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# Towards Reconciling Gravitational Theories

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## Abstract

The theories of emergent gravity and of the gravitational effects of gravitationally polarised dipoles and associated concepts are described. Evidence is offered to suggest that they are equivalent, and this equivalence is used to show the dependence of the virtual particle density responsible for dark matter on the Hubble scale and Planck's constant and to offer further evidence that they consist of closely packed pions.

## 1 Introduction

Zwicky in 1933 [1] observed anomalous velocity dispersion in the Coma cluster of galaxies, and Rubin in 1970 [2] discovered that at large radii galactic material rotates faster than predicted according to Newton's Laws. Both these observations suggested the presence of stronger than expected gravitational fields. The two most popular theories to explain these, and other galactic phenomena, are the presence of undiscovered massive matter (dark matter) or Bekenstein and Milgrom's MOND (Modified Newtonian Dynamics) theory [3] of the modification of the laws of gravity in the vicinity of a large mass. There are also two less popular but promising theories. The theory of emergent gravity is based upon a melange of the theories of thermodynamics, entropy, black holes, information, and elasticity, and string theory [4]. Gravity is shown to emerge from the quantum bits of information which characterise the discrete particles of space-time, and is, therefore, not a fundamental property of matter. According to the second theory [5], the additional gravitational force needed to account for the increased gravity associated with galaxies is contributed by the partial alignment of the gravitationally polarised gravitational dipoles of virtual particle pairs in regions of the galactic gravitational field. The virtual particle pairs consist of particles and their antiparticles, which spontaneously appear very briefly and then self-annihilate, according to Heisenberg's Uncertainty Principle and Einstein's equation of equivalence between matter and energy,  $E = mc^2$ . These antiparticles are assumed to have negative mass, as well as negative charge. None of these theories have so far been shown to account for all the gravitational affects, while despite extensive searching, dark matter has yet to be found. However, where tested, the emergent gravity and gravitational dipole theories have met with some successes, and it is these that are considered in this paper. It is shown here that,

based on a spherical galactic model, the two theories predict dark matter to occur at the same location. By combining formulaic results from the two theories, it is shown that the virtual particle density in the galactic halo depends upon the Planck wavelength and the size of the universe.

The emergent gravity theory and the polarised dipole gravitational theory are first reviewed together with some underlying concepts. The new contribution appears in Sections 9 and 10.

## **2 Dark matter**

Dark matter is the name given to the unknown matter postulated to exist in the vicinity of galaxies to explain the anomalously high gravitational fields needed there to explain some of their behaviour. The universe does not contain enough known matter to offer sufficient gravitational attraction to prevent galaxies flying apart or to permit the high rotational velocities of outer stars of individual galaxies. The proposal is that unknown dark matter must make up this deficit. Since it is undetectable, but must have high mass, the existence of massive, weakly-interacting, cold, dark particles, which neither emit nor reflect light, has been proposed. The location of these particles, assuming they exist, has been determined by computer modelling.

Models show that every galaxy is surrounded by a halo of dark matter. These dark matter haloes have been shown to have a constant central surface density [6]. Also studies of the dwarf galaxies Fornax and Sculptor show dark matter distributed uniformly within the central region [7]. Thus a uniform density is characteristic of dark matter galaxial haloes. Further evidence for possible dark matter is offered by the lensing effect of the Bullet Cluster of colliding galaxies. Measurements show that there are two centres of mass, that of the baryonic material and that of dark matter [8], and that the two types of matter are behaving differently in collisions. Evidence of the existence of dark matter has also been deduced from the cosmic background microwave radiation (CMBR) [9]. However, the existence of dark matter has not been proved.

## **3 Vacuum energy and virtual particles**

According to the theory of quantum chromo dynamics the vacuum of space is full of electromagnetic radiation of all wavelengths. This is associated with energy, known as the quantum vacuum energy. By Heisenberg's Uncertainty Principle there are constant fluctuations in this energy, and decreases in energy are associated with the appearance of mass, and vice versa, according to  $E=mc^2$ . The mass is created as particle-antiparticle pairs. These mutually annihilate each other after about  $10^{-22}$  to  $10^{-15}$  s, returning the energy to the vacuum. Because of their impermanence, the particles are termed virtual particles. The effect of these particles has been observed in the Casimir Effect, in which parallel conducting plates in a vacuum are caused to

move by the force exerted [10]. They are also responsible for the Lamb shift in which their interaction with the electrons of the hydrogen atom separate one of the energy levels into two separate levels [11]. Thus, they are known to exist.

