



HAL
open science

Attitude tracking control of a flexible spacecraft under angular velocity constraints

Laurent Burlion, Jean-Marc Biannic, Tarek Ahmed-Ali

► **To cite this version:**

Laurent Burlion, Jean-Marc Biannic, Tarek Ahmed-Ali. Attitude tracking control of a flexible spacecraft under angular velocity constraints. *International Journal of Control*, 2017, 92 (7), pp.1524-1540. 10.1080/00207179.2017.1398841 . hal-01664704

HAL Id: hal-01664704

<https://hal.science/hal-01664704>

Submitted on 11 Jun 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Interval based control of a flexible spacecraft under angular velocity constraints

L. Burlion^{a*}, J.-M. Biannic^a and T. Ahmed-Ali^b

^aONERA, The French Aerospace Lab, F-31055 Toulouse, France; ^bLaboratoire GREYC, UMR CNRS 6072, Université de Caen Basse-Normandie / ENSICAEN, Caen, France

(v1.0 released February 2017)

In this paper, a new technique is proposed for trajectory tracking of a flexible spacecraft subject to angular velocity constraints. The problem is addressed using an Output to Input Saturation Transformation (OIST) which converts the prescribed bounds into state-dependent saturations on the control input signals. It is shown that an interval observer can be used in combination with the OIST technique to ensure that the constraints remain satisfied despite unmeasured flexible modes and torque disturbances. Some realistic simulations conclude the paper and validate our approach.

Keywords: constrained control; interval observers; anti-windup; spacecraft; attitude stabilisation; time-domain constraint

1. Introduction

State constraints arise in numerous engineering applications. Among several approaches summarized in G.C. Goodwin and de Dona (2004), Glattfelder and Schaufelberger (2003), a popular idea consists of modifying a nominal controller whenever the constraints are about to be violated. Following this line, the Output to Input Saturation Transformation (OIST) approach was recently proposed in Burlion (2012); Chambon, Burlion, and Apkarian (2015a) to replace the state constraints by some state-dependent control saturations. Such an approach is attractive because it enables use of the vast literature on control saturation problems and may especially lead to consider anti-windup loops which are now well understood (see e.g Galeani, Tarbouriech, Turner, and Zaccarian (2009); Tarbouriech and Turner (2009)).

On the other hand, the spacecraft attitude pointing and tracking control problems have been extensively studied in the past decades (see e.g Akella, Thakur, and Mazenc (2015); Chen and Huang (2009); Di Gennaro (2002); Gennaro (2003); H.-H. Zhang and Trivailo. (2008); Lovera and Astolfi (2004); Tayebi (2008); Tréguët, Arzelier, Peaucelle, Pittet, and Zaccarian (2015); Zhang and Cheng (2012) and the references therein). Most of the papers dealing with satellite attitude control subject to constraints use Model Predictive Control (MPC) techniques (see e.g Gupta, Kalabic, Cairano, Bloch, and Kolmanovsky (2015); Hegrenaes, Gravdahl, and Tondel (2005)). Although MPC provides very efficient solutions, the full state is most of the time assumed to be available. As an alternative to MPC, only a few papers deal with state-constraints, especially with the problem of limiting the angular velocity of a spacecraft as in Hu (2009); Luzi, Peaucelle, Biannic, Pittet, and Mignot (2014). This is a significant problem because the limitations of the satellite actuators require a response with a limited speed when the pointing error is too large.

In this paper, the problem of spacecraft attitude reference trajectory tracking is considered. To cope with angular velocity limitations, a nominal controller is combined with the so-called OIST methodology. Since in its preliminary form, this technique requires the full state to be measured, an

*Corresponding author. Email: lburlion@onera.fr

interval observer is proposed developing the ideas introduced in Chambon, Burlion, and Apkarian (2015b). This observer is used to compute upper and lower bounds of the torque induced by the unmeasured flexible modes and external disturbances. Finally the stability of the closed-loop system is established using an anti-windup design.

The paper is organized as follows. In Section 2, the mathematical model of the flexible spacecraft is recalled, while in Section 3, the control problem is stated. In Section 4, a novel Output to Input Saturation Transformation which uses a time-varying interval observer is proposed to deal with the angular velocity constraints. The obtained closed-loop saturated system is studied in Section 5 : it is demonstrated that asymptotic stability properties are achieved when a nonlinear anti-windup loop is combined with a nominal control law. Finally, the effectiveness of the proposed approach is illustrated by numerical simulations in Section 6.

2. Preliminaries

2.1 Definitions and notation

Notation 1: The classical unit quaternion representation for satellite orientation with respect to an inertial frame is denoted $q = [q_0, q_v^T]^T$, where $q_0 \in \mathcal{R}$ is the scalar component and $q_v \in \mathcal{R}^3$ is the vector component. The quaternion is unit when $q_0^2 + q_v^T q_v = 1$. The inverse of q is denoted $q^{-1} = [q_0, -q_v^T]^T$ and the product of two quaternions q_1 and q_2 is denoted $q_1 * q_2$. The following notation will be useful when developing the equations for satellite attitude dynamics:

$$S(v) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad E(q) = \begin{bmatrix} -q_v^T \\ q_0 I_3 + S(q_v) \end{bmatrix} \quad (1)$$

$$R(q) = (q_0^2 - q_v^T q_v) I_3 + 2q_v q_v^T - 2q_0 S(q_v)$$

where $v = [v_1, v_2, v_3]^T$. Note that $S(\cdot)$ is the skew symmetric operator and that $E(q) \in \mathcal{R}^{4 \times 3}$ is classically used in the expression of \dot{q} . Moreover, $R(q) \in SO(3)$ is the rotation matrix that corresponds with q .

Notation 2: Given two vectors $x_1, x_2 \in \mathbb{R}^n$, the relation $x_1 \leq x_2$, is understood element-wise.

Notation 3: For any vector $x \in \mathcal{R}^n$, and any matrix $A \in \mathcal{R}^{m \times n}$, $|x|$ and $|A|$ denote respectively the Euclidean norm of x and the induced norm of A . The Frobenius norm of is denoted by $|A|_F := \sqrt{\text{trace}(A^T A)}$. Moreover, given any positive definite matrix Q , $\lambda_{\min}(Q)$ (resp. $\lambda_{\max}(Q)$) denotes the minimal (resp. maximal) eigenvalue value of Q .

Definition 1: A square matrix $M = (M_{ij}) \in \mathcal{R}^{n \times n}$ is said to be Metzler if $M_{ij} \geq 0, \forall i \neq j$.

Definition 2: Given two scalar functions $\underline{u}, \bar{u} : \mathcal{R} \rightarrow \mathcal{R}$ such that $\forall t, \underline{u}(t) \leq \bar{u}(t)$, the following saturation operator with time-varying bounds is defined:

$$\forall u \in \mathcal{R}, \quad \text{Sat}_{\underline{u}(t)}^{\bar{u}(t)}(u) = \max\{\underline{u}(t), \min\{\bar{u}(t), u\}\} \quad (2)$$

When applied to a vector, this operator is understood component-wise.

2.2 Flexible spacecraft modeling

In this paper, we consider the flexible spacecraft model used in Di Gennaro (2002):

$$\dot{q}_e = \frac{1}{2}E(q_e)\omega_e \tag{3a}$$

$$J_{mb}\dot{\omega}_e = -N(\omega, \omega_e, z_e, \omega_r^b) + u + d + C_z z_e + D_z \omega_e - J_{mb}\dot{\omega}_r^b \tag{3b}$$

$$\dot{z}_e = A_z z_e + B_{1,z}\omega_e + B_{2,z}\dot{\omega}_r^b \tag{3c}$$

where:

- $q_e = q * q_r^{-1} = [q_{e0} \ q_{ev}^T]^T$ is the error between the spacecraft quaternion q and the reference quaternion q_r
- $\omega_e = \omega - \omega_r^b \in \mathcal{R}^3$ is the error between the angular velocity of the spacecraft ω and ω_r^b which is the reference angular velocity ω_r expressed in the body fixed frame. In other words, ω_r^b is defined by:

$$\omega_r^b = R(q_e)\omega_r = R(q)R(q_r)^T\omega_r \tag{4}$$

- $u \in \mathcal{R}^3$ are the control inputs produced by gas jets.
- $d \in \mathcal{R}^3$ are the torque disturbances.
- $z_e = z - z_r$ where $z = [\eta ; \psi] = [\eta ; \dot{\eta} + \delta\omega]$ ($\eta \in \mathcal{R}^N$ being the vector of the modal displacements) and $z_r := [O_{N,1} ; \delta\omega_r^b]$.
- $J_{mb} = J - \delta^T\delta$ is the main body symmetric inertia matrix where $J \in \mathcal{R}^{3 \times 3}$ is the symmetric inertia matrix of the undeformed structure and $\delta \in \mathcal{R}^{N \times 3}$ is the coupling matrix between elastic and rigid dynamics
- $(A_z, B_{1,z}, B_{2,z}, C_z, D_z)$ are matrices expressed with respect to the damping (resp. stiffness) matrix $C = \text{diag}\{2\xi_i\omega_i, i = 1..N\}$ (resp. $K = \text{diag}\{\omega_i^2, i = 1..N\}$) of the N flexible modes:

$$\begin{aligned} A_z &= \begin{bmatrix} O_N & I_N \\ -K & -C \end{bmatrix}, & B_{2,z} &= -\begin{bmatrix} O_N \\ I_N \end{bmatrix}\delta, & B_{1,z} &= A_z B_{2,z} \\ C_z &= \delta^T [K \ C], & D_z &= -\delta^T C \delta \end{aligned} \tag{5}$$

- the function N is defined by the following relation:

$$\forall e, f, g \in \mathcal{R}^3, \forall z \in \mathcal{R}^{2N}, N(e, f, z, h) = S(e)(J_{mb}f + G_z z + Jh) \tag{6}$$

where $G_z = [O_{3,N} \ \delta^T]$

Remark 1: The expression retained for z_r corresponds to the absence of modal displacements. Indeed, z_r can be rewritten as $z_r := [\eta_r ; \dot{\eta}_r + \delta\omega_r^b]$ where $\eta_r = \dot{\eta}_r = O_{N,1}$. Moreover, considering the expressions of $A_z, B_{1,z}$ and $B_{2,z}$ (5), it can be easily shown that z_r verifies $\dot{z}_r = A_z z_r + B_{1,z}\omega_r^b - B_{2,z}\dot{\omega}_r^b$.

