

1 **Scale-by-scale turbulent energy budget in the intermediate wake of two-dimensional**
2 **generators**

3 F. Thiesset,^{1, a)} R. A. Antonia,¹ and L. Danaila²

4 ¹⁾*School of Engineering, University of Newcastle, NSW 2308 Callaghan,*
5 *Australia*

6 ²⁾*CORIA, UMR 6614, Avenue de l'Université, BP 12,*
7 *76801 Saint Etienne du Rouvray, France*

It is first established, on the basis of new X-wire measurements, that the equilibrium similarity of the terms in the scale-by-scale (s-b-s) budget of the turbulent energy $\overline{q^2}$ is reasonably well approximated on the axis of the intermediate wake of a circular cylinder. The similarity, which scales on the Taylor microscale λ and $\overline{q^2}$, is then used to determine s-b-s energy budgets from the data of Antonia, Zhou & Romano¹ for 5 different two-dimensional wake generators. In each case, the budget is reasonably well closed, using the locally isotropic value of the mean energy dissipation rate, except near separations comparable to the wavelength of the coherent motion (CM). The influence of the initial conditions is first felt at a separation L_c identified with the cross-over between the energy transfer and large scale terms of the s-b-s budget. When normalized by $\overline{q^2}$ and L_c , the mean energy dissipation rate is found to be independent of the Taylor microscale Reynolds number. The CM enhances the maximum value of the energy transfer, the latter exceeding that predicted from models of decaying homogeneous isotropic turbulence.

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^{a)}Electronic mail: [Email address : Fabien.Thiesset@newcastle.edu.au](mailto:Fabien.Thiesset@newcastle.edu.au)

9 I. INTRODUCTION

10 The concept of self-preservation, which effectively assumes that the flow is governed by
11 single length and velocity scales, has an important role in describing the spatial evolution
12 of turbulent flows. Implicit in this concept is the notion of forgetfulness or disentanglement
13 from initial conditions. There is now however abundant evidence to suggest that initial
14 conditions can influence the type of self-preservation that can be achieved in several types
15 of turbulent flows, in particular free shear flows such as jets and wakes. The latter flows are
16 especially interesting in view of the invariably strong organisation, in the form of vortical
17 coherent structures, which is first observed immediately downstream of the wake-generating
18 obstacle and which subsequently influences the streamwise development of the flow (*e.g.*
19 Refs. 1–6).

20 Although the conditions for self-preservation, or sometimes self-similarity, have been in-
21 vestigated by considering the mean momentum and turbulent energy equations at one point
22 in space, it is of course more powerful and general to consider the self-preservation of the two-
23 point velocity correlation tensor. The precursor of this approach was outlined by Ref. 7 who
24 obtained a similarity solution of the two-point velocity correlation for homogeneous isotropic
25 turbulence when the Reynolds number is very large. George⁸ proposed an equilibrium simi-
26 larity solution of Lin’s spectral equation, equivalent to the Kármán-Howarth equation, which
27 is valid at any Reynolds number. This solution was tested in grid turbulence by Antonia
28 *et al.*⁹, in the context of the transport equation for the turbulent energy structure function
29 $\overline{(\Delta q)^2} = \overline{(\Delta u)^2} + \overline{(\Delta v)^2} + \overline{(\Delta w)^2}$, where Δu , Δv , Δw are the velocity increments for u , v , w
30 between two points separated by a distance r (*e.g.* $\Delta u = u(x+r) - u(x)$), and the overbar
31 denotes time-averaging. A plausible interpretation of the transport equation of $\overline{(\Delta q)^2}$ is
32 that it represents an energy budget at each scale, or a scale-by-scale (hereafter s-b-s) en-
33 ergy budget. There have been several attempts at investigating s-b-s budgets, using either
34 experimental or numerical data, in other types of flows, *e.g.* in homogeneous shear flow^{10–12},
35 in the central region of a channel flow¹³, in a circular jet^{14–16}, in wall-bounded flows^{17,18}. In
36 general, closure of the budget has been satisfactory validated by the data, allowing for the
37 assumptions that are made when deriving the transport equations. In these previously cited
38 flows, different physical phenomena are at play, for example grid turbulence is dominated by
39 the advection of energy whilst diffusion is prominent near the centreline of a channel flow.

40 On the centreline of an axisymmetric jet, the major source of (large scale) inhomogeneity
41 is associated with the streamwise decay of turbulent energy with only a small contribution
42 from the production of energy arising from the difference in the Reynolds normal stresses.

43 In this paper, we examine the s-b-s energy budget in the intermediate wake of different
44 obstacles. We focus on this region primarily because we expect the energy transfer in this
45 region of the flow to be dominated, as in the case of grid turbulence and along the axis of
46 a circular jet, by the streamwise decay of turbulent energy. However, unlike the latter two
47 flows, one also expects that the transfer may feel the influence of the organised motion or
48 coherent structures, the latter depending clearly on the type of initial condition, *viz.* the
49 geometry of the wake-generating obstacle. Detailed velocity and vorticity measurements
50 were made by Antonia, Zhou & Romano¹, hereafter AZR, at $x/d = 70$, in wakes generated
51 by 5 different bluff-bodies, each with the same characteristic dimension d at nominally the
52 same Reynolds number based on the Taylor microscale $R_\lambda \approx 200$. Unfortunately, there was
53 no attempt in Ref. 1 at measuring the one point energy budget or establishing if similarity
54 applied in this region of the flow. This would have enabled the construction of the the s-b-s
55 energy budget. To remedy this shortcoming, we present new measurements made at several
56 streamwise locations in the intermediate wake of a circular cylinder. Experimental details
57 are given in section II. The one-point energy budget (section III) is then obtained and the
58 equilibrium similarity is tested (section IV) and exploited in section V to examine the effect
59 of initial conditions on the s-b-s energy budget. Finally, conclusions are drawn in section
60 VI.

