

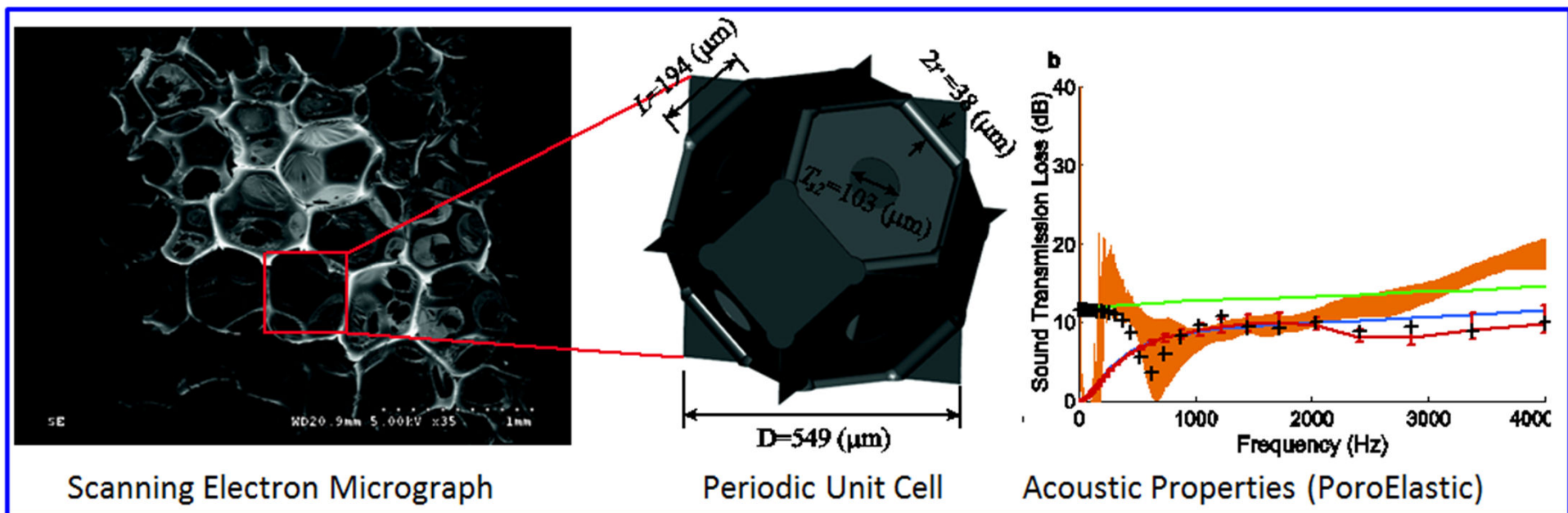
MULTI-SCALE ACOUSTICS OF PARTIALLY OPEN-CELL POROELASTIC FOAMS: LINEAR ELASTIC PROPERTIES DERIVATION FROM MICROSTRUCTURES REPRESENTATIVE OF TRANSPORT PARAMETERS

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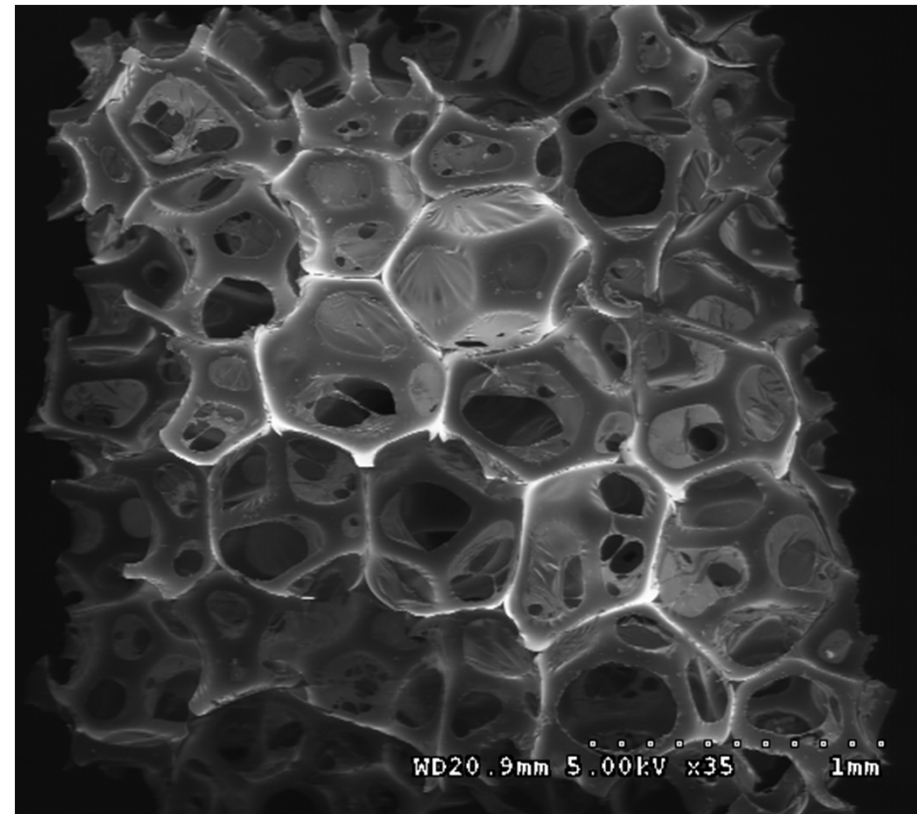


- **General Objective.** Optimization of the long-wavelengths acoustic properties of real sound proofing foam samples.
- **Methodology.** Use the method of Periodic Unit Cell (PUC) reconstruction of porous media, which consists of 2 main steps:
 - 1) **Identification** of the critical local geometry features governing long-wavelengths acoustics of porous media.
 - 2) **Generation** of 3D parameterized PUCs suitable for optimization.
 - Transport properties are obtained by ensemble averages of velocity and thermal fields, solutions of the local boundary value problems.
 - Elastic properties are derived from micromechanical techniques.

