

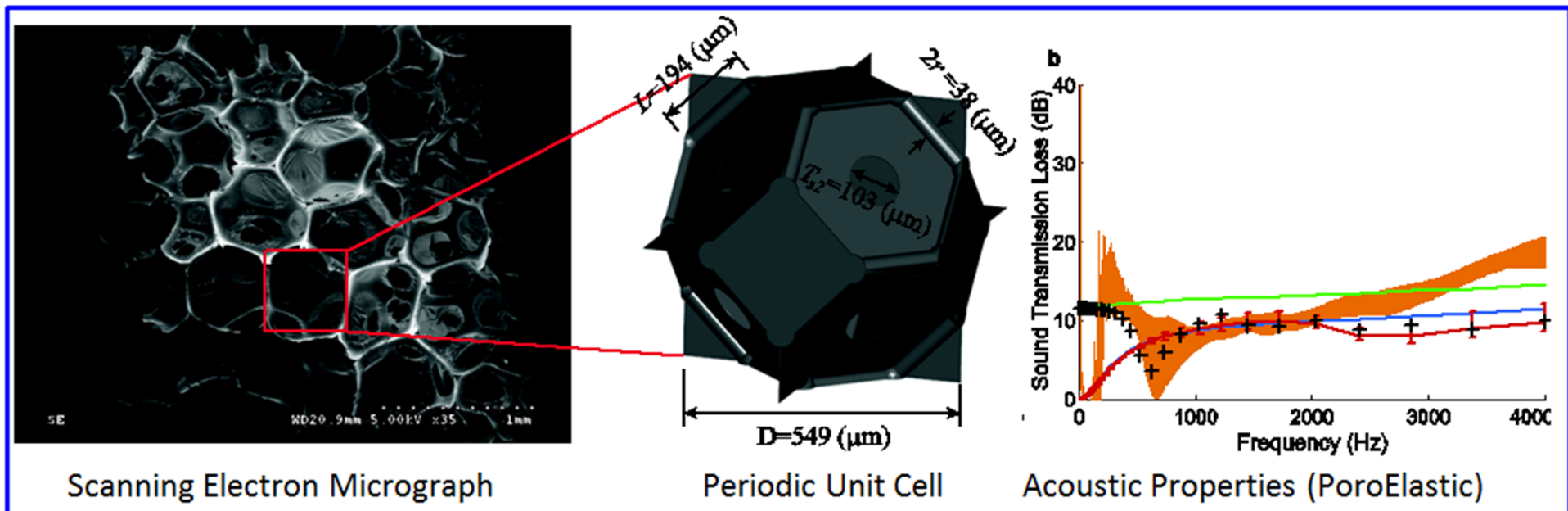
MULTI-SCALE ACOUSTICS OF CELLULAR FOAM SAMPLES

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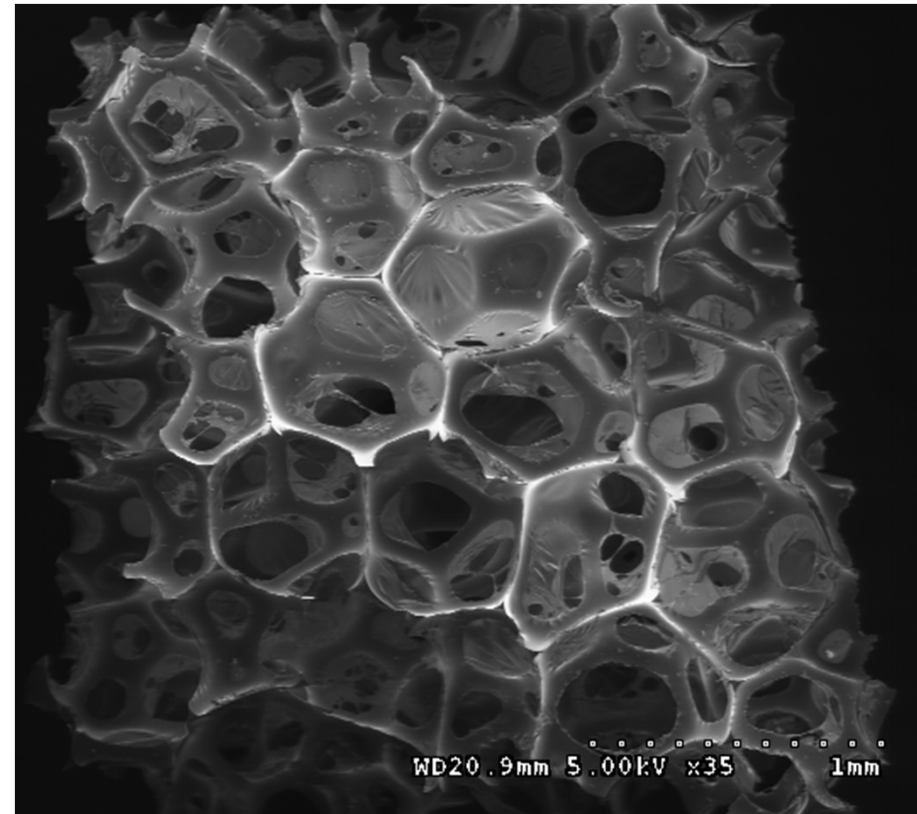


- **General Objective.** Optimization of the long-wavelengths acoustic properties of real sound proofing foam samples.
- **Methodology.** Use the method of Periodic Unit Cell (PUC) reconstruction of porous media, which consists of 2 main steps:
 - 1) **Identification** of the critical local geometry features governing long-wavelengths acoustics of porous media.
 - 2) **Generation** of 3D parameterized PUCs suitable for optimization.
 - Transport properties are obtained by ensemble averages of velocity and thermal fields, solutions of the local boundary value problems.
 - Elastic properties are derived from micromechanical techniques.

Aim of this work

- To present techniques devised to account explicitly for the role played by membranes in transport phenomena.
- To perform numerical homogenization simulations of linear elastic properties through the finite element technique in solid foams containing membranes.

