phi} \left( g + \ddot{z}_1^d - \beta_1 \dot{e} - k_1 |s|^{1/2} \text{sign}(s) \right. \\ &\quad \left. - k_2 \int_0^t \text{sign}(s) d\tau \right) \end{aligned} \quad (3)$$

and with this controller the closed loop error dynamics are rewritten as

$$\dot{s} = -k_1 |s|^{1/2} \text{sign}(s) - k_2 \int_0^t \text{sign}(s) d\tau + \xi \quad (4)$$

Defining new variables  $x_1$  and  $x_2$  as

$$\begin{aligned} x_1 &= s \\ x_2 &= -k_2 \int_0^t \text{sign}(s) d\tau + \xi \end{aligned}$$

the sliding surface close loop dynamics (4) are given by

$$\begin{aligned} \dot{x}_1 &= -k_1 |x_1|^{1/2} \text{sign}(x_1) + x_2 \\ \dot{x}_2 &= -k_2 \text{sign}(x_1) + \dot{\xi} \end{aligned} \quad (5)$$

**Theorem 1.** *Suppose that for system (1) the derivative of the perturbation is globally bounded by  $|\dot{\xi}| < \xi_1$  with  $\xi_1 > 0$ , and the gains  $k_1$  and  $k_2$  are selected according to*

$$\begin{aligned} k_1 &> 0 \\ k_2 &> k_1 \frac{6\xi_1 + 4\left(\frac{\xi_1}{k_1}\right)^2}{2k_1} \end{aligned} \quad (6)$$

*Then, the Super-Twisting control law (3) yields finite-time convergence of the sliding surface  $s = 0$  and the tracking errors  $e$  and  $\dot{e}$  will asymptotically converge to zero [7],[30].*

### B. Modified Super Twisting-SMC

The proposed Modified Super Twisting-SMC strategy is given by the following equation

$$\begin{aligned} u_{MST} &= \frac{m}{\cos \theta \cos \phi} \left( g + \ddot{z}_1^d - \beta_1 \dot{e} - k_1 |s|^{1/2} \text{sign}(s) - k_2 s \right. \\ &\quad \left. - k_3 \int_0^t \text{sign}(s) d\tau - k_4 \int_0^t s d\tau \right) \end{aligned} \quad (7)$$

and the closed loop error dynamics  $\dot{s}$  are given by

$$\begin{aligned} \dot{s} &= -k_1 |s|^{1/2} \text{sign}(s) - k_2 s - k_3 \int_0^t \text{sign}(s) d\tau \\ &\quad - k_4 \int_0^t s d\tau + \xi \end{aligned}$$

Defining new variables  $x_1$  and  $x_2$  as

$$\begin{aligned} x_1 &= s \\ x_2 &= -k_3 \int_0^t \text{sign}(s) d\tau - k_4 \int_0^t s d\tau + \xi \end{aligned}$$

it can be obtained the following closed loop dynamic corresponding to the surface sliding

$$\begin{aligned} \dot{x}_1 &= -k_1 |x_1|^{1/2} \text{sign}(x_1) - k_2 x_1 + x_2 \\ \dot{x}_2 &= -k_3 \text{sign}(x_1) - k_4 x_1 + \dot{\xi} \end{aligned} \quad (8)$$

**Theorem 2.** *Suppose that for system (1) the derivative of the perturbation is globally bounded by  $|\dot{\xi}| < \xi_1 + \xi_2 |x_1|$  with  $\xi_1 > 0$  and  $\xi_2 > 0$ , assuming that the gains  $k_1, k_2, k_3, k_4$  are selected according to*

$$\begin{aligned} k_1 &> \sqrt{\xi_1} \\ k_2 &> \frac{1}{2} \sqrt{8\xi_2} \\ k_3 &> \xi_1 \\ k_4 &> \frac{k_1 \left[ \frac{1}{2} k_1^3 (2k_2 - \xi_2) + \left( \frac{5}{2} k_2^2 + \xi_2 \right) p_1 \right]}{k_1 \left( p_1 - \frac{1}{2} k_1^3 \right)} \end{aligned}$$

where

$$p_1 = k_1 \left( \frac{1}{4} k_1^2 - \xi_1 \right) + \frac{1}{2} k_1 \left( 2k_3 + \frac{1}{2} k_1^2 \right)$$

*Then, the Modified Super Twisting SMC (7) yields finite-time convergence of the sliding surface  $s = 0$  and the tracking errors  $e$  and  $\dot{e}$  will asymptotically converge to zero [7],[30].*

### C. Nonsingular Terminal Sliding Mode Control

In the development of the Nonsingular Terminal Super Twisting-SMC a *nonlinear* sliding mode surface  $s$  is defined as [31]

$$s = \dot{e} + \beta_1 e + \beta_2 \exp^{-\lambda t} e^{-2\alpha} \quad (9)$$

The proposed Nonsingular Terminal Super Twisting-SMC strategy is given as

$$u_{NSTST} = \frac{m}{\cos \theta \cos \phi} \left( g + \ddot{z}_1^d - \beta_1 \dot{e} - \beta_2 [-\exp^{-\lambda t} e^{-2\alpha} (\lambda + 2\alpha \dot{e})] - k_1 |s|^{1/2} \text{sign}(s) - k_2 \int_0^t \text{sign}(s) d\tau \right) \quad (10)$$

with  $\beta_1 > 0$ ,  $\beta_2 > 0$ ,  $\alpha > 0$  and  $\lambda > 0$ . The closed loop error dynamics  $\dot{s}$  are given by

$$\dot{s} = -g + \frac{\cos \theta \cos \phi}{m} u - \ddot{z}_1^d + \beta_1 \dot{e} + \beta_2 [-\exp^{-\lambda t} e^{-2\alpha} (\lambda + 2\alpha \dot{e})] + \xi \quad (11)$$

Replacing the control law (10) in the dynamics (11) we obtain

$$\dot{s} = -k_1 |s|^{1/2} \text{sign}(s) - k_2 \int_0^t \text{sign}(s) d\tau + \xi \quad (12)$$

The closed loop error dynamics (12) correspond to the ones given in equation (4) for the Super Twisting-SMC. Therefore, the stability analysis corresponding to Nonsingular Terminal Super Twisting-SMC is the same given by the Theorem 1.