Aim of this work

- To present techniques devised to account explicitly for the role played by membranes in transport phenomena.
- To perform numerical homogenization simulations of linear elastic properties through the finite element technique in solid foams containing membranes.

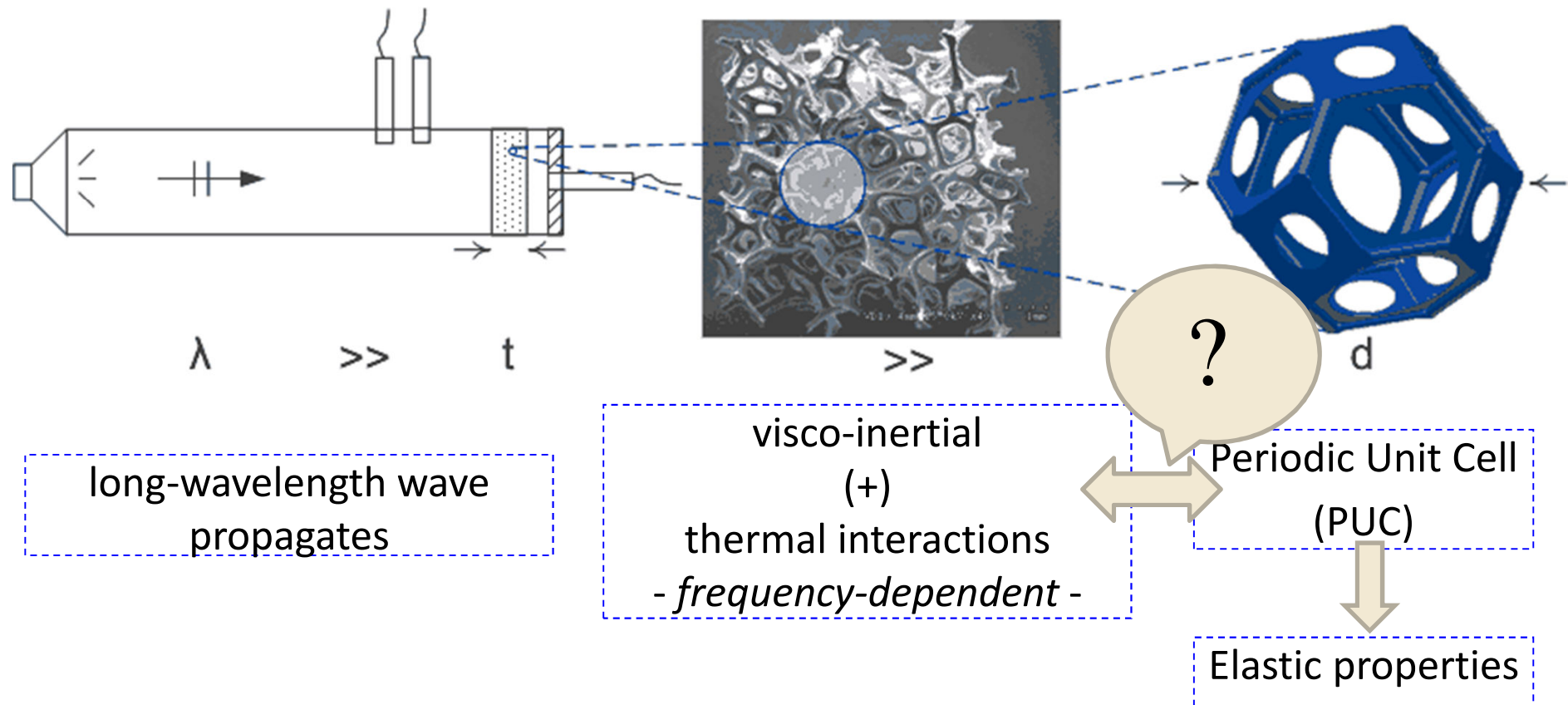
- Solid films or membranes in porous materials such as polyurethane or metal foams only account for a **very small fraction in the overall mass** of the porous media.
- Yet, their role might be of **primary importance** in the understanding of **transports, elastic, and acoustical** properties of these *poroelastic* foams.