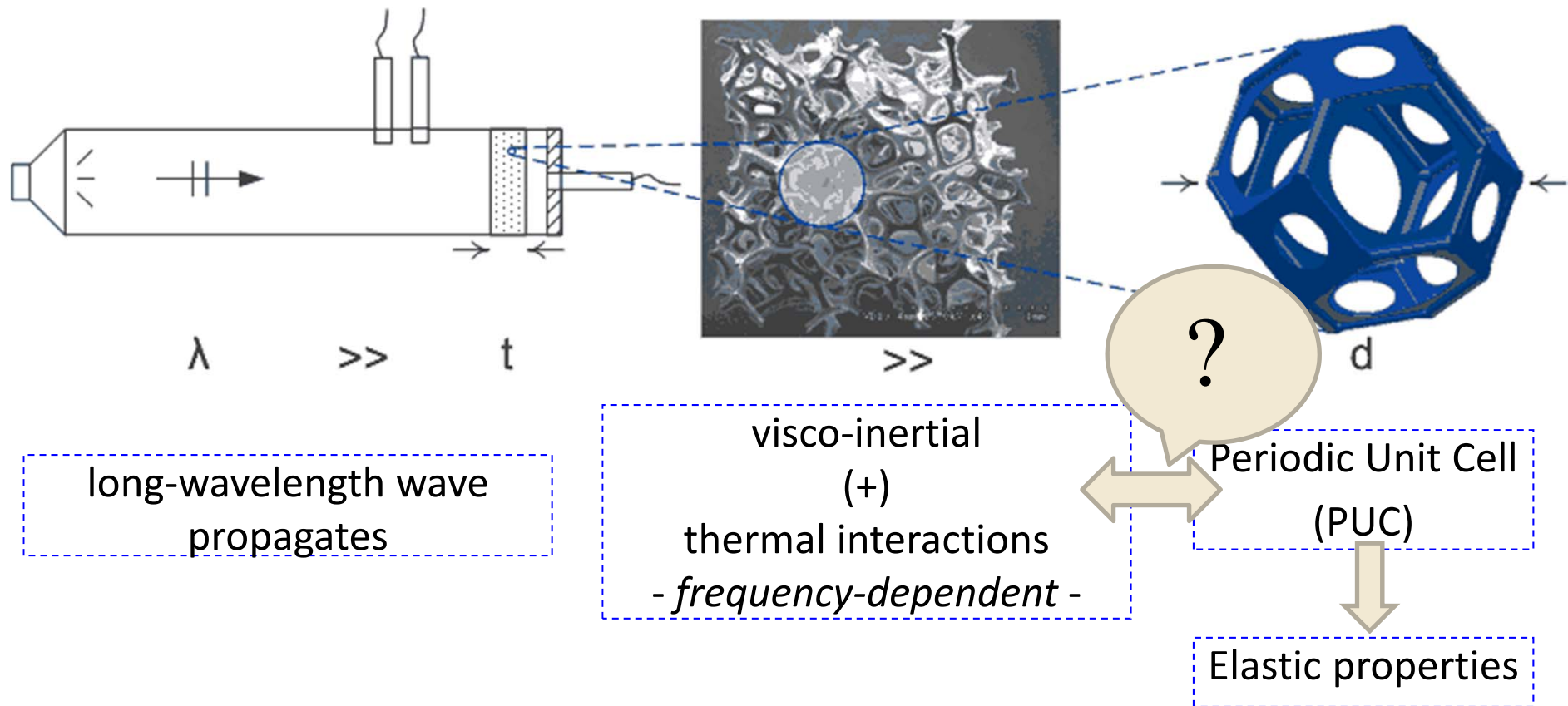
- Solid films or membranes in porous materials such as polyurethane or metal foams only account for a **very small fraction in the overall mass** of the porous media.
- Yet, their role might be of **primary importance** in the understanding of **transports, elastic, and acoustical** properties of these *poroelastic* foams.



I. INTRODUCTION

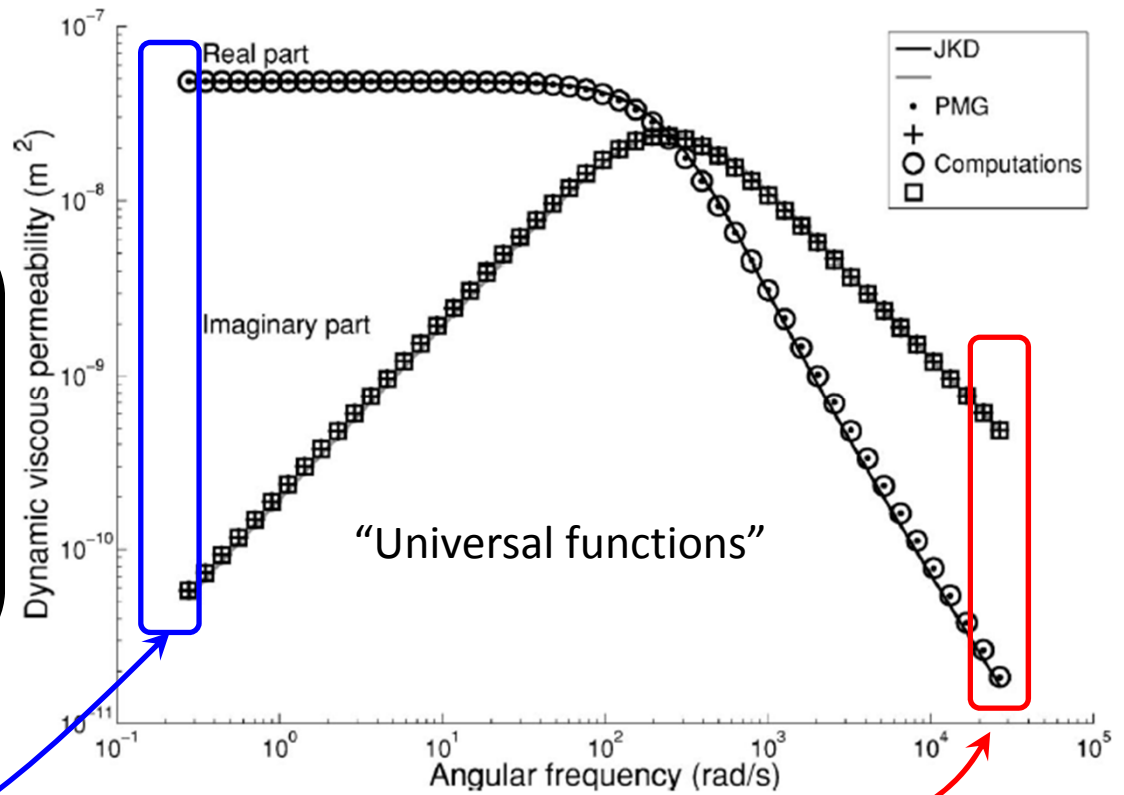
I. 3) Specific Objectives

Multi-Scale Acoustics of Poro-Elastic Foams



A fundamental physical challenge to the **microstructural identification of features** which are **characteristic** of the **overall transports and elastic properties**

Part of the solution of this problem lies in the fact that the overall dissipation of the porous material is expected to be dominated by a few key linkages.