61 II. EXPERIMENTAL DETAILS

62 X-wire measurements were made in the wake of a circular cylinder in the CORIA wind
63 tunnel, which is of recirculating type. The residual turbulence level is smaller than 0.2%.
64 The square test section is $(0.4 \times 0.4\text{m}^2)$ has a length of 2.5m. The mean pressure gradient
65 was set to zero. A circular cylinder of diameter $d = 10\text{mm}$, placed horizontally downstream
66 the contraction, spanned the width of the test section. The upstream velocity U_0 was
67 6.5ms^{-1} , corresponding to a Reynolds number Re_d , based on the cylinder diameter, of 4333.
68 Measurements of velocity fluctuations u and v were made at several streamwise locations
69 in the range $40 \leq x/d \leq 110$. The X-wire probe (Dantec 55P51) was calibrated using a

70 look-up table technique (*e.g.* Refs. 19 and 20) with velocity and angle increment of 1ms^{-1}
71 and 5° respectively. The hot wires (Pt-W, $5\mu\text{m}$ diameter, 1.25mm length) were operated
72 with a Dantec constant temperature bridge at an overheat ratio of 1.6. Voltage signals were
73 passed through buck and amplifiers (SRS SIM983) and low-pass filtered (SRS SIM965) at a
74 frequency f_c close to the Kolmogorov frequency $f_K \equiv \bar{U}/2\pi\eta$ (\bar{U} is the local mean velocity
75 and the Kolmogorov length-scale $\eta = (\nu^3/\bar{\epsilon})^{1/4}$). The sampling frequency $f_s = 20\text{kHz}$ and
76 the sampling period was about 4min . The air temperature in the wind tunnel was kept
77 constant during the calibration and measurements using a regulating cross-flow water-air
78 cooler system which allowed the mean temperature to be controlled to $\pm 0.5^\circ\text{C}$.

79 Measurements made at the University of Newcastle in the wakes downstream of 5 different
80 two dimensional obstacles, each with the same characteristic dimension $d = 25.4\text{mm}$ have
81 been detailed in Refs. 1 and 21. Only a brief description is given below (see Fig. 1).

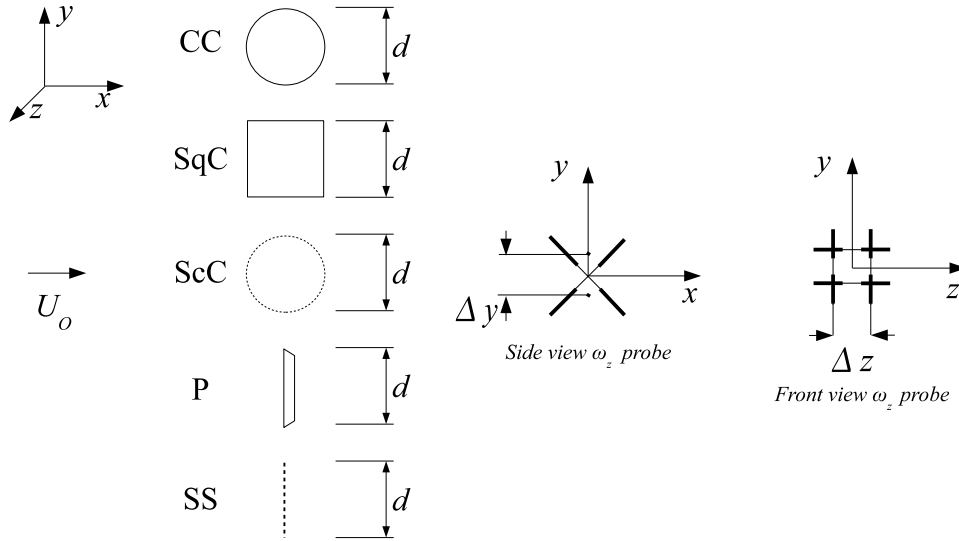


FIG. 1. Cross section of the five different generators used for the experiments carried out in Newcastle by Ref. 1. From top to bottom: CC, SqC, ScC, P, SS. Also sketched is a one-component vorticity probe in the ω_z configuration (ω_z is the vorticity component in the z direction).

82 The data were obtained in an open circuit wind tunnel with a 2.4m long square test
83 section ($0.35 \times 0.35\text{m}^2$). The five generators (a circular cylinder CC, a square cylinder SqC,
84 a screen cylinder ScC, a solid plate P normal to the flow and a screen strip SS, Fig. 1) were
85 located 20cm downstream of the end of the contraction and spanned the width of the test

86 section. All measurements were made on the centreline at $x = 70d$ at several values of Re_d .
87 A one-component vorticity probe (Fig. 1), which included an X-wire straddled by 2 parallel
88 hot wires, was used to determine either ω_z when the X-wire was in the plane (x,y) or ω_y
89 when the X-wire was in the (x,z) plane. All the hot wires were etched from Pt-10%Rh to a
90 diameter of $2.5\mu\text{m}$ and an active length of approximately 0.5mm (length to diameter ratio
91 ≈ 200). They were operated at an overheat ratio of 1.5 with in-house constant temperature
92 anemometers. The anemometer output signals were passed through buck and gain circuits,
93 prior to low-pass filtering at a frequency $f_c \approx f_K$. The filtered signals were then digitized
94 at a frequency $f_s = 2f_c$ into a PC using a 12bit A-D converter. The sampling period was
95 in the range 45 to 100s. The mean velocity U_0 , upstream of the wake generator, was varied
96 so that the Taylor microscale Reynolds number R_{λ_u} ($\equiv \overline{u^2}^{1/2} \lambda_u / \nu$, where $\lambda_u = 15\nu \sqrt{\overline{u^2}/\bar{\epsilon}}$ is
97 the Taylor microscale) was in the range $150 \lesssim R_{\lambda_u} \lesssim 300$ at $x/d = 70$.

98 III. ONE-POINT ENERGY BUDGET

99 It is important that the one-point energy budget is known relatively accurately since it
100 represents the limiting state, as the separation r approaches the integral scale, of the s-b-s
101 energy budgets. Here, we focus on the region along the axis of the intermediate wake. The
102 general formulation of the one-point energy budget is

$$\frac{1}{2} \overline{U}_i \frac{\partial \overline{q^2}}{\partial x_i} + \overline{u_i u_j} \frac{\partial \overline{U}_i}{\partial x_j} + \frac{1}{2} \frac{\partial}{\partial x_j} \left(2\overline{u_j p} + \overline{u_j q^2} \right) + \bar{\epsilon} = 0. \quad (1)$$

103 (the summation convention applies). The first term on left hand side of Eq. (1) represents the
104 advection of turbulent kinetic energy by the mean velocity. The second term is interpreted
105 as the production of kinetic energy through the mean velocity gradient. The third term
106 corresponds to the diffusion of turbulent energy while the last term is the mean energy
107 dissipation rate.