**Remark 1.** *The Lyapunov stability proof of Theorems 1 and 2 has been presented in [7] [4], [30]. Therefore, only a sketch of the stability analysis is presented in Appendix A.*

### III. SIMULATION RESULTS

In this section, the altitude dynamical model of the quadrotor UAS given by Equation (1) is used to test the validity and performance of the proposed Second Order SMCs (ST, MST, NSTST) when faced with external disturbances. The control laws employed in the simulation are given by Equations (3), (7) and (10). The reference signal employed in all the simulations consists in a square waveform with magnitude equal to  $1m$ , offset of  $1m$  and frequency of  $0.0333Hz$ . The disturbance signal used in the simulations is given by  $\xi = \frac{1}{2} \cos(t)$ . It is worth mentioning that the gains parameter used for the three SMCs are the same both in simulation and experimentation. A complete list of parameters employed is shown in Table I.

TABLE I: Simulation and Experimental Parameters.

Parameters	ST	MST	NTSMC
Mass of the quadrotor, (m) [kg]	1.4	1.4	1.4
Gravitational acceleration, (g) [ $m/s^2$ ]	9.81	9.81	9.81
$k_1$	3.5	3	3
$k_2$	6	4.5	4.5
$k_3$	-	0.18	-
$k_4$	-	0.18	-
$\beta_1$	0.25	0.25	0.25
$\beta_2$	-	-	0.5
$\lambda$	-	-	0.35
$\alpha$	-	-	0.5

In Figure 2 the altitude control performance of the first order sliding mode controller is shown. It can be appreciated that

the UAS presents an tracking error in steady state. Moreover, the chattering effect is appreciated in the signal control and this phenomenon might excite unstable system dynamics and will degraded the overall controller performance in real-time implementation.

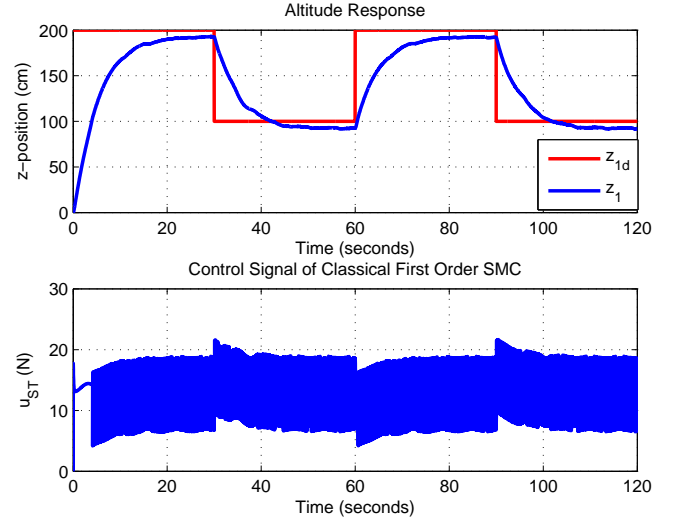


Fig. 2: Altitude response tracking simulation test using *First Order Sliding Mode control*.

The simulation results of the altitude control for the Second Order SMCs (ST, MST, NSTST) are presented in Figure 3. It can be seen that NSTST controller presents a faster response compared with ST and MST controllers. The ST-SMC and MST-SMC have a similar performance in terms of transient response and steady state. Should be noted that the three controllers follows efficiently the reference signal.

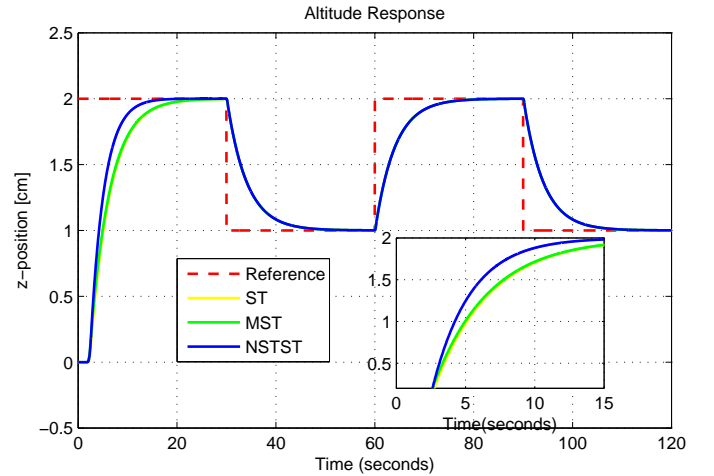


Fig. 3: Altitude response tracking simulation test using *ST, MST, NSTST*.

Figures 4-6 show the control signal of Super Twisting, Modified Super Twisting and Nonsingular Terminal Super Twisting Sliding Mode Controllers, respectively. From the Figures the reduction of the chattering effected is noted in the

right boxes and in the left boxes is shown the control signal in the first 4 seconds of simulation, where it is appreciated that NSTST controller present a larger action control generating a faster response. In Figure 7 the sliding surface for the three controllers is presented. From Figures 4-6 it can be observed the substantial chattering reduction in the signal control in comparison with the control signal of Classical First Order SMC presented in Figure 2 .