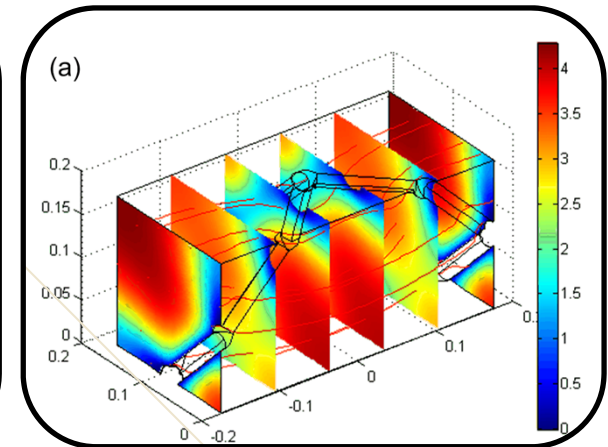
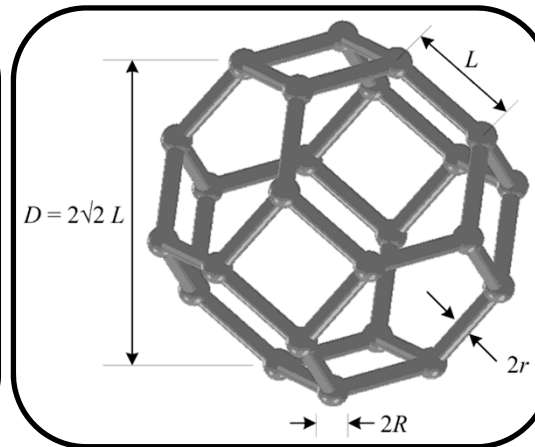
Key linkages

- Low frequency { Porosity, ϕ [-]
Permeability, k_0 [m²]
- High frequency { Viscous length, Λ [m]
Tortuosity, α_∞ [-]

References

[1] D. L. Johnson *et al.*, J. Fluid Mech. **176**: 379 (1987)
 [2] P. Sheng and M.-Y. Zhou, PRL **61**: 1591 (1988)

... and in the **identification** of an open-cell responsible for the **viscous dissipations**



Experiments & 3D Numerical simulations

Key linkages

- Low frequency { Porosity, ϕ [-]
Permeability, k_0 [m²]
- High frequency { Viscous length, Λ [m]
Tortuosity, α_∞ [-]

References

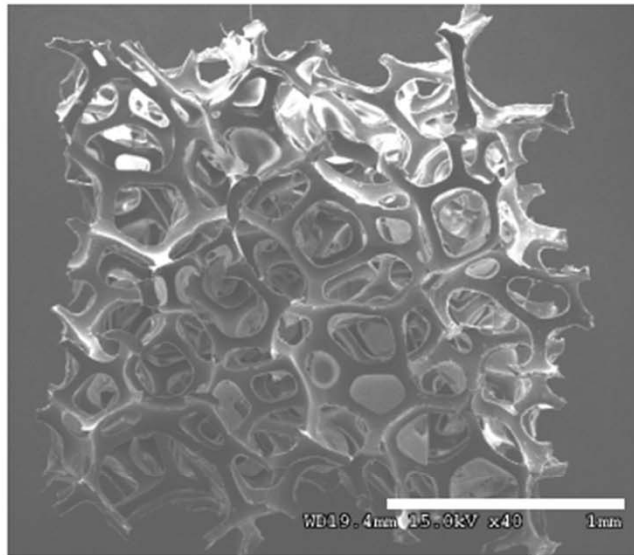
- [3] C. Perrot *et al.*, J. Appl. Phys. **111**: 014911 (2012)
 “Microstructure, transport, and acoustic properties of open-cell foam samples: Experiments and three-dimensional numerical simulations”