108 Profiles along the y axis of $\overline{vq^2}$ approximated by $2\overline{vu^2} + \overline{v^3}$ indicated that the diffusion
109 of $\overline{q^2}$ by v is negligible along the axis since $\overline{vq^2} \approx \text{constant}$ on either side of the axis. A
110 similar conclusion was arrived at by examining the data supplied to us by Zhou²². The
111 latter measurements were made with a 3-component vorticity probe at $x/d = 10, 20$ and
112 40 . These data, with simultaneous information for u, v and w , indicated that $\overline{vq^2}$ was more
113 closely approximated by $2\overline{vu^2} + \overline{v^3}$ than by $\overline{vu^2} + 2\overline{v^3}$, *i.e.* axisymmetry with respect to y is

114 superior to that with respect to x . The latter axisymmetry along the y axis for large-scale
 115 statistics contrasts with Ref. 23 who observed a local axisymmetry with respect to the x axis
 116 at the level of the smallest scales. However, the present result appears reasonable since the
 117 influence of the Bénard-von Kármán street (a large-scale effect), which induces flow along
 118 the y direction cannot be neglected in this region of the wake.

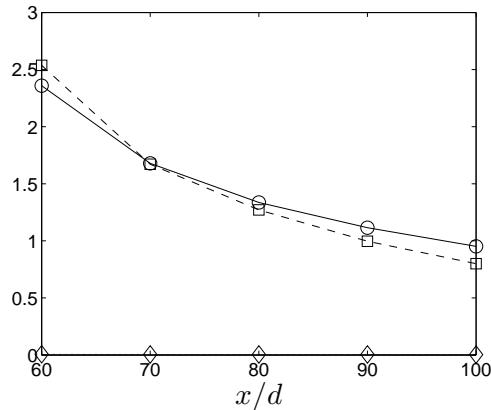


FIG. 2. Terms of the one-point kinetic energy budget (Eq. (2)) on the center-line of the wake of a circular cylinder normalized by U_0^3/d . \square $-\frac{1}{2}\bar{U}\frac{\partial\bar{q}^2}{\partial x}$, \circ $\bar{\epsilon}$, \diamond $(\overline{u^2} - \overline{v^2})\frac{\partial\bar{U}}{\partial x}$.

119 With the assumption that diffusion by the pressure p is also negligible on the axis, the
 120 one-point energy budget Eq. (1) reduces to

$$\frac{1}{2}\bar{U}\frac{\partial\bar{q}^2}{\partial x} + (\overline{u^2} - \overline{v^2})\frac{\partial\bar{U}}{\partial x} + \bar{\epsilon} = 0 \quad (2)$$

121 The distributions (Fig. 2) indicate that the production term associated with the difference
 122 in the Reynolds normal stresses is negligible. Consequently, Eq. (2) further simplifies to

$$\frac{1}{2}\bar{U}\frac{\partial\bar{q}^2}{\partial x} + \bar{\epsilon} = 0 \quad (3)$$

123 Although Fig. 2 shows that the advection and energy dissipation rate data crossover
 124 at $x/d = 70$, the magnitude of the two distributions are sufficiently close to each other to
 125 claim that the energy budget over this range of x/d is quite similar to that for decaying
 126 homogeneous isotropic turbulence (or HIT), as approximated by a turbulent grid flow. This
 127 is a considerable simplification in the context of estimating the s-b-s energy budget since
 128 only the decay term needs to be taken into account when r is sufficiently large.

129 IV. SCALE-BY-SCALE ENERGY BUDGETS

130 In view of the analogy with decaying turbulence HIT, the s-b-s budget is in essence
 131 provided by the Kármán-Howarth equation⁷ as detailed in Ref. 9. The latter authors
 132 assumed equilibrium similarity, as proposed by Ref. 8. The similarity scales are λ and $\overline{q^2}$. $\overline{q^2}$
 133 decays according to a power-law, whose exponent depends on the initial conditions. Second-
 134 and third-order structure functions, as they appear in the following transport equation^{24,25}

$$-\overline{\Delta u(\Delta q)^2} + 2\nu \frac{\partial}{\partial r} \overline{(\Delta q)^2} - \frac{\overline{U}}{r^2} \int_0^r s^2 \frac{\partial}{\partial x} \overline{(\Delta q)^2} ds = \frac{4}{3} \epsilon r, \quad (4)$$

135 may be written as

$$f_q(\tilde{r}) = \frac{\overline{(\Delta q)^2}}{\overline{q^2}}, \quad g_q(\tilde{r}) = -\overline{\Delta u(\Delta q)^2} \frac{3^{1/2} R_\lambda}{(\overline{q^2})^{3/2}} \quad (5)$$

136 where $\tilde{r} = r/\lambda$ and s in Eq. (4) is a dummy integration variable. Note that f_q and g_q depend
 137 only on \tilde{r} . Using Eq.(5), Eq. (4) can be rewritten as

$$g_q + 2f'_q - \left(\frac{5\Gamma_{q1}}{m_q} - 10\Gamma_{q2} \right) \tilde{r}^{-2} = \frac{20}{3} \tilde{r} \quad (6)$$

138 where

$$\Gamma_{q1} = \int_0^{\tilde{r}} \tilde{s}^3 f'_q d\tilde{s}, \quad \Gamma_{q2} = \int_0^{\tilde{r}} \tilde{s}^2 f_q d\tilde{s}. \quad (7)$$

139 The prime denotes differentiation with respect to \tilde{r} . The parameter m_q is the power-law
 140 decay exponent of $\overline{q^2}$, *i.e.*

$$\overline{q^2} \propto (x - x_0)^{m_q} \quad (8)$$

141 where x_0 is a virtual origin. Eq. (3) implies that $\overline{\epsilon}$ will also decay in a power-law fashion

$$\overline{\epsilon} \propto (x - x_0)^{m_\epsilon} \quad (9)$$

142 with $m_\epsilon = m_q - 1$. Since $\lambda^2 = 5\nu\overline{q^2}/\overline{\epsilon}$, it follows that the exponent m_λ , *viz.*

$$\lambda \propto (x - x_0)^{m_\lambda} \quad (10)$$

143 should be equal to 0.5. The expected linearity of λ^2 with respect to x was used to estimate
 144 x_0 ($\approx 8d$). The distributions, along the wake centreline of $\overline{u^2}$, $\overline{q^2}$, $\overline{\epsilon}$, λ and $\overline{U_d^2}$ ($\overline{U_d}$ is the
 145 maximum velocity defect) are shown in Fig. 3. Note that m_{U_d} ($\overline{U_d^2} \propto (x - x_0)^{m_{U_d}}$) is -0.68,
 146 which is substantially different from the value of -1, expected in the far wake. Also R_λ

