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# Possible refinements of RECIFE-MILP

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In this document we propose formulations of additional constraints or alternative objective functions which can be added to RECIFE-MILP. The aim of these refinements is acquiring the possibility of representing additional features of the railway traffic system and alternative traffic management strategies.

## 1 RECIFE-MILP

In this section, we detail the MILP formulation that we use in RECIFE-MILP [1], in the next sections referred as to standard RECIFE-MILP. It describes the infrastructure in terms of track-circuits, that is, in terms of track sections on which the presence of a train is automatically detected. In addition to the existing track-circuits, two dummy ones  $tc_0$  and  $tc_\infty$  are also considered. They represent the entry and the exit locations of the infrastructure considered, respectively. Sequences of track-circuits are grouped into block sections, which are opened by a signal indicating their availability.

Before a train can enter (start the occupation of) a block section, all the track-circuits belonging to the same block section must be reserved for the train itself. In the following, we will name *utilization* time the sum of reservation and occupation time. If a train starts its trip at null speed from a platform, then we consider the beginning of the occupation to correspond to the moment in which the train starts moving. If it remains still at the platform, its actual utilization will be ensured through reservation. Each block section is reserved by the train some time before its entering, to allow the route formation, and it remains reserved after its leaving, to allow the route release.

We define the routes in terms of sequence of track-circuits and by the intermediate stops. Hence, as an example, a sequence of track-circuits that can be traversed performing or not intermediate stops defines two different routes. The running times for a route with intermediate stops include the appropriate deceleration and acceleration times, but not the dwell times.

In the MILP formulation, we use the following notation:

$T$  set of trains,

$w_t$	weight associated to train $t$ 's delay,
$ty_t$	type corresponding to train $t$ (indicating train characteristics),
$init_t, sched_t$	earliest time at which train $t$ can be operated given the timetable and the primary delay, and earliest time at which train $t$ can reach its destination given $init_t$ , the route assigned to $t$ in the timetable and the intermediate stops,
$i(t', t)$	indicator function: 1 if trains $t'$ and $t$ use the same rolling stock and $t$ results from the turnaround, join or split of train $t'$ , 0 otherwise,
$ms$	minimum separation time between the arrival of a train and the departure of another train which uses the same rolling stock,
$R_t, TC_t$	set of routes and track-circuits which can be used by train $t$ ,
$TC^r$	set of track-circuits composing route $r$ ,
$OTC_{ty_t, r, tc}$	set of consecutive track-circuits preceding $tc$ which are occupied by $t$ traveling along route $r$ if its head is on $tc$ , depending on $t$ 's and $tc$ 's length,
$TC(tc, tc', r)$	set of track-circuits between $tc$ and $tc'$ along route $r$ ,
$p_{r, tc}, s_{r, tc}$	track-circuits preceding and following $tc$ along route $r$ ,
$rt_{ty_t, r, tc}, ct_{ty_t, r, tc}$	running and clearing time of $tc$ along route $r$ for a train of type $ty_t$ ,
$ref_{r, tc}$	reference track-circuit for the reservation of $tc$ along route $r$ ,
$e(tc, r)$	indicator function: 1 if track-circuit $tc$ belongs to an extreme (either the first or the last) block section on route $r$ , 0 otherwise,
$bs_{r, tc}$	block section including track-circuit $tc$ along route $r$ ,
$for_{bs}, rel_{bs}$	formation and release time for block section $bs$ ,
$S_t, TCS_{t, s}$	set of stations where train $t$ has a scheduled stop and set of track-circuits that can be used by $t$ for stopping at station $s$ ,
$dw_{t, s}, arr_{t, s}, dep_{t, s}$	minimum dwell time, scheduled arrival and scheduled departure times for train $t$ at station $s$ ,
$M$	large constant.