II. MATERIALS & METHODS

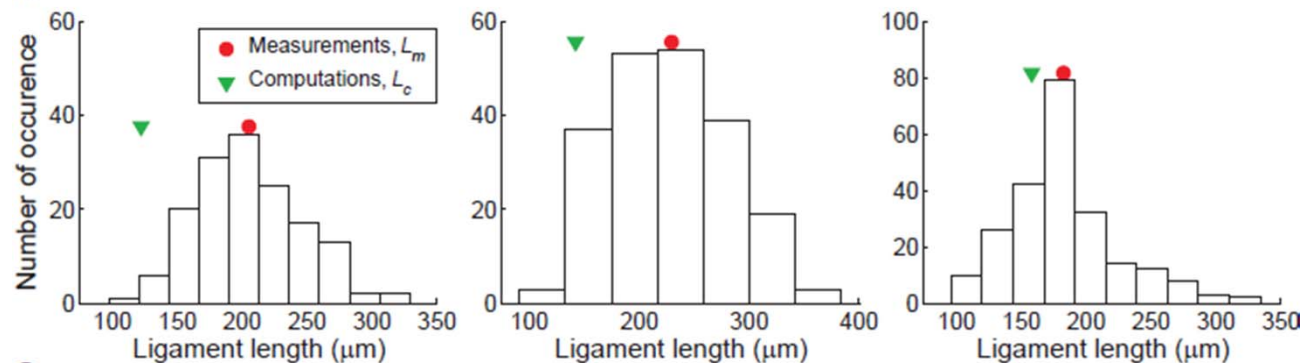
II. 3) A missing ingredient

Multi-Scale Acoustics of Poro-Elastic Foams

a



b

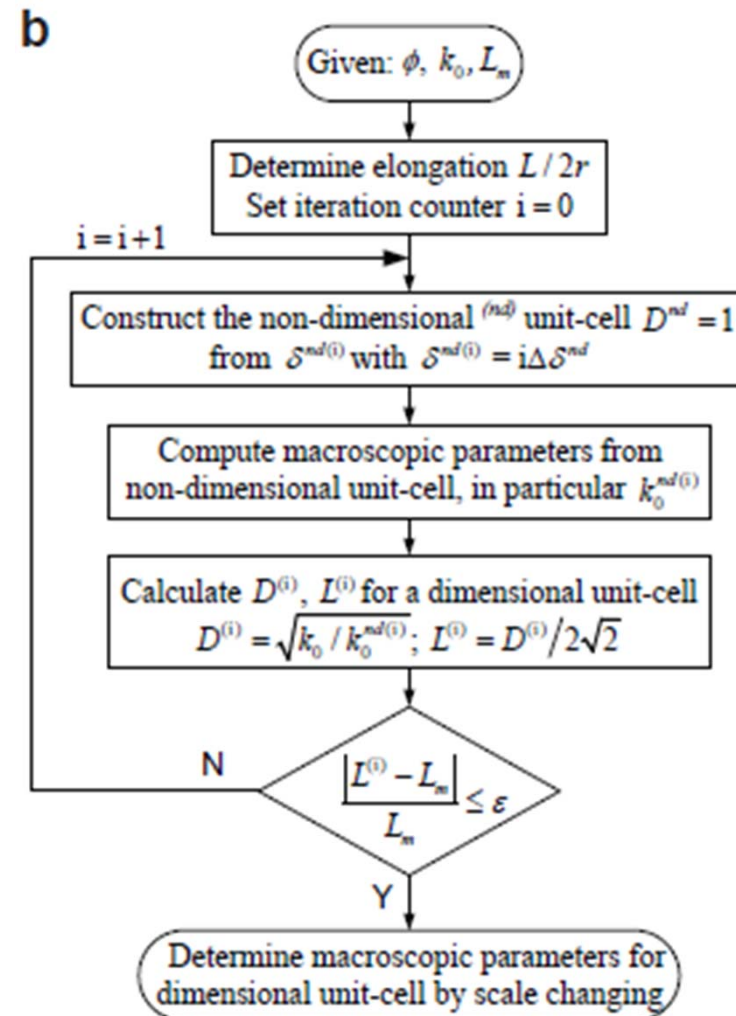
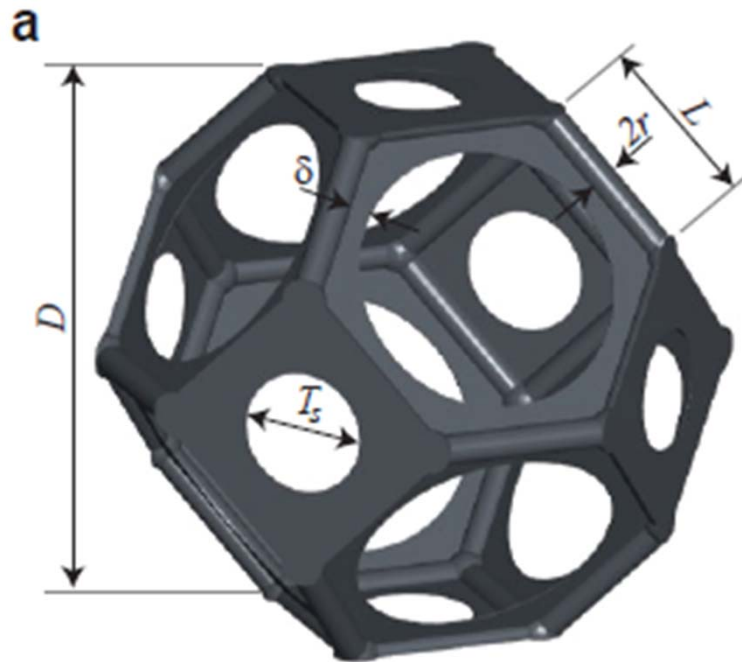


The superimposed average values of the experimental measurements (L_m , ●) and ligament length of the open-cell solid foam structure with the same average permeability (L_c , ▼) convey the impression of a **missing microstructural ingredient** in the PUC

II. MATERIALS & METHODS

II. 4) Iterative approach

Multi-Scale Acoustics of Poro-Elastic Foams



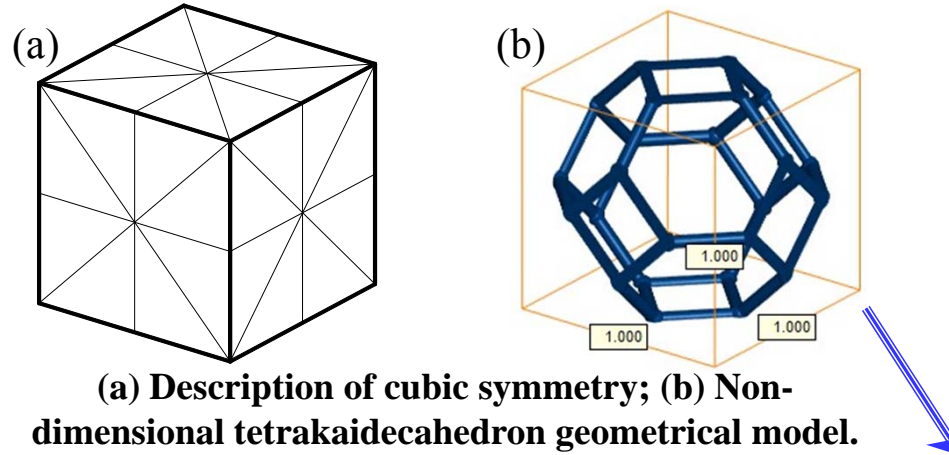
References

- [4] M. T. Hoang and C. Perrot, J. Appl. Phys. **112**, 054911 (2012)
"Solid films and transports in cellular foams"
- [5] M. T. Hoang and C. Perrot, J. Appl. Phys. **113**, 084905 (2013)
"Identifying local characteristic lengths governing sound wave
properties in solid foams"

The iterative procedure illustrates the importance of porosity ϕ , permeability k_0 ; and average ligament lengths length L_m measurements from which all the transport parameters are computed **without any adjusted constant**.

Material with cubic symmetry

- ❖ What is a cubic symmetry?
 => A structure which contains 9 planes of symmetry



(a) Description of cubic symmetry; (b) Non-dimensional tetrakaidecahedron geometrical model.