Remark 2: Control inputs u are merely produced by gas jets in the retained model of Di Gennaro (2002). Considering additional control inputs like reaction wheels and magnetorquers is postponed to future studies

3. Problem formulation

3.1 Main assumptions

In the remainder of the paper, the following not so restrictive assumptions have been made:

(A1) Although it might be poorly damped, the spacecraft structure is assumed strictly stable so that A_z is Hurwitz (all eigenvalues lie strictly inside the open left half plane),

(A2) The torque disturbance signal is bounded. More precisely, we assume that:

$$d \in \mathcal{L}_2 \cap \mathcal{L}_\infty \tag{7}$$

which implies:

$$\exists \bar{d} > 0 / d(t) \in [-\bar{d}, \bar{d}] \tag{8}$$

(A3) The initial state vector $z(0) \in \mathbb{R}^{2N}$ associated to the flexible modes is assumed to belong to a bounded interval. Then, there exist $z_0^-, z_0^+ \in \mathbb{R}^{2N}$ such that $z_0^- \leq z(0) \leq z_0^+$ or equivalently:

$$z(0) \in [z_0^-, z_0^+] \tag{9}$$

(A4) The reference angular velocity ω_r and the acceleration $\dot{\omega}_r$ are assumed to be bounded. More precisely, we will consider:

$$\omega_r \in \mathcal{L}_\infty \quad ; \quad \dot{\omega}_r \in \mathcal{L}_2 \cap \mathcal{L}_\infty \tag{10}$$

3.2 Problem statement

The starting point of the paper is the following nominal control law which does not take into account the angular velocity constraints and requires the whole state to be measured:

Lemma 1: *Under assumptions (A2) and (A4) and further considering that the whole state is measurable, let the following static state-feedback controller be applied to system (3a)-(3c)*

$$\begin{aligned} u = u_{nom}(q_{e0}, x_e, \omega_r^b, \dot{\omega}_r^b) = & -k_p q_{ev} - k_d \omega_e - \frac{1}{2} J_{mb}(q_{e0} I_3 + S(q_{ev})) \omega_e + N(\omega, \omega_e, z_e, \omega_r^b) \\ & - C_z z_e - D_z \omega_e + J_{mb} \dot{\omega}_r^b \end{aligned} \tag{11}$$

where the controller gains k_p and k_d are strictly positive scalars. Then, the error state $x_e = [q_{ev} ; \omega_e ; z_e]$ asymptotically converges to the origin for any initial condition $x(0)$.

Proof. The control law is similar to the one of [Theorem 1, Di Gennaro (2002)]. There are two slight differences in the proof of the result:

- (1) in our modeling, we consider a torque disturbance d which introduces an additional term in the derivative of the Lyapunov function V . This is treated in the same manner that the $\dot{\omega}_r$ term. Indeed, using assumption (A2), one can still conclude the proof by invoking Barbalat's lemma.
- (2) we do not impose any lower-bound on k_d since the proof is unchanged when P is replaced by εP where the positive real ε is an extra degree of freedom which can be chosen sufficiently small so that $Q > 0$ no matter the value of $k_d > 0$.

□

Remark 3: Note that q_{ev} tends to 0 while q_{e0} is driven to ± 1 which in all cases make $R(q_e)$ converge to the same rotation I_3 . Akella et al. (2015) proposes to adopt the notion of 'almost' global asymptotic stability for this problem.

In this paper, the problem at hand is thus to redesign the nominal full state feedback control law (11) in the case where

- the modal displacements η and their time derivatives $\dot{\eta}$ are not measured, which means that z is no longer available for feedback,
- the angular velocity vector ω must satisfy the following asymmetric constraints

$$\forall t, \quad \underline{\omega} \leq \omega(t) \leq \bar{\omega} \quad (12)$$

with

$$\underline{\omega} < 0 < \bar{\omega} \quad (13)$$

4. Output to Input Saturation Transformation extended for Robustness (OISTeR)

Before stating our main results and for clarity of presentation, we first recall the OIST technique when both the full state and the disturbance are measured. Then, we propose an extended technique using the upper and lower bounds of the unmeasured states which are thus provided by the interval observer. This new technique called OISTeR means OIST extended with Robustness properties (with respect to both the uncertain initial condition $z(0) \in [z_0^-, z_0^+]$ and uncertainties on $d \in [-\bar{d}, \bar{d}]$).

4.1 The 'classical' OIST methodology

For clarity of presentation, let us first recall the OIST methodology when the full state and the disturbance d are measured.

Lemma 2: *Let k_O be a strictly positive real number. Let $u \in \mathcal{L}_2$. Suppose that the whole state and the disturbance torque d are available for feedback. Suppose that $\omega(0) \in [\underline{\omega}, \bar{\omega}]$ and consider the following input saturated system:*

$$J_{mb}\dot{\omega} = -N(\omega, \omega, z, 0) + OISat(u) + d + C_z z + D_z \omega \quad (14a)$$

$$\dot{z} = A_z z + B_{1,z} \omega \quad (14b)$$

where:

$$OISat(u) := J_{mb} \left(Sat_{k_O \underline{\omega}}^{k_O \bar{\omega}}(\alpha + k_O \omega) - k_O \omega \right) + N(\omega, \omega, z, 0) - d - C_z z - D_z \omega \quad (15)$$

with:

$$\alpha = J_{mb}^{-1} (-N(\omega, \omega, z, 0) + u + d + C_z z + D_z \omega) \quad (16)$$

Then, $\forall t \geq 0, \omega(t) \in [\underline{\omega}, \bar{\omega}]$

Proof. The proof is straightforward using the OIST technique as defined in Burlion (2012); Chambon et al. (2015a) in the relative degree 1 case (i.e when u appears in the expression of the first derivative of ω with respect to time). For clarity, we develop the basic computations. Using (14a)-(15), it is readily seen that ω undergoes the following dynamics:

$$\dot{\omega} = Sat_{k_O \underline{\omega}}^{k_O \bar{\omega}}(\alpha + k_O \omega) - k_O \omega \quad (17)$$

which implies that $\forall t$,

$$-k_O(\omega - \underline{\omega}) \leq \dot{\omega} \leq -k_O(\omega - \bar{\omega}) \quad (18)$$

from this, the result is easily deduced using the fact that $\omega(0) \in [\underline{\omega}, \bar{\omega}]$ and $k_O > 0$. \square

Remark 4: 'OISat(u)' denotes a saturation of the control input u . This notation is preferred to 'Sat' since it is a state dependent saturation element which comes from an Output to Input Saturation transformation.

4.2 The OISTeR methodology

The OISTeR methodology is now developed : it extends the above result when neither z nor d are no longer measured. Following the preliminary ideas of Chambon et al. (2015b), we first build an interval observer in order to obtain lower and upper bounds of the missing variables. Then, we propose a novel 'OISat' function which exploits these bounds. Note that the use of an interval observer is the key idea to respect the angular velocity constraints (using an estimation of z in combination with the OIST methodology presented in Lemma 1 would not be sufficient to make sure that these constraints are fulfilled).

4.2.1 Interval observer synthesis

Let us introduce the following quantity:

$$\dot{\hat{z}} = A_z \hat{z} + B_{1,z} \omega \quad (19a)$$

$$\hat{z}(0) = \frac{z_0^+ + z_0^-}{2} \quad (19b)$$

Let us denote $e_z = z - \hat{z}$ and consider the following system:

$$\dot{e}_z = A_z e_z \quad (20a)$$

$$e_y = J_{mb}^{-1} (-S(\omega)G_z e_z + C_z e_z + d) \quad (20b)$$

Lemma 3: *Let the damping ratios verify $0 < \xi_i < 1$ for all $i \in \{1, N\}$. Then, under assumptions (A2) and (A3), there exist a Hurwitz Metzler matrix $M \in \mathcal{R}^{2N \times 2N}$, $Q_0 \in \mathcal{R}^{2N \times 2N}$ and an operator $\Pi : \mathcal{R}^4 \rightarrow \mathcal{R}^{2N \times 2N}$ such that:*

$$\forall t \geq 0, |e_y| \leq \bar{e}_y \quad (21)$$

with

$$\bar{e}_y = \Pi(t, \omega) \zeta^+ + |J_{mb}^{-1}| \bar{d} \quad (22a)$$

$$\dot{\zeta}^+ = M \zeta^+ \quad (22b)$$

$$\zeta(0) = |Q_0| \frac{z_0^+ - z_0^-}{2} \quad (22c)$$

$$(22d)$$

Proof. the proof of this result is reported in Appendix A. \square

Remark 5: As evidenced by the proof, the result is obtained by using a time-varying interval observer for system (20a). Such an observer is necessarily chosen here since the state matrix A_z exhibits a poor damping. Let us indeed rewrite the latter in the appropriate basis such that $A_z \sim \text{blkdiag}(J_i, i = 1..N)$ with

$$J_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix} \quad (23)$$

Next, compute the following key mapping

$$\mathcal{F}(J_i) = \text{tr}(J_i)^2 - 2\det(J_i) = 4\xi_i^2\omega_i^2 - 2\omega_i^4 \quad (24)$$

As a consequence of [Thm 2, Mazenc and Bernard (2010)], when the following condition (25) holds,

$$\mathcal{F}(J_i) < 0 \Leftrightarrow \xi_i < \frac{\sqrt{2}}{2}\omega_i \quad (25)$$

there does not exist any linear time-invariant change of coordinates transforming $\dot{N}_i = J_i N_i$ to a system for which a 'classical' exponentially stable framer can be used. In that case which usually corresponds to poorly damped systems – thus involving flexible modes – Mazenc and Bernard (2010) suggests to use a time-varying interval observer.

