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Scale-by-scale energy budgets which account for the coherent motion

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Abstract. Scale-by-scale energy budget equations for coherent motion and turbulent kinetic energy have been written in flows with coherent motions. The general and the locally isotropic formulations are both provided. In particular, the contribution to the production, diffusion and energy transfer terms associated with the coherent motion are emphasized. Preliminary results are shown in the intermediate wake of a cylinder, for the phase-scale second- and third-order structure functions.

1. Introduction

Characterizing and understanding turbulent flows are necessary steps before modelling and predicting their statistics, and the dynamical behaviour of these statistics. Many studies aimed at characterizing turbulent flows revealed, since the pioneer studies of Townsend (1976) that the organized, coherent motion (CM) persists in many turbulent shear flows. Since CM is an important player of turbulence dynamics, a lot of effort has been devoted to the extraction of coherent structures, with the aim to learn more about their dynamical nature and their contribution to the total statistics. Extraction of CM signifies establishing clear criteria in defining it, and therefore distinguishing CM from the mean flow and the random, turbulent motion (TM). Since the big challenge is predicting turbulent flows statistics, a first (quite important) step would be predicting interactions between CM and TM. The process of turbulence birth starts with mean velocity gradients (stagnation points or regions), followed by instabilities further leading to CM, further followed by TM birth. For decaying flows such as wakes, there is now abundant experimental and numerical support of the persisting influence of the CM far downstream of the turbulent energy injection (Cimbala *et al.* (1988), Bisset *et al.* (1990)).

Turbulent flows are generally represented by energy at a wide range of scales. A first feature which distinguishes CM from TM is the scale(s) at which these two entities live. CM is generally characterized by a single scale, or a very restricted range of scales (a narrow peak on energy spectra) whereas TM is represented by a wide range of scales. Kolmogorov (1941) stipulated that there is a scale beyond which the influence of the anisotropic/coherent large scales is not perceptible anymore and, as a consequence, the velocity statistics become locally isotropic. Presumably for this reason, the influence of CM on TM was not considered at the level of an arbitrary scale, implicitly supposing that the effect of CM is confined at the level of

its characteristic scale. Or, both the nonlocality of the cascade and the finite Reynolds number (FRN) effects lead to the fact that CM might play a role on TM at a given, arbitrary scale.

The aim of this study is twofold.

- First, we derive scale-by-scale energy budgets for both CM and TM which mathematically emphasize their interaction at a vectorial scale \vec{r} separating two spatial points. The general (Section 2.1), as well as the locally isotropic formulations (Section 2.2), are provided. These equations evidence additional terms corresponding to the transport, production, diffusion and forcing of TM by CM.
- Second, we turn our attention on the particular case of the wake behind a cylinder (Section 3), in which the scale-phase second- and third-order structure functions are calculated from hot-wire measurements with phase reference. We particularly focus on this flow because it is characterized by a persisting Bénard-Von Kármán street, even in the so-called 'far field' (Cimbala *et al.* (1988), Bisset *et al.* (1990)). Here, we mainly focus on TM and the effect of CM on energy transfer process at a scale. We show that the scale-by-scale budget of the random field is well supported by experimental data on the wake centreline.

2. Analytical development

2.1. General formulation

The starting point is the triple decomposition $U_i = \overline{U}_i + \tilde{u}_i + u'_i$, (U_i is the instantaneous velocity in the i^{th} direction, $\overline{[\cdot]}$, $\tilde{[\cdot]}$ and $[\cdot]'$ denote respectively the mean, the CM and TM). Since CM is characterized by a time/scale periodicity, it is useful to introduce the operation of phase-averaging (i.e. averaging over the CM period), hereafter noted as $\langle \cdot \rangle$. The result is statistics which depend on the phase (normalized such that they vary over a 2π period). The objective is to emphasize the behaviour of such a quantity conditioned by a particular phase of the CM.