The MILP formulation includes the following non-negative continuous variables:

- for all triplets of train  $t \in T$ , route  $r \in R_t$  and track-circuit  $tc \in TC^r$ :

$o_{t, r, tc}$  : time at which  $t$  starts the occupation of  $tc$  along  $r$ ,  
 $l_{t, r, tc}$  : longer stay of  $t$ 's head on  $tc$  along route  $r$ ,  
due to dwell time and scheduling decisions (delay);

- for all pairs of train  $t \in T$  and track-circuit  $tc \in TC_t$ :

$sU_{t, tc}$  : time at which  $tc$  starts being utilized by  $t$ ;  
 $eU_{t, tc}$  : time at which  $tc$  ends being utilized by  $t$ ;

- for all pairs of train  $t \in T$  and track-circuit  $tc \in TC_t$   $tc = tc_\infty$  or  $\exists s \in S_t, tc \in TCS_{t, s}$ :

$D_{t, tc_\infty}$  : delay suffered by train  $t$  when exiting the infrastructure considered.  
 $D_{t, tc}$  : delay suffered by train  $t$  when exiting  $tc$ .

In addition the MILP formulation includes the following binary variables:

- for all pairs of train  $t \in T$  and route  $r \in R_t$ :

$$x_{t,r} = \begin{cases} 1 & \text{if } t \text{ uses } r, \\ 0 & \text{otherwise,} \end{cases}$$

- for all triplets of train  $t, t' \in T$  such that the index  $t$  is smaller than the index  $t'$ , and track-circuit  $tc \in TC_t \cap TC_{t'}$ :

$$y_{t,t',tc} = \begin{cases} 1 & \text{if } t \text{ utilizes } tc \text{ before } t' (t \prec t'), \\ 0 & \text{otherwise } (t \succ t'). \end{cases}$$

The objective function to be minimized is the total weighted delays suffered by trains at their exit from the infrastructure:

$$\min \sum_{t \in T} w_t D_{t,tc_\infty}. \quad (1)$$

The sets of constraints considered impose the following conditions:

- A train  $t$  cannot be operated earlier than  $init_t$ :

$$o_{t,r,tc} \geq init_t x_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r. \quad (2)$$

- The start time of track-circuit occupation along a route is zero if the route itself is not used:

$$o_{t,r,tc} \leq M x_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r. \quad (3)$$

- A train starts occupying track-circuit  $tc$  along a route after spending in the preceding track-circuit its running time, if the route is used, and its longer stay.

$$o_{t,r,tc} = o_{t,r,p_{r,tc}} + l_{t,r,p_{r,tc}} + r t_{r,ty_1,p_{r,tc}} x_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r; \quad (4)$$

remark that these constraints imply that  $l_{t,r,p_{r,tc}}$  equals 0 if  $t$  does not use  $r$ .

- A train  $t$  with a scheduled stop at station  $s$  and using route  $r$  does not enter the track-circuit following  $tc$  before the scheduled departure time from  $s$  if  $tc$  is in  $TCS_{t,s}$ :

$$o_{t,r,s_r,tc} \geq \sum_{s \in S_t: tc \in TCS_{t,s} \cap TC^r} dep_{t,s} x_{t,r} \quad \forall t \in T, r \in R_t, tc \in \bigcup_{s \in S_t} TCS_{t,s}. \quad (5)$$

- A train  $t$  with a scheduled stop at station  $s$  and using route  $r$  has a longer stay in  $tc$  of at least  $dw_{t,s}$  if  $tc$  is in  $TCS_{t,s}$ :

$$l_{t,r,s_r,tc} \geq \sum_{s \in S_t: tc \in TCS_{t,s} \cap TC^r} dw_{t,s} x_{t,r} \quad \forall t \in T, r \in R_t, tc \in \bigcup_{s \in S_t} TCS_{t,s}. \quad (6)$$

- A train  $t$  must use exactly one route:

$$\sum_{r \in R_t} x_{t,r} = 1 \quad \forall t \in T. \quad (7)$$

- The value of a delay  $D_{t,tc_\infty}$  cannot be less than the difference between the actual and the scheduled arrival times at the exit of the infrastructure:

$$D_{t,tc_\infty} \geq \sum_{r \in R_t} o_{t,r,tc_\infty} - sched_t \quad \forall t \in T. \quad (8)$$

- The value of a delay  $D_{t,tc}$  cannot be less than the difference between the actual and the scheduled arrival times at station  $s$  such that  $tc \in TCS_{t,s}$ :

$$D_{t,tc} \geq \sum_{r \in R_t: tc \in TC^r} (o_{t,r,tc} + rt_{r,ty_t,tc} x_{t,r}) - arr_{t,s} \quad \forall t \in T, tc \in TC_t \exists s \in S_t, tc \in TCS_{t,s} \quad (9)$$