4.2.2 Design of a novel 'OISat' function

Lemma 4: Let k_O be a strictly positive real number. Let $d \in \mathcal{L}_2$. Let assumptions (A2) and (A3) hold and suppose that $\omega(0) \in [\underline{\omega}, \bar{\omega}]$. Consider the input saturated system (14a)-(14b) with:

$$\text{OISat}(u) := J_{mb} \left(\text{Sat}_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha + k(t)\omega) - k(t)\omega \right) + N(\omega, \omega, \hat{z}, 0) - C_z \hat{z} - D_z \omega \quad (26)$$

and:

$$\dot{\alpha} = J_{mb}^{-1} (-N(\omega, \omega, \hat{z}, 0) + u + C_z \hat{z} + D_z \omega) \quad (27a)$$

$$k(t) = k_O + \frac{2\bar{e}_y}{\bar{\omega} - \underline{\omega}} \quad (27b)$$

$$\dot{\hat{z}} = A_z \hat{z} + B_{1,z} \omega \quad (27c)$$

$$\hat{z}(0) = \frac{z_0^- + z_0^+}{2} \quad (27d)$$

Then,

$$\forall t \geq 0, \quad \omega(t) \in [\underline{\omega}, \bar{\omega}] \quad (28)$$

Proof. Using (14a)-(14b)-(20b)-(44b)-(19a), it is readily checked that ω undergoes the following dynamics:

$$\dot{\omega} = \text{Sat}_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha + k(t)\omega) - k(t)\omega + e_y \quad (29)$$

Note that the saturation function is well defined because (27b) implies that,

$$\forall t \geq 0, \quad k(t)\bar{\omega} - \bar{e}_y \geq k(t)\underline{\omega} + \bar{e}_y \quad (30)$$

it follows from (29) that

$$\forall t \geq 0, \quad -k(t)(\omega - \underline{\omega}) + e_y + \bar{e}_y \leq \dot{\omega} \leq e_y - \bar{e}_y - k(t)(\omega - \bar{\omega}) \quad (31)$$

which implies that

$$\forall t \geq 0, \quad -k(t)(\omega - \underline{\omega}) \leq \dot{\omega} \leq -k(t)(\omega - \bar{\omega}) \quad (32)$$

Recalling that $\omega(0) \in [\underline{\omega}, \bar{\omega}]$ and noting from (13,27b) that $k(t) \geq k_O > 0$, the final result (28) immediately follows from (32) which concludes the proof. \square

5. Main result : attitude tracking under velocity constraints

Based on the notation introduced in subsection 4.2.1 dedicated to interval observer design, and more specifically on equations (19a)-(20b), the flexible spacecraft model (3a)-(3c) may be rewritten as follows:

$$\dot{q}_e = \frac{1}{2}E(q_e)\omega_e \quad (33a)$$

$$J_{mb}\dot{\omega}_e = -N(\omega, \omega_e, \hat{z}_e, \omega_r^b) + u + C_z \hat{z}_e + D_z \omega_e + J_{mb} \left(e_y - \dot{\omega}_r^b \right) \quad (33b)$$

$$\dot{\hat{z}}_e = A_z \hat{z}_e + B_{1,z} \omega_e + B_{2,z} \dot{\omega}_r^b \quad (33c)$$

$$\dot{e}_z = A_z e_z \quad (33d)$$

with

$$\hat{z}_e = \hat{z} - z_r \quad (34)$$

$$e_z = z - \hat{z} \quad (35)$$

$$e_y = J_{mb}^{-1} (-S(\omega)G_z e_z + C_z e_z + d) \quad (36)$$

and the main result of the paper, summarized in the following theorem, may now be stated.

Theorem 1: *Let k_O, k_a be strictly positive real numbers. Let $d \in \mathcal{L}_2$. Let assumptions (A2) to (A4) hold and suppose that $\omega(0) \in [\underline{\omega}, \bar{\omega}]$. Consider a closed-loop system consisting of the plant (33a)-(33d) and the dynamic control law:*

$$u = OISat(\tilde{u}_n + u_a) \quad (37)$$

$$= J_{mb} \left(Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha + k(t)\omega) - k(t)\omega \right) + N(\omega, \omega, \hat{z}, 0) - C_z \hat{z} - D_z \omega \quad (38)$$

$$\dot{q}_a = \frac{1}{2}E(q_a)\omega_a := \frac{1}{2}E(q_a)R(q_e)R(q_a)^T \omega_a^{be} \quad (39)$$

$$J_{mb}\dot{\omega}_a^{be} = \left(N(\tilde{\omega} + \omega_r^b, \tilde{\omega}, \hat{z}, \omega_r^b) - N(\omega, \omega_e, \hat{z}_e, \omega_r^b) \right) + C_z z_a + D_z \omega_a^{be} \quad (40)$$

$$+ OISat(\tilde{u}_n + u_a) - \tilde{u}_n \quad (41)$$

$$\dot{z}_a = A_z z_a + B_{1,z} \omega_a^{be} \quad (42)$$

$$(43)$$

with:

$$\tilde{u}_n = u_{nom}(\tilde{q}_0, \hat{\tilde{x}}, \omega_r^b, \dot{\omega}_r^b) \quad (44a)$$

$$\alpha = J_{mb}^{-1}(-N(\omega, \omega, \hat{z}, 0) + \tilde{u}_n + u_a + C_z \hat{z} + D_z \omega) \quad (44b)$$

$$u_a = J_{mb}(-k_a q_{av} - k(t)\omega_a^{be}) + N(\omega, \omega_a^{be}, z_a, 0) - C_z z_a - D_z \omega_a^{be} \quad (44c)$$

where

- $\hat{\tilde{z}} = \hat{z}_e - z_a$, $\tilde{\omega} = \omega_e - \omega_a^{be}$, $\tilde{q} = q_e * q_a^{-1}$.
- \bar{e}_y (resp. $k(t)$) is given by (22a) (resp. (27b)).

Then, $\forall t \geq 0$, $\omega(t) \in [\underline{\omega}, \bar{\omega}]$ and there exists \underline{k}_O such that the error $x_e = [q_{ev}; \omega_e; z_e]$ is asymptotically stabilized to the origin for $k_O \geq \underline{k}_O$.

Remark 6: Note that the dynamic part of the control law described by equations (39)-(42), is the anti-windup loop.

Proof. The proof of the result is divided into two parts. In the first part, it is shown that the error system whose state $\tilde{x} = [\tilde{q}_v; \tilde{\omega}; \tilde{z}]$ asymptotically converges to 0. The second part of the proof establishes that the anti-windup state denoted by $x_a = [q_{av}; \omega_a^{be}; z_a]$ asymptotically converges to the origin as well. As a consequence, the state of the error system $x_e = [q_{ev}; \omega_e; z_e]$ tends asymptotically to 0.

Part 1. Let us consider the error system whose state is \tilde{x} . After straightforward computations, one obtains:

$$\begin{cases} \dot{\tilde{q}} &= \frac{1}{2}E(\tilde{q})\tilde{\omega} \\ J_{mb}\dot{\tilde{\omega}} &= -N(\tilde{\omega} + \omega_r^b, \tilde{\omega}, \hat{\tilde{z}}, \omega_r^b) + u_{nom}(\tilde{q}_0, \hat{\tilde{x}}, \omega_r^b, \dot{\omega}_r^b) + C_z \tilde{z} + D_z \tilde{\omega} + J_{mb}(e_y - \dot{\omega}_r^b) \\ \dot{\tilde{z}} &= A_z \tilde{z} + B_{1,z} \tilde{\omega} + B_{2,z} \dot{\omega}_r^b \end{cases} \quad (45)$$

Following the ideas of the proof of [Theorem 1, Di Gennaro (2002)], we first consider the following Lyapunov function:

$$V(t, \tilde{x}) = (k_p + k_d)[(1 - \tilde{q}_0)^2 + |\tilde{q}_v|^2] + \frac{1}{2}(\tilde{q}_v + \tilde{\omega})^T J_{mb}(\tilde{q}_v + \tilde{\omega}) + \frac{\varepsilon}{2} \tilde{z}^T P_z \tilde{z} + \gamma e_z^T P_z e_z \quad (46)$$

where $\varepsilon, \gamma > 0$ and $P_z = P_z^T > 0$ is a solution to:

$$P_z A_z + A_z^T P_z = -2Q_z \quad (47)$$

for any fixed $Q_z = Q_z^T > 0$. (P_z exists because A_z is strictly Hurwitz stable).