As an example, Reynolds & Hussain (1972) described analytically the interaction between turbulence and an organized wave. These authors derived dynamical and one-point energy budgets equations, for both the coherent and the random components of the velocity field. They proposed using the phase averaging operator which allows assessing the dynamical process associated with coherent motion, one step before the temporal averaging. These authors Reynolds & Hussain (1972) obtained the dynamical equations for both the random and the coherent motion, viz.

$$\frac{\partial \tilde{u}_i}{\partial t} + \overline{U}_j \frac{\partial \tilde{u}_i}{\partial x_j} + \tilde{u}_j \frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j - \overline{\tilde{u}_i \tilde{u}_j}) + \frac{\partial}{\partial x_j} (\langle u'_i u'_j \rangle - \overline{u'_i u'_j}) = -\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2}; \quad (1)$$

$$\frac{\partial u'_i}{\partial t} + \overline{U}_j \frac{\partial u'_i}{\partial x_j} + \tilde{u}_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial \overline{U}_i}{\partial x_j} + u'_j \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (u'_i u'_j - \langle u'_i u'_j \rangle) = -\frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_j^2}. \quad (2)$$

In (1) and (2), ν is the kinematic viscosity, p is the pressure divided by density, and double indices signify summation. Following the procedure established by Antonia *et al.* (1997) and Danaila *et al.* (2004), (1) and (2) are each written at the two points \vec{x} and $\vec{x}^+ = \vec{x} + \vec{r}$, where \vec{r} is the separation vector between the two points, and the superscript '+' hereafter denotes quantities considered at the point $\vec{x} + \vec{r}$. Subtraction of one from the other yields the transport equations of the organized and random velocity increment, $\Delta \tilde{u}_i = \tilde{u}_i^+ - \tilde{u}_i$ and $\Delta u'_i = u'^+ - u'_i$

respectively, viz.

$$\begin{aligned} & \frac{\partial \Delta \tilde{u}_i}{\partial t} + \Delta \left(\bar{U}_j \frac{\partial \tilde{u}_i}{\partial x_j} \right) + \Delta \left(\tilde{u}_j \frac{\partial \bar{U}_i}{\partial x_j} \right) + \Delta \left(\frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j - \overline{\tilde{u}_i \tilde{u}_j}) \right) \\ & + \Delta \left(\frac{\partial}{\partial x_j} (\langle u'_i u'_j \rangle - \overline{u'_i u'_j}) \right) = - \left(\frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \Delta \tilde{p} + \nu \left(\frac{\partial^2}{\partial x_j \partial x_j} + \frac{\partial^2}{\partial x_j^+ \partial x_j^+} \right) \Delta \tilde{u}_i; \end{aligned} \quad (3)$$

and

$$\begin{aligned} & \frac{\partial \Delta u'_i}{\partial t} + \Delta \left(\bar{U}_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left(\tilde{u}_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left(u'_j \frac{\partial \bar{U}_i}{\partial x_j} \right) + \Delta \left(u'_j \frac{\partial \tilde{u}_i}{\partial x_j} \right) \\ & + \Delta \left(\frac{\partial}{\partial x_j} (u'_i u'_j - \langle u'_i u'_j \rangle) \right) = - \left(\frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \Delta p' + \nu \left(\frac{\partial^2}{\partial x_j \partial x_j} + \frac{\partial^2}{\partial x_j^+ \partial x_j^+} \right) \Delta u'_i. \end{aligned} \quad (4)$$

At this stage, the statistics at the two points \vec{x} and \vec{x}^+ are considered as being independent. Then, considering the gradient with respect to the mid-point $\vec{X} = \frac{1}{2}(\vec{x} + \vec{x}^+)$ (Hill (2002), Danaila *et al.* (2004))

$$\frac{\partial}{\partial x_j} = -\frac{\partial}{\partial r_j} + \frac{1}{2} \frac{\partial}{\partial X_j}, \quad \frac{\partial}{\partial x_j^+} = \frac{\partial}{\partial r_j} + \frac{1}{2} \frac{\partial}{\partial X_j}, \quad (5)$$

by multiplying (3) and (4) with $2\Delta \tilde{u}_i$ and $2\Delta u'_i$ respectively, applying a phase followed by a time averaging, and finally noting that

$$\langle \Delta u'_j \Delta u'_i \rangle = \langle \Delta u_i \Delta u_j \rangle - \Delta \tilde{u}_i \Delta \tilde{u}_j \quad (6)$$

$$\overline{\langle \Delta u_j \Delta q^2 \rangle} = \overline{\Delta \tilde{u}_j \Delta \tilde{q}^2} + \overline{\Delta \tilde{u}_j \langle \Delta q'^2 \rangle} + \overline{\langle \Delta u'_j \Delta q'^2 \rangle} + 2\Delta \tilde{u}_i \overline{\langle \Delta u'_j \Delta u'_i \rangle}, \quad (7)$$