I. INTRODUCTION

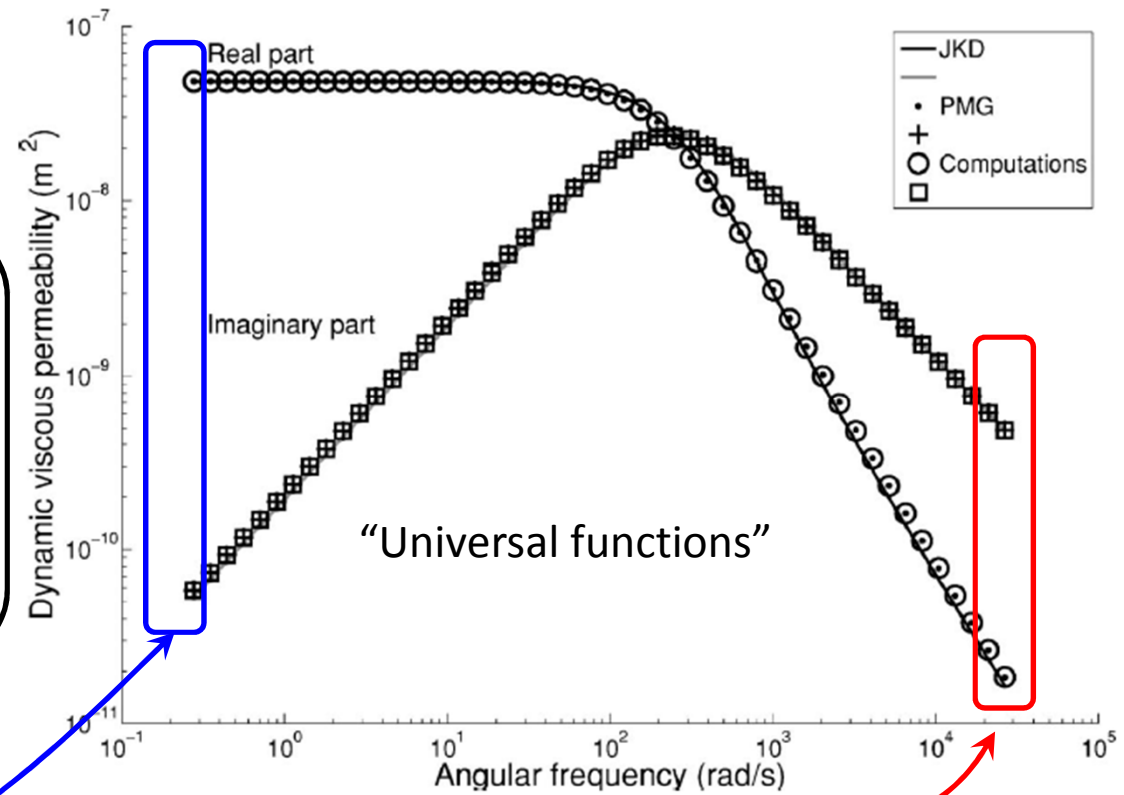
I. 3) Specific Objectives

Multi-Scale Acoustics of Poro-Elastic Foams



A fundamental physical challenge to the **microstructural identification of features** which are **characteristic** of the **overall transports and elastic properties**

Part of the solution of this problem lies in the fact that the overall dissipation of the porous material is expected to be dominated by a few key linkages.



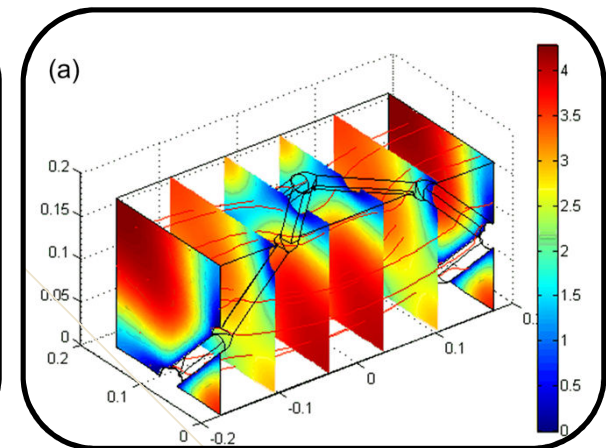
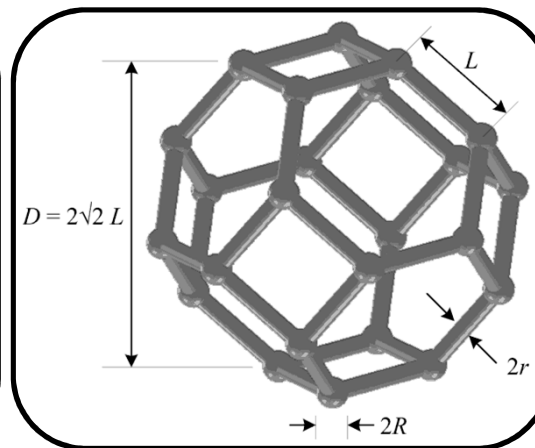
Key linkages

- Low frequency { Porosity, ϕ [-]
Permeability, k_0 [m²]
- High frequency { Viscous length, Λ [m]
Tortuosity, α_∞ [-]

References

[1] D. L. Johnson *et al.*, J. Fluid Mech. **176**: 379 (1987)
 [2] P. Sheng and M.-Y. Zhou, PRL **61**: 1591 (1988)

... and in the **identification** of an open-cell responsible for the **viscous dissipations**



Experiments & 3D Numerical simulations

Key linkages

- Low frequency { Porosity, ϕ [-]
Permeability, k_0 [m²]
- High frequency { Viscous length, Λ [m]
Tortuosity, α_∞ [-]

References

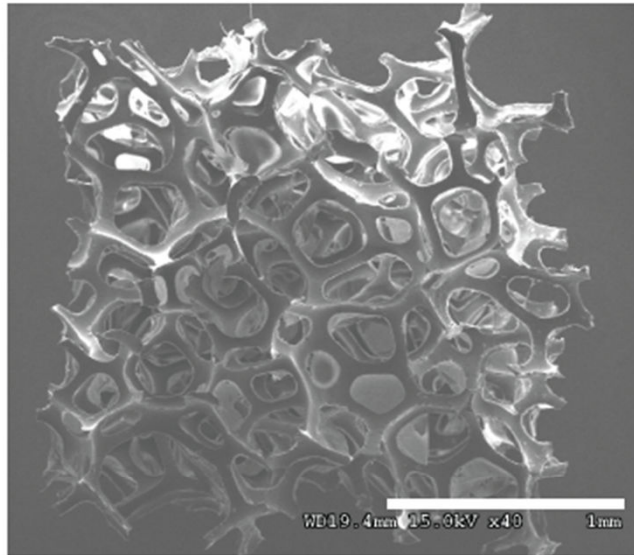
- [3] C. Perrot *et al.*, J. Appl. Phys. **111**: 014911 (2012)
 “Microstructure, transport, and acoustic properties of open-cell foam samples: Experiments and three-dimensional numerical simulations”