Computing the derivative of V , one gets:

$$\dot{V}(t, \tilde{x}) = -\tilde{x}^T Q \tilde{x} - \gamma e_z^T Q_z e_z + (\tilde{q}_v + \tilde{\omega})^T (e_y - \dot{\omega}_r^b) \quad (48)$$

with

$$Q = \begin{bmatrix} k_p I_3 & O & O \\ O & k_d I_3 & \frac{\varepsilon}{2} B_{1,z}^T P_z \\ O & \frac{\varepsilon}{2} P_z B_{1,z} & \varepsilon Q_z \end{bmatrix} \quad (49)$$

which, for sufficiently small $\varepsilon > 0$, is positive definite.

Then, using (36) and the boundedness of ω (which follows from (38) and Lemma (3)), there exist $\lambda_1, \lambda_2, \lambda_3 > 0$ such that:

$$\dot{V}(t, \tilde{x}) = -\lambda_{\min}(Q)|\tilde{x}|^2 - \gamma e_z^T Q_z e_z + \lambda_1 |\tilde{x}| |e_z| + \lambda_2 |\tilde{x}| |d| + \lambda_3 |\tilde{x}| |\dot{\omega}_r| \quad (50)$$

which, for $\gamma > 0$ chosen sufficiently large, yields:

$$\dot{V}(t, \tilde{x}) \leq -\frac{\lambda_{\min}(Q)}{2} |\tilde{x}|^2 - \frac{\gamma}{2} e_z^T Q_z e_z + \lambda_2 |\tilde{x}| |d| + \lambda_3 |\tilde{x}| |\dot{\omega}_r| \quad (51)$$

where $d, \dot{\omega}_r \in \mathcal{L}_2$ in virtue of Assumptions (A2) and (A4). The proof of this part ends by invoking Barbalat's Lemma as in the proof of [Theorem 1, Di Gennaro (2002)].

Remark 7: As a result, \tilde{x} is uniformly bounded. Furthermore, ω being bounded implies that $\omega_a^b := \omega - \tilde{\omega} - \omega_r^{ba}$ is bounded. From Assumption (A1) and equation (42), it readily follows that z_a is bounded. Moreover, q_a being a unit quaternion, q_{av} is bounded. As a consequence, there exist well defined positive constants \tilde{c}, c_a such that :

$$\forall t, \quad |\tilde{x}(t)| \leq \tilde{c} \quad |x_a(t)| \leq c_a \quad (52)$$

Part 2.

On the other hand, the nonlinear anti-windup loop (39)-(42) is now rewritten as follows:

$$\begin{cases} \dot{q}_a &= \frac{1}{2} E(q_a)(\omega_a^{be} + \Delta_q) \\ J_{mb} \dot{\omega}_a^{be} &= (-N(\omega, \omega_a^{be}, z_a, 0) + C_z z_a + D_z \omega_a^{be}) + OISat(u_a) + \Delta_\omega \\ \dot{z}_a &= A_z z_a + B_{1,z} \omega_a^{be} \end{cases} \quad (53)$$

where:

$$\Delta_q = (R(\tilde{q})^T - I_3) \omega_a^{be} \quad (54)$$

$$\Delta_\omega = OISat(u_a + \tilde{u}_n) - OISat(u_a) - \tilde{u}_n + N(\tilde{\omega} + \omega_r^b, \tilde{\omega}, \hat{z}, \omega_r^b) \quad (55)$$

$$+ \left(N(\omega, \omega_a^{be}, z_a, 0) - N(\omega, \omega_e, \hat{z}_e, \omega_r^b) \right) \quad (56)$$

$$OISat(u_a) = J_{mb} \left(Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha_a + k(t)\omega_a^{be}) - k(t)\omega_a^{be} \right) \quad (57)$$

$$+ N(\omega, \omega_a^{be}, z_a, 0) - C_z z_a - D_z \omega_a^{be} \quad (58)$$

$$\alpha_a = J_{mb}^{-1} \left(-N(\omega, \omega_a^{be}, z_a, 0) + u_a + C_z z_a + D_z \omega_a^{be} \right) \quad (59)$$

Substituting (58) in (53) results in the following subsystem:

$$\begin{cases} \dot{q}_a &= \frac{1}{2} E(q_a)(\omega_a^{be} + \Delta_q) \\ J_{mb} \dot{\omega}_a^{be} &= J_{mb} \left(Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha_a + k(t)\omega_a^{be}) - k(t)\omega_a^{be} \right) + \Delta_\omega \\ \dot{z}_a &= A_z z_a + B_{1,z} \omega_a^{be} \end{cases} \quad (60)$$

Let us now apply the control law u_a defined by (44c). It readily follows from (59) and (60) that:

$$\begin{cases} \dot{q}_a &= \frac{1}{2}E(q_a)(\omega_a^{be} + \Delta_q) \\ J_{mb}\dot{\omega}_a^{be} &= J_{mb} \left(Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(-k_a q_{av}) - k(t)\omega_a^{be} \right) + \Delta_\omega \\ \dot{z}_a &= A_z z_a + B_{1,z}\omega_a^{be} \end{cases} \quad (61)$$

Now considering the time-varying bounds of the saturation operator and substitute $k(t)$ as in (27b), one obtains:

$$\begin{cases} k(t)\bar{\omega} - \bar{e}_y &= k_O\bar{\omega} + \frac{\bar{\omega} + \underline{\omega}}{\bar{\omega} - \underline{\omega}}\bar{e}_y \\ k(t)\underline{\omega} + \bar{e}_y &= k_O\underline{\omega} + \frac{\bar{\omega} + \underline{\omega}}{\bar{\omega} - \underline{\omega}}\bar{e}_y \end{cases} \quad (62)$$

From (22a)-(22b), it readily follows that \tilde{y} tends asymptotically to $|J_{mb}^{-1}|\bar{d}$. Thus there exists $T \geq 0$ such that :

$$\forall t \geq T, |J_{mb}^{-1}|\bar{d} \leq \bar{e}_y \leq 2|J_{mb}^{-1}|\bar{d} \quad (63)$$

which together with (62) implies that:

$$\forall t \geq T, \begin{cases} k(t)\bar{\omega} - \bar{e}_y \geq k_O\bar{\omega} + \frac{\bar{\omega} + \underline{\omega}}{\bar{\omega} - \underline{\omega}}|J_{mb}^{-1}|\bar{d} \\ k(t)\underline{\omega} + \bar{e}_y \leq k_O\underline{\omega} + 2\frac{\bar{\omega} + \underline{\omega}}{\bar{\omega} - \underline{\omega}}|J_{mb}^{-1}|\bar{d} \end{cases} \quad (64)$$

Thus, choosing

$$k_O > \max \left\{ 0, \frac{1}{\bar{\omega}} \left(k_a - \frac{\bar{\omega} + \underline{\omega}}{\bar{\omega} - \underline{\omega}}|J_{mb}^{-1}|\bar{d} \right), \frac{-1}{\underline{\omega}} \left(k_a + 2\frac{\bar{\omega} + \underline{\omega}}{\bar{\omega} - \underline{\omega}}|J_{mb}^{-1}|\bar{d} \right) \right\} \quad (65)$$

one gets:

$$\forall t \geq T, \begin{cases} k(t)\bar{\omega} - \bar{e}_y \geq k_a \\ k(t)\underline{\omega} + \bar{e}_y \leq -k_a \end{cases} \quad (66)$$

As a consequence,

$$\forall t \geq T, Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(-k_a q_{av}) = -k_a q_{av} \quad (67)$$

Hence, the solutions being bounded for all time in virtue of (52), the asymptotic stability of x_a is now proven by studying the following system on $t \in [T, +\infty[$:

$$\begin{cases} \dot{q}_a &= \frac{1}{2}E(q_a)(\omega_a^{be} + \Delta_q) \\ \dot{\omega}_a^{be} &= -k_a q_{av} - k(t)\omega_a^{be} + J_{mb}^{-1}\Delta_\omega \\ \dot{z}_a &= A_z z_a + B_{1,z}\omega_a^{be} \end{cases} \quad (68)$$

To this aim, we consider the following function:

$$V_a(t, x_a) = k_a[(1 - q_{a0})^2 + |q_{av}|^2] + \frac{1}{2}|\omega_a^{be}|^2 + \frac{\varepsilon}{2}z_a^T P_z z_a + \mu q_{av}^T \omega_a^{be} \quad (69)$$

where ε is a small positive constant to be clarified next and $\mu > 0$ is a tuning parameter. From a simple completion of squares:

$$-\frac{\mu}{2}|q_{av}|^2 - \frac{\mu}{2}|\omega_a^{be}|^2 \leq \mu q_{av}^T \omega_a^{be} \leq \frac{\mu}{2}|q_{av}|^2 + \frac{\mu}{2}|\omega_a^{be}|^2 \quad (70)$$

it is readily checked that V_a is a Lyapunov candidate function since it verifies:

$$k_a(1-q_{a0})^2 + \frac{k_a}{2}|q_{av}|^2 + \frac{1}{4}|\omega_a^{be}|^2 + \frac{\varepsilon}{2}z_a^T P_z z_a \leq V_a \leq k_a(1-q_{a0})^2 + \frac{3k_a}{2}|q_{av}|^2 + \frac{3}{4}|\omega_a^{be}|^2 + \frac{\varepsilon}{2}z_a^T P_z z_a \quad (71)$$

when $0 < \mu \leq \min\{k_a, \frac{1}{2}\}$.