we obtain the energy budgets for the organized and random motions. These are

$$\begin{aligned} & \underbrace{\frac{\partial}{\partial t} \overline{\Delta \tilde{q}^2}}_{\mathcal{I}_c} + \underbrace{\bar{U}_j \frac{\partial \overline{\Delta \tilde{q}^2}}{\partial x_j}}_{\mathcal{A}_{cm}} + \underbrace{\frac{1}{2} \left(\frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \left[\overline{(\tilde{u}_j + \tilde{u}_j^+) \Delta \tilde{q}^2} + 2 \overline{\langle (u'_j + u'_j^+) \Delta u'_i \rangle \Delta \tilde{u}_i} \right]}_{\mathcal{D}_{cc} + \mathcal{D}_{cr}^1} \\ & + \underbrace{2 \overline{\Delta \tilde{u}_i \Delta \tilde{u}_j} \frac{\partial \bar{U}_i}{\partial x_j}}_{\mathcal{P}_{cm}} - \underbrace{\overline{\langle (u'_j + u'_j^+) \Delta u'_i \rangle \left(\frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \Delta \tilde{u}_i}}_{\mathcal{P}_{rc}} + \underbrace{2 \left(\frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \overline{\Delta \tilde{u}_i \Delta \tilde{p}}}_{\mathcal{D}_{cp}} \\ & \quad \quad \quad + \underbrace{\frac{\partial}{\partial r_j} \overline{\Delta \tilde{u}_j \Delta \tilde{q}^2}}_{\mathcal{T}_c} + \underbrace{2 \Delta \tilde{u}_i \frac{\partial}{\partial r_j} \overline{\langle \Delta u'_i \Delta u'_j \rangle}}_{\mathcal{F}_c} \\ & + \nu \underbrace{\left[- \left(2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \overline{\Delta \tilde{q}^2} - 4 \left(\frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial \tilde{u}_j}{\partial x_i} + \frac{\partial \tilde{u}_i^+}{\partial x_j^+} \frac{\partial \tilde{u}_j^+}{\partial x_i^+} \right) \right]}_{\mathcal{V}_c} + 2(\tilde{\epsilon} + \tilde{\epsilon}^+) = 0; \end{aligned} \quad (8)$$

and

$$\begin{aligned}
& \underbrace{\frac{\partial \overline{\Delta q'^2}}{\partial t}}_{\mathcal{I}_r} + \underbrace{\overline{U}_j \frac{\partial \overline{\Delta q'^2}}{\partial x_j}}_{\mathcal{A}_{rm}} + \underbrace{\frac{1}{2} \left(\frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \left[\overline{(u'_j + u'^+_j) \Delta q'^2} + \overline{(\tilde{u}_j + \tilde{u}_j^+) \langle \Delta q'^2 \rangle} \right]}_{\mathcal{D}_{rr} + \mathcal{D}_{rc}^2} \\
& + \underbrace{2 \overline{\Delta u'_i \Delta u'_j} \frac{\partial \overline{U}_i}{\partial x_j}}_{\mathcal{P}_{rm}} + \underbrace{\left\langle (u'_j + u'^+_j) \Delta u'_i \right\rangle \left(\frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \Delta \tilde{u}_i}_{\mathcal{P}_{rc}} + \underbrace{2 \left(\frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \overline{\Delta u'_i \Delta p'}}_{\mathcal{D}_{rp}} \\
& \quad + \underbrace{\frac{\partial}{\partial r_j} \langle \Delta u_j \Delta q^2 \rangle}_{\mathcal{T}} - \underbrace{\frac{\partial}{\partial r_j} \overline{\Delta \tilde{u}_j \Delta \tilde{q}^2}}_{\mathcal{T}_c} - \underbrace{2 \Delta \tilde{u}_i \frac{\partial}{\partial r_j} \langle \Delta u'_i \Delta u'_j \rangle}_{\mathcal{F}_c} \\
& + \nu \underbrace{\left[- \left(2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \overline{\Delta q'^2} - 4 \left(\frac{\overline{\partial u'_i \partial u'_j}}{\partial x_j \partial x_i} + \frac{\overline{\partial u'^+_i \partial u'^+_j}}{\partial x_j^+ \partial x_i^+} \right) \right]}_{\mathcal{V}_r} + 2(\epsilon' + \epsilon'^+) = 0, \quad (9)
\end{aligned}$$

where $\overline{\Delta \tilde{q}^2} = \overline{\Delta \tilde{u}_i \Delta \tilde{u}_i}$ and $\overline{\Delta q'^2} = \overline{\Delta u'_i \Delta u'_i}$ are respectively the CM and TM kinetic energies at a given scale. Quantities $\tilde{\epsilon} = \frac{\nu}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)^2$ and $\epsilon' = \frac{\nu}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)^2$ are the mean energy dissipation rates of the coherent and the random motions, respectively.