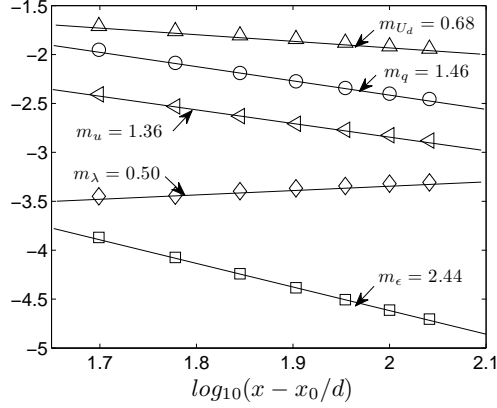


FIG. 3. Decay exponents on the wake center-line of the circular cylinder wake ; $\circ \log_{10}(\overline{q^2}/U_0^2)$, $\triangleleft \log_{10}(\overline{u^2}/U_0^2)$, $\square \log_{10}(\overline{\epsilon d}/U_0^3)$, $\diamond \log_{10}(10^{-1}\lambda/d)$, $\triangle \log_{10}(U_d^2/U_0^2)$, solid lines are linear fits to the data.

147 must decay with x through the intermediate wake ($R_\lambda \propto (x - x_0)^{-0.23}$) whereas it should be
 148 independent of x in the far wake.

149 For convenience, Eqs.(6) can be rewritten as

$$G_q + D_q + I_q = \frac{4}{3} \quad (11)$$

150 where $G_q = g_q \tilde{r}^{-1}/5$, $D_q = 2f'_q \tilde{r}^{-1}/5$ and $I_q = -(\Gamma_{q1}/m_q - 2\Gamma_{q2}) \tilde{r}^{-3}$. Each term can be
 151 estimated from measurements made at one spatial location provided m_q is available.

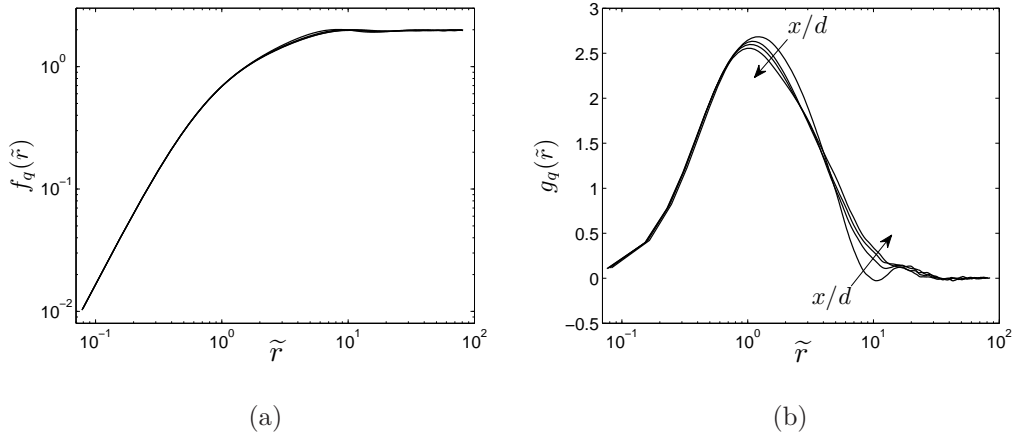


FIG. 4. (a) f_q and (b) g_q/\tilde{r} as a function of \tilde{r} , for x between $50d$ and $110d$ of the circular cylinder wake.

152 Measured distributions of f_q in Fig. 4(a) confirm that the collapse, based on λ and
 153 $\overline{q^2}$, is quite satisfactory. Note that f_q approaches its limiting value ($= 2$) at large \tilde{r} after
 154 overshooting it, due mainly to the contribution from $\overline{(\Delta v)^2}$, which is affected by the coherent
 155 motion²¹. The quality of the collapse for g_q (Fig. 4(b)) is not as good as that in Fig. 4(a); it
 156 is however at least as good, if not better, than that reported by Ref. 9 for grid turbulence.
 157 Note also that the curve of g_q at $x = 50d$ differs quite significantly from those beyond
 158 $x = 70d$ for which the collapse is much more satisfactory irrespectively of the downstream
 159 distance. Particularly, at a separation $\tilde{r} \approx 10$, g_q reaches negative values which is most likely
 160 due to the presence of the organized motion. This may highlight that a transition in the
 161 behaviour of the coherent motion occurs between $x = 50d$ and $70d$.

162 V. RESULTS FOR SCALE-BY-SCALE ENERGY BUDGETS IN THE 5 163 WAKES

164 Since all measurements in the five different wakes were made¹ at only one streamwise
 165 location ($x/d = 70$), equilibrium similarity, which was verified in the previous section for
 166 the wake of the circular cylinder, will be used, together with the value of m_q reported in
 167 Fig. 3, to construct the s-b-s budgets for these 5 wakes. Although one would expect some
 168 dependence of m_q on both Re_d and the initial conditions, we have assumed that it remains
 169 constant for the present purpose, since its influence on the s-b-s energy budget, especially
 170 for $R_\lambda \approx 200$ is likely to be small.