Next, differentiating V_a with respect to time and using (60) yields $\forall t \geq T$

$$\dot{V}_a = k_a q_{av}^T (\omega_a^{be} + \Delta_q) + (\omega_a^{be} + \mu q_{av})^T \left(-k_a q_{av} - k(t)\omega_a^{be} + J_{mb}^{-1} \Delta_\omega \right) - \varepsilon z_a^T Q_z z_a \quad (72)$$

$$+ \varepsilon z_a^T P_z B_{1,z} \omega_a^{be} + \frac{\mu}{2} \left((q_{a0} I_3 + S(q_{av})) \omega_a^{be} \right)^T \omega_a^{be} \quad (73)$$

$$\leq -\mu k_a |q_{av}|^2 - k(t) |\omega_a^{be}|^2 - \varepsilon z_a^T Q_z z_a - \mu k(t) q_{av}^T \omega_a^{be} \quad (74)$$

$$+ \varepsilon z_a^T P_z B_{1,z} \omega_a^{be} + \frac{\mu}{2} |\omega_a^{be}|^2 + k_a q_{av}^T \Delta_q + (\omega_a^{be} + \mu q_{av})^T J_{mb}^{-1} \Delta_\omega \quad (75)$$

Considering equations (62) and (63), it is easily checked that

$$\forall t \geq T, \quad k_{\min} \leq k(t) \leq k_{\max} \quad (76)$$

with

$$k_{\min} = k_O + \frac{2}{\bar{\omega} - \underline{\omega}} |J_{mb}^{-1}| \bar{d}, \quad k_{\max} = k_O + \frac{4}{\bar{\omega} - \underline{\omega}} |J_{mb}^{-1}| \bar{d} \quad (77)$$

Combining (75) with (76) yields $\forall t \geq T$:

$$\dot{V}_a \leq -\mu k_a |q_{av}|^2 - k_{\min} |\omega_a^{be}|^2 - \varepsilon z_a^T Q_z z_a + \mu k_{\max} |q_{av}| |\omega_a^{be}| \quad (78)$$

$$+ \varepsilon z_a^T P_z B_{1,z} \omega_a^{be} + \frac{\mu}{2} |\omega_a^{be}|^2 + k_a q_{av}^T \Delta_q + (\omega_a^{be} + \mu q_{av})^T J_{mb}^{-1} \Delta_\omega \quad (79)$$

$$\leq -x_a^T Q_a x_a + k_a q_{av}^T \Delta_q + (\omega_a^{be} + \mu q_{av})^T J_{mb}^{-1} \Delta_\omega \quad (80)$$

where:

$$Q_a = \begin{bmatrix} \mu k_a & \frac{\mu k_{\max}}{2} & O \\ \frac{\mu k_{\max}}{2} & k_{\min} - \frac{\mu}{2} & -\frac{\varepsilon}{2} P_z B_{1,z} \\ O & -\frac{\varepsilon}{2} P_z B_{1,z} & \varepsilon Q_z \end{bmatrix} \quad (81)$$

which is positive definite for small values of μ and $\varepsilon > 0$.

Finally, it is convenient to bound the coupling terms between the \tilde{x} and x_a subsystems, which is the scope of the following lemma (its proof is detailed in appendix B)

Lemma 5: *there exists $\lambda > 0$ such that $\forall t \geq T$*

$$k_a q_{av}^T \Delta_q + (\omega_a^{be} + \mu q_{av})^T J_{mb}^{-1} \Delta_\omega \leq \lambda |x_a| (|\tilde{x}| + |e_z| + |\omega_r|) \quad (82)$$

Consequently, $\forall t \geq T$

$$\dot{V}_a \leq -x_a^T Q_a x_a + \lambda |x_a| (|\tilde{x}| + |e_z| + |\omega_r|) \quad (83)$$

ω_r (resp. \tilde{x} and e_z) being in \mathcal{L}_2 from Assumption (A4) (resp. from the proof of part 1), it readily follows from the application of Barbalat's Lemma that x_a asymptotically converges to the origin. The proof is now completed. □

6. Application to a flexible spacecraft attitude control problem

A realistic application to a flexible spacecraft attitude control problem of the proposed design methodology is now presented.

6.1 Numerical data

In this application, the spacecraft model described in subsection 2.2 is considered with the following numerical data (borrowed from Hu (2009)):

$$J_{mb} = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} kg.m^2, \quad \delta = \begin{bmatrix} 6.46 & 1.28 & 2.16 \\ -1.26 & 0.92 & -1.67 \\ 1.12 & 2.49 & -0.84 \end{bmatrix} kg^{1/2}.m.s^{-2} \quad (84)$$

The characteristics of the three flexible modes are summarized below in Table 1.

| | pulsation | damping ratio |
|------------------|---------------------------------|------------------|
| Flexible mode #1 | $\omega_1 = 0.77 \text{ rad/s}$ | $\xi_1 = 0.0056$ |
| Flexible mode #2 | $\omega_2 = 1.10 \text{ rad/s}$ | $\xi_2 = 0.0086$ |
| Flexible mode #3 | $\omega_3 = 1.87 \text{ rad/s}$ | $\xi_3 = 0.0130$ |

Table 1.: Characteristics of the flexible modes

and the angular velocity constraints are as follows:

$$\bar{\omega} = -\underline{\omega} = [6 \quad 15 \quad 10]^T \text{ deg/s} \quad (85)$$

6.2 Scenario

The simulation starts from the initial state below:

$$\begin{cases} q(0) = [0.173648 & -0.263201 & 0.789603 & -0.526402]^T \\ \omega(0) = \eta(0) = \dot{\eta}(0) = [0 \quad 0 \quad 0]^T \end{cases}$$

The flexible satellite attitude shall track a reference signal $q_r(t)$ defined by its initial value $q_r(0) = [1 \quad 0 \quad 0 \quad 0]^T$ and its time-domain variation ω_r generated by a two-steps response of a first order filter $F_r(s) = (1 + 0.3s)^{-1}$.

Moreover, a periodic disturbance torque is applied to the satellite:

$$d(t) = [0.3 \cos(0.01t) + 0.1 \quad 0.15 \sin(0.02t) + 0.3 \cos(0.025t) \quad 0.3 \sin(0.01t) + 0.1]^T Nm \quad (86)$$

In our interval observer design, we will then use the bound $\bar{d} = [0.5 \ 0.5 \ 0.5]^T$ Nm.

6.3 Controller gains tuning

For comparison purpose, we consider the nominal (state-feedback) control law resulting from Lemma 1 and the enhanced (output-feedback) version resulting from Theorem 1:

- the nominal control law gains are simply chosen by poles placement of the following system:

$$\dot{q}_{ev} = \omega_e \quad (87a)$$

$$J_{mb}\dot{\omega}_e = -k_p q_{ev} - k_d \omega_e \quad (87b)$$

which simply consists of a linearization of the rigid part of the closed loop system (3a)-(3c)-(11). Here, a satisfying pole placement was obtained for:

$$k_p = 0.25J_{mb}, \quad k_d = 0.1J_{mb} \quad (88)$$

- the enhanced control law is tuned by choosing the gains $k_O, k_a > 0$ which must satisfy inequalities (65). Given $\bar{d}, \bar{\omega}, \underline{\omega}$, we selected $k_a = 2$ in order to obtain $k_O \cong k_d(1, 1)$.

6.4 Nonlinear simulations results

The spacecraft model with both nominal and enhanced control laws has been implemented in SIMULINKTM files. Following the scenario presented in subsection 6.2, nonlinear simulations have been performed on a time range from 0 to 120 s. The results are presented below on Figures 1, 2 and 3.

As is clarified in the paper, the use of an interval observer of the unmeasured disturbance torque is essential in our approach to enforce the angular rates constraints thanks to the OISTeR methodology. The behavior of this observer is illustrated by Figure 1. The tightness of the computed bounds as a function of time is clearly visualized.

Next, the angular velocities on the three axes are displayed in Figure 2 where a comparison between the nominal and enhanced controllers is proposed. As expected, every time a step input is applied, the angular velocities exceed the constraints on each axis when the nominal controller is used. This is however no longer the case with the enhanced control system. In this case indeed, the velocities are always kept in the prescribed limits.

The comparison of the nominal and enhanced controllers is further investigated in Figure 3.

- The first subplot (upper) illustrates the time-domain evolution of the tracking error (via the norm of the quaternion error $\|q_{ev}(t)\|$). It is easily observed that the nominal controller outperforms the OISTeR-based solution. But this is not surprising since velocities constraints are not respected in the first case. More precisely, when comparing both plots, the price to pay for constraints satisfaction is clearly visualized between $t \cong 50$ s and $t \cong 65$ s. During this time interval the reference quaternion is no longer tracked by the enhanced system. Next, the error starts to decrease as soon as the anti-windup loop becomes active.
- The second subplot (middle) illustrates the activity of the flexible mode. Quite interestingly, it is observed that the excitation of the flexible modes is considerably lower when the enhanced controller is used. Here again, this is not surprising since the angular velocities are now controlled.
- The last and third subplot (lower) illustrates the control activity which, as expected also, is significantly lower when the OISTeR-based system is used.