- A minimum separation time  $ms$  must separate the arrival and departure of trains using the same rolling stock:

$$\sum_{\substack{r \in R_t, tc \in TC^r: \\ p_{r,tc} = tc_0}} o_{t,r,tc} \geq \sum_{\substack{r \in R_{t'}, tc \in TC^r: \\ s_{r,tc} = tc_\infty}} o_{t',r,tc} + (ms + rt_{r,ty_{t'},tc}) x_{t',r} \quad \forall t, t' \in T : i(t', t) = 1. \quad (10)$$

- If trains  $t'$  and  $t$  use the same rolling stock and  $t$  results from the turnaround, join or split of train  $t'$ , the track-circuit  $tc$  where the turnaround, join or split takes place must be utilized by the two trains and it must be such for the whole time between  $t'$ 's arrival and  $t$ 's departure. Thus,  $tc$  starts being reserved by  $t$  at the latest when  $t'$  ends its utilization.

$$\sum_{r \in R_t: tc \in TC^r} x_{t,r} = \sum_{r' \in R_{t'}: tc \in TC^{r'}} x_{t',r'} \quad \forall t, t' \in T : i(t', t) = 1, tc \in \cup_{r \in R_t} \{s_{r,tc_0}\}, \quad (11)$$

$$\sum_{tc \in TC_t: \exists r \in R_t, p_{r,tc} = tc_0} sU_{t,tc} \leq \sum_{tc \in TC_{t'}: \exists r \in R_{t'}, s_{r,tc} = tc_\infty} eU_{t',tc} \quad \forall t, t' \in T : i(t', t) = 1. \quad (12)$$

Here, the inequality must be imposed since, in case of a join, two trains arrive and are connected to become a single departing one. The utilization of the departing train must then immediately follow the utilization of the first train arriving, being strictly smaller than the one of the second train.

- A train's utilization of a track-circuit starts as soon as the train starts occupying the track-circuit  $ref_{r,tc}$  along one of the routes including it, minus the formation time:

$$sU_{t,tc} = \sum_{r \in R_t: tc \in TC^r} (o_{t,r,ref_{r,tc}} - for_{bs_{r,tc}} x_{t,r}) \quad \forall t \in T, tc \in TC_t : (\nexists t' \in T : i(t', t) = 1) \vee (\forall r \in R_t : ref_{r,tc} \neq s_{r,tc_0}). \quad (13)$$

Constraints (13) are imposed as inequalities ( $\leq$ ) when they concern a track-circuit of the first block sections of the route ( $ref_{r,tc} = s_{r,tc_0}$ ) and the train  $t$  results from the turnaround, join or split of one or more other trains. This fact is a consequence of the need of keeping platforms utilized. Indeed, if  $t$  results from the turnaround of  $t'$ , Constraints (12) ensure that the track-circuit where the turnaround takes place starts being reserved by  $t$  as soon as  $t'$  arrives. However,  $t$  needs to wait at least for a time  $ms$  before departing. The occupation of the track-circuit by  $t$  is starting from its actual departure, for guaranteeing the coherence of the occupation variables and the running time (Constraints (4)). Hence,  $t$ 's reservation starts much earlier than its occupation.

- The utilization of a track-circuit  $tc$  lasts till the train utilizes it along any route, plus the formation and the release time:

$$eU_{t,tc} = \sum_{r \in R_t: tc \in TC^r} o_{t,r,ref_{r,tc}} + (for_{bs_{r,tc}} + rel_{bs_{r,tc}})x_{t,r} + ul_{t,r,tc} \quad \forall t \in T, tc \in TC_t. \quad (14)$$

Here  $ul_{t,r,tc}$  is the total utilization time:

$$ul_{t,r,tc} = \sum_{tc' \in TC(ref_{r,tc}, tc, r)} (rt_{r,ty_r,tc'}x_{t,r} + l_{t,r,tc'}) + \sum_{tc' \in TC_t: tc \in OTC_{ty_r,tc'}} l_{t,r,tc'} + ct_{r,ty_r,tc}x_{t,r}.$$

It includes: the running time of all track-circuits between  $ref_{r,tc}$  and  $tc$ , the longer stay of the train's head on each of these track-circuits  $l_{t,r,tc}$  and the clearing time of  $tc$ . Moreover, it includes the longer stay on all track-circuits  $tc'$  such that  $tc \in OTC_{ty_r,tc'}$ . As mentioned in the definition of  $OTC_{ty_r,tc'}$ , if the head of the train is on one of these track-circuits, then its tail has not yet exited  $tc$ : the train is longer than  $tc'$ , or of the sequence of track-circuits between  $tc$  and  $tc'$ . Hence, if the train suffers a longer stay when its head is on one of these track-circuits, such a longer stay must be counted in the utilization time of  $tc$ .