It has been suggested that the gravitational polarisation of these particles accounts for the dark matter explained by the MOND model [12], [13], as well as for dark matter and dark energy [14], [15]. These ideas are taken further in the theory of the gravity of dark matter by gravitational polarisation of the virtual dipoles.

#### 4 Entropy and Information

In thermodynamics entropy is a measure of the amount of energy which becomes unavailable when work is done. It is a measure of uncertainty or randomness. It is measured as the number of different states or arrangements which the constituents of matter can occupy, and is given by the well-known Boltzmann equation

$$S = k_B \ln W \quad (1)$$

In which  $k_B$  is Boltzmann's constant,  $W$  is the number of possible states, and  $\ln$  denotes logarithm.

In information theory information is conveyed in binary codewords, that is in strings of  $N$  1s and 0s. The mean amount of information per codeword, also known as the information entropy, is given by

$$S = \log_2 2^N \quad (2)$$

These equations are identical in form, showing that the number of possible states and the number of information bits needed to represent them are equivalent, that is, entropy and information are interchangeable.

Microscopically, in the domain of quantum mechanics, the binary information bits become qubits taking any value between 0 and 1. The wave function may represent two states simultaneously. For example, these might be spin up and spin down. These states are said to be entangled. Thus, their representative qubits are also entangled. In the theory of black holes [16]

$$S = \frac{k_B A}{4G\hbar} c^3 = \frac{k_B A}{4\lambda_c^2} \quad (3)$$

represents the entropy of a black hole of area  $A$ , and may be regarded as the number of associated entangled qubits. Here  $c$  is the velocity of light,  $G$  is the gravitational constant,  $\hbar$  is the reduced Planck constant, and  $\lambda_c$  is the Compton wavelength.

An increase in entropy,  $\Delta S$ , is associated with the work done,  $F\Delta x$ , by the application of a force,  $F$ , in moving an object through a distance  $\Delta x$ . The energy increase in entropy terms is  $T\Delta S$ , where  $T$  is the temperature. Equating these gives

$$T\Delta S = F\Delta x \quad (4)$$

## 5 Dark energy, matter, and emergent gravity

Verlinde presented a theory of emergent gravity [17], and a non-mathematical description of his theory has been given by Visser and Vonk [18]. According to Verlinde an empty space is filled with dark energy, which provides a volume contribution to the entropy, and gives the space a positive curvature (de Sitter space). It also acts as an expansive force, which drives the expansion of the universe. If some of this energy is converted into ordinary baryonic matter, the entropy change produces a force, according to equation (4). The universe is considered an elastic medium and this force results in elastic strain, which slowly relaxes over a very long timescale. Viewed from the standpoint of information bits, the creation of matter displaces them, leaving a vacuum in the vicinity of the mass. The information bits push to fill the vacuum, which creates an additional gravitational force. This phenomenon is significant whenever the mass is sufficiently large. This results in the large gravitational fields in the vicinity of galaxies, which have been ascribed to dark matter. Thus, gravity is regarded as an emergent macro property of the microscopic behaviour of the information qubits.

## 6 Newton and emergent gravity

Verlinde has demonstrated how Newton's Laws may be derived from his theory of emergent gravity [19]. Use is made of the holographic principle [20], [21], which for a three dimensional volume, such as the universe, states that in a quantum theory of gravity all the information necessary to describe the volume, can be represented as information bits on its two-dimensional surface.

Consider a particle of mass  $m$  in emerged space-time approaching a two dimensional screen, which separates it from a non-emerged region. After Bekenstein [16], when the particle is one Compton wavelength away from the screen it may be represented by information bits on the screen. The corresponding number of bits, or its entropy, is given by equation (3). Verlinde effectively regarded the information bits as occupying an area  $8\pi\lambda_c^2$  on the screen in choosing the change in information as

$$\Delta S = 2\pi k_B \quad (5)$$

The distance the particle moves to the screen is  $\Delta x$  equated to one Compton wavelength and so

$$\Delta x = \frac{\hbar}{mc} \quad (6)$$

One more piece of information is needed, and this was provided by Unruh [22] and is the relationship between the temperature of an observer in an accelerated frame and the acceleration  $a$ , which is

$$k_B T = \frac{\hbar a}{2\pi c} \quad (7)$$

Substitution of equations (5) and (6) as well as for  $T$  from equation (7) into (4) gives

$$F = ma \quad (8)$$

which is Newton's equation for the force on an accelerated body, derived from the emergent gravity theory.