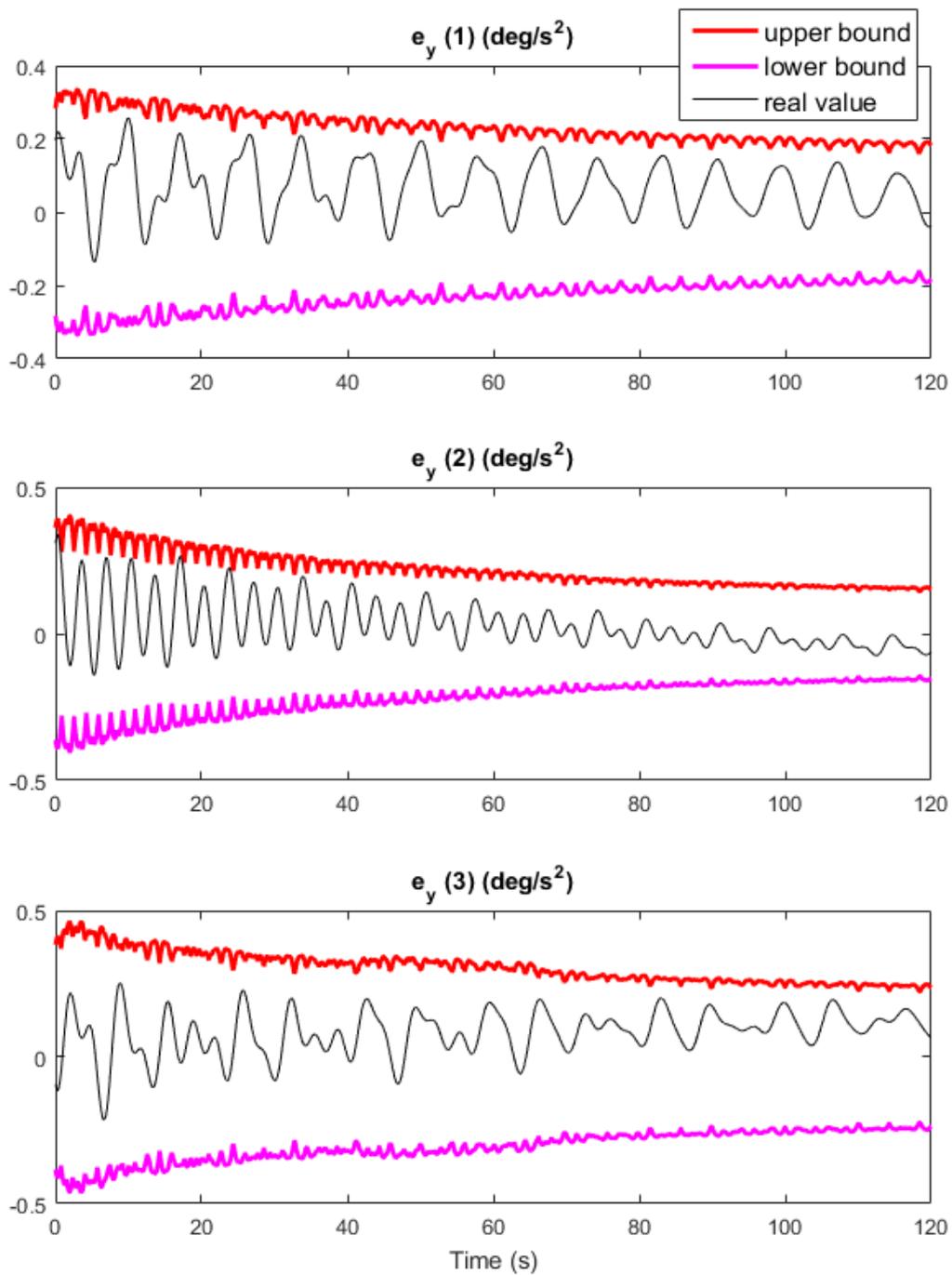


Figure 1.: Interval observer (bounds detection on the unmeasured disturbance torque e_y)

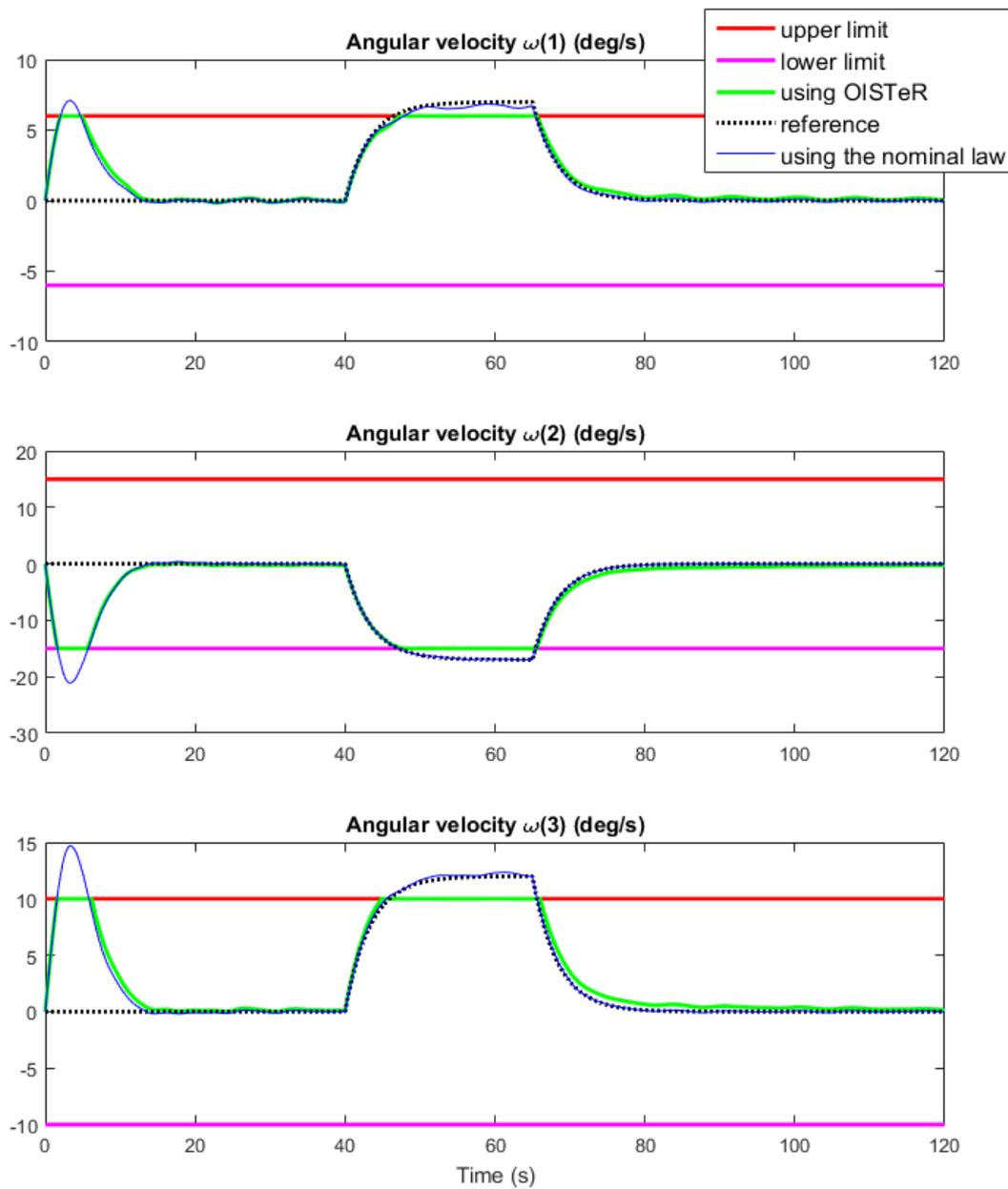


Figure 2.: Comparison of the angular velocities on each axis with the nominal and enhanced (OISTeR-based) control systems