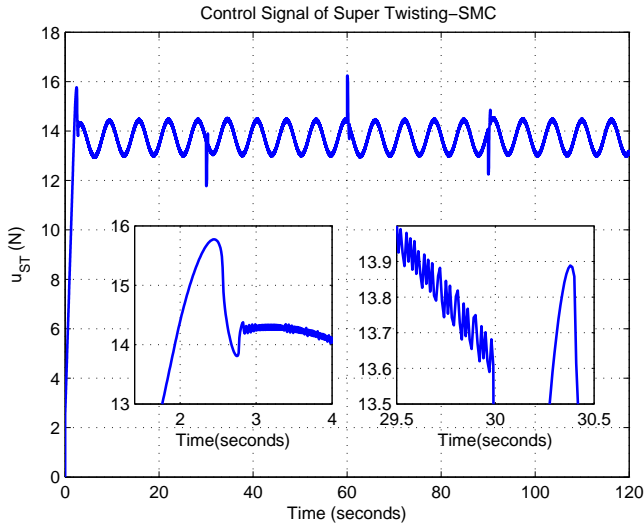


Fig. 4: Control signal of Super Twisting controller.

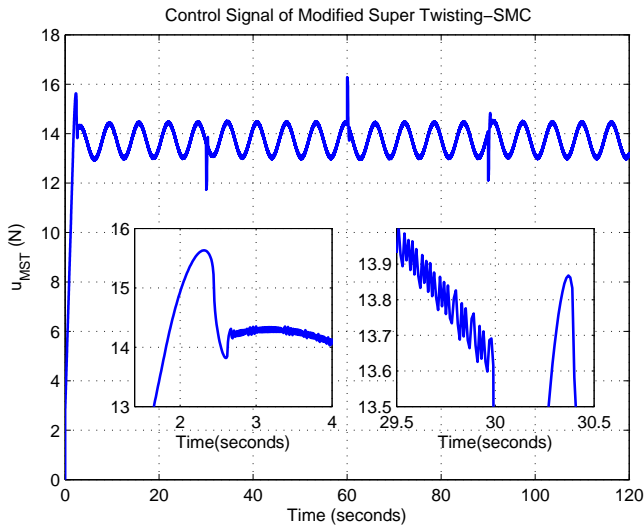


Fig. 5: Control signal of Modified Super Twisting controller.

#### IV. EXPERIMENTAL RESULTS

In this section we present the results obtained when applying the different control techniques proposed in the main results section. For this objective, we use a small Quad-rotor aircraft with a laser sensor placed at the bottom (see Figure 9) in order to obtain the actual distance that the vehicle is flying to compare and observe the behaviour on the z-dynamic (*altitude*) for the aerial vehicle using different controls such as

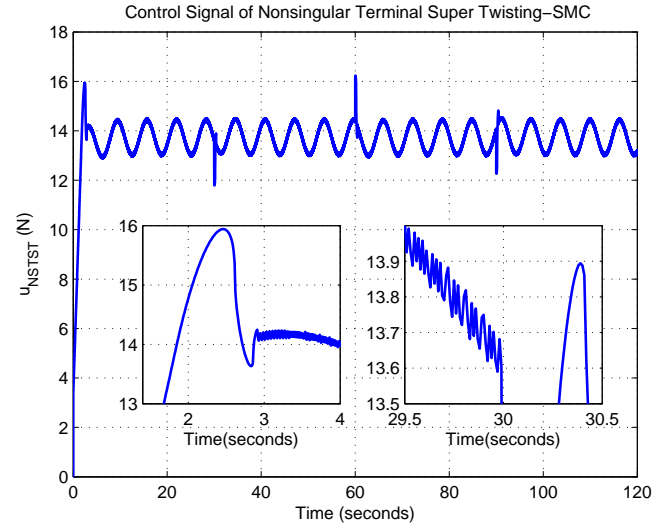


Fig. 6: Control signal of Nonsingular Terminal Super Twisting controller.

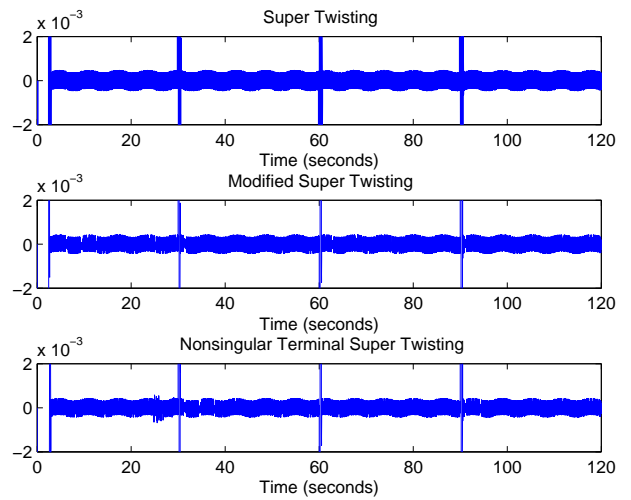


Fig. 7: Sliding Surface for Second Order Sliding Mode Controllers.