171 We first consider the distributions of f_q (Fig. 5) for the five different wakes. The main
 172 point which emerges from this figure is that the effect of initial conditions cannot be dis-
 173 counted, at least when \tilde{r} is sufficiently large. The collapse at small \tilde{r} is almost as good as
 174 that reported at small r^* ($\equiv r/\eta$) in Fig. 7 of¹, where $\overline{(\Delta u^*)^2}$ and $\overline{(\Delta v^*)^2}$ were plotted as
 175 functions of r^* (the asterisk stands for the normalization by the Kolmogorov scales). How-
 176 ever, although Kolmogorov scaling may be superior in the dissipative range, scaling on λ
 177 and $\overline{q^2}$ extends the collapse to a wider range of scales. In general, distributions for different
 178 wakes depart from each other at $\tilde{r} \approx 2$. Note also the significant overshoot in $f_q = \overline{(\Delta q)^2}/\overline{q^2}$
 179 for the screen strip (SS) with a maximum at $\tilde{r} = 12$. Included in Fig. 5 are the distributions
 180 (thin and bold curves) obtained from the models of Kurien & Sreenivasan²⁶ (hereafter KS)
 181 and Aivalis *et al.*²⁷ (henceforth A). The latter models rely on a parametric equation for

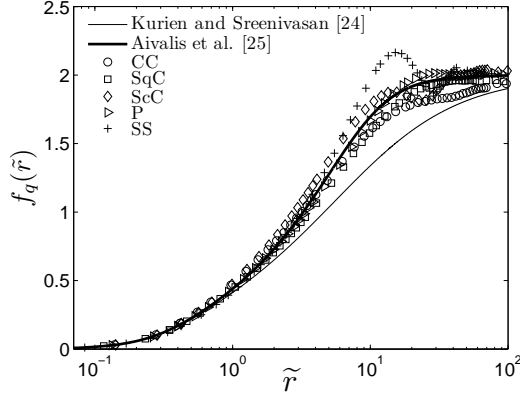


FIG. 5. Measured distributions of f_q as a function of \tilde{r} for the five different wakes. Also shown is f_q inferred from KS²⁶ (thin line) and A²⁷ (bold line). The Taylor microscale Reynolds number is approximatively the same in each case, *viz.* $R_{\lambda_u} \approx 195$. \circ CC, \square SqC, \diamond ScC, \triangleright P, $+$ SS.

182 second-order structure functions

$$\overline{(\Delta u)^{*2}}|_K = \frac{r^{*2} (1 + \beta r^{*})^{(2c-2)}}{15 (1 + \alpha r^{*2})^c} \quad (12)$$

$$\overline{(\Delta u)^{*2}}|_A = \frac{r^{*2} (1 + (\beta r^{*})^2)^{(c-1)}}{15 (1 + \alpha r^{*2})^c}, \quad (13)$$

183 where the subscript K and A refer to the models of KS²⁶ and A²⁷ respectively. In Eqs. (12)
 184 and (13), $\beta = L^{*-1}$ where $L^* = C_\epsilon R_\lambda^{3/2} 15^{-3/4}$ ($C_\epsilon = 1$ as in Refs. 9 and 28) is a measure of
 185 the integral length-scale. $\alpha = (15C_u)^{-3/2}$ ($C_u = 2$ was chosen^{9,28}) designates the cross-over
 186 between the dissipative and inertial range. Finally, $c = 1 - \zeta_u/2$ where ζ_u is the inertial
 187 range exponent of the longitudinal second order structure function. In contrast to Danaila
 188 *et al.*²⁹ or Thiesset *et al.*³⁰, for which ζ_u is function of the turbulent Reynolds number R_λ ,
 189 we assume here $\zeta_u = 2/3$ as in Refs. 9 and 28. Then, $\overline{(\Delta q)^2}$ is calculated using the isotropic
 190 relation

$$\overline{(\Delta q)^{*2}}|_{K,A} = (3 + r^* \frac{\partial}{\partial r^*}) \overline{(\Delta u)^{*2}}|_{K,A}. \quad (14)$$

191 After inserting Eq. (14) in Eq. (11), one can calculate the third-order structure function by
 192 difference. These models are thus only parametrized by the Reynolds number R_λ and the
 193 decay exponent m_q .

194 A comparison of Eqs.(13) and (12) indicates that the two models differ only through their

195 numerator, *i.e.*

$$\frac{\overline{(\Delta u)^{*2}}|_K}{\overline{(\Delta u)^{*2}}|_A} = \left[\frac{1 + 2\beta r^* + (\beta r^*)^2}{1 + (\beta r^*)^2} \right]^{(c-1)} \quad (15)$$

196 As $c < 1$, $\overline{(\Delta q)^{*2}}|_K < \overline{(\Delta q)^{*2}}|_A$ for every separation r as observed in Fig. 5. Thus, $I_q|_K < I_q|_A$
 197 and so $G_q|_K > G_q|_A$. Nevertheless, in the limit of large scales

$$\lim_{r \rightarrow \infty} \overline{(\Delta q)^{*2}}|_K = \lim_{r \rightarrow \infty} \overline{(\Delta q)^{*2}}|_A = \frac{6R_\lambda}{15^{1/2}} \quad (16)$$

198 and for small separations

$$\lim_{r \rightarrow 0} \overline{(\Delta q)^{*2}}|_K = \lim_{r \rightarrow 0} \overline{(\Delta q)^{*2}}|_A = \frac{r^{*2}}{3} \quad (17)$$

199 Therefore, the two models lead to a same asymptotic distributions of energy in the limit
 200 of small and large separations. The main difference between the two models occurs at
 201 intermediate scales.

202 We now turn our attention to the different terms in the s-b-s energy budgets displayed in
 203 Fig. 6. All the terms in Eq. (11) are shown in this figure. Differences in the budgets between
 204 the 5 wakes tend to occur when $\tilde{r} \geq 2$. This separation corresponds approximatively to the
 205 cross-over L_c (identified by vertical arrow in Fig. 6) between the transfer term and the large
 206 scale decay or inhomogeneous term. It follows that the energy distribution becomes more
 207 sensitive to the initial conditions the larger \tilde{r} is by comparison to L_c . This distribution is
 208 more influenced by the contribution of the relatively organized large scales which are more
 209 likely to remember the initial conditions.

210 G_q is much more affected by whether the wake generator is porous or solid than the other
 211 terms. For the porous wakes, the magnitude of G_q remains larger than that predicted by the
 212 two models when $\tilde{r} \geq 2$. This reflects the relatively strong organization in these two wakes
 213 and hence the stronger influence of the initial conditions. Note that the A²⁷ model fails
 214 to adequately represent G_q in these two cases (for $\tilde{r} \geq 2$) whereas it represents adequately
 215 the other terms (I_q and D_q) and is superior to the KS model²⁶. The sum $G_q + I_q + D_q$ is
 216 close to the expected value of $4/3$ for the solid generators. There is however a significant
 217 departure, typically in the range $2 \leq \tilde{r} \leq 10$, for the porous generators and especially SS. For
 218 these two wakes, the local anisotropy is greater than for the other 3 wakes²¹, reflecting the
 219 larger degree of organisation in the porous generator wakes than in the impervious generator
 220 wakes²¹.