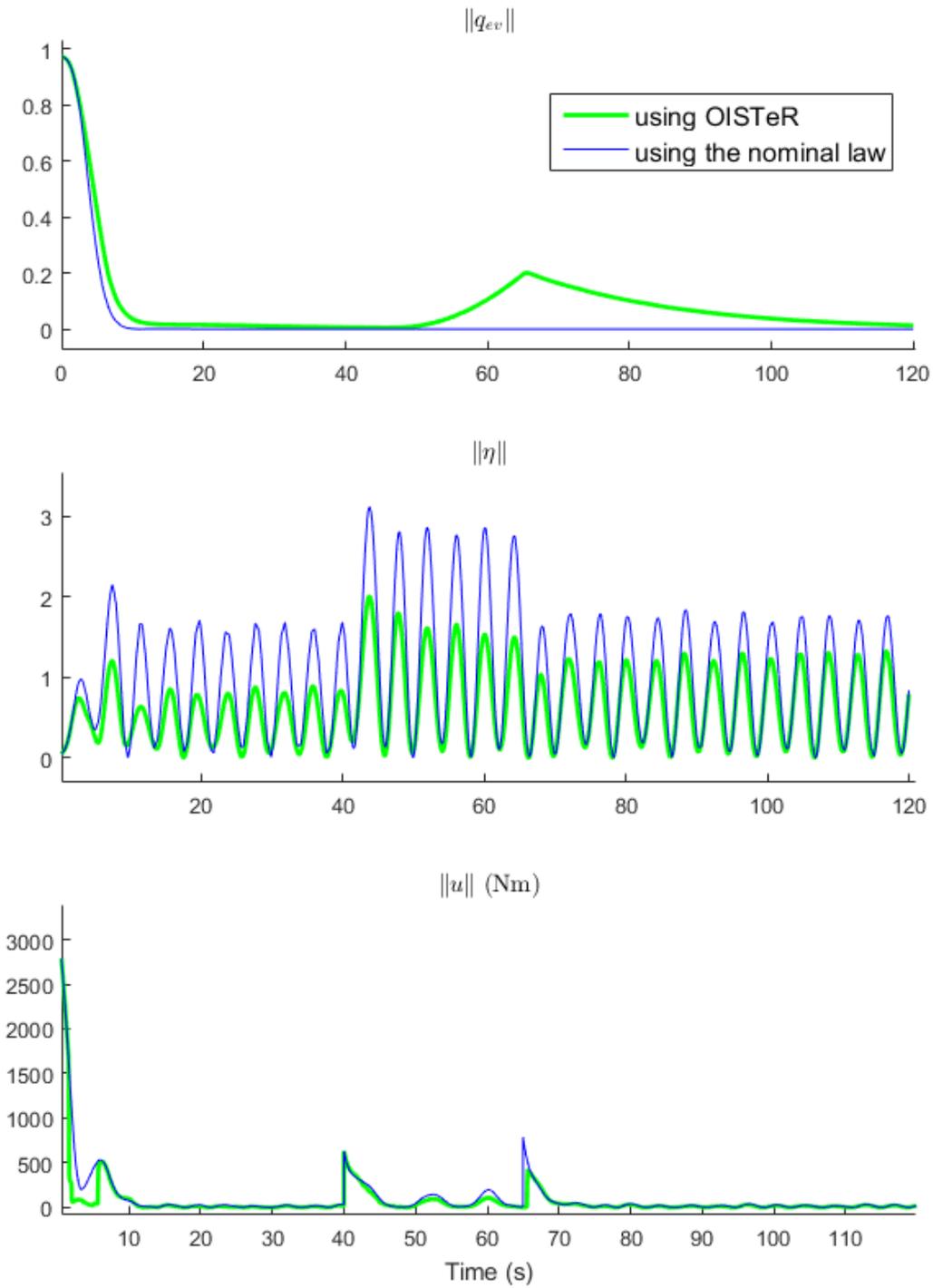


Figure 3.: Further comparison of the nominal and enhanced controllers through the tracking error ($\|q_{ev}(t)\|$), the activity of the flexible modes ($\|\eta(t)\|$) and the control effort ($\|u(t)\|$).

7. Conclusions

The problem of output-feedback trajectory tracking of flexible spacecraft attitude under angular velocity constraints has been addressed in this paper. Given a nominal controller which solves the tracking problem in the full state-feedback case without velocity constraints, it is proven that the general problem can be solved by saturating the nominal controller by an Output to Input Saturation. Such a saturated control law is then applied in combination with a nonlinear anti-windup loop. Note that one key feature of this result is to use an interval observer not only to estimate the flexible modes but also to provide at each instant their lower and upper bounds, which is crucial to ultimately guarantee the satisfaction of the velocity constraints. In future research, we plan to extend this work to handle parametric uncertainties and control torque limitations.

References

- Akella, M., Thakur, D., & Mazenc, F. (2015). Partial Lyapunov strictification: Smooth angular velocity observers for attitude tracking control. *Journal of Guidance, Control, and Dynamics*, *38*(3), 442-451.
- Burlion, L. (2012). A new saturation function to convert an output constraint into an input constraint. In *20th Mediterranean Conference on Control Automation (MED)* (p. 1217-1222).
- Chambon, E., Burlion, L., & Apkarian, P. (2015a). Output to input saturation transformation: Demonstration and application to disturbed linear systems. In *IEEE 54th Annual Conference on Decision and Control (CDC)* (p. 7566-7571).
- Chambon, E., Burlion, L., & Apkarian, P. (2015b). Robust output interval constraint using O/I saturation transformation with application to uncertain linear launch vehicle. In *European Control Conference (ECC)* (p. 1802-1807).
- Chen, Z., & Huang, J. (2009). Attitude tracking and disturbance rejection of rigid spacecraft by adaptive control. *IEEE Transactions on Automatic Control*, *54*(3), 600-605.
- Di Gennaro, S. (2002). Output attitude tracking for flexible spacecraft. *Automatica*, *38*(10), 1719 - 1726.
- Galeani, S., Tarbouriech, S., Turner, M., & Zaccarian, L. (2009). A tutorial on modern anti-windup design. *European Journal of Control*, *15*(34), 418 - 440.
- G.C. Goodwin, M. S., & de Dona, J. (2004). *Constrained control and estimation: an optimization approach*. Springer.
- Gennaro, S. D. (2003). Output stabilization of flexible spacecraft with active vibration suppression. *IEEE Transactions on Aerospace and Electronic Systems*, *39*(3), 747-759.
- Glattfelder, A., & Schaufelberger, W. (2003). *Control systems with input and output constraints*. Springer.
- Gupta, R., Kalabic, U. V., Cairano, S. D., Bloch, A. M., & Kolmanovsky, I. V. (2015, July). Constrained spacecraft attitude control on SO(3) using fast nonlinear model predictive control. In *American Control Conference (ACC)* (p. 2980-2986).
- Hegrenaes, O., Gravdahl, J., & Tondel, P. (2005). Spacecraft attitude control using explicit model predictive control. *Automatica*, *41*(12), 2107-2114.
- H.-H. Zhang, F. W., & Trivailo., P. (2008). Spin-axis stabilisation of underactuated rigid spacecraft under sinusoidal disturbance. *International Journal of Control*, *81*(12), 1901-1909.
- Hu, Q. (2009). Robust adaptive backstepping attitude and vibration control with L_2 -gain performance for flexible spacecraft under angular velocity constraint. *Journal of Sound and Vibration*, *327*(35), 285 - 298.
- Lovera, M., & Astolfi, A. (2004). Spacecraft attitude control using magnetic actuators. *Automatica*, *40*(8), 1405 - 1414.
- Luzi, A.-R., Peaucelle, D., Biannic, J.-M., Pittet, C., & Mignot, J. (2014). Structured adaptive attitude control of a satellite. *International Journal of Adaptive Control and Signal Processing*, *28*(7-8), 664-685.
- Mazenc, F., & Bernard, O. (2010, Feb). Asymptotically stable interval observers for planar systems with complex poles. *IEEE Transactions on Automatic Control*, *55*(2), 523-527.
- Tarbouriech, S., & Turner, M. (2009). Anti-windup design: an overview of some recent advances and open problems. *Control Theory Applications, IET*, *3*(1), 1-19.

Tayebi, A. (2008). Unit quaternion-based output feedback for the attitude tracking problem. *IEEE Transactions on Automatic Control*, 53(6), 1516-1520.

Tréguët, J.-F., Arzelier, D., Peaucelle, D., Pittet, C., & Zaccarian, L. (2015). Reaction wheels desaturation using magnetorquers and static input allocation. *IEEE Transactions on Control Systems Technology*, 23(2), 525-539.

Zhang, X., & Cheng, D. (2012). Nonlinear internal model based attitude tracking and disturbance rejection. *Asian Journal of Control*, 14(5), 1397-1402.

Appendix A. Proof of Lemma 2

First of all, let us introduce the following notation. Note that this notation (which is very specific and merely used in this proof) was not given before to avoid breaking the flow of the paper derivations.

Notation 4: we introduce the following two functions (as in Mazenc and Bernard (2010)):

$$\mathcal{L}(s) = \max\{0, s\}, \quad \mathcal{M}(s) = \min\{0, s\} \quad (\text{A1})$$

Then, given a time-varying matrix $\Pi(t)$, we define:

$$\Pi_p(t) = \left(\mathcal{L}(\Pi_{i,j}(t)) \right), \quad \Pi_m(t) = \left(\mathcal{M}(\Pi_{i,j}(t)) \right), \quad (\text{A2})$$

As a consequence of $|s| = \mathcal{L}(s) - \mathcal{M}(s)$, one has $|\Pi(t)| = \Pi_p(t) - \Pi_m(t)$, such an equality being understood element-wise.

Let us now prove Lemma 2:

Proof. First, let us consider:

$$\dot{e}_z = A_z e_z \quad (\text{A3})$$

where $|e_{z,0}| \leq \frac{z_0^+ - z_0^-}{2}$.

Let us note $\mathcal{N} = [N_1^T, \dots, N_N^T]^T$ where

$$N_i = [e_{z,i}, e_{z,i+N}]^T, \quad \forall i \in [1, N] \quad (\text{A4})$$

These relations define a change of coordinates which is now denoted as follows:

$$\mathcal{N} = P e_z \quad (\text{A5})$$

It is readily seen that the N_i 's verify:

$$\dot{N}_i = J_i N_i, \quad i \in [1, N] \quad (\text{A6})$$

where

$$J_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix}$$

Following Mazenc and Bernard (2010), we apply the change of coordinate $\zeta_i = \lambda_i(t)N_i$ where

$$\lambda_i(t) = \begin{bmatrix} \cos(\omega_i t) - \xi_i \sin(\omega_i t) & -\frac{1}{\omega_i} \sin(\omega_i t) \\ \omega_i \sin(\omega_i t) & \cos(\omega_i t) + \xi_i \sin(\omega_i t) \end{bmatrix} \quad (\text{A7})$$

which gives:

$$\dot{\zeta}_i = -\kappa_i \zeta_i, \quad i \in [1, N] \quad (\text{A8})$$

where $\kappa_i = \xi_i \omega_i$.