II. MATERIALS & METHODS

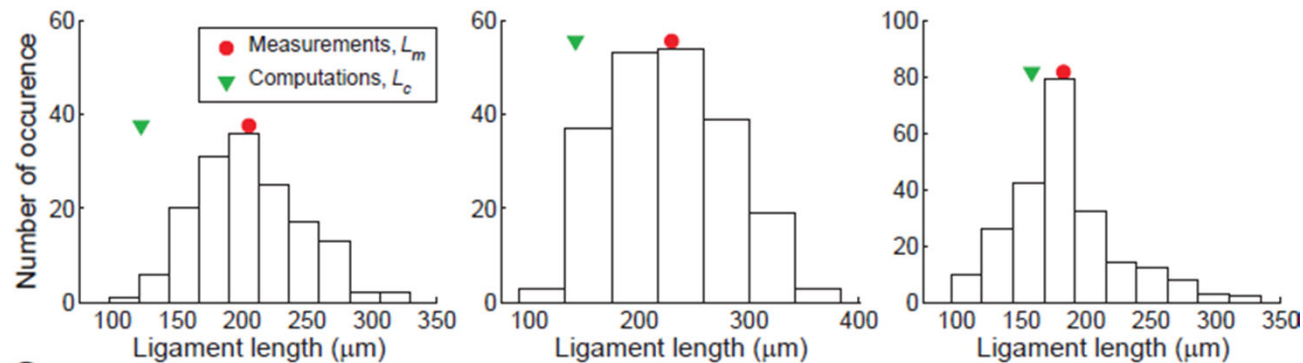
II. 3) A missing ingredient

Multi-Scale Acoustics of Poro-Elastic Foams

a



b

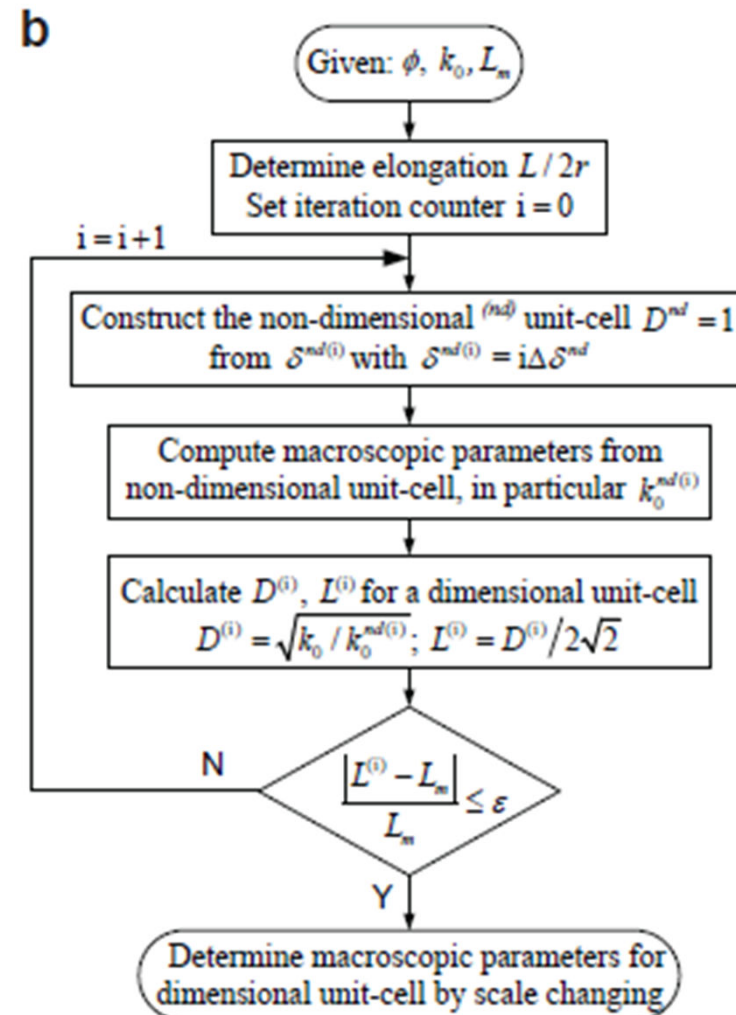
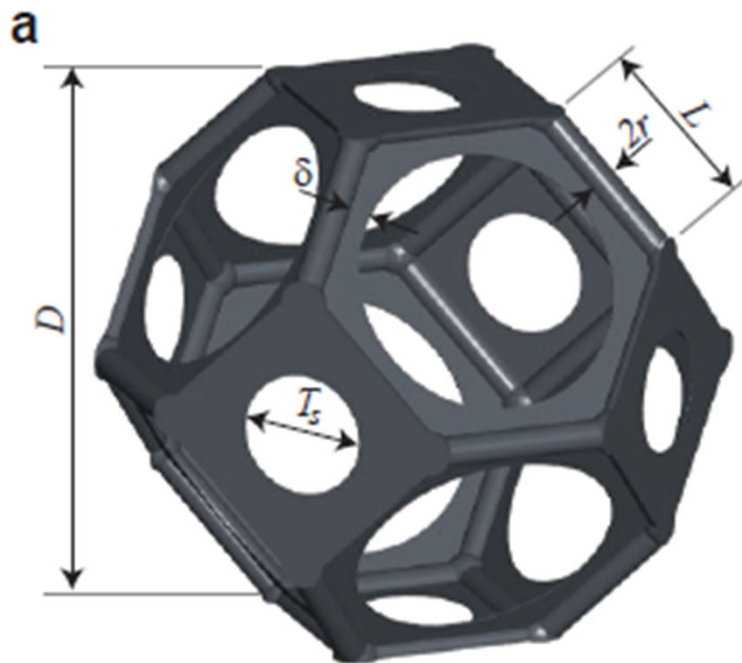


The superimposed average values of the experimental measurements (L_m , ●) and ligament length of the open-cell solid foam structure with the same average permeability (L_c , ▼) convey the impression of a **missing microstructural ingredient** in the PUC

II. MATERIALS & METHODS

II. 4) Iterative approach

Multi-Scale Acoustics of Poro-Elastic Foams



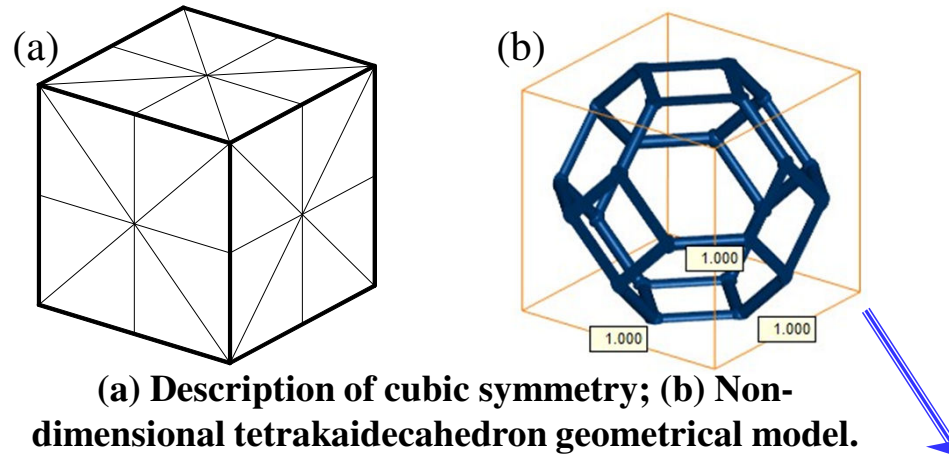
References

- [4] M. T. Hoang and C. Perrot, J. Appl. Phys. **112**, 054911 (2012)
"Solid films and transports in cellular foams"
- [5] M. T. Hoang and C. Perrot, J. Appl. Phys. **113**, 084905 (2013)
"Identifying local characteristic lengths governing sound wave
properties in solid foams"