Our geometrical model satisfies the criteria for a cubic symmetry

- ❖ Elasticity matrix of a material with cubic symmetry:

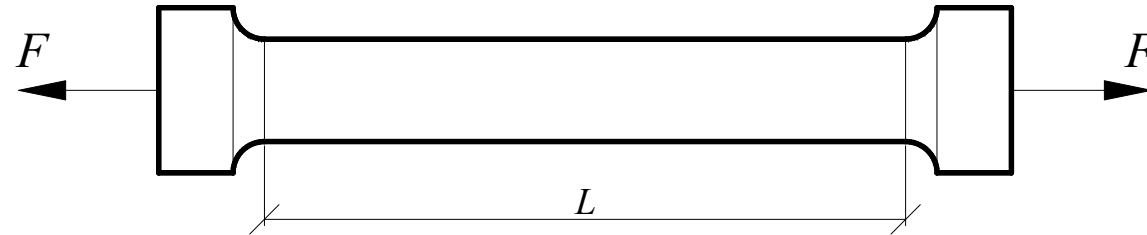
$$\tilde{C} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & sym & & & C_{44} & 0 \\ & & & & & C_{44} \end{pmatrix} \quad (7)$$

=> 3 independent elastic coefficients

- ❖ Non-zero constants of compliance matrix:

$$\begin{cases} S_{11} = S_{22} = S_{33} = \frac{C_{11} + C_{12}}{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2} \\ S_{12} = S_{21} = S_{13} = S_{31} = S_{23} = S_{32} = \frac{-C_{12}}{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2} \\ S_{44} = S_{55} = S_{66} = \frac{1}{C_{44}} \end{cases} \quad (8)$$

Elastic longitudinal modulus and Poisson's ratio



Basic description of tensile testing

❖ **Hook's law:**

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \sqrt{2}\varepsilon_{23} \\ \sqrt{2}\varepsilon_{31} \\ \sqrt{2}\varepsilon_{12} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{44} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (9)$$

sym

❖ **Strain constants:**

$$\begin{cases} \varepsilon_{11} = S_{11}\sigma_{11} \\ \varepsilon_{22} = S_{12}\sigma_{11} \\ \varepsilon_{33} = S_{13}\sigma_{11} \\ \varepsilon_{23} = \varepsilon_{31} = \varepsilon_{12} = 0 \end{cases} \quad (10)$$

❖ **Elastic longitudinal modulus and Poisson's ratio were deduced:**

❖ **We deduce:**

$$\begin{cases} \sigma_{11} = \frac{1}{S_{11}} \varepsilon_{11} \\ \varepsilon_{22} = \frac{S_{12}}{S_{11}} \varepsilon_{11} \\ \varepsilon_{33} = \frac{S_{13}}{S_{11}} \varepsilon_{11} \end{cases} \quad (11)$$

$$\begin{cases} E_L = \frac{1}{S_{11}} = \frac{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2}{C_{11} + C_{12}} \\ \nu_{12} = -\frac{S_{12}}{S_{11}} = \frac{C_{12}}{C_{11} + C_{12}} \\ \nu_{13} = -\frac{S_{13}}{S_{11}} = \frac{C_{12}}{C_{11} + C_{12}} \end{cases} \quad (12)$$

Macroscopic parameters: Comparison between computations and experiments

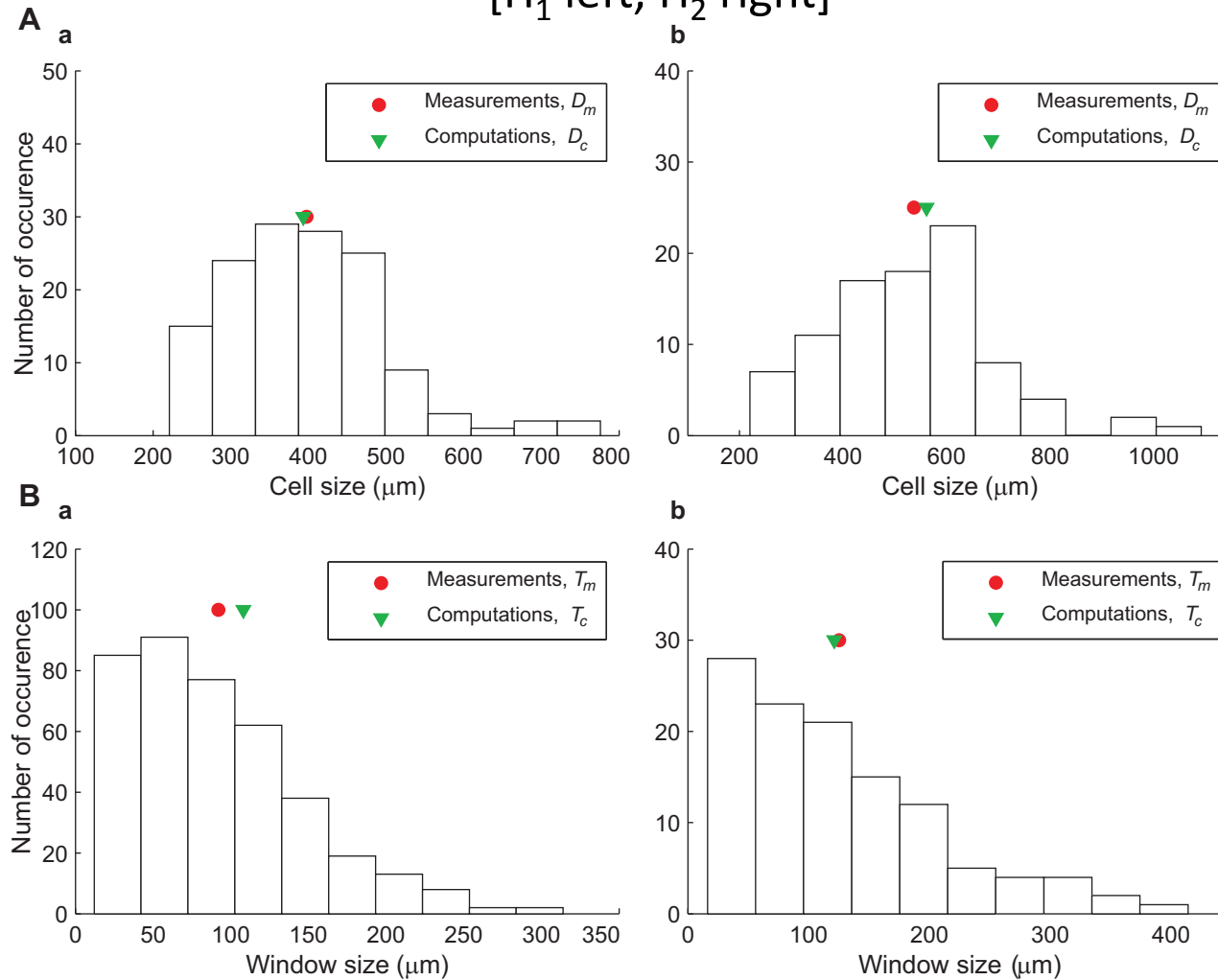
Experimental results include direct measurements of porosity ϕ and permeability k_0 (plus cell size), and inverse characterization of the remaining parameters.