(Note that the change of coordinate is well defined ; indeed, $\det(\lambda_i(t)) = \cos^2(\omega_i t) + (1 - \xi_i^2) \sin^2(\omega_i t) \neq 0$ since the damping ratios are assumed to satisfy $0 < \xi_i < 1$)

Finally, noting $M = \text{diag}(\{-\kappa_i I_2, i = 1..N\})$ and $\lambda(t) = \text{diag}(\{\lambda_i(t), i = 1..N\})$, system (A3) is equivalent to:

$$\dot{\zeta} = M\zeta \quad (\text{A9a})$$

$$\zeta = \lambda(t)Pe_z := Q(t)e_z \quad (\text{A9b})$$

It follows from $|e_{z,0}| \leq \frac{z_0^+ - z_0^-}{2}$ and from (A9b) that

$$\zeta(0) \in [\zeta_0^-, \zeta_0^+] \quad (\text{A10})$$

where

$$\begin{cases} \zeta_0^+ &= Q_p(0) \frac{z_0^+ - z_0^-}{2} - Q_m(0) \frac{z_0^+ - z_0^-}{2} = |Q(0)| \frac{z_0^+ - z_0^-}{2} \\ \zeta_0^- &= Q_m(0) \frac{z_0^+ - z_0^-}{2} - Q_p(0) \frac{z_0^+ - z_0^-}{2} = -|Q(0)| \frac{z_0^+ - z_0^-}{2} \end{cases} \quad (\text{A11})$$

M being Hurwitz Metzler, the following system defines an interval observer for system (A9a)

$$\begin{cases} \dot{\zeta}^+ &= M\zeta^+ \\ \dot{\zeta}^- &= M\zeta^- \end{cases} \quad (\text{A12})$$

where $\zeta^+(0) = \zeta_0^+$ (resp. $\zeta^-(0) = \zeta_0^-$).

Finally, such an interval observer can be used to bound the output e_y since:

$$e_y = J_{mb}^{-1}(-S(\omega)G_z e_z + C_z e_z + d) \quad (\text{A13})$$

$$= J_{mb}^{-1}((-S(\omega)G_z + C_z)Q(t)^{-1}\zeta + d) \quad (\text{A14})$$

$$:= \Pi(t, \omega)\zeta + J_{mb}^{-1}d \quad (\text{A15})$$

Indeed, it is readily seen that $e_y \in [e_y^-, e_y^+]$ where:

$$\begin{cases} e_y^+ &:= \Pi_p(t, \omega)\zeta^+ + \Pi_m(t, \omega)\zeta^- + |J_{mb}^{-1}| \bar{d} \\ e_y^- &:= \Pi_m(t, \omega)\zeta^+ + \Pi_p(t, \omega)\zeta^- - |J_{mb}^{-1}| \bar{d} \end{cases} \quad (\text{A16})$$

which simplifies as follows:

$$\begin{cases} e_y^+ &:= |\Pi(t, \omega)|\zeta^+ + |J_{mb}^{-1}| \bar{d} \\ e_y^- &:= -e_y^+ \end{cases} \quad (\text{A17})$$

Indeed, it readily follows from (A11)-(A12) that $\forall t, \zeta^+(t) = -\zeta^-(t)$. The proof of the Lemma is then completed. \square

Appendix B. Proof of Lemma 5

The proof of the Lemma readily follows from the proofs of the two following inequalities:

(1) there exists $\lambda_4, \lambda_5, \lambda_6 > 0$ such that $\forall t \geq T$:

$$(\omega_a^b + \mu q_{av})^T \Delta_\omega \leq \lambda_4 |x_a| |\tilde{x}| + \lambda_5 |x_a| |e_z| + \lambda_6 |x_a| |\omega_r| \quad (\text{B1})$$

(2) there exists $\lambda_7 > 0$ such that

$$q_{av}^T \Delta_q \leq \lambda_7 |x_a| |\tilde{x}| \quad (\text{B2})$$

B.1 Part 1

Starting from equation (56), we obtain (B4) by using (11)-(44a)-(44c)-(58) :

$$\begin{aligned} (\omega_a^b + \mu q_{av})^T \Delta_\omega &= (\omega_a^b + \mu q_{av})^T \left(OISat(u_a + \tilde{u}_n) - OISat(u_a) - \tilde{u}_n + N(\tilde{\omega} + \omega_r^b, \tilde{\omega}, \hat{\tilde{z}}, \omega_r^b) \right. \\ &\quad \left. + \left(N(\omega, \omega_a^{be}, z_a, 0) - N(\omega, \omega_e, \hat{z}_e, \omega_r^b) \right) \right) \end{aligned} \quad (\text{B3})$$

$$\leq 3(1 + \mu) |x_a| (\lambda_{\max}(J_{mb}) |\Delta_S| + |\tilde{\Delta}|) \quad (\text{B4})$$

where

$$\Delta_S := Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha + k(t)\omega) - Sat_{k(t)\underline{\omega} + \bar{e}_y}^{k(t)\bar{\omega} - \bar{e}_y}(\alpha_a + k(t)\omega_a^{be}) \quad (\text{B5})$$

$$\begin{aligned} \tilde{\Delta} &:= J_{mb}(-k(t)\tilde{\omega}) - C_z \tilde{z} - D_z \tilde{\omega} - \tilde{u}_n - N(\omega, \tilde{\omega}, \hat{\tilde{z}}, \omega_r^b) \\ &\quad - C_z(z_r - e_z) - (J_{mb}k(t) + D_z)\omega_r^b \end{aligned} \quad (\text{B6})$$

Looking at (44b)-(59), we observe that

$$\begin{aligned} \alpha + k(t)\omega &= \alpha_a + k(t)\omega_a^{be} + k(t)(\tilde{\omega} + \omega_r^b) + J_{mb}^{-1} \left(N(\omega, \omega_a^{be}, z_a, 0) - N(\omega, \omega, \hat{z}, 0) \right. \\ &\quad \left. + \tilde{u}_n + C_z(\tilde{z} - e_z + z_r) + D_z(\tilde{\omega} + \omega_r) \right) \end{aligned} \quad (\text{B7})$$

Substituting (B7) in (B5) and using the fact that *Sat* is 1-Lipschitz yields

$$\begin{aligned} |\Delta_S| &\leq \left| k(t)(\tilde{\omega} + \omega_r^b) + J_{mb}^{-1} \left(N(\omega, \omega_a^{be}, z_a, 0) - N(\omega, \omega, \hat{z}, 0) \right. \right. \\ &\quad \left. \left. + \tilde{u}_n + C_z(\tilde{z} - e_z + z_r) + D_z(\tilde{\omega} + \omega_r) \right) \right| \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} &\leq \left| k(t)(\tilde{\omega} + \omega_r^b) + J_{mb}^{-1} \left(-N(\omega, \tilde{\omega} + \omega_r^b, \hat{\tilde{z}} + z_r, 0) \right. \right. \\ &\quad \left. \left. + \tilde{u}_n + C_z(\tilde{z} - e_z + z_r) + D_z(\tilde{\omega} + \omega_r) \right) \right| \end{aligned} \quad (\text{B9})$$

Looking at the expressions (6)(resp. (11)-(44a)) of N (resp. \tilde{u}) and bearing in mind the fact that ω is bounded and $k(t)$ is dominated by a constant $k_{\max} > 0$ for $t \geq T$, it is not difficult to see that $|\Delta_S|$ (resp. $|\tilde{\Delta}|$) given by (B9)(resp. (B6)) is Lipschitz with respect to the variables \tilde{x}, e_z and ω_r for all $t \geq T$. The proof of (B1) readily follows from this fact plus the use of (B4).

B.2 Part 2

Starting from equation (54), one can successively prove that:

$$\begin{aligned} q_{av}^T \Delta_q &= q_{av}^T (R(\tilde{q})^T - I_3) \omega_a^{be} \\ &= q_{av}^T ((\tilde{q}_0^2 - \tilde{q}_v^T \tilde{q}_v - 1)I_3 + 2\tilde{q}_v \tilde{q}_v^T - 2\tilde{q}_0 S(\tilde{q}_v)) \omega_a^{be} \\ &= q_{av}^T (-2\tilde{q}_v^T \tilde{q}_v I_3 + 2\tilde{q}_v \tilde{q}_v^T - 2\tilde{q}_0 S(\tilde{q}_v)) \omega_a^{be} \\ &= q_{av}^T (2S(\tilde{q}_v)^2 - 2\tilde{q}_0 S(\tilde{q}_v)) \omega_a^{be} \end{aligned} \tag{B10}$$

$$\begin{aligned} &= 2q_{av}^T (S(\tilde{q}_v) - \tilde{q}_0 I_3) S(\tilde{q}_v) \omega_a^{be} \\ &\leq 2|q_{av}|(\sqrt{2}|\tilde{q}_v| + 1)\sqrt{2}|\tilde{q}_v| |\omega_a^{be}| \end{aligned} \tag{B11}$$

$$\begin{aligned} &\leq 2(2 + \sqrt{2})|\tilde{q}_v| |\omega_a^{be}| \\ &\leq 8|\tilde{x}| |x_a| \end{aligned} \tag{B12}$$

where (B10) is obtained by using the relation $S(x)^2 = -(x^T x)I_3 + xx^T$ where (B11) is obtained by using the relation $|S(q_v)| \leq \sqrt{2|q_v|^2} = \sqrt{2}|q_v|$ which readily follows from the well known relation $|A| \leq |A|_F = \sqrt{\text{trace}(A^T A)}$ (substituting A for $S(q_v)$). The proof is thus completed for any $\lambda_7 \geq 8$.