The iterative procedure illustrates the importance of porosity ϕ , permeability k_0 ; and average ligament lengths length L_m measurements from which all the transport parameters are computed **without any adjusted constant**.

Material with cubic symmetry

- ❖ What is a cubic symmetry?
 => A structure which contains 9 planes of symmetry



(a) Description of cubic symmetry; (b) Non-dimensional tetrakaidecahedron geometrical model.

Our geometrical model satisfies the criteria for a cubic symmetry

- ❖ Elasticity matrix of a material with cubic symmetry:

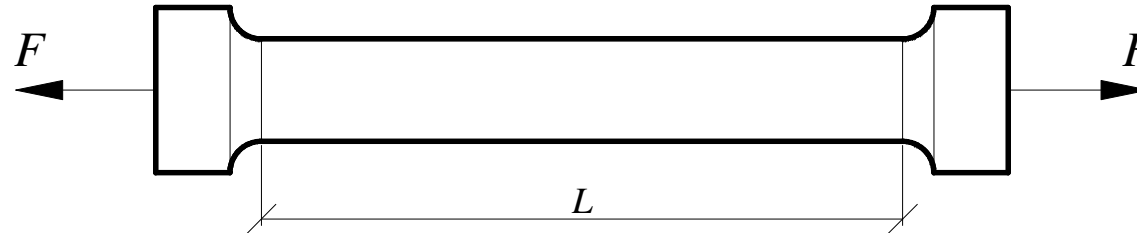
$$\underline{\underline{\zeta}} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & \text{sym} & & & C_{44} & 0 \\ & & & & & C_{44} \end{pmatrix} \quad (7)$$

=> 3 independent elastic coefficients

- ❖ Non-zero constants of compliance matrix:

$$\begin{cases} S_{11} = S_{22} = S_{33} = \frac{C_{11} + C_{12}}{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2} \\ S_{12} = S_{21} = S_{13} = S_{31} = S_{23} = S_{32} = \frac{-C_{12}}{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2} \\ S_{44} = S_{55} = S_{66} = \frac{1}{C_{44}} \end{cases} \quad (8)$$

Elastic longitudinal modulus and Poisson's ratio



Basic description of tensile testing

❖ **Hook's law:**

$$\begin{pmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \sqrt{2}\epsilon_{23} \\ \sqrt{2}\epsilon_{31} \\ \sqrt{2}\epsilon_{12} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{44} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (9)$$

sym

❖ **Strain constants:**

$$\begin{cases} \epsilon_{11} = S_{11}\sigma_{11} \\ \epsilon_{22} = S_{12}\sigma_{11} \\ \epsilon_{33} = S_{13}\sigma_{11} \\ \epsilon_{23} = \epsilon_{31} = \epsilon_{12} = 0 \end{cases} \quad (10)$$

❖ **Elastic longitudinal modulus and Poisson's ratio were deduced:**

❖ **We deduce:**

$$\begin{cases} \sigma_{11} = \frac{1}{S_{11}}\epsilon_{11} \\ \epsilon_{22} = \frac{S_{12}}{S_{11}}\epsilon_{11} \\ \epsilon_{33} = \frac{S_{13}}{S_{11}}\epsilon_{11} \end{cases} \quad (11)$$

$$\begin{cases} E_L = \frac{1}{S_{11}} = \frac{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2}{C_{11} + C_{12}} \\ \nu_{12} = -\frac{S_{12}}{S_{11}} = \frac{C_{12}}{C_{11} + C_{12}} \\ \nu_{13} = -\frac{S_{13}}{S_{11}} = \frac{C_{12}}{C_{11} + C_{12}} \end{cases} \quad (12)$$