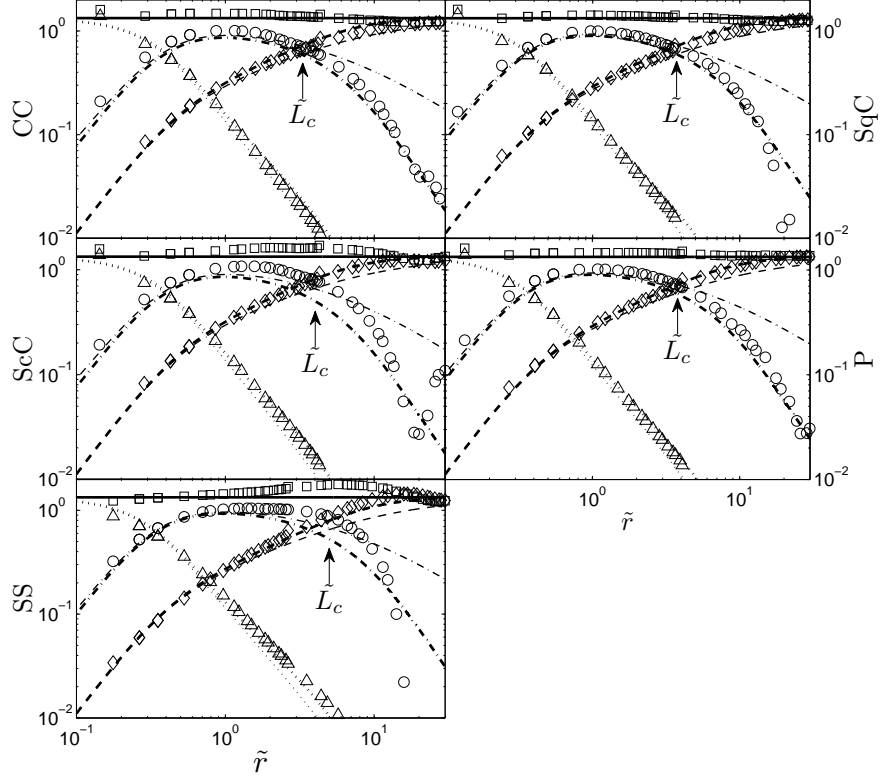


FIG. 6. Scale-by-scale energy budgets Eq. (11) in the five different wakes. Thin curves correspond to the model of KS²⁶ and bold curves to the model of A²⁷. Symbols denote the experimental data. \circ , $-\cdot-$: G_q ; \triangleright , \cdots : D_q ; \diamond , $- - -$: I_q ; \square , $—$: $(G_q + D_q + I_q)$. The thick horizontal line corresponds to $4/3$.

221 The crossover length scale L_c has been identified with the separation at which $G_q = I_q$.
 222 From Eq. (6), we can thus write

$$\tilde{L}_c = \left[\frac{1}{g_q(\tilde{L}_c)} \left(\frac{5\Gamma_{q1}(\tilde{L}_c)}{m_q} - 10\Gamma_{q2}(\tilde{L}_c) \right) \right]^{1/2}. \quad (18)$$

223 One expects \tilde{L}_c to depend on initial conditions since g_q , Γ_{q1} , Γ_{q2} and m_q should all be affected
 224 by the initial conditions. A plausible interpretation of L_c is that it represents a length scale
 225 at which the energy injected in the system starts being transferred towards small scales. It
 226 thus seems appropriate to normalize the mean energy dissipation rate $\bar{\epsilon}$ by L_c and $\overline{q^2}$, *i.e.*

$$\bar{\epsilon} = C_{ec} \frac{\left(\overline{q^2}/3 \right)^{3/2}}{L_c}. \quad (19)$$

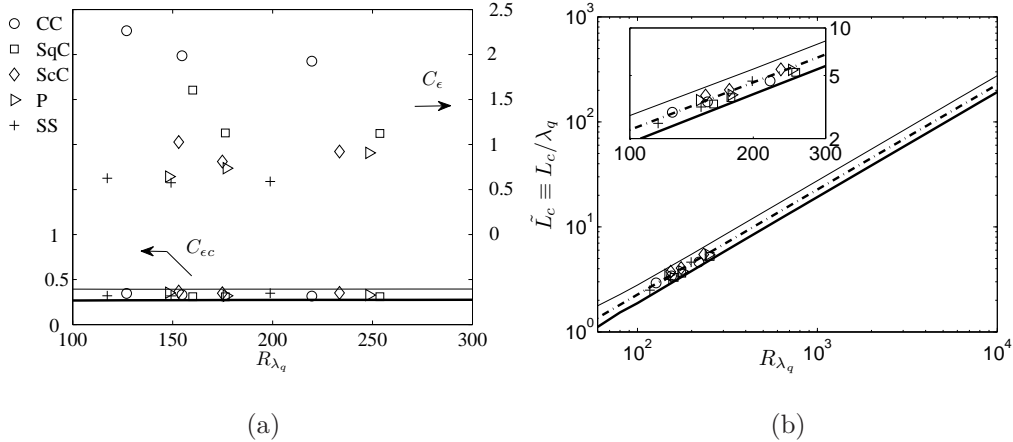


FIG. 7. (a) Evolution of the normalized dissipation rate parameters C_ϵ and $C_{\epsilon c}$ with R_λ for the five wakes. Also presented is the expectation for $C_{\epsilon c}$ using either the model of A²⁷ (thick lines) or KS²⁶ (thin lines). (b) Evolution of the crossover length scale L_c normalized by λ with the Taylor microscale Reynolds number R_λ . Models; — A²⁷ (thick lines), — KS²⁶ (thin lines), — · — $L_s/\lambda_q = C_{\epsilon c} R_{\lambda q}/15$ using $C_{\epsilon c} = 0.34$. Experiments \circ CC, \square SqC, \diamond ScC, \triangleright P, $+$ SS.