Foam	Method	ϕ	Λ' (μm)	k_0 ($\times 10^{-10} \text{ m}^2$)	Λ (μm)	α_∞	k_0' ($\times 10^{-10} \text{ m}^2$)
H ₁	Computation		146 ± 22		55 ± 6	1.40 ± 0.26	28 ± 12
	Measurements	0.93 ± 0.01		5.35 ± 0.42			
	Characterization		143 ± 57		33 ± 4	1.05 ± 0.08	55 ± 28
H ₂	Computation		179 ± 46		53 ± 9	2.40 ± 0.55	48 ± 26
	Measurements	0.97 ± 0.01		2.56 ± 0.60			
	Characterization		424 ± 92		13 ± 6	1.58 ± 0.64	53 ± 16

The purely geometrical macroscopic properties and transport parameters computed from the course of this multi-scale approach are in a good agreement with experimental data, especially when standard deviations are taken into account

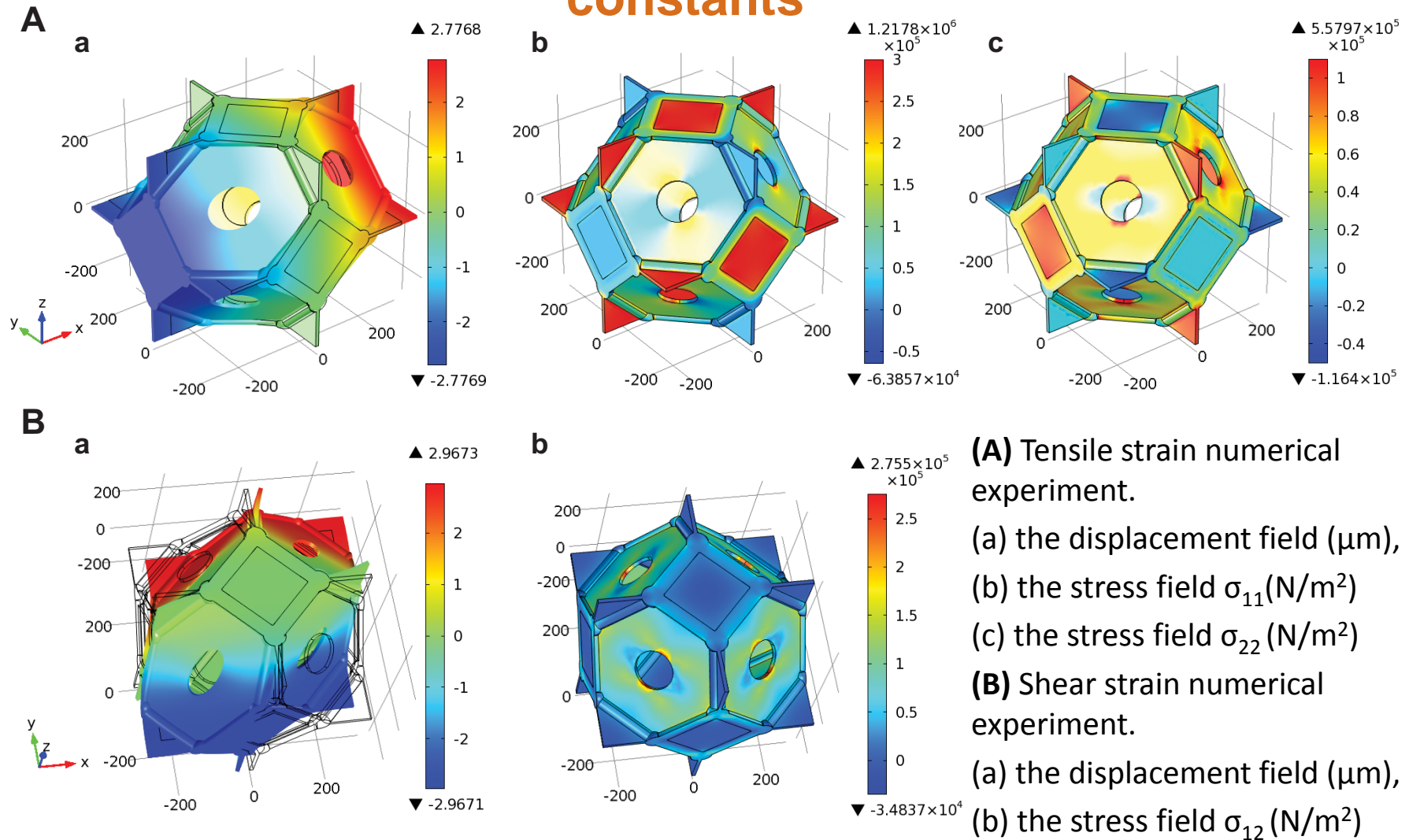
Analysis of the representativity of the microstructure from SEM imaging

[H₁ left, H₂ right]



Our approach is also consistent with microstructural results

Numerical experiments allowing to identify the elastic constants



III. RESULTS AND DISCUSSION

Multi-Scale Acoustics of Poro-Elastic Foams

III. 2.2) Elastic properties

Computed non-dimensional elastic properties of foam samples H1 and H2. In these simulations, the membrane thickness is equal to 1.7 μm .