Macroscopic parameters: Comparison between computations and experiments

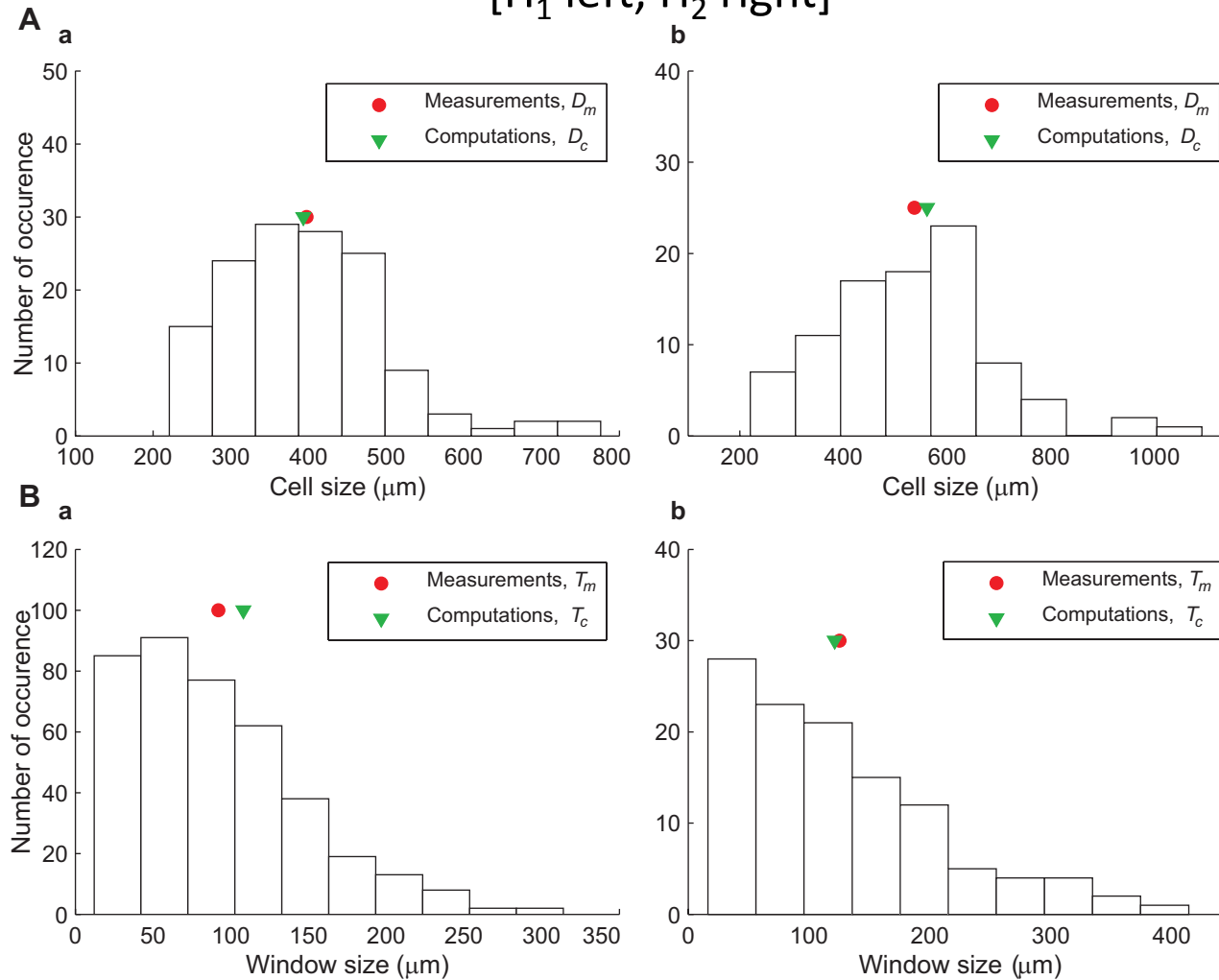
Experimental results include direct measurements of porosity ϕ and permeability k_0 (plus cell size), and inverse characterization of the remaining parameters.

Foam	Method	ϕ	Λ' (μm)	k_0 ($\times 10^{-10} \text{ m}^2$)	Λ (μm)	α_∞	k_0' ($\times 10^{-10} \text{ m}^2$)
H ₁	Computation		146 ± 22		55 ± 6	1.40 ± 0.26	28 ± 12
	Measurements	0.93 ± 0.01		5.35 ± 0.42			
	Characterization		143 ± 57		33 ± 4	1.05 ± 0.08	55 ± 28
H ₂	Computation		179 ± 46		53 ± 9	2.40 ± 0.55	48 ± 26
	Measurements	0.97 ± 0.01		2.56 ± 0.60			
	Characterization		424 ± 92		13 ± 6	1.58 ± 0.64	53 ± 16

The purely geometrical macroscopic properties and transport parameters computed from the course of this multi-scale approach are in a good agreement with experimental data, especially when standard deviations are taken into account

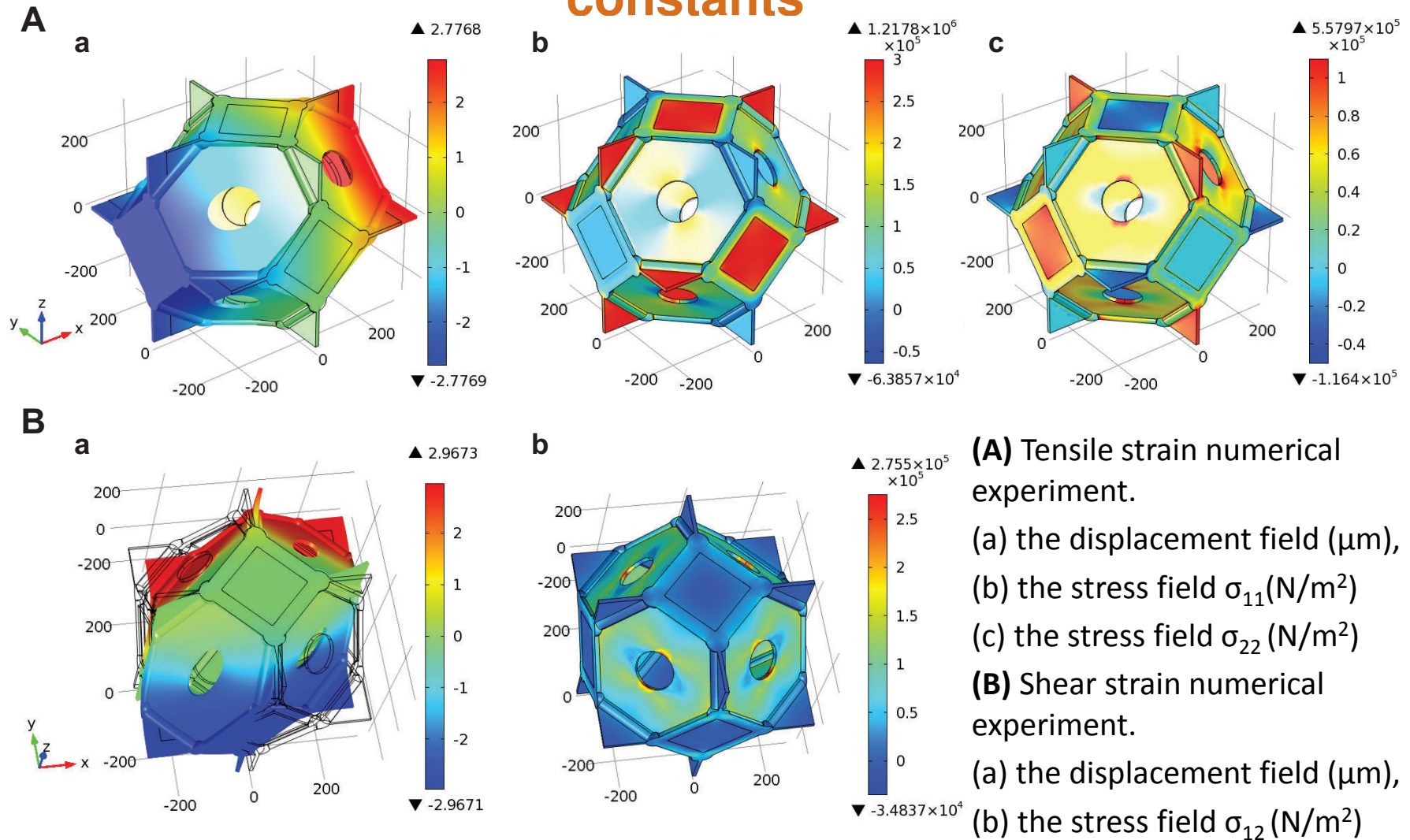
Analysis of the representativity of the microstructure from SEM imaging