- The track-circuit utilizations by two trains must not overlap.

$$eU_{t,tc} - M(1 - y_{t,t',tc}) \leq sU_{t',tc} \quad \forall t, t' \in T, \text{index } t < \text{index } t', tc \in TC_t \cap TC_{t'} : \\ i(t, t') \sum_{r \in R_t} e(tc, r) = 0 \wedge i(t', t) \sum_{r \in R_{t'}} e(tc, r) = 0, \quad (15)$$

$$eU_{t',tc} - My_{t,t',tc} \leq sU_{t,tc} \quad \forall t, t' \in T, \text{index } t < \text{index } t', tc \in TC_t \cap TC_{t'} : \\ i(t, t') \sum_{r \in R_t} e(tc, r) = 0 \wedge i(t', t) \sum_{r \in R_{t'}} e(tc, r) = 0. \quad (16)$$

These last constrains are disjunctive. Similar conditions can be found, as an example, in [3]. Remark that, if  $t$  chooses a route which includes  $tc$  and  $t'$  does not, then  $y_{t,t',tc}$  must be equal to 0: the utilization variables  $sU_{t,tc}$  and  $eU_{t,tc}$  will be equal to the time at which  $t$  actually utilizes  $tc$  while  $sU_{t',tc}$  and  $eU_{t',tc}$  will be set to 0 for Constraints (3), (13) and (14). Conversely, if only  $t'$  chooses a route which includes  $tc$ , then  $y_{t,t',tc}$  must be equal to 1. If no train uses  $tc$  both values 0 and 1 are feasible for  $y_{t,t',tc}$ , since the utilization start and end will be 0 for both trains.

In [2], we proposed a boosting method named *topology* to the formulation presented in [1], which we consider and we extend in this paper. It is based on the reduction of the number of binary  $y$ -variables, i.e., the variables appearing in Constraints (15) and (16) that define the precedence relation between couples of trains utilizing the same track-circuit. This reduction exploits the infrastructure topology. Specifically, it exploits the fact that the topology of a physical network frequently imposes that the precedence relation between couples of trains must be identical on different track-circuits. Consider as an example a bidirectional track without any overpassing area. In

a preprocessing phase, for each couple of trains  $t, t'$  and on the basis of the physical network topology, we first partition the set of shared track-circuits  $TC_t \cap TC_{t'}$  into subsets defined as follows: two track-circuits belong to the same subset  $\hat{TC}$  if the same precedence relation between  $t$  and  $t'$  must hold on both the track-circuits, disregard the routes chosen by the trains. Then, for each  $\hat{TC}$ , a single  $y_{t,t',\hat{tc}}$  variable is associated to the  $tc \in \hat{TC}$  with the lowest index, and we replace  $y_{t,t',tc}$  with  $y_{t,t',\hat{tc}}$  in Constraints (15) and (16).

## 2 Refinements

In this section we present different additional features of the railway system and different traffic management strategies which may be integrated to RECIFE-MILP, together with their MILP formalization.

### 2.1 Rolling-stock assignment

In the classic RECIFE-MILP, we take the rolling-stock assignment as an input and we impose the consistency between arrivals and departures of trains operated with the same rolling-stock through Constraints (10) to (12).

Another possibility is to consider the assignment as a traffic management decision.