For the sake of simplicity, (8) and (9) can be formally written as

$$\mathcal{I}_c + \mathcal{A}_{cm} + \mathcal{D}_{cc} + \mathcal{D}_{cr}^1 + \mathcal{P}_{cm} - \mathcal{P}_{rc} + \mathcal{D}_{cp} + \mathcal{T}_c + \mathcal{F}_c + \mathcal{V}_c + 2(\tilde{\epsilon} + \tilde{\epsilon}^+) = 0; \quad (10)$$

$$\mathcal{I}_r + \mathcal{A}_{rm} + \mathcal{D}_{rr} + \mathcal{D}_{rc}^2 + \mathcal{P}_{rm} + \mathcal{P}_{rc} + \mathcal{D}_{rp} + \mathcal{T} - \mathcal{T}_c - \mathcal{F}_c + \mathcal{V}_r + 2(\epsilon' + \epsilon'^+) = 0, \quad (11)$$

where \mathcal{I} , \mathcal{A} , \mathcal{D} , \mathcal{P} , \mathcal{T} , \mathcal{F} et \mathcal{V} denote respectively the non stationarity, advection, diffusion, production, transfer, forcing and viscous terms. The subscripts m , c , r correspond to the mean, coherent and random motions, and \mathcal{D}_p indicates the pressure diffusion.

Equations (8) and (9) are the general formulations of the scale-by-scale budgets which account for the coherent motion in which each term depends on the separation vector \vec{r} . For homogeneous flows and in the limit of large separations, the scale-by-scale budgets (8) and (9) are fully consistent with one-point energy budget provided by Reynolds & Hussain (1972) (Eqs. (3.2b) and (3.2c) p. 266 in Reynolds & Hussain (1972)). Each term can be evaluated by DNS (Direct Numerical Simulations), without any other assumption. Equations (8) and (9) constitute the general fundamental basis to unravel the physics of the coherent and random fields interaction.

In comparison with Danaila *et al.* (2004), some additional terms are to be emphasized. Particularly, the terms \mathcal{P}_{rc} , \mathcal{T}_c and \mathcal{F}_c , identified as the production of random fluctuations by the coherent motion, the coherent kinetic energy transfer and the forcing associated by the presence of a coherent motion are emphasized. Both these terms are present in equations (8) and (9), but their sign differs from one to the other. This means that what represents a loss of energy for CM (8), constitutes an energetic gain for TM (9). Furthermore, we put light on the transport of random statistical quantities by the organized motion \mathcal{D}_{rc}^1 and \mathcal{D}_{rc}^2 (?? je ne comprends pas ..).

2.2. The locally homogeneous and isotropic context

We now turn our attention to the derivation of (8) and (9) in a locally homogeneous and isotropic context. This assumption yields to considerable simplifications of the analytical development

and experimental measurements (Danaila *et al.* (1999), Danaila *et al.* (2001)). Considering first a locally homogeneous turbulent flow, the viscous term simplifies such as

$$\nu \left[- \left(2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \overline{\Delta q'^2} - 4 \left(\frac{\overline{\partial u'_i \partial u'_j}}{\partial x_j \partial x_i} + \frac{\overline{\partial u'^+_i \partial u'^+_j}}{\partial x^+_j \partial x^+_i} \right) \right] = 2\nu \frac{\partial^2}{\partial r_j^2} \overline{\Delta q'^2}, \quad (12)$$

since $\frac{\overline{\partial u'_i \partial u'_j}}{\partial x_j \partial x_i} = 0$ and $\frac{\partial^2}{\partial X_j^2} = 0$, Hill (1997). The same simplification holds for the coherent motion. Then, in the context of local isotropy, the divergence and the Laplacian operators are expressed as:

$$\frac{\partial}{\partial r_j} = \frac{2}{r} + \frac{\partial}{\partial r}, \quad \frac{\partial^2}{\partial r_j^2} = \left(\frac{2}{r} + \frac{\partial}{\partial r} \right) \frac{\partial}{\partial r}. \quad (13)$$