- Classical First Order Sliding Mode Control
- Super Twisting Sliding Mode Control
- Modified Super Twisting Sliding Mode Control
- Nonsingular Terminal Super Twisting Sliding Mode Control

We additionally use a Futaba 2.4GHz FASST radio system for transmitting the command signals to activate different corresponding altitude controls for the purpose of observe their performance. We implemented the control laws on a embedded high-performance autopilot-on-module system called *pixhawk* at 100 MHz, manufactured by 3D Robotics to test the efficiency of the proposed algorithms control for the altitude. The benefits of the Pixhawk system include integrated



multithreading, a Unix/Linux-like programming environment for applications in real-time. The physical parameters of the used aerial vehicle are given in Table II.

TABLE II: Quad-rotor aircraft physical parameters

Parameters	Value
Mass of the Quad-rotor aircraft, (m) [kg]	1.00
Gravitational acceleration, (g) [ $m/s^2$ ]	9.81
Distance, (d) [cm]	30
$I_{xx}$ , ( $kg \cdot m^2$ )	0.00029
$I_{yy}$ , ( $kg \cdot m^2$ )	0.00038
$I_{zz}$ , ( $kg \cdot m^2$ )	0.00079

Furthermore, in order to obtain a better measurement of the height ( $z$ -position) of our aerial vehicle was used a compact high performance optical sensor by PulsedLight model *LIDAR-Lite v2*, this sensor is shown in Figure 8. The laser sensor can be interfaced via I2C or PWM to the Quad-rotor aircraft, in Figure 9 it can be observed the sensor mounted in the aerial vehicle. Finally, its operating range is up to 40 meters which is feasible for the development of our experiments outdoor.



Fig. 8: (*LIDAR-Lite v2*) laser sensor used to measure the altitude of the Quad-rotor aircraft outdoor.

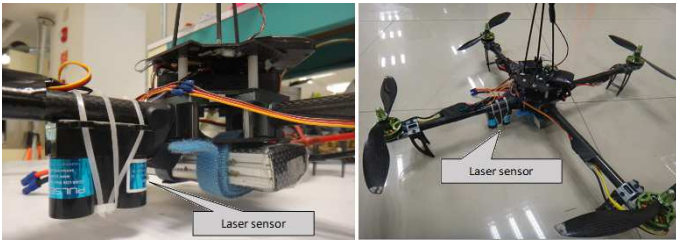


Fig. 9: Photo of the Quad-rotor aircraft setup with the laser sensor mounted used for testing.

To analyze the effectiveness and the robustness of the proposed second order sliding mode controllers, four set of experiments have been performed on the Quad-rotor aircraft. Experiment IV-A involves the aircraft altitude stabilization at  $2m$ . Initial conditions are set for the aerial vehicle altitude as  $z_0 = 1m$ . In Experiment IV-B, altitude tracking test was carried out. The reference signals are of square form with magnitude equal to  $1m$  and a frequency of  $0.0125Hz$ . In Experiment IV-C, altitude control is placed at an initial height of 2 meters for about 40 seconds then the control is disabled until the aircraft increase or decrease its height as the case may, and afterwards the control is enabled to observe the behavior of the rate of response of different

controls. Finally, in Experiment IV-D the altitude is externally disturbed to explore the disturbance rejection performance in stabilization around the desired altitude. For this application the gains picked for the controllers are described in the Table I.

We also carried out several real-time experiments to stabilize the altitude of aerial vehicle outdoor. The controllers parameters were tuned by simulations results, until obtaining a better responses performance of the system. Figures 10 to 22 presents all experimental results obtained by applying the different controllers previously described.

#### A. Hovering experiment

In this experiment, the Classical First Order SMC and the three Second Order SMC were implemented in hovering mode in order to observe the performance in outdoor environment. To start, Figure 10 illustrates the behavior of a typical sliding mode control with its corresponding control signal. The main disadvantage of this controller is that it has a poor performance due to the so-called *chattering phenomenon* (the effects of the discontinuous nature of the control). On the other hand, Figures 11 to 13 show the altitude response and the applied control signals. The desired reference changes from 1 meter to 2 meters in height and the signals corresponding to the control inputs applied at each instant. One can notice the effectiveness of the proposed second order sliding mode controllers compared with the typical sliding mode control above described.

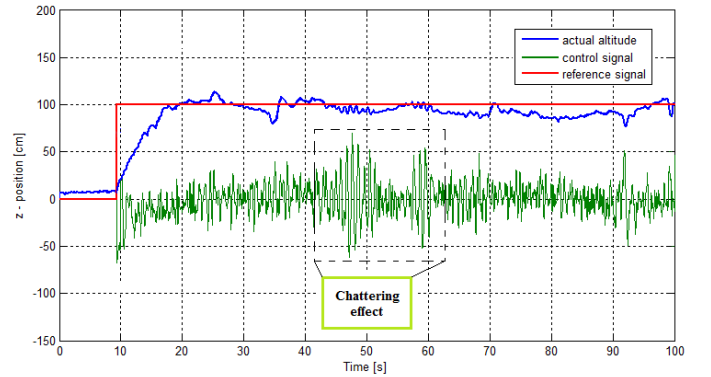


Fig. 10: Real-time altitude control response at a height of  $1m$  using a typical *sliding mode algorithm* in outdoor environment.

#### B. Tracking experiment

Altitude tracking experiment has been carried out in outdoor environment. Figures 14 to 16 show the altitude response when the reference signal is a square wave with a period equal to  $T = 80s$ . The obtained results clearly show the effectiveness controllers. Moreover, we can notice that the applied control signal decrements in each case because the gains are bounded by the terms defined in the control laws (3), (6) and (9). On the other hand, we can see that the phenomenon of chattering persists in the control signal even when the integral action in each of the controls is added.