To obtain Newton's Law of gravity, Verlinde assumed the screen to become a sphere with the emerged space on the outside [19]. The amount of volume information stored on its surface is given by equation (3). It was assumed that the number of bits stored,  $N$ , is proportional to the surface area of the sphere,  $A$ . With this assumption, and inspired by equation (3), equation (9) may be written:

$$N = \frac{k_B A c^3}{4G'\hbar} \quad (9)$$

where  $G'$  is some constant. By the principle of equipartition of energy, each bit will have energy  $k_B T$  and so the average energy for  $N$  bits is

$$E = \frac{1}{2} N k_B T \quad (10)$$

and, for the mass,  $M$ , that would emerge from the sphere,

$$E = M c^2 \quad (11)$$

Equations (10) and (11) can be solved for  $T$ . Equations (5) and (6) allow equation (12) to be written:

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x \quad (12)$$

$T$  and equation (12) are now substituted into equation (4), which is solved for  $F$  to leave

$$F = G' \frac{Mm}{k_B R^2} \quad (13)$$

Putting  $G'/k_B = G$  gives finally

$$F = G \frac{Mm}{R^2} \quad (14)$$

which is Newton's Law of Gravity, derived from the theory of emergent gravity. Thus, the theories of emergent and Newtonian gravity are compatible.

## 7 Gravitationally polarised virtual particles

Hajdukovic produced the theory of gravitationally polarised virtual particles to explain the phenomenon of dark matter [23]. The idea of gravitational effects due to gravitational polarisation in the vacuum of space [14], [15] was extended. The virtual particle-antiparticle pairs are assumed to have opposite gravitational charges and to form gravitational dipoles in which their gravitational charges repel each other. The concept of matter and antimatter being mutually repulsive has been investigated theoretically [24] with the conclusion that antigravity is predicted by the theory of general relativity assuming charge, parity, and time reversal symmetry. See also [25]. Hajdukovic also assumes that the virtual particle pairs constitute a gas of virtual pions, which are quark-antiquark pairs [26]. This conforms to the suggestion that the mass of a pion is the smallest possible in nature [27], and Hajdukovic argues that pions may dominate the content of the quantum vacuum [28]. While the opposite gravitational charges of the virtual dipoles repel each other, their opposite electrical charges attract, and the separation between the poles  $\cong \lambda_c = \hbar/mc$ , their Compton wavelength, at their equilibrium separation. Further, according to quantum field theory, a virtual particle pair should occupy the volume  $\lambda_c^3$  [23]. Thus, the particle pair density in the vacuum is  $N = 1/\lambda_c^3$ .

These polarised particle pairs become aligned in a gravitational field to an extent which depends upon the field strength, just as electric dipoles align in an electric field. Hajdukovic used a simple spherical model of a galaxy and calculated the radial gravitational charge density. The evidence for a uniform matter density in regions of dark matter has already been mentioned above [6],[7], and so regions of uniform gravitational charge density were taken to represent dark matter regions. In fact the radial distance from the centre at which dark matter was to be found was determined to be

$$R_0 = \lambda_c \sqrt{\frac{M_b}{m_c}} \quad (15)$$

in which  $M_b$  is the baryonic mass. Hajdukovic took  $\lambda_c = \lambda_\pi$  and  $m_c = m_\pi$  because he was considering pions.

## 8 Gravitational theories – the evidence

Table 1 gives a summary of the successes,  $\checkmark$ , failures, X, and unknown predictions, ?, for dark matter, as known by the author at present, by the different theories in different instances.