[H₁ left, H₂ right]



Our approach is also consistent with microstructural results

Numerical experiments allowing to identify the elastic constants



III. RESULTS AND DISCUSSION

III. 2.2) Elastic properties

Multi-Scale Acoustics of Poro-Elastic Foams

Computed non-dimensional elastic properties of foam samples H1 and H2. In these simulations, the membrane thickness is equal to 1.7 μm .

Foam	Method	Σ_{11}^{nd} ($\times 10^{-3}$)	Σ_{22}^{nd} ($\times 10^{-3}$)	Σ_{12}^{nd} ($\times 10^{-3}$)	C_{11}^{nd} ($\times 10^{-3}$)	C_{12}^{nd} ($\times 10^{-3}$)	C_{44}^{nd} ($\times 10^{-3}$)	E_{TI}^{nd} ($\times 10^{-3}$)	ν_{TI} (-)	E_I^{nd} ($\times 10^{-3}$)	ν_I (-)
H ₁	Without membranes	0.13 ± 0.02	0.09 ± 0.01	0.03 ± 0.01	13.06 ± 2.50	9.33 ± 1.36	3.15 ± 0.94	5.28 ± 1.57	0.42 ± 0.01	4.79 ± 1.40	0.43 ± 0.01
	With membranes	0.18 ± 0.03	0.10 ± 0.01	0.05 ± 0.01	17.62 ± 2.57	10.00 ± 1.3	5.34 ± 1.07	10.38 ± 1.71	0.36 ± 0.01	8.43 ± 1.56	0.38 ± 0.01
H ₂	Without membranes	0.04 ± 0.02	0.04 ± 0.01	0.00 ± 0.00	4.24 ± 1.88	3.72 ± 1.45	0.47 ± 0.38	0.76 ± 0.62	0.47 ± 0.01	0.71 ± 0.57	0.47 ± 0.01
	With membranes	0.09 ± 0.02	0.05 ± 0.01	0.03 ± 0.01	8.77 ± 2.33	5.03 ± 1.38	3.27 ± 0.68	5.10 ± 1.51	0.36 ± 0.02	4.73 ± 1.12	0.37 ± 0.01

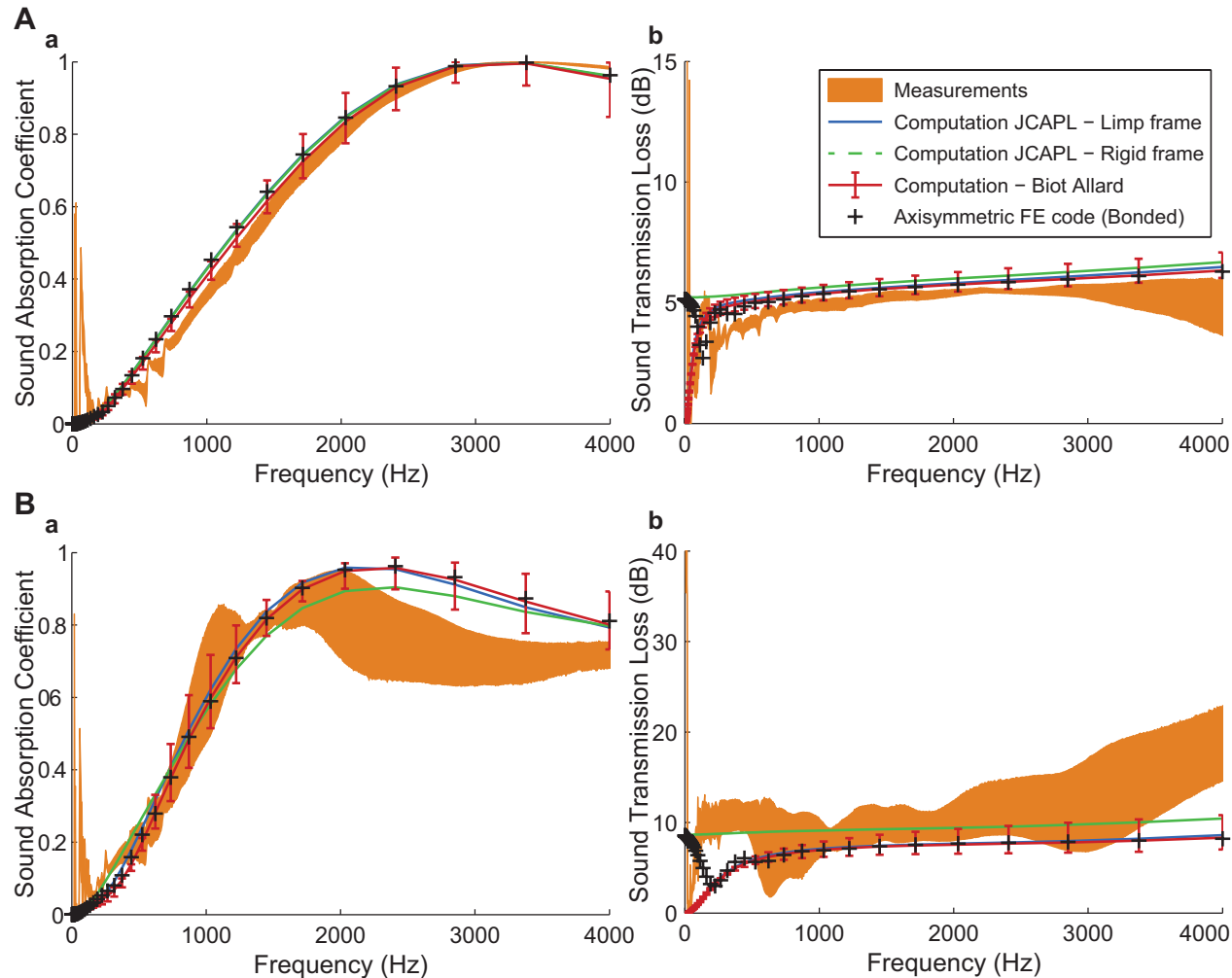
Very significant effect of membranes on the elastic properties of foam samples

