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# Scale-by-scale energy budgets which account for the coherent motion

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**Abstract.** The scale-by-scale energy budget equation for turbulent kinetic energy has been written in flows with coherent motions. The general and the locally isotropic formulations are both provided. In particular, the contribution to the production, diffusion and energy transfer terms associated with the coherent motion are emphasized. Preliminary results are shown in the intermediate wake of a cylinder for the phase-scale second- and third-order structure function.

## 1. Introduction

Since the studies of the pioneer Townsend (1976), it is now well established that an organized motion persists in many turbulent shear flows (jets, wakes, shear layers,...). Recent efforts have been thus devoted to the extraction of coherent structures in the idea of learning more about their dynamical nature and their contribution to the Reynolds stresses. There are now many experimental and numerical support of the persisting influence of the organized motion far downstream the injection of turbulent energy (Cimbala *et al.* (1988), Bisset *et al.* (1990)).

The work of Reynolds & Hussain (1972) is the first analytical attempt to describe turbulent flow revealing an organized wave. They derived dynamical equations and one-point energy budgets for both the coherent and the random part of the velocity field. They proposed the phase averaging operator which allows to assess the dynamical process associated with coherent structures one step beyond the temporal averaging.

Nevertheless, at least two points - with one moving relatively to the other - are needed to define a scale and account for the wide and continuous variety of scales which characterize a turbulent flow. For that reason, two-point statistics (correlations of structure functions) were originally considered by Taylor (1935), Kármán & Howarth (1938), Kolmogorov (1941), Yaglom (1949) and many others since yet. These last stipulate that there is a scale beyond which the influence of the anisotropic/coherent large scales is no more perceptible and as a consequence the velocity statistics become isotropic. For that reason the organized motion were not considered.

Now, for most of turbulent flows, the separation between large and small scales is not wide enough for this supposition to be valid. The energy contribution of the largest scales - whether coherent or not - is still perceptible even on the smallest scales. In the light of previous remarks, it is therefore necessary to dissociate the energy contribution of the coherent motion from the rest of the purely turbulent field. The aim of this study is to derive the scale-by-scale budgets which

account for the coherent motion. The general as well as the locally isotropic formulations are provided. These equations put light on some additional terms corresponding to the transport, production, forcing of random fluctuations by the organized motion.

Then, we turn our attention on the particular case of the wake behind a cylinder in which the scale-phase second- and third- order structure functions are calculated from hot wire experiments. We particularly focus on this flow because it is reputed to reveal a turbulent but still persisting Bénard-Von Kármán street even in the so-called far field (Cimbala *et al.* (1988), Bisset *et al.* (1990)). The effect of the organized wave on the kinetic energy distribution and its dynamical influence on the energy transfer is evidenced. Also the scale-by-scale budget of the random field is well supported by experimental data.

## 2. Analytical background

### 2.1. General formulation

Starting from the triple decomposition  $U_i = \bar{U}_i + \tilde{u}_i + u'_i$ , ( $U_i$  is the instantaneous velocity in the  $i^{\text{th}}$  direction,  $\bar{[\cdot]}$ ,  $[\tilde{\cdot}]$  and  $[\cdot]'$  denote respectively the mean, the organized and the random motion) Reynolds & Hussain (1972) obtained the dynamical equations for both the random and the coherent motion :

$$\frac{\partial \tilde{u}_i}{\partial t} + \bar{U}_j \frac{\partial \tilde{u}_i}{\partial x_j} + \tilde{u}_j \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j - \bar{\tilde{u}_i \tilde{u}_j}) + \frac{\partial}{\partial x_j} (\langle u'_i u'_j \rangle - \bar{u'_i u'_j}) = -\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} \quad (1)$$

$$\frac{\partial u'_i}{\partial t} + \bar{U}_j \frac{\partial u'_i}{\partial x_j} + \tilde{u}_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial \bar{U}_i}{\partial x_j} + u'_j \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (u'_i u'_j - \langle u'_i u'_j \rangle) = -\frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_j^2} \quad (2)$$

In (1) and (2),  $\langle \cdot \rangle$  is the phase averaging operation and  $\nu$  is the kinematic viscosity and  $p$  is the pressure divided by the density. Following the procedure established by Antonia *et al.* (1997) and Danaïla *et al.* (2004), (1) and (2) are both written at a point  $\vec{x}$  and  $\vec{x}^+ = \vec{x} + \vec{r}$  and subtracted ( $\vec{r}$  is the separation vector between the two points and the superscript  $+$  hereafter denotes quantities considered at the point  $\vec{x} + \vec{r}$ ). Yields the transport equations of the