227 This differs from the classical definition of the normalized dissipation rate parameter C_ϵ

$$\bar{\epsilon} = C_\epsilon \frac{(\overline{u^2})^{3/2}}{L_u}, \quad (20)$$

228 where L_u is the longitudinal integral length-scale. Fig. 7(a) shows the evolution of the nor-
 229 malized dissipation rate parameter, defined with either Eq. (20) or (19). As was emphasized
 230 by Ref. 1, C_ϵ reaches very different values depending on the type of obstacle. Indeed, C_ϵ
 231 is equal to about 2 for CC wake and has a quite different value (≈ 0.6) in the SS wake.
 232 On the other hand, when the normalization of $\bar{\epsilon}$ is based on L_c and $\overline{q^2}$, $C_{\epsilon c}$ appears to be
 233 remarkably universal, *i.e.* it is independent of both the type of obstacle and the Reynolds
 234 number. The constancy of $C_{\epsilon c}$ is also reasonably well reproduced by the two models. The
 235 measured value of $C_{\epsilon c}$ is equal to 0.34 whereas the values predicted by the KS and A models
 236 are equal to 0.28 and 0.4 respectively.

237 It is evident that $C_{\epsilon c}$ has a much stronger claim to universality than C_ϵ . This is due in part
 238 to the fact that L_c is much more precisely determined than L_u but more importantly because
 239 L_c is an intermediate scale between that at which energy is injected into the flow and that
 240 ($r \sim \lambda$) at which the transfer of energy is maximum. In other words, the cross-over between
 241 G_q and I_q defines the scale at which the energy associated with all the inhomogeneous large

242 scales, arising from the way energy is injected into the flow, is exactly balanced by the energy
 243 transferred to the dissipative scales.

244 The use of L_c and $\overline{q^2}$ as the normalization scales for the normalized dissipation rate
 245 parameter has two main practical advantages. First, replacing $\overline{u^2}$ by $\overline{q^2}$ allows the large-
 246 scale anisotropy to be accounted for. Second, the estimation of L_c is much less arbitrary
 247 than that of L_u , the latter being usually inferred by integrating the longitudinal correlation
 248 function to its first zero crossing.

249 It seems more appropriate to consider the ratio L_c/λ since λ is the scale at which the
 250 energy transfer is maximum. This ratio can be thought of as a measure of the separation
 251 between large scale and inertial range effects. By using Eq. (19) and since $\lambda_q = 5\nu\overline{q^2}/\overline{\epsilon}$, we
 252 obtain

$$L_c/\lambda_q = \frac{1}{15}C_{cc}R_{\lambda_q}. \quad (21)$$

253 The magnitude of \tilde{L}_c , identified with the values of \tilde{r} for which $G_q = I_q$ is shown in Fig.
 254 7(b). Also included in this figure are the values of \tilde{L}_c which correspond to the two models. \tilde{L}_c
 255 is always smaller for the A model²⁷ than for the KS model²⁶. The measured values of \tilde{L}_c lie
 256 between the two models; although the range of R_λ is small, all the data are quite consistent
 257 with a linear variation of \tilde{L}_c with R_λ and are very close to the line $L_c/\lambda_q = C_{cc}R_{\lambda_q}/15$.

258 The interpretation for L_c provided here is similar to that for the shear length-scale L_S ,
 259 first introduced by Corrsin³¹ in the context of shear-dominated flows, and further physically
 260 interpreted in Refs. 11, 17, 18, and 32. Indeed, the authors emphasized that $L_S \equiv \sqrt{S^3/\overline{\epsilon}}$ ³¹
 261 corresponds to the crossover between contributions from the production term due to the
 262 mean shear S and the nonlinear transfer term in the spherically averaged scale-by-scale
 263 kinetic energy budget. Note also that the present authors have recently generalized the def-
 264 inition of L_S in order to account for the additional effect of the coherent strain using similar
 265 arguments as in Ref. 11, *i.e.* based on scale-by-scale budgets (see Ref. 21). This suggests
 266 that there exists a large variety of different relevant injection length-scales depending on
 267 the flow under consideration. For example, as far as wake flows are concerned, L_c and L_S
 268 coexist, L_c being the crossover between the decay term and the transfer term and L_S the
 269 crossover between the production (due to both the mean shear and the coherent shear) and
 270 the transfer term.

271 In addition, even though we observe that L_c restores universality for the dissipation rate,

272 its applicability for modeling the dissipation rate has the shortcoming that this applies only
 273 to decaying flows. In shear-dominated turbulence, L_S is likely to be the relevant injection
 274 length-scale for normalizing the dissipation rate. This point is worth investigating in the
 275 future.

276 Finally, L_c represents the largest length-scale that is in essence affected by the shape of
 277 the obstacle. Therefore, as shown in Ref. 32 for shear flows, for a proper implementation of
 278 a Large Eddy Simulation (LES), L_c represents the threshold scale that has to be simulated,
 279 scales smaller than L_c having the best prospect of being modeled by a universal function.

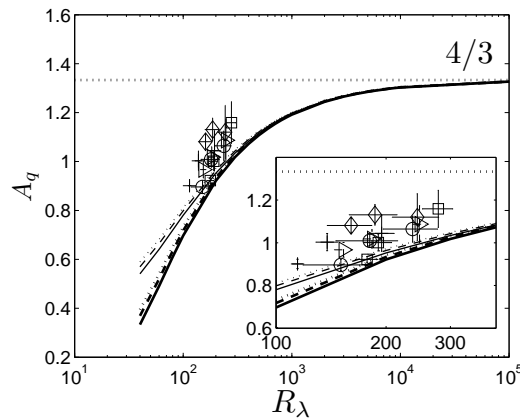


FIG. 8. Evolution of A_q with R_λ and the effect of m_q on A_q . Thick and thin curves correspond to the models of A²⁷ and KS²⁶ respectively. —: $m_q = 1.25$, - - -: $m_q = 1.5$, ·····: $m_q = 1.75$.
 ○ CC, □ SqC, ◇ ScC, ▷ P, + SS.