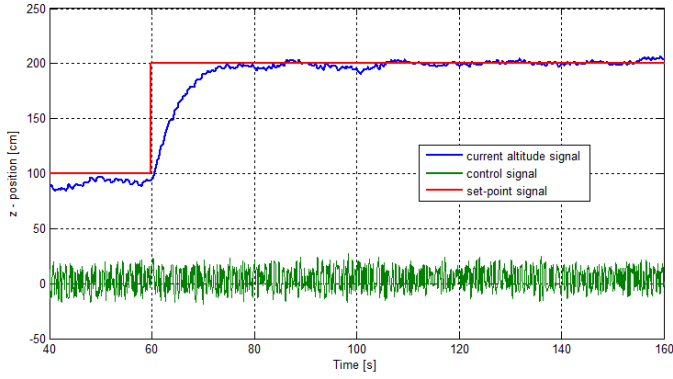


Fig. 11: Altitude response stabilization in hover-mode using *super twisting-SMC* in outdoor environment.

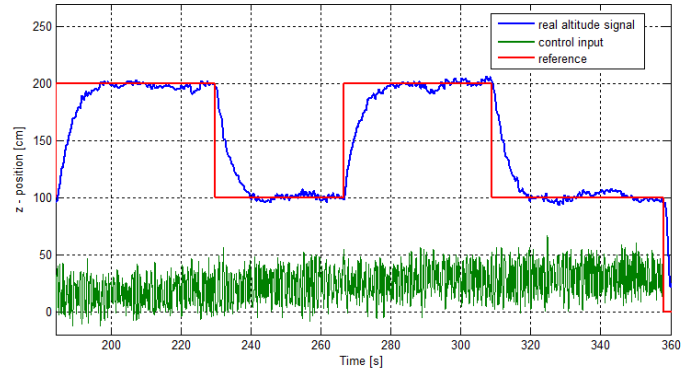


Fig. 14: Altitude response tracking test using *super twisting-SMC* in outdoor environment.

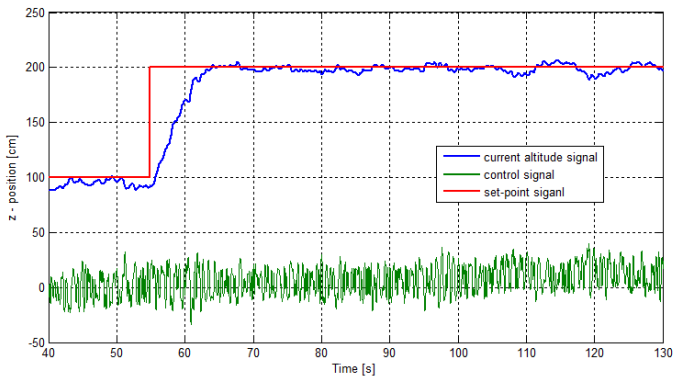


Fig. 12: Altitude response stabilization in hover-mode using *modified super twisting-SMC* in outdoor environment.

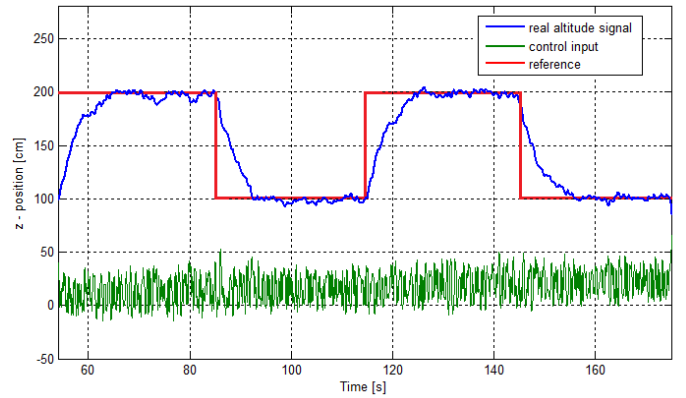


Fig. 15: Altitude response tracking test using *modified super twisting-SMC* in outdoor environment.

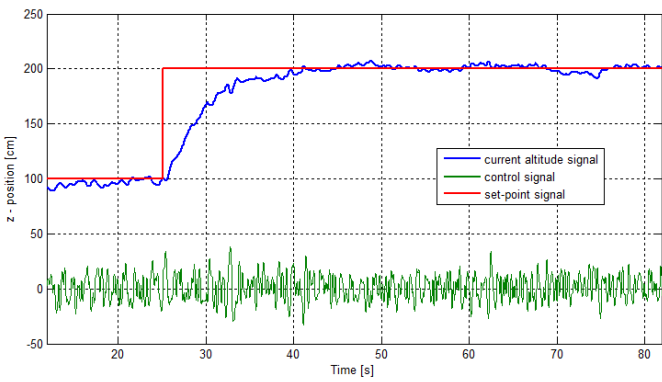


Fig. 13: Altitude response stabilization in hover-mode using *nonsingular terminal sliding mode control* in outdoor environment.

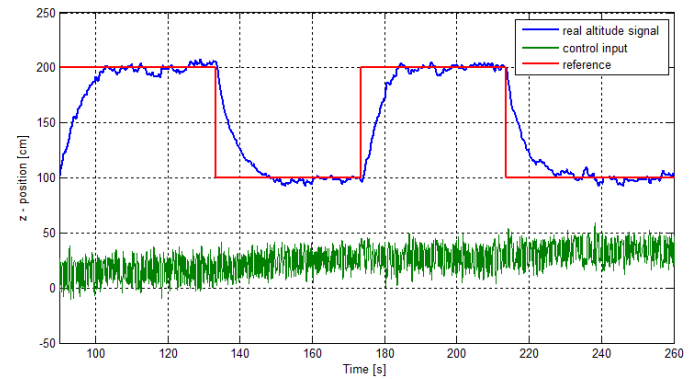


Fig. 16: Altitude response tracking test using *nonsingular terminal sliding mode control* in outdoor environment.



