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A Note on Dirichlet's Theorem and Goldbach's Conjecture Proof

Raúl Alberto R. C. de Sousa

1 Abstract

This article presents an alternative and simple proof to the Dirichlet's Theorem and a proof for the Goldbach's Strong Conjecture, using the Complex Wave Model and its properties. The Complex Wave Model allows the definition of an α function based on a sines product. As a consequence of the properties of trigonometric functions, with emphasis to its periodicity and symmetry, some of the questions regarding the prime numbers distribution, k-tuples and prime numbers arithmetic progressions, including the Dirichlet's Theorem and Goldbach's Conjecture, can be solved.

2 Keywords

Dirichlet's Theorem, Goldbach's Strong Conjecture, Proof, Prime Numbers, Complex Wave model, Green-Tao's Theorem.

3 AMS subject classifications

11A41, 11A51, 11P32, 35Q15, 35Q99, 81S99, 81V99

4 Introduction

As was shown in the article "Prime Numbers: the Particle in the Box and the Complex Wave Model" (Sousa, 2015), it is possible to define a function $\alpha(x)$ which detects prime numbers and coprime numbers relatively to all integers included in the definition of $\alpha(x)$, thus excluding other integers from being prime numbers. This function never generates false negatives, regarding primality testing, and within the \sqrt{x} limit it also doesn't generate false positives.

It was also identified patterns in the distribution of primes and coprime, that thins out towards infinity. These patterns are periodic with its smaller period represented by the product of the first prime factor of each integer included in *alpha's* definition. If only and all prime number are used up to a certain N , then the product will be equal to the primorial of N .

It was also stated that the patterns generated by α function can represent any k-tuples, arithmetic progressions and all kind of polynomial that generates primes. In this article, α function will be used to provide an alternative and simple proof to the Dirichlet's theorem and to provide a definitive prove to the Goldbach's strong conjecture.

5 Dirichlet's Theorem

In 1837, Dirichlet put forward one of his most famous theorem stating that:

"For any two positive coprime integers a and d , there are infinitely many primes of the form $a + nd$, where n is a non-negative integer."

Dirichlet used the characters of finite abelian groups and L-series to prove the theorem presented above. The proof that Dirichlet presented is extremely technical and long. An alternative proof has been presented by Haifeng Xu (Xu, 2016), through sieving analysis and induction. This inductive analysis allows the identification of periodicity and symmetry in prime number candidates. These properties were formally demonstrated by Raúl de Sousa

through a alternative method using a sines product named Complex Wave Model.(Sousa, 2015). The α function periodicity is of great importance in providing an alternative and simpler proof for the Dirichlet's Theorem, as it will be demonstrated.

Proof. Let us consider an α function defined as follows:

For all x, k and $N \in$ integers,

$$\alpha(x) : \prod_{m=k}^l \sin\left(\frac{\pi}{m}x\right) \quad (1)$$

It is evident that $\alpha(x)$ is a periodic function due to the definition of $\alpha(x)$ as an sines product, with its period been:

$$\tau = p_1^x p_2^y \dots p_n^z \quad (2)$$

x, y, \dots, z represents the number of times that its p_i is the lowest prime factor of every integer from k to l .

A note for τ definition and its coincidence with the Fundamental Theorem of Arithmetic, that states every positive integer $n > 1$ can be represented in an unique product of prime powers. This will be useful further on in this article, regarding the Goldbach's Strong Conjecture.

If we use all PN from 2 up to N in $\alpha(x)$, then τ is the primorial of N :

$$\tau = N\# \quad (3)$$

If we use in $\alpha(x)$, any number N , then τ is the radical of N , being the radical of any integer the product of the distinct prime numbers dividing N :

$$\tau = Rad(N) \quad (4)$$

If $k = 2$ and all integers up to l are included in the $\alpha(x)$, all α 's solutions different from zero will signal all primes for $l < x \leq l^2$ and candidates to primes for $x > l^2$. These solutions are coprimes to all integers from k to l , by definition, and include all prime numbers besides the ones in k to l , as well. Another way to stated the same is stating that all candidates to prime are coprime to τ since τ is an integer product from k up to l .