To do so, we need first of all to define an indicator function  $roll(t', t)$  which has value 1 if it is possible to perform  $t'$  and  $t$  with the same rolling-stock, with  $t' \prec t$ , and 0 otherwise. Moreover, for each train  $t \in T$  we need to know the necessary number of train units  $tu_t$ . Finally, let  $orig(t)$  be an indicator function being 1 if we need to control the rolling-stock at  $t$ 's origin, and 0 otherwise. We do not have to control the train's origin if its route either starts out of the control area, or starts within the control area but  $t$  uses a rolling-stock which was located at the route start before the beginning of the time horizon considered<sup>1</sup>. Similarly  $dest(t)$  be an indicator function telling whether we need to control the rolling-stock at the destination of train  $t$ .

Then, for each pair of trains such that  $roll(t', t) = 1$ , we can define binary variables

$$ass_{t',t} = \begin{cases} 1 & \text{if the rolling-stock of } t' \text{ is assigned to } t, \\ 0 & \text{otherwise.} \end{cases}$$

First of all, we use these variables to ensure that the trains compositions are correctly done:

$$\sum_{t' \in T: roll(t',t)=1} ass_{t',t} tu_{t'} = tu_t \quad \forall t \in T : orig(t) = 1, \quad (17)$$

$$\sum_{t' \in T: roll(t,t')=1} ass_{t,t'} tu_{t'} = tu_t \quad \forall t \in T : dest(t) = 1. \quad (18)$$

Then, to ensure that a minimum separation time  $ms$  separates the arrival and depar-

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<sup>1</sup>It is possible to omit this function and simply consider in the constraints only the trains for which a  $t'$  exists such that  $roll(t', t)$ .

ture of trains using the same rolling stock we set:

$$\sum_{\substack{r \in R_t, tc \in TC^r \\ p_{r,tc} = tc_0}} o_{t,r,tc} \geq \sum_{\substack{r \in R_{t'}, tc \in TC^r \\ s_{r,tc} = tc_\infty}} o_{t',r,tc} + (ms + rt_{r,ty_{t'},tc})x_{t',r} - M(1 - ass_{t',t})$$

$$\forall t, t' \in T : roll(t', t) = 1. \quad (19)$$

Moreover, to ensure the spatial coherence between the arrival and departure, we set:

$$\sum_{r \in R_t, tc \in TC^r} x_{t,r} - \sum_{r' \in R_{t'}, tc \in TC^{r'}} x_{t',r'} \leq M(1 - ass_{t',t})$$

$$\forall t, t' \in T : roll(t', t) = 1, tc \in \cup_{r \in R_t} \{s_{r,tc_0}\}, \quad (20)$$

$$\sum_{r \in R_t, tc \in TC^r} x_{t,r} - \sum_{r' \in R_{t'}, tc \in TC^{r'}} x_{t',r'} \geq M(ass_{t',t} - 1)$$

$$\forall t, t' \in T : roll(t', t) = 1, tc \in \cup_{r \in R_t} \{s_{r,tc_0}\}. \quad (21)$$

Furthermore, if trains  $t'$  and  $t$  use the same rolling stock, then the track-circuit  $tc$  where the turnaround, join or split takes place must be utilized by at least one of the two trains for the whole time between  $t'$ 's arrival and  $t$ 's departure.

$$\sum_{tc \in TC_t : \exists r \in R_t, p_{r,tc} = tc_0} sU_{t,tc} \leq \sum_{tc \in TC_{t'} : \exists r \in R_{t'}, s_{r,tc} = tc_\infty} eU_{t',tc} + M(1 - ass_{t',t})$$

$$\forall t, t' \in T : roll(t', t) = 1. \quad (22)$$

Finally, we must slightly modify the disjunctive constraints. In the classic RECIFE-MILP if two trains use the same rolling-stock, the constraints are not imposed on the extreme track-circuits of the routes, to be coherent with Constraints (12). Since in the case considered in this section, we do not know a priori if two trains use the same rolling stock or not, we cannot decide whether to impose the constraints or neglect them. Hence, we need to set all the constraints and make them trivially satisfied (and hence ineffective) if the model decides that the two trains use the same rolling-stock.