Dark matter phenomenon	MOND	Emergent gravity	Gravitational dipoles
Galactic haloes	$\checkmark$	$\checkmark$	$\checkmark$
Dwarf galaxies	$\checkmark$	?	?
Gravitational lensing	X	$\checkmark$	$\checkmark$
Milky Way disc	X	?	$\checkmark$

**Table 1** The theoretical predictions compared to the locations of known dark matter

The MOND theory predicted that the Milky Way would exhibit a disc of dark matter, while the theory of gravitational dipoles did not predict one. Attempts to find it were unsuccessful [29]. This casts doubt on the MOND theory. The emergent gravity theory has been successfully tested for the case of gravitational lensing [30]. While much more work remains to be done, the emergent gravity and gravitational dipole theories look promising, whilst the MOND theory failed in two instances.

## 9 Predicted location of dark matter

Since the emergent gravity and gravitational dipole theories both describe the locations of dark matter in two or three of the above cases, then the radial distance from the galactic centre at which dark matter is found should be similar when calculated for each theory. This can be done easily for assumed spherical galaxies. For the gravitational dipole theory the distance was given in equation (15). The corresponding formula for emergent gravity may be derived from Verlinde's equations [17]. Thus, with  $A$  as the surface area of a sphere of radius  $R$  surrounding the galactic centre, beyond which dark matter is predicted, Verlinde's equation (4) in [17] describes the situation by,

$$\frac{A}{4G\hbar} > \frac{2\pi M}{\hbar a_0} \quad (16)$$

where  $M$  is the enclosed mass, and  $a_0$  is the cosmological acceleration scale, given by

$$a_0 = \frac{c^2}{L} \quad (17)$$

where  $L$  is the Hubble scale, which is taken as the radius of the universe. Upon substituting equation (17) into equation (16), putting  $A = 4\pi R^2$ , and rearranging gives

$$R > \sqrt{2MLG}/c \quad (18)$$

Taking the case of our galaxy, the Milky Way,  $M = 1.6 \times 10^{42}$  kg, while  $L = 4.4 \times 10^{25}$  m,  $G = 6.67 \times 10^{-11}$  m<sup>3</sup> kg<sup>-1</sup>, and  $c = 3 \times 10^8$  m s<sup>-1</sup> giving  $R > 3.2 \times 10^{20}$  km.

Now we calculate  $R_0$  from equation (15), substituting  $\hbar/mc$  for  $\lambda_c$ , putting  $m = 2.4 \times 10^{-28}$  kg as the mass of a pion, and using  $M_b = 1.6 \times 10^{42}$  kg again, obtaining  $R_0 = 1.2 \times 10^{20}$  km. Thus,  $R \cong 2.7R_0$ . Since the calculation of  $R$  depends upon the measured mass of the pion, while  $R_0$  depends upon the estimated value of the size of the universe, while both depend upon the estimated mass of the universe, this may be regarded as a good agreement.

## 10 Combining theories for dark matter

The evidence in Table 1, and the agreement between  $R$  and  $R_0$  suggests the emergent gravity and polarised dipole theories are compatible. Since equations (15) and (18) give essentially the same result, they may be equated to give the new equation

$$\lambda_c \sqrt{\frac{M}{m_c}} = \sqrt{2MLG}/c \quad (19)$$

Substituting  $m_c = \hbar/\lambda_c c$  and  $N = 1/\lambda_c^3$ , it is found that

$$NL = \frac{c^3}{2G\hbar} \quad (20)$$

showing that the particle density at  $R_0$  only depends upon known constants and the radius of the spherical universe. The Planck wavelength,  $\lambda_p$ , is

$$\lambda_p = \sqrt{\frac{\hbar G}{c^3}} \quad (21)$$

Combining equations (20) and (21) gives

$$NL = \frac{1}{2\lambda_p^2} = \frac{\pi/2}{\pi\lambda_p^2} = \frac{\pi/2}{\text{Planck area}} \quad (22)$$

Showing that  $NL$  is inversely proportional to the smallest meaningful area of space. The particle density at  $R_0$  can be estimated from equation (20) as

$$N = \frac{c^3}{2G\hbar L} \quad (23)$$

Substituting values gives  $N = 4.4 \times 10^{43}$  particles  $\text{m}^{-3}$ .