By further using (13), multiplication of (8) and (9) with $r^2 = r_j r_j$, and integration with respect to r and division by r^2 , yields

$$\begin{aligned} & \frac{1}{r^2} \int_0^r s^2 (\mathcal{A}_{cm} + \mathcal{D}_{cc} + \mathcal{D}_{rc}^1 + \mathcal{P}_{cm} + \mathcal{P}_{rc} + \mathcal{D}_{cp}) ds \\ & + \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \overline{\langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds - 2\nu \frac{\partial}{\partial r} \overline{\Delta \tilde{q}^2} + \frac{4}{3} \tilde{\epsilon} r = 0; \end{aligned} \quad (14)$$

$$\begin{aligned} & \frac{1}{r^2} \int_0^r s^2 (\mathcal{A}_{rm} + \mathcal{D}_{rr} + \mathcal{D}_{rc}^2 + \mathcal{P}_{rm} - \mathcal{P}_{rc} + \mathcal{D}_{rp}) ds \\ & + \overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle} - \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} - \frac{2}{r^2} \int_0^r \Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \overline{\langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds - 2\nu \frac{\partial}{\partial r} \overline{\Delta q'^2} + \frac{4}{3} \epsilon' r = 0. \end{aligned} \quad (15)$$

Equations (14) and (15) are the scale-by-scale energy budgets of the organized and random motions respectively, in locally homogeneous and isotropic context. Here, s is a dummy variable and the subscript \parallel denotes the direction parallel to the separation vector. When the spatial separation is inferred by invoking Taylor hypothesis, this direction coincides with the direction of the mean flow.

The first line of Eqs. (14) and (15) represents the energy contribution of the largest scales (Danaila *et al.* (2004)). The main difference with respect to the extended form of Kolmogorov equation Antonia *et al.* (1997) equation, consists in several extra terms due to the presence of CM. The effective energy transfer of the random velocity component is explicit and is thus constituted of the total energy transfer $\overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle}$ (including the coherent and random contributions), from which is subtracted the coherent energy transfer $\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2}$ and the forcing term $\frac{2}{r^2} \int_0^r \Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \overline{\langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$.

3. Results in the wake of a circular cylinder

3.1. Measurements

The analytical considerations previously developed are now used to assess the essential physics and particularly the dynamical nature associated with the presence of the organized motion. Hot-wire measurements previously made at the University of Newcastle Zhou *et al.* (2003) are used to calculate two-point statistics of CM and TM. The measurements were conducted in an open-circuit wind tunnel with a square working section of $0.35 \times 0.35m$ and $2.4m$ long. The

cylinder of diameter $d = 12.7\text{mm}$ was placed horizontally to generate the wake flow. The three vorticity components were measured simultaneously by means of a four-X-wire probe, Zhou *et al.* (2003). The downstream location investigated here is $40d$, sufficiently far from the energy injection to expect the local isotropy to be verified and close enough to accurately extract the organized motion. The free stream velocity U_0 is 3m.s^{-1} corresponding to a Reynolds number based on the cylinder diameter and the upstream velocity of $Re_d = 2525$ and a turbulent Reynolds number based on a typical fluctuation and the Taylor micro-scale of 70 on the wake centerline. For the estimates of velocity derivatives, the separations $\Delta x \approx \Delta y \approx \Delta z$ are set equal to 6η . This spatial resolution leads to an attenuation of velocity derivatives which were corrected by using the spectral method proposed by Zhu & Antonia (1996). The mean dissipation rate is estimated using the isotropic relation $\epsilon = 15\nu\overline{\left(\frac{\partial u}{\partial x}\right)^2}$ in which $\overline{\left(\frac{\partial u}{\partial x}\right)^2}$ was corrected following the procedure of Zhu & Antonia (1996). The spatial separation Δx is calculated by means of the Taylor hypothesis, $\Delta x = -U_c\Delta t$, where U_c is the average convection velocity of vortices. This velocity is defined as the mean velocity at the vortices center location and is equal to $U_c = 0.92U_0$ at $40d$ downstream the cylinder (Zhou *et al.* (2003)).