$$eU_{t,tc} - M(1 - y_{t,t',tc}) \leq sU_{t',tc}$$

$$\forall t, t' \in T, \text{index } t < \text{index } t', tc \in TC_t \cap TC_{t'} :$$

$$roll(t, t') \sum_{r \in R_t} e(tc, r) = 0 \wedge roll(t', t) \sum_{r \in R_{t'}} e(tc, r) = 0, \quad (23)$$

$$eU_{t',tc} - My_{t,t',tc} \leq sU_{t,tc}$$

$$\forall t, t' \in T, \text{index } t < \text{index } t', tc \in TC_t \cap TC_{t'} :$$

$$roll(t, t') \sum_{r \in R_t} e(tc, r) = 0 \wedge roll(t', t) \sum_{r \in R_{t'}} e(tc, r) = 0, \quad (24)$$

$$eU_{t,tc} - M(1 - y_{t,t',tc}) \leq sU_{t',tc} + M ass_{t',t}$$

$$\forall t, t' \in T, \text{index } t < \text{index } t', tc \in TC_t \cap TC_{t'} :$$

$$roll(t, t') \sum_{r \in R_t} e(tc, r) = 1 \wedge roll(t', t) \sum_{r \in R_{t'}} e(tc, r) = 1, \quad (25)$$

$$eU_{t',tc} - My_{t,t',tc} \leq sU_{t,tc} + M ass_{t',t}$$

$$\forall t, t' \in T, \text{index } t < \text{index } t', tc \in TC_t \cap TC_{t'} :$$

$$roll(t, t') \sum_{r \in R_t} e(tc, r) = 1 \wedge roll(t', t) \sum_{r \in R_{t'}} e(tc, r) = 1. \quad (26)$$



To slightly strengthen the model, it is possible to link precedence and rolling-stock assignment variables: if the rolling-stock of  $t'$  is re-utilized for  $t$ , then  $t' \prec t$  on all the common track-circuits:

$$y_{t,t',tc} + ass_{t',t} \leq 1 \quad \forall t, t' \in T : roll(t', t) = 1, tc \in TC_t \cap TC_{t'}. \quad (27)$$

## 2.2 Compulsory and optional connections

To include a connection we need to add constraints very similar to (12). In particular, let  $c(t', t, s)$  be the indicator function assuming value 1 if  $t'$  is in connection with  $t$  at station  $s \in S_t \cap S_{t'}$ , with  $t'$  feeding  $t$ . Moreover, let  $mc_{t',t,s}$  be the minimum separation between the arrival of train  $t'$  at  $s$  and the departure of  $t$ . The connection constraint is as follows:

$$\sum_{t' \in R_{t'}, tc \in TCS_{t',s} \cap TC_{t'}} o_{t',t',tc} + (rt_{t',ty_t,tc} + mc_{t',t,s})x_{t',t'} \leq \sum_{r \in R_t, tc \in TCS_{t,s} \cap TC_r} o_{t,r,sr,tc} \quad \forall t, t' \in T, s \in S_t \cap S_{t'} : c(t', t, s) = 1. \quad (28)$$

If connections are optional, we need to add a binary variable for each of them:

$$con_{t',t,s} = \begin{cases} 1 & \text{if the connection between } t' \text{ and } t \text{ at } s \text{ is cancelled,} \\ 0 & \text{otherwise.} \end{cases}$$

Then, Constraints (28) becomes:

$$\sum_{t' \in R_{t'}, tc \in TCS_{t',s} \cap TC_{t'}} o_{t',t',tc} + (rt_{t',ty_t,tc} + mc_{t',t,s})x_{t',t'} \leq \sum_{r \in R_t, tc \in TCS_{t,s} \cap TC_r} o_{t,r,sr,tc} + Mcon_{t',t,s} \quad \forall t, t' \in T, s \in S_t \cap S_{t'} : c(t', t, s) = 1. \quad (29)$$

The cancelling of connections can be penalized in the objective function, or it can be function of the delays. For example, it may be compulsory if the delay of the departing train is larger than a threshold *thre*. To do so, we need Constraints (30):

$$\sum_{t' \in R_{t'}, tc \in TCS_{t',s} \cap TC_{t'}} o_{t',t',tc} + rt_{t',ty_t,tc}x_{t',t'} \leq arr_{t,s} + thre + Mcon_{t',t,s} \quad \forall t, t' \in T, s \in S_t \cap S_{t'} : c(t', t, s) = 1. \quad (30)$$

Several other functions of the delay may be thought of, we do not list others here for sake of brevity.

## 2.3 Train cancellation

To allow train cancellation, we can simply add a dummy route  $\hat{r}$  to the set of routes available for each train. This dummy route does not include any track-circuit. If a train uses this route, then it means it is cancelled.