Been $\alpha(x)$ a periodic function, we have that $\alpha(x) = \alpha(\tau + x)$, meaning that all solutions of $\alpha(x) \neq 0$ from $x = 0$ to ∞ are coprime to the set k, \dots, N and have the form:

$$x + n\tau \text{ and } x, \tau \text{ are coprimes for every } n \in \text{Integers.} \quad (5)$$

Since it has stated in the second Complex Wave Theorem that all primes numbers are included in the $\alpha(x) \neq 0$ solutions. It implies that all prime numbers have the Dirichlet's Theorem form 5 and that there are infinite n for which that the linear polynomial $x + n\tau$ produces prime numbers, thus demonstrating the Dirichlet's Theorem. \square

Regarding Green-Tao's Theorem, it is evident that we can choose any x and any τ so that, we can obtain arbitrarily, but finite, long arithmetic progressions of prime numbers interrupted by non-primes, that are coprimes to x and every factor in τ .

From the periodicity property of $\alpha(x)$, a very significant conclusion is dawned. All prime numbers have to be in the form $P_x + k\tau = P_y$ where τ is coprime to P_x and P_y , although not all numbers in the $P_x + k\tau$ are prime numbers.

6 GoldBach's Strong Conjecture

The Goldbach's Strong, or Binary, conjecture stated by Euler asserts that all positive even integers greater than 4, represented by N , can be expressed as the sum of two prime numbers P_y, P_x : $P_y + P_x = N$. Although the

Goldbach's Weak, or Ternary, conjecture has been proved by H. A. Helfgott, the Strong conjecture remains to be proven.

Another way to put the Goldbach's Strong conjecture is to state that, for all even N there exists at least one set of two prime numbers, P_y and P_x , at the same distance from $\frac{N}{2}$. In other words, there exist at least one pair of prime numbers symmetric to $\frac{N}{2}$. Please note that $P_y = P_x$ is also a solution, which implies that $P_y = P_x = \frac{N}{2}$. The symmetry pattern in the prime numbers location identified by the $\alpha(x) \neq 0$, can demonstrate the existence of the required type of symmetry in the prime numbers distribution, for all even numbers equal or above 4, as follows.

Proof. As showed above, $\alpha(x)$ is periodic. Also, because $\alpha(x)$ is a product of sines, it is implied that $\alpha(x)$ is symmetric to $\frac{\tau}{2}$. This symmetry property was also been pointed by Haifeng Xu(Xu, 2016), by inductive analysis of the Eratosthenes Sieve. Therefore, the prime numbers candidates, which includes all prime numbers besides the ones included in $\alpha(x)$, therefore in τ , are symmetric to $\frac{\tau}{2}$. Since that the possible locations for prime numbers through the pattern repetition can not change, there must be at least one prime number among the prime numbers candidates on the second half of τ , symmetrical to a prime number in the first half. If it was not the case, that would mean that there were not anymore locations for further prime numbers, thus implying the false conclusion that the Prime Numbers' set was to be finite.

It is possible to classify $\alpha(x)$ as a even function if the set cardinal $\#a, \dots, N$ is even and as an odd function if the set cardinal is odd. Since we are only interested in the center of symmetry, we can generalize and state that:

$$|\alpha(x)| = |\alpha(\tau - x)|, \quad (6)$$

with the center of symmetry been $\frac{\tau}{2}$. As already stated, one implication of this symmetry is that for every x solution to $\alpha \neq 0$, exists a $\tau - x$ that is also solution to $\alpha \neq 0$ including all prime numbers that are not used in the $\alpha(x)$

With a simple observation of the τ product, it is easy to verify that it has the form stated in the Arithmetic Fundamental Theorem, and in its short period is equal to the radical of an integer 4. A consequence of that observation is that any number can be chosen to be the τ and, therefore, to construct an $\alpha(x)$ function.

So, we can write the unique integer factorization of any even number N and use those factors to define the set $\{P_x, P_y, \dots\}$ for the $\{k, \dots, l\}$ of an $\alpha(x)$, so that τ is equal to the chosen even number N with $\alpha(x)$ defined as $\alpha_{P_x, P_y, \dots}(x)$. Every x that is solution for $\alpha \neq 0$ is coprime to the set $\{P_x, P_y, \dots\}$ as it is, also, its symmetric $\tau - x$ to the period $\tau = N$. Since all prime numbers have to be solution to $\alpha(x) \neq 0$ and none is excluded, it implies that for every even N there exists $N = P_x + P_y$.

Recalling eq. 4, it can be defined an $\alpha_{Rad(N)}(x)$ so that the solutions for $\alpha_{Rad(N)}(x) \neq 0$ includes, at least, one set of prime numbers that are symmetric to $\frac{\tau}{2}$, proving the Golbach's Strong Conjecture. \square

From the symmetry property of $\alpha(x)$, a very significant conclusion is dawned. All prime numbers have to be in the form $\tau - P_x = P_y$ where τ is coprime to P_x and P_y , although not all numbers in the $\tau - P_x$ are prime numbers.

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