To calculate phase averaged statistics, the transverse velocity signal v is bandpass filtered at frequency centered on Strouhal frequency. The Hilbert transform h of the filtered signal v_f is calculated and the phase is inferred from $\phi(t) = \text{Atan}\left(\frac{h(t)}{v_f(t)}\right)$. Finally, the phase is divided into 32 segments and phase averaged statistics are calculated for each class. Statistics were calculated over 750 integral time-scales, with a correct convergence.

3.2. Phase-scale second-and third-order structure functions, on the wake centerline

The essential difference between our approach and the classical energy budget equations, is the phase averaging operation which allows us to assess the temporal dynamics associated with the CM, one step before time averaging. Second-and third-order structure functions are as usually functions of r , but specific to our methodology, they are also functions of the phase ϕ of the CM, before being time-integrated.

Fig1(a) and 1(b) represent the phase-scale distributions of the kinetic energy of the organized and random motions respectively. In Fig1(a), one can note a strong temporal periodicity of period $\phi = \pi$ of the coherent motion kinetic energy. This periodicity might also be observable on the r axis, at scales characteristic of the organized motion (not shown). This emphasizes the spatio-temporal periodicity of the Von Kármán street. Concerning the energy distribution of random fluctuations (Fig1(b)), the influence of the CM is less perceptible. Its shape in scale r is very similar to that of the 'classical' time-integrated second-order structure function.

The non linear transfer term $-\langle\Delta u_{\parallel}\Delta q^2\rangle$ divided by ϵr is displayed on Fig1(c). The temporal periodicity is strongly discernible, and reveals two maxima at a phase location $\phi = \pm\pi/2$ and a scale $r \approx L_v/10 \approx \lambda$, where λ is the Taylor micro-scale. Furthermore, at $\phi = 0$ this term reveals some negative values, which might a priori be associated with a local, inverse cascade. Noticeable are also maximum values larger than $4/3$, presumably signifying very accelerate cascade.

In Fig1(d), it is represented the phase-averaged vorticity spanwise component. The Von Kármán street is thus magnified (??), and reveals a negative sign vortex centred at $\phi = -\pi/2, y = 0.75d$. Its partner of positive sign, not visible on the figure, is located at $\phi = \pi/2, y = -0.75d$. Therefore, the two maxima of the non-linear transfer term coincide with the vortex centers. There is evidence that the organized motion induces a strong temporal dynamics on the kinetic energy transfer at a scale r .

3.3. Scale-by-scale budget

We now turn our attention on time averaged structure functions. In Fig. 2, it is displayed the total non linear transfer $-\langle\overline{\Delta u_{\parallel}\Delta q^2}\rangle$, the additional coherent transfer and forcing due to the