Indeed, the delay suffered by these trains must be modified. Constraints (8) and (9) become:

$$D_{t,tc_\infty} \geq \sum_{r \in R_t \setminus \{\hat{r}\}} o_{t,r,tc_\infty} - sched_t(1 - x_{t,\hat{r}}) \quad \forall t \in T. \quad (31)$$

$$D_{t,tc} \geq \sum_{r \in R_t: tc \in TC^r} (o_{t,r,tc} + rt_{r,ty_t,tc} x_{t,r}) - arr_{t,s}(1 - x_{t,\hat{r}}) \\ \forall t \in T, tc \in TC_t \exists s \in S_t, tc \in TCS_{t,s}. \quad (32)$$

The number of trains being cancelled (using  $\hat{r}$ ) can be, for example, minimized or penalized in the objective function. Moreover, similarly to what done for the optional connections, we can add constraints to make the train cancellation function of the delay if suitable.

If the possibility of cancelling trains is combined with other features, as the rolling-stock assignment decision or the connections consideration, the suitable adjustments to the concerned constraints must be made.

## 2.4 Train speed dynamics approximation

Being RECIFE-MILP a fixed-speed model, the train speed dynamics of the trains needing to brake due to a restrictive signal aspect are not modelled. In the classic RECIFE-MILP, no consideration is made on the fact that, if some delay is suffered by a train on one track-circuit, then the delay will necessarily spread to other neighbour ones. In particular, if a train suffers a delay in a track-circuit, it may have started braking on a preceding one and it will have to re-accelerate in one or more following ones. Let  $spread(t, r, tc, tc')$  be an indicator function equal to 1 if a delay suffered by train  $t$  along route  $r$  on track-circuit  $tc$  spreads to track-circuit  $tc'$ . Let  $spt_{t,r,tc,tc'}$  be the delay accumulated on  $tc'$  in this case. To make things easy, we suppose that if a train suffers a delay in a track-circuit, then it started braking at the planned speed and does so up to its stop. If it is not necessary to stay in a track-circuit longer, then it will start accelerating immediately: we simulate a zero meter visibility distance of signals. Hence, if a train suffers a delay on a track-circuit along a route, then this delay needs to be of at least  $minDel_{t,r,tc}$  seconds.

To impose this, we need a binary variable for each train, each route and each track-circuit such that

$$delay_{t,r,tc} = \begin{cases} 1 & \text{if } t \text{ suffers a delay in } tc \text{ along } r, \\ 0 & \text{otherwise.} \end{cases}$$

This variable needs to be set to 1 as soon as  $l_{t,r,tc}$  is positive:

$$l_{t,r,tc} \leq M \, delay_{t,r,tc} \quad \forall t \in T, r \in R_t, tc \in TC^r. \quad (33)$$

Moreover, if  $delay_{t,r,tc} = 1$ , the minimum delay must be imposed:

$$l_{t,r,tc} \geq minDel_{t,r,tc} \, delay_{t,r,tc} \quad \forall t \in T, r \in R_t, tc \in TC^r. \quad (34)$$

Finally, if a track-circuit  $tc$  is such the delay suffered on another one  $tc'$  spreads on  $tc$ , then Constraint (4) becomes

$$o_{t,r,s_{r,tc}} = o_{t,r,tc} + l_{t,r,tc} + rt_{r,ly_t,tc}x_{t,r} + \sum_{\substack{tc' \in TC^r: \\ spread(t,r,tc,tc')=1}} spt_{t,r,tc',tc} delay_{t,r,tc'} \quad \forall t \in T, r \in R_t, tc \in TC^r; \quad (35)$$

Similarly, the utilization time becomes

$$\begin{aligned} ul_{t,r,tc} = & \sum_{tc' \in TC(ref_{r,tc,tc'})} (rt_{r,ly_t,tc'}x_{t,r} + l_{t,r,tc'} + \sum_{\substack{tc'' \in TC^r: \\ spread(t,r,tc'',tc')=1}} spt_{t,r,tc'',tc'} delay_{t,r,tc''}) + \\ & + \sum_{tc' \in TC_t: tc \in OTC_{ly_t,r,tc'}} (l_{t,r,tc'} + \sum_{\substack{tc'' \in TC^r: \\ spread(t,r,tc'',tc')=1}} spt_{t,r,tc'',tc'} delay_{t,r,tc''}) + ct_{r,ly_t,tc}x_{t,r}. \end{aligned}$$

Indeed, the delay spread possibly increases the running time on all track-circuits between the reference one and  $tc$  itself: it must hence be in the first parenthesis. Then, if  $tc$  remains occupied when the head of the train is delayed on another track-circuit ( $tc \in OTC_{ly_t,r,tc'}$ ), then also the delay spread on this other track-circuit matters and it must be in the second parenthesis.