### C. Disable/Enable control experiment

Altitude disable/enable switch control experiment has been carried out in outdoor environment. Figures 17 to 19 show the altitude response when the control is disabled and subsequently is taken up again the control (enabled) in order to see if this control is able to return the aerial vehicle to the given reference. From these Figures, it is clear that the Quad-rotor aircraft has a better behavior towards the reference when the control strategy considers the integral term, attenuating the chattering effect as in the previous cases. Notice that, in Figure 18 you can see the immediate response of the control input when the control is triggered in order to increase or decrease the energy depending on the case to carry the aircraft to the reference at 2 meters.

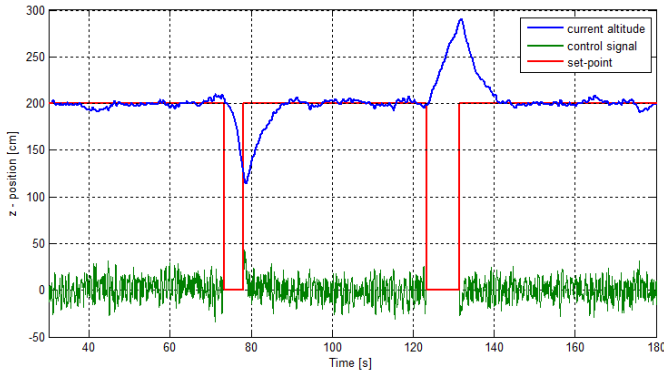


Fig. 17: Altitude response test with disable/enable switch control using *super twisting-SMC* in outdoor environment.

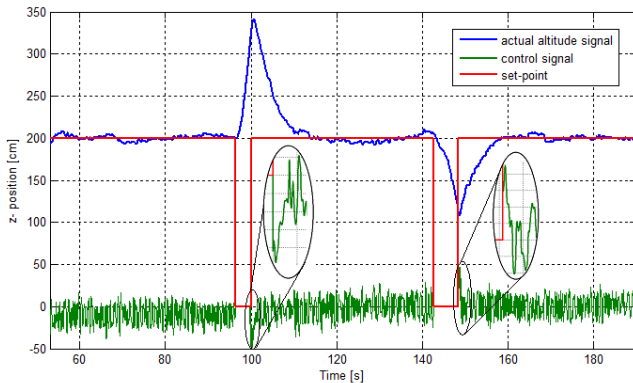


Fig. 18: Altitude response test with disable/enable switch control using *modified super twisting-SMC* in outdoor environment.

### D. Disturbance rejection experiment

In this experiment, Figures 20 and 21 illustrate the altitude response in different references given under induced disturbances in outdoor environment. As shown in Figure 20 the disturbances are externally applied at time  $t_1 = 108s$  and  $t_2 = 128s$ . We can clearly see a good performance for robust controller immediately compensate such disturbances.

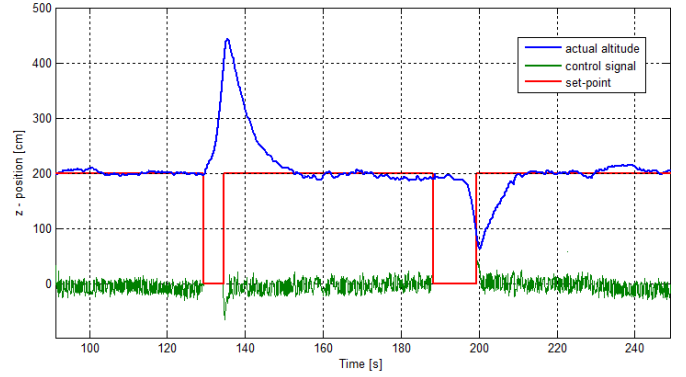


Fig. 19: Altitude response test with disable/enable switch control using *nonsingular terminal sliding mode control* in outdoor environment.

As a particular case of different control techniques used in this article, Figures 21 and 22 show the behavior of the altitude using the super-twisting controller. The response is quite satisfactory because the disturbance is rejected almost immediately by the control so that the tracking altitude corresponds to the reference given demonstrating the effectiveness of real-time controller to external disturbances (in this case: the wind). Notice that, all previous experiments were performed autonomously, only one selector position of a radio was used to indicate the start of each test.

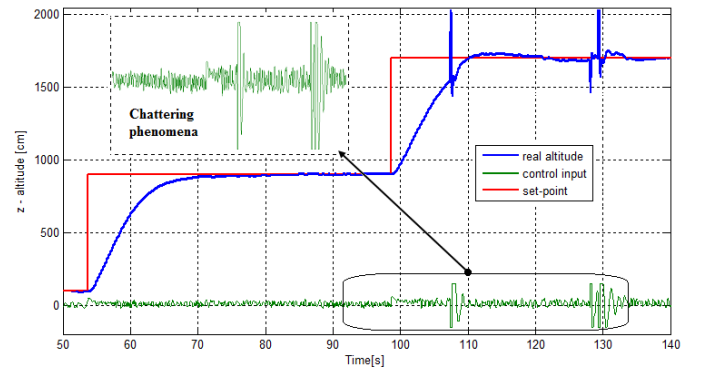


Fig. 20: Altitude hover response test at 9m and 17m using *super twisting-SMC* under disturbances in outdoor environment.