So far the type of particle has not been defined. If we assume, like Hajdukovic [23], that they are close neighbours with  $N = 1/\lambda_c^3$  then  $\lambda_c = N^{-1/3} = 2.84 \times 10^{-15}$  m. This is close in value to  $\lambda_c$  for the electron ( $2.82 \times 10^{-15}$  m) and for the pion ( $1.46 \times 10^{-15}$  m). Uncertainties in the values mean that either of these is a possibility, but it is thought that the quantum vacuum is dominated by pions [23]. This agreement is further evidence that the two theories yield similar results and that equation (19) is valid.

In deriving the expression for  $R_0$ , equation (15), the spacing between the virtual particle and its antiparticle was taken to be approximately  $\lambda_c$  according to quantum field theory, and the virtual dipole pairs were assumed tightly packed so that the particle density is  $N = 1/\lambda_c^3$ . This second assumption can now be tested. Let the particle density be  $N = 1/x^3$  where  $x \geq \lambda_c$ . Then from equation (19)

$$x = \frac{\sqrt{2LGm_c}}{c} \quad (24)$$

If pions are assumed,  $m_c = 2.4 \times 10^{-28}$  kg, whence  $x = 3.2 \times 10^{-15}$  m, compared with  $\lambda_\pi = 1.46 \times 10^{-15}$ . The factor of 2.7 difference is probably insignificant and the pions are indeed tightly packed. This is more evidence for the compatibility of the two theories.

## 11 Conclusion

The predicted agreements with the “observations” of dark matter, and the agreement between the predicted radial locations of dark matter suggest the theories of gravitationally polarised dipoles and of emergent gravity are equivalent. By equating the two formulae for the radial location of dark matter it is shown that the virtual particle density in this region depends only upon the Hubble scale and the Planck area,  $\pi\lambda_p^2$ . The number density of virtual particle pairs was found to be  $4 \times 10^{43}$   $\text{m}^{-3}$ , and their separation by approximately one Compton wavelength corresponded to that of tightly packed pions or electrons.

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## References

- [1] F. Zwicky, *Die Rotverschiebung von extragalaktischen Nebeln*, Helvetica Physica Acta, **6**, 110, (1933), doi:<http://adsabs.harvard.edu/abs/1933AchPh...6..110Z>
- [2] V. Rubin, W. K. Ford, *Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions*, The Astrophysical Journal, **159**, 379, (1970), doi:10.1086/150317.
- [3] J. Bekenstein and M. Milgrom, *Does the missing mass problem signal the breakdown of Newtonian Gravity?* Astrophysical Journal, **286**, 7, Nov. 1, (1984), doi: 10.1086/162570
- [4] E. P. Verlinde, *Emergent Gravity and the Dark Universe*, SciPost Phys. 2, 016 (2017), doi:10.21468/SciPostPhys.2.3.016
- [5] D. S. Hajdukovic, *Is dark matter an illusion created by the gravitational polarization of the quantum vacuum?* Astrophys Space Sci **334**, 215, (2011), doi: 10.1007/s10509-011-0744-4
- [6] F. Donato, G. Gentile, P. Salucci, F. Martins, M. I. Wilkinson, G. Gilmore, E. K. Grebel, A. Koch, and R. Wyse, *A constant dark matter halo surface density in galaxies*, Mon. Not. R. Astron. Soc., **397**, 1169, (2009), doi: 10.1111/j.1365-2966.2009.15004.x
- [7] M. Walker and J. Peñarrubia, *A method for measuring (slopes of) the mass profiles of dwarf spheroidal galaxies*, The Astrophysical Journal, 742, Number 1, 1, (2011), doi:10.1088/0004-637X/742/1/20
- [8] D. Clowe, M. Bradač, A. H. Gonzalez, Maxim Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, *A direct empirical proof of the existence of dark matter*, The Astrophysical Journal Letters, **648**, No. 2, 109, (2006), doi:10.1086/508162
- [9] J Einasto, *Dark Matter*, Baltic Astronomy, **20**, 231, (2011), doi:10.1515/astro-2017-0287
- [10] H.B.G. Casimir, *On the attraction between two perfectly conducting plates*, Proc. Kon. Ned. Akad. Wetensch, **B51**, 793 (1948)
- [11] W. E. Lamb, and R. C. Retherford, *"Fine Structure of the Hydrogen Atom by a Microwave Method"*, Physical Review, **72**, No. 3, 241, (1947), doi: 10.1103/PhysRev.72.241.
- [12] L. Blanchet, L: *Gravitational polarization and the phenomenology of MOND*, Class. and Quant.Grav., **24**, No. 14, 3, (2007),doi:10.1088/0264-9381/24/14/001
- [13] L. Blanchet, *Dipolar particles in general relativity*, Class. and Quant. Grav., **24**, No. 14, 3, (2007), doi:10.1088/0264-9381/24/14/002