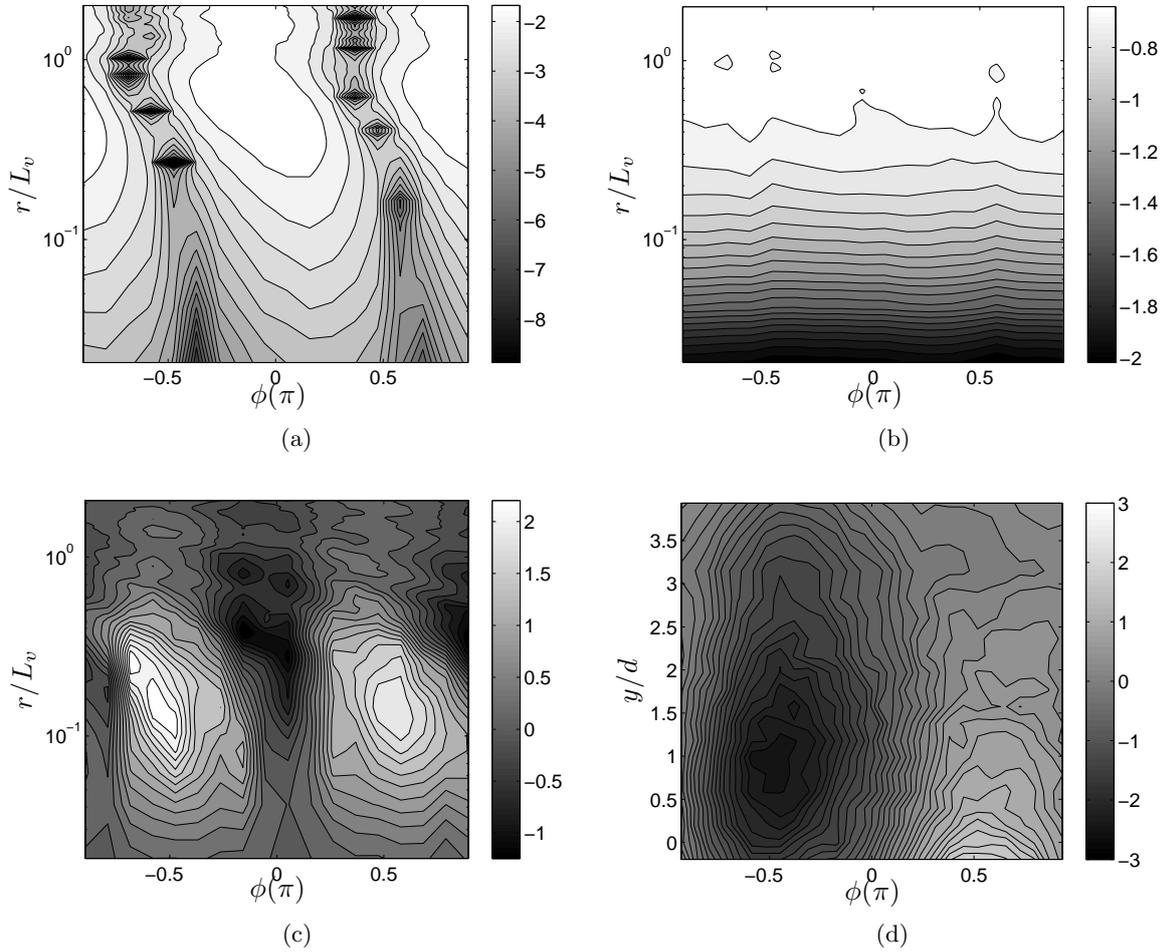


Figure 1. (a) $\log_{10} (\Delta \tilde{q}^2)$ function of r and ϕ on the wake center line at $x = 40d$, r is normalized by $L_v = 4.2d$, the streamwise distance between two consecutive vortices. (b) $\log_{10} (\langle \Delta q^2 \rangle)$ function of r and ϕ on the wake center line at $x = 40d$. (c) Transfer term $-\langle \Delta u_{\parallel} \Delta q^2 \rangle / \epsilon' r$ function of r and ϕ on the wake center line at $x = 40d$. (d) Phase averaging of the spanwise vorticity component $\tilde{\omega}_z$ normalized by d/U_0 in the plan (ϕ, y) .

coherent motion $\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$ and the effective transfer inferred from their difference, in function of the separation r/L_v .

For weakly turbulent flows the non linear transfer term is smaller than $\frac{4}{3}\epsilon r$, because of the cross-over between viscous and large-scale effects (Danaila *et al.* (1999), Danaila *et al.* (2004)). Here, $-\langle \Delta u_{\parallel} \Delta q^2 \rangle / \epsilon r \approx 0.93$. The additional energy transfer associated with the coherent motion is negative, its value is quite small, but nonnegligible. Its contribution is non zero for all separation with a maximum contribution located at about 2λ . Finally, the effective transfer of the random motion is quite smaller than the total transfer of about 12% at the maximum location.