Finally, Figure 23 shows the satisfactory performance when add an external weight at helicopter, the mass added is of 12% of the Quad-rotor total mass. It can be seen in this Figure at instants  $t = 70s$ ,  $t = 100$  and  $t = 140s$  when induce the external mass to the Quad-rotor aircraft effectively recover up to the given reference.

In order to evaluate the performance of the Second Order SMCs implemented, a comparative study based in the error analysis was realized for the first 50 seconds in the set of *Hovering experiments*. The performance indexes used in this study are: Integral of Absolute Error (IAE), Integral of Time multiplied Absolute Error (ITAE), Integral of Squared Error (ISE) and Integral of Time multiplied Squared Error (ITSE). Table III shows the results obtained for each index, based on

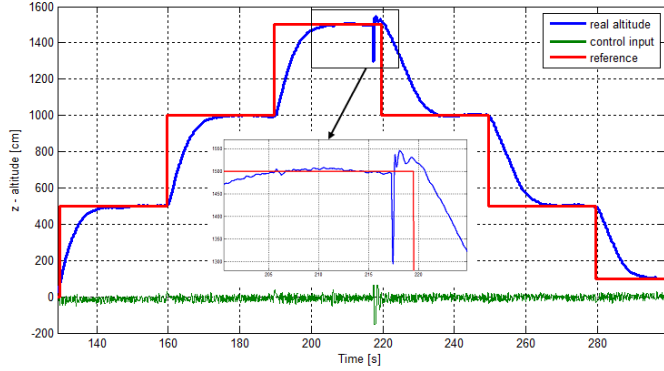


Fig. 21: Altitude response tracking test up to 15m using *super twisting-SMC* under disturbances added in outdoor environment.

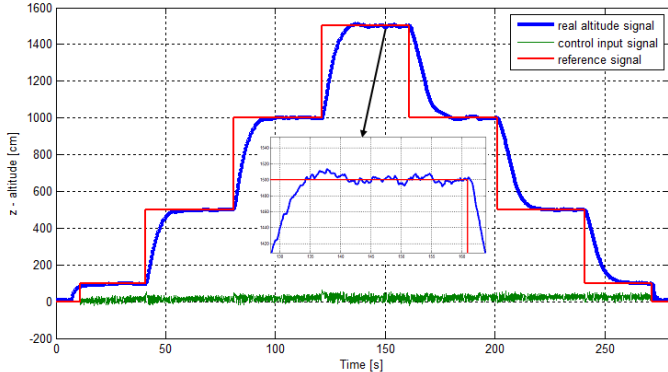


Fig. 22: Altitude response tracking test up to 15 meters using *super twisting-SMC* in outdoor environment.

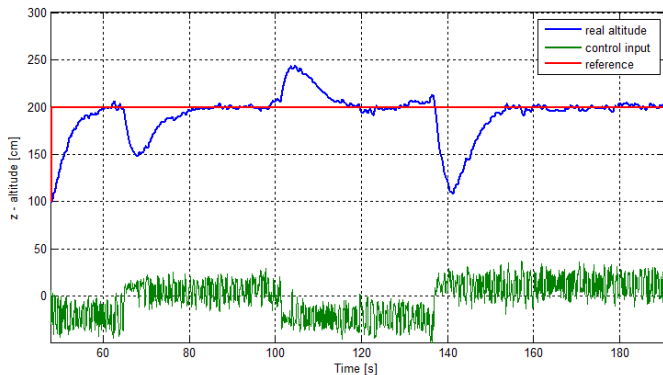


Fig. 23: Altitude response under disturbances induced (mass added).

these results it can be concluded that Modified Super Twisting-SMC has the best performance.

TABLE III: Performance index for Hovering experiments.

Performance Index	ST	MST	NTSMC
IAE	559.8688	500.8595	611.1740
ITAE	$4.9569 \times 10^3$	$3.7224 \times 10^3$	$5.3738 \times 10^3$
ISE	$2.9768 \times 10^4$	$2.7308 \times 10^4$	$3.1499 \times 10^4$
ITSE	$7.0600 \times 10^4$	$6.1070 \times 10^4$	$8.5957 \times 10^4$

In Figure 24 an error histogram for *Hovering experiments* is presented. It can be observed that the greater number of samples with near zero error are obtained with the Modified Super Twisting-SMC proving its best performance. A video of the experiments carried out can be seen at <https://youtu.be/T5KAzwhH9Ig>

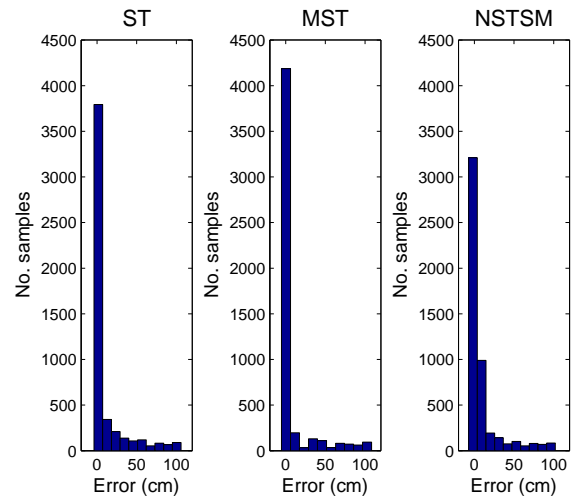


Fig. 24: Performance of error histogram for *Hovering experiments*.