- [14] L. Blanchet, and A. Tiec, *Model of dark matter and dark energy based on gravitational polarization*, Phys.Rev., **D78**, 024031, (2008),  
doi: 10.1103/PhysRevD.78.024031
- [15] L. Blanchet, and A. Tiec, *Dipolar dark matter and dark energy*, Phys.Rev., **D80**, 023524, (2009), doi: 10.1103/PhysRevD.80.023524
- [16] J. D. Bekenstein, *Black Holes and Entropy*, Phys. Rev., D **7**, 2333, (1973),  
doi: 10.1103/PhysRevD.7.2333
- [17] E. P. Verlinde, *Emergent Gravity and the Dark Universe*, SciPost Phys. 2, 016 ,  
(2017), doi: 10.21468/SciPostPhys.2.3.016
- [18] M. Visser and M. Vonk, *Emergent gravity and the dark universe*, Quantum Universe, 20/02/2017, 23?05
- [19] E. Verlinde, *On the origin of gravity and the laws of Newton*, Journal of High Energy Physics, **29**, 1, 2011, doi:10.1007/JHEP04(2011)029  
ArXiv ePrint:1001.0785
- [20] L. Susskind, *The world as a hologram*, Journal of Mathematical Physics, **36**, 6377, (1995), doi:10.1063/1.531249
- [21] G. 't Hooft, *Dimensional Reduction in Quantum Gravity*, 20 March 2009  
arXiv:gr-qc/9310026v2
- [22] W. G. Unruh, *Notes on black-hole evaporation*, Phys. Rev. D, **14**, 870, (1976),  
doi. 10.1103/PhysRevD.14.870
- [23] D. Hajdukovic, *Is dark matter an illusion created by the gravitational polarization of the quantum vacuum?*, Astrophysics and Space Science, **334**, No. 2, 215, (2011),  
doi: 10.1007/s10509-011-0744-4
- [24] M. Villata, *CPT symmetry and antimatter gravity in general relativity*, EPL (Europhysics Letters), **94**, No. 2, 20001, (2011),  
doi:10.1209/0295-5075/94/20001
- [25] D. Hajdukovic, *Dark matter, dark energy and gravitational properties of antimatter* , (2009), arXiv:0810.3435v3 [physics.gen-ph]
- [26] D. S. Hajdukovic, *On the relation between mass of a pion, fundamental physical constants and cosmological parameters*, Europhysics Letters, **89**, 1, No. 4, (2010),  
Doi:10.1209/0295-5075/89/49001

[27] C.G.Böhmer and T.Harko, *Does the cosmological constant imply the existence of a minimum mass?*, Physics Letters B, **630**, 73, (2005), doi:10.1016/j.physletb.2005.09.071

[28] D. S. Hajdukovic, *On the relation between mass of a pion, fundamental physical constants and cosmological parameters*, Europhysics Letters, **89**, No. 4, 49001, (2010), doi:10.1209/0295-5075/89/49001

[29] C. Moni Bidin, G. Carraro, R. A. Méndez, and W. F. van Altena, *No evidence for a dark matter disk within 4 kpc from the galactic plane*, The Astrophysical Journal Letters, **724**, No. 1,122, (2010), doi:10.1088/2041-8205/724/1/L122

[30] M. M. Brouwer, M. R. Visser, A. Dvornik, H. Hoekstra, K. Kuijken, E. A. Valentijn, M. Bilicki, C. Blake, S. Brough, H. Buddelmeijer, T. Erben, C. Heymans, H. Hildebrandt, B. W. Holwerda, A. M. Hopkins, D. Klaes, J. Liske, J. Loveday, J. McFarland, R. Nakajima, C. Sifón, E. N. Taylor, *First test of Verlinde's theory of emergent gravity using weak gravitational lensing measurements*, Monthly Notices of the Royal Astronomical Society, **466**, No. 3, 2547, (2017), doi:10.1093/mnras/stw3192