III. RESULTS AND DISCUSSION

III. 2.3) Acoustic properties

Multi-Scale Acoustics of Poro-Elastic Foams

Acoustical properties of two different poroelastic foam samples, H1 (A) and H2 (B)



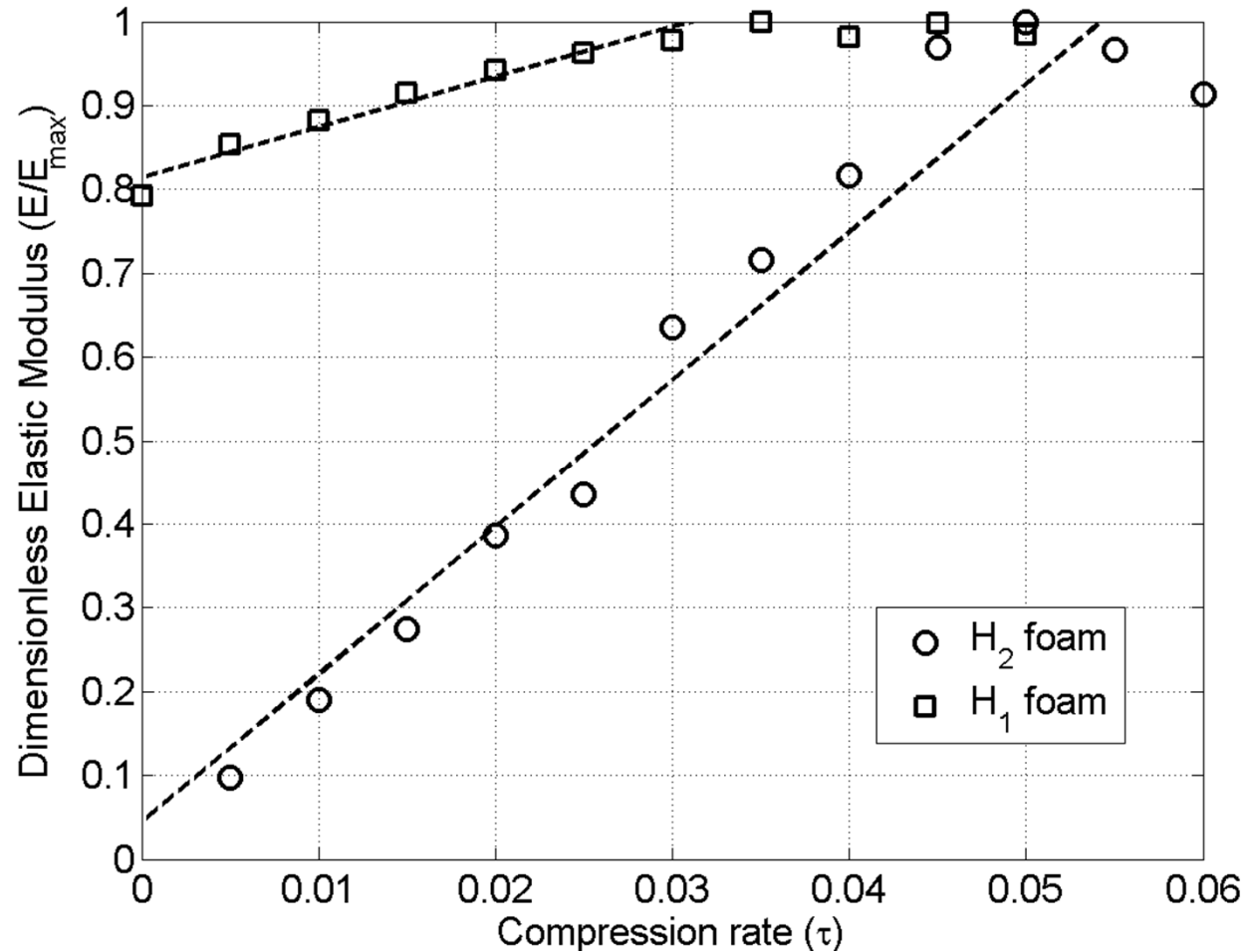
Estimates (Biot-Johnson) close to laboratory measurements of sound absorption
(Poroelastic FE formulation to properly model the boundary condition for STL)

III. RESULTS AND DISCUSSION

III. 2.4) Acoustic/Elastic properties

Multi-Scale Acoustics of Poro-Elastic Foams

Elastic properties depend on the compression rate



The method estimates linear elastic properties (no compression rate)

- The proposed micromechanical method can provide **reasonable estimates of linear elastic properties** for poroelastic foams including the *significant effects of membranes' closure rate and thickness*.
- The method is an **idealized periodic cell method**, based on the use of a simplified cellular morphology with identified local characteristic lengths.
- Further systematic investigation on the **sensitivity** of the results with regard to choice of particular features of the **cellular morphology** should be carried out.
- Accurate values for the Young's modulus and Poisson ratio of the **base material** are difficult to obtain because of the variability of the base material itself.
- Extending this multi-scale method to real life sound insulation optimization problems is not straightforward but the present **methodology should readily be extended**.