280 We now turn our attention to the maximum value of the energy transfer $A_q \equiv \max(\overline{(\Delta u)(\Delta q)^2}/\bar{\epsilon}r)$
 281 depicted in Fig. 8. Vertical and horizontal error bars are also given in order to appraise
 282 uncertainties in assessing A_q . Vertical error bars correspond to the standard deviation
 283 between different (isotropic) estimations of the dissipation rate. From the one-component
 284 vorticity probe oriented in either the $x - y$ plane or $x - z$ plane, one can calculate $\bar{\epsilon}$ from

285 the following six expressions

$$\bar{\epsilon}_{iso} = 15\nu \overline{\left(\frac{\partial u}{\partial x}\right)^2} \quad (22)$$

$$\bar{\epsilon}_q = 3\nu \left[\overline{\left(\frac{\partial u}{\partial x}\right)^2} + \overline{\left(\frac{\partial v}{\partial x}\right)^2} + \overline{\left(\frac{\partial w}{\partial x}\right)^2} \right] \quad (23)$$

$$\bar{\epsilon}_{\omega_z} = \nu \left(2\overline{\omega_z^2} + \overline{\omega_y^2} \right) \quad (24)$$

$$\bar{\epsilon}_{\omega_y} = \nu \left(\overline{\omega_z^2} + 2\overline{\omega_y^2} \right) \quad (25)$$

$$\bar{\epsilon}_{xy} = \nu \left[6\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 3\overline{\left(\frac{\partial u}{\partial y}\right)^2} + 2\overline{\left(\frac{\partial v}{\partial x}\right)^2} + \overline{\left(\frac{\partial u}{\partial y} \frac{\partial v}{\partial x}\right)} \right] \quad (26)$$

$$\bar{\epsilon}_{xz} = \nu \left[6\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 3\overline{\left(\frac{\partial u}{\partial z}\right)^2} + 2\overline{\left(\frac{\partial w}{\partial x}\right)^2} + \overline{\left(\frac{\partial u}{\partial z} \frac{\partial w}{\partial x}\right)} \right]. \quad (27)$$

286 Vertical error bars can thus be interpreted as a measure of anisotropy at the level of
 287 the smallest scales. A_q is calculated using $\bar{\epsilon}_{iso}$ which appears to be close to the mean value
 288 between these 6 different surrogates. Horizontal error bars are calculated from the difference
 289 between $R_{\lambda_u} = \sqrt{u^2}\lambda_u/\nu$ and $R_{\lambda_q} = \sqrt{q^2}\lambda_q/\sqrt{3}\nu$, where $\lambda_u^2 = 15\nu\overline{u^2}/\bar{\epsilon}_{iso}$ and $\lambda_q^2 = 5\nu\overline{q^2}/\bar{\epsilon}_{iso}$.
 290 The ratio between R_{λ_u} and R_{λ_q} is thus

$$\frac{R_{\lambda_u}}{R_{\lambda_q}} = \sqrt{\frac{3\overline{u^2}}{\overline{q^2}}} \left(\frac{\lambda_u}{\lambda_q} \right) = \frac{3\overline{u^2}}{\overline{q^2}} \quad (28)$$

291 which gives a measure of the anisotropy at the largest scales. Measured values of A_q are also
 292 compared together with those inferred from the A and KS models, for three different values
 293 of m_q (1.25, 1.5 and 1.75). Note that variations in the magnitude of m_q has virtually no
 294 effect on the way A_q approaches the asymptote of 4/3. The influence of the decay exponent
 295 is more discernible when R_λ is small, as the contribution from the large scale term becomes
 296 more pronounced. For R_λ larger than 100, the differences are not perceptible and the choice
 297 of a single value of m_q for the 5 wakes seems therefore reasonable.

298 Measured values of A_q are systematically larger than those predicted from the models
 299 of decaying isotropic turbulence by on average 15%. The two porous obstacles, for which
 300 CM is more prominent, appear to produce a higher maximum value of the energy transfer,
 301 independently of the Reynolds number. This observation indicates that the energy transfer
 302 is affected by the presence of the coherent motion, whose effect is to locally enhance the
 303 cascade mechanism. A similar observation was made by Ref. 33, for the wake of a circular
 304 cylinder at $x = 40d$ and a smaller Reynolds number ($R_\lambda \approx 70$).

305 VI. CONCLUSIONS

306 There are three major conclusions which arise from this work, all pertaining to the inter-
 307 mediate region of a turbulent wake.

308 (i) Firstly, the X-wire measurements at various streamwise locations along the axis of
 309 the wake indicate that equilibrium similarity, based on the Taylor microscale λ and the
 310 turbulent energy $\overline{q^2}$, is verified satisfactorily. In particular, the collapse of the energy transfer
 311 term $\overline{\Delta \tilde{u}(\Delta \tilde{q})^2} R_\lambda \overline{q^2}^{-3/2} \tilde{r}^{-1}$ appears to be of better quality than that previously obtained for
 312 decaying turbulence downstream a grid⁹. In the intermediate wake and for large separations,
 313 the one-point energy budget is closely approximated by a balance between advection and
 314 energy dissipation rate. This is substantially different to what is generally observed in the
 315 far field³⁴ in which the diffusion term has also to be taken into account. This difference
 316 between the intermediate and far wake might be related to a reorganization or rescaling
 317 of the large-scale structures^{4,35,36}, sometimes referred to as the secondary vortex street.
 318 Further investigations of velocity statistics and corresponding length-scales obtained over a
 319 sufficiently large distance between the near and far fields are needed to confirm this.

320 (ii) Secondly, the equilibrium similarity is used to estimate the s-b-s budgets, also along
 321 the centreline, for wakes generated by five two-dimensional bodies of different shapes. The
 322 closure of the budgets is less satisfactory for the two porous bodies (screen strip and screen
 323 cylinder) than for the solid bodies (cylinder, plate and square), reflecting the persistence
 324 of the coherent motion arising from the Kelvin-Helmholtz instability in the shear layers
 325 associated with the porous bodies.

326 Initial conditions are first felt at a separation L_c , which corresponds to the intersection
 327 point between the energy transfer and the large scale terms. For $r < L_c$, the shape of the
 328 generator is not important and the statistics are similar to those for decaying homogeneous
 329 isotropic turbulence. The parameter C_{ec} which involves normalization by L_c and $\overline{q^2}$, is
 330 remarkably independent of R_λ and almost unaffected by the initial conditions. Clearly,
 331 the ramification of this universality and its possible extension to other flows deserves to be
 332 investigated further. L_c/λ varies linearly with R_λ , a result in accord with existing models
 333 of $\overline{(\Delta q)^2}$ as well as our measurements.

334 (iii) Finally, the maximum value of the energy transfer term is systematically larger
 335 than that predicted by models for decaying HIT. This reflects the presence of the coherent

336 motion. The latter observation is important in the context of providing some insight into the
337 way the coherent motion interacts with small scales. Some progress has been made on this
338 interaction³³, by quantifying the contribution of the coherent motion to the energy transfer
339 after writing the s-b-s budgets for both the coherent and random motions.

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