## V. CONCLUSIONS AND FUTURE WORKS

The altitude in a quadrotor helicopter is a great issue due to their dynamic include the mass and the roll and pitch angles. The disturbances in altitude could be provided from the many factors, the rotational dynamics, change of mass, wind gusts, etc. We presented a comparative study of three second-order sliding mode controllers in real-time application for the altitude of a quadrotor rotorcraft. In the practice we increase the performance in outdoors flight due to attenuation the chattering effects and the rejection of disturbances even of mass changes. We presented an error comparative study in order to show that the best performance is the modified super twisting algorithm in a hover outdoors flights. The future work could be included the X and Y positions in order to compensate the GPS noisy and perturbation like wind gusts.

## ACKNOWLEDGEMENTS

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APPENDIX A  
STABILITY PROOF

A sketch of the proof of Theorem 1 is presented below, the complete proof of the theorem can be found on [7], [30]. Introducing a new state vector  $\nu$  defined as

$$\nu = [\nu_1 \quad \nu_2]^T = [|x_1|^{1/2} \text{sign}(x_1) \quad x_2]^T$$

the system (5) is rewritten as follows

$$\begin{aligned} \dot{\nu}_1 &= \frac{1}{|\nu_1|} \left( -\frac{1}{2} k_1 \nu_1 + \frac{1}{2} \nu_2 \right) \\ \dot{\nu}_2 &= \frac{1}{|\nu_1|} (-k_2 \nu_1) + \dot{\xi} \end{aligned} \quad (13)$$

The system (13) is written in matrix form as follow

$$\begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \frac{1}{|\nu_1|} \underbrace{\begin{bmatrix} -\frac{1}{2} k_1 & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix}}_{A(\nu_1)} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \dot{\xi} \end{bmatrix}}_{G(\dot{\xi})} \quad (14)$$

*Remark.* If  $\nu_1, \nu_2 \rightarrow 0$  in finite time, then  $x_1, x_2 \rightarrow 0$  in finite time.

The following Lyapunov function candidate is introduced in order to prove the convergence of  $\nu_1, \nu_2$  in finite time [30]

$$V(\nu) = 2k_2 \nu_1^2 + \frac{1}{2} \nu_2^2 + \frac{1}{2} (k_1 \nu_1 - \nu_2)^2$$

which can be rewritten as

$$V(\nu) = \nu^T P \nu \quad (15)$$

with  $P = P^T$  defined by

$$P = P^T = \begin{bmatrix} 2k_2 + \frac{1}{2} k_1^2 & -\frac{1}{2} k_1 \\ -\frac{1}{2} k_1 & 1 \end{bmatrix}$$

the matrix  $P$  is positive definite if  $k_2 > 0$ .

**Remark 2.** *The function  $V$  is continuous but not locally Lipschitz, for this reason, the usual second method of Lyapunov is not valid. In [30] all the conditions required by the Zubov's theorem were obtained for the proposed Lyapunov candidate function. Therefore, we can continue the Lyapunov analysis following these ideas. For those points where the derivative does not exist the arguments presented in [32],[33] are employed.*

The temporal derivative of (15) is given has

$$\begin{aligned} \dot{V}(\nu) &= \frac{1}{|\nu_1|} \nu^T [A^T(\nu_1) P + P A(\nu_1)] \nu + 2\nu^T P G(\dot{\xi}) \\ &= -\frac{1}{|\nu_1|} \nu^T Q \nu + \dot{\xi} q_1 \nu \end{aligned}$$

with  $Q = Q^T$  and  $q_1$  defined as

$$Q = \begin{bmatrix} \frac{k_1}{2} (k_1^2 + 2k_2) & -\frac{k_1^2}{2} \\ -\frac{k_1^2}{2} & \frac{k_1}{2} \end{bmatrix}, \quad q_1 = [-k_1 \quad 2]$$

Assuming that the perturbation term of the system (5) is globally bounded by

$$|\dot{\xi}| \leq \xi_1$$

it can be shown that

$$\dot{V}(\nu) \leq -\frac{1}{|\nu_1|} \nu^T \tilde{Q} \nu \quad (16)$$

where

$$\tilde{Q} = \begin{bmatrix} \frac{k_1}{2} (k_1^2 + 2k_2) - k_1 \xi_1 & -\frac{k_1^2}{2} - \xi_1 \\ -\frac{k_1^2}{2} - \xi_1 & \frac{k_1}{2} \end{bmatrix}$$

From Equation(16) it can be observed that  $\dot{V}(\nu)$  is negative definite if  $\tilde{Q}$  is definite positive, this fact is ensure if the gains  $k_1$  and  $k_2$  are chosen as in Equation (6).

Considering the inequalities

$$\lambda_{\min}\{P\} \|\nu\|^2 \leq V(\nu) \leq \lambda_{\max}\{P\} \|\nu\|^2 \quad (17)$$

$$|\nu_1| \leq \|\nu\| \leq \frac{V^{1/2}(\nu)}{\lambda_{\min}^{1/2}\{P\}} \quad (18)$$

equation (16) can be rewritten as

$$\dot{V}(\nu) \leq -\gamma V^{1/2}(\nu) \quad (19)$$

with  $\gamma$  defined as

$$\gamma = \frac{\lambda_{\min}^{1/2}\{P\} \lambda_{\min}\{Q\}}{2\lambda_{\max}\{P\}}$$

From equation (19) is obtained that  $V(\nu)$  and  $\nu$  converge to zero in finite time at most after  $T = \frac{2V^{1/2}(\nu(t_0))}{\gamma}$  units of time. If the condition on the gains  $k_1$  and  $k_2$  are satisfied,  $s = 0$  in finite time. Therefore, according to  $s = c_1 e + \dot{e}$  it can be concluded that

$$\lim_{t \rightarrow \infty} e = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} \dot{e} = 0$$

providing asymptotic convergence of the tracking error.

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