## 2.5 Measure of the number of delays at stations

To measure the number of delays at stations, we need to introduce binary variables for each train  $t$  and station  $s \in S_t$ :

$$delayed_{t,s} = \begin{cases} 1 & \text{if } t \text{ arrives late at } s, \\ 0 & \text{otherwise.} \end{cases}$$

These variables are set as:

$$\sum_{tc \in TC_{S_t,s}} D_{t,tc} \leq M \, delayed_{t,s} \quad \forall t \in T, s \in S_t. \quad (36)$$

The sum of  $delayed_{t,s}$  over  $t \in T$  and  $s \in S_t$  can be included in the objective function with the aim of minimizing it.

## 2.6 Measure of the number of delayed trains

If we wish to measure the number of trains which suffer some delay at least at one of their stops or at their exit from the infrastructure, we can use the same variables and constraints defined in Section 2.5. Moreover, we shall add the delay at the exit of the infrastructure using variables

$$delayed_{t,tc_\infty} = \begin{cases} 1 & \text{if } t \text{ exits late from the infrastructure,} \\ 0 & \text{otherwise.} \end{cases}$$

and constraints

$$D_{t,tc_\infty} \leq M \, delayed_{t,tc_\infty} \quad \forall t \in T. \quad (37)$$

Then, by adding variables

$$delayed_t = \begin{cases} 1 & \text{if } t \text{ suffers a delay at one of its stops or at its exit from the infrastructure,} \\ 0 & \text{otherwise.} \end{cases}$$

and constraints

$$delayed_t \geq delayed_{t,s} \forall t \in T, s \in S_t, \quad (38)$$

$$delayed_t \geq delayed_{t,tc_\infty} \forall t \in T. \quad (39)$$

$$(40)$$

It is possible to wish to minimize this number, which is done by including the sum over  $t$  of variables  $delayed_t$  in the objective function.

## 2.7 Measure of recovery time

Let the recovery time be equal to the latest between the last delayed exit of a train from the infrastructure and the last delayed arrival of a train at a station. To compute it we use variables and constraints introduced in Sections 2.5 and 2.6. Then, the recovery time is a non-negative continuous variable  $latest$  which is set by imposing:

$$latest \geq \sum_{r \in R_t: tc \in TC^r} (o_{t,r,tc} + rt_{r,ty_t,tc} x_{t,r}) - M(1 - delayed_{t,s}) \forall t \in T, s \in S_t, \quad (41)$$

$$latest \geq \sum_{r \in R_t} o_{t,r,tc_\infty} - M(1 - delayed_{t,tc_\infty}) \forall t \in T, s \in S_t, \quad (42)$$

$$(43)$$

Of course if the train can be cancelled a slight modifications as the one shown in Section 2.3 must be applied.

A possible use of this measure is its inclusion in the objective function to minimize the time to recover.

## 2.8 Measure of the maximum delay

To measure the maximum delay suffered by any train at a station or at its exit from the infrastructure, we only need to define a continuous non-negative variable  $D$  and set the following constraints:

$$D \geq D_{t,tc_\infty} \forall t \in T, \quad (44)$$

$$D \geq D_{t,tc} \forall t \in T, tc \in TC_t \exists s \in S_t, tc \in TCS_{t,s}. \quad (45)$$

If we rather want to measure the maximum total delay suffered by any train, we only need to set

$$D \geq D_{t,tc_\infty} + \sum_{tc \in TC_t \exists s \in S_t, tc \in TCS_{t,s}} D_{t,tc} \quad \forall t \in T. \quad (46)$$

By minimizing the value of this variable in the objective function, we can minimize the maximum tardiness, from a scheduling perspective.

## References

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