Foam	Method	Σ_{11}^{nd} ($\times 10^{-3}$)	Σ_{22}^{nd} ($\times 10^{-3}$)	Σ_{12}^{nd} ($\times 10^{-3}$)	C_{11}^{nd} ($\times 10^{-3}$)	C_{12}^{nd} ($\times 10^{-3}$)	C_{44}^{nd} ($\times 10^{-3}$)	E_{TI}^{nd} ($\times 10^{-3}$)	ν_{TI} (-)	E_I^{nd} ($\times 10^{-3}$)	ν_I (-)
H ₁	Without membranes	0.13 ± 0.02	0.09 ± 0.01	0.03 ± 0.01	13.06 ± 2.50	9.33 ± 1.36	3.15 ± 0.94	5.28 ± 1.57	0.42 ± 0.01	4.79 ± 1.40	0.43 ± 0.01
	With membranes	0.18 ± 0.03	0.10 ± 0.01	0.05 ± 0.01	17.62 ± 2.57	10.00 ± 1.3	5.34 ± 1.07	10.38 ± 1.71	0.36 ± 0.01	8.43 ± 1.56	0.38 ± 0.01
H ₂	Without membranes	0.04 ± 0.02	0.04 ± 0.01	0.00 ± 0.00	4.24 ± 1.88	3.72 ± 1.45	0.47 ± 0.38	0.76 ± 0.62	0.47 ± 0.01	0.71 ± 0.57	0.47 ± 0.01
	With membranes	0.09 ± 0.02	0.05 ± 0.01	0.03 ± 0.01	8.77 ± 2.33	5.03 ± 1.38	3.27 ± 0.68	5.10 ± 1.51	0.36 ± 0.02	4.73 ± 1.12	0.37 ± 0.01

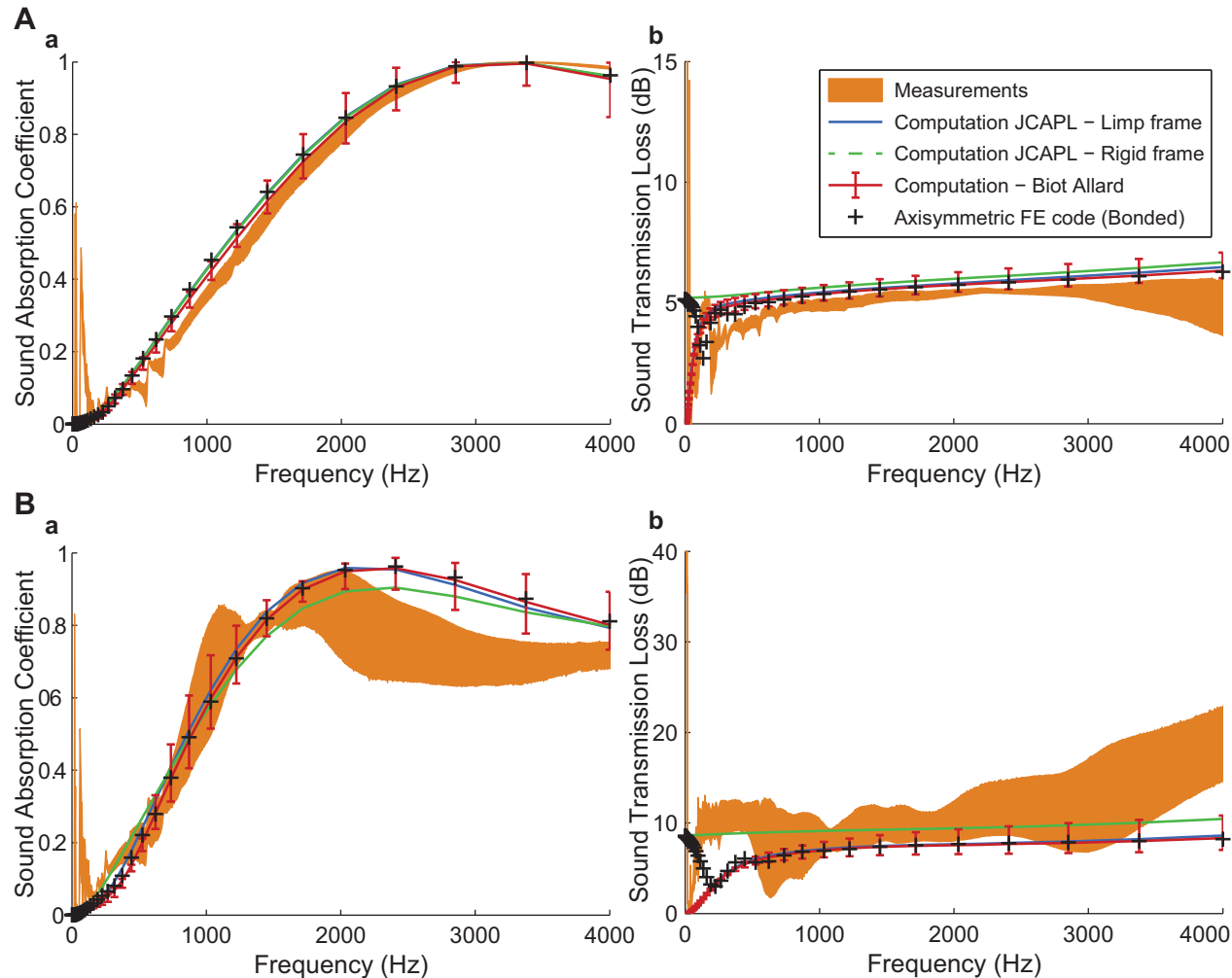
Very significant effect of membranes on the elastic properties of foam samples