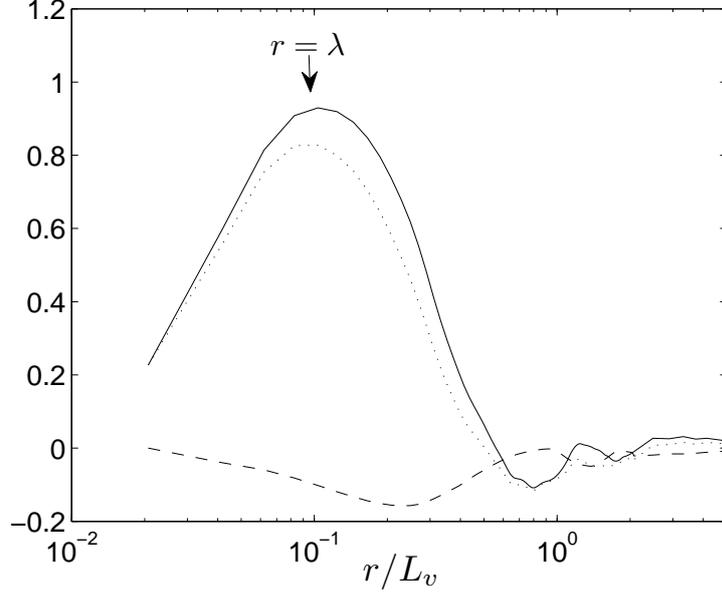


Figure 2. Non linear transfer term divided by ϵr . — total transfer term $-\overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle}$, - - - coherent transfer and forcing term $\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$, difference $-\overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle} + \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$.

On the wake centerline, the isotropic scale-by-scale budget of the random motion is:

$$\begin{aligned}
 & -\frac{1}{r^2} \int_0^r s^2 \mathcal{A}_{rm} ds - \overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle} \\
 & + \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds + 2\nu \frac{\partial}{\partial r} \overline{\Delta q'^2} = \frac{4}{3} \epsilon' r,
 \end{aligned} \tag{16}$$

which means that in the limit of large scales, the advection term is almost entirely compensated by the energy dissipation only. All terms of the scale-by-scale budget (16) are shown on Fig. 3.

The balance between the right- and left-hand sides of (16) is relatively well equilibrated at all scales. The degree of accuracy of the experimental validation of (16) is as precise as the local isotropy is achieved. Local isotropy tests are checked against experimental data in the far-field of a wake and presented elsewhere. Therefore, the experimental investigation in the cylinder intermediate wake supports the analytical considerations provided in Section 1.

4. Concluding remarks

The scale-by-scale budgets of the kinetic energy of the organized and random field are derived in a general and isotropic formulation. It emphasized some additional diffusion, production, transport terms as well as some extra energy transfer and forcing associated with the presence of the coherent motion. These considerations explicitly evidence that the effective energy transfer of the random motion is constituted by the total energy transfer from which it is subtracted the coherent energy transfer and the forcing by the organized structures. The hot wire measurements

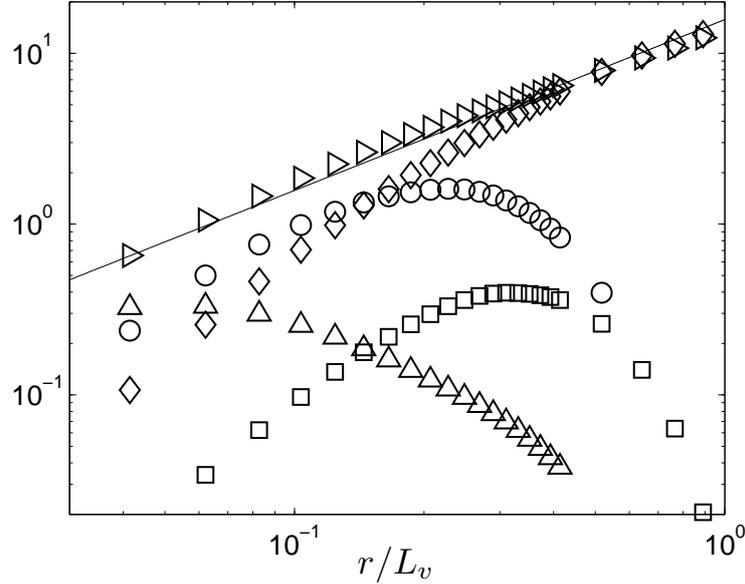


Figure 3. Term in equation (16) divided by ϵ' . — : $\frac{4}{3}r$, \diamond : $-\frac{1}{r^2} \int_0^r s^2 \mathcal{A}_{rm} ds$, \circ : $-\overline{\Delta u_{\parallel} \Delta q^2}$, \square : $-\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} - \frac{2}{r^2} \int_0^r \Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \overline{\Delta u'_{\parallel} \Delta u'_i} ds$, \triangle : $2\nu \frac{\partial}{\partial r} \overline{\Delta q^2}$, \triangleright Left hand side of (16).

in the cylinder wake are then employed to emphasize the temporal dynamics associated with the presence of the organized motion. It is shown that the energy transfer is clearly influenced by the coherent motion, revealing two maxima located in the vortices centers. The time-integrated scale-by-scale budget (16) is also well supported by experimental data.

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