III. RESULTS AND DISCUSSION

III. 2.3) Acoustic properties

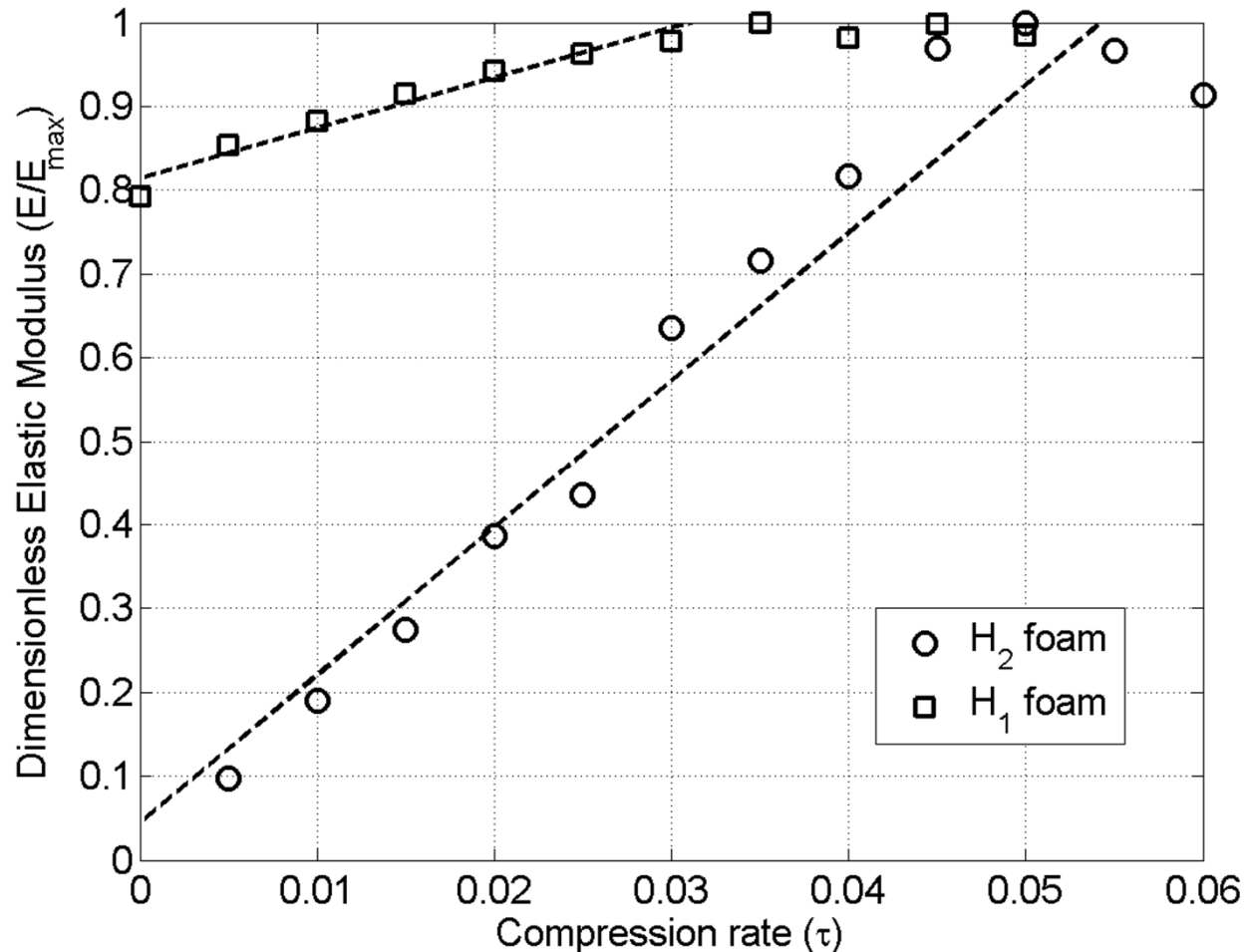
Multi-Scale Acoustics of Poro-Elastic Foams

Acoustical properties of two different poroelastic foam samples, H1 (A) and H2 (B)



Estimates (Biot-Johnson) close to laboratory measurements of sound absorption
(Poroelastic FE formulation to properly model the boundary condition for STL)

Elastic properties depend on the compression rate



The method estimates linear elastic properties (no compression rate)

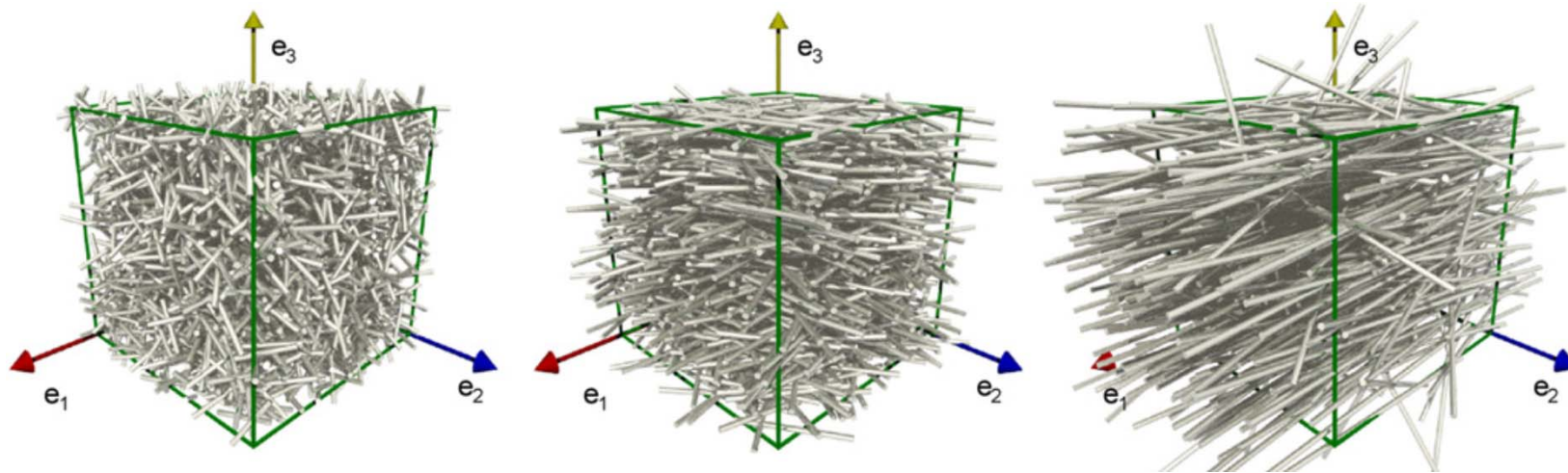
- The proposed micromechanical method can provide **reasonable estimates of linear elastic properties** for poroelastic foams including the *significant effects of membranes' closure rate and thickness*.
- The method is an **idealized periodic cell method**, based on the use of a simplified cellular morphology with identified local characteristic lengths.
- Further systematic investigation on the **sensitivity** of the results with regard to choice of particular features of the **cellular morphology** should be carried out.
- Accurate values for the Young's modulus and Poisson ratio of the **base material** are difficult to obtain because of the variability of the base material itself.
- Extending this multi-scale method to real life sound insulation optimization problems is not straightforward but the present **methodology should readily be extended**.

Reference

[6] M. T. Hoang, G. Bonnet, H. T. Luu, C. Perrot, J. Acoust. Soc. Am. **135(6)**, 3172-3185 (2014)

"Linear elastic properties derivation from microstructures representative of transport parameters"

- **Generalization** of the proposed approach to fibrous materials (i.e. glass wools)
- **Generation of REV**s corresponding to a specified **fiber orientation tensor**:
(a) 3D random, (b) planar random, and (c) nearly unidirectional structures:



- **Experimental identification** of the orientation tensors for real fibrous samples.
- Modeling **contacts between fibers**.

Reference

[7] J-P Vassal, L Orgéas and D. Favier, Modelling Simul. Mater. Sci. Eng. **16**, 035007 (2008)

“Modelling microstructure effects on the conduction in fibrous materials with fibre–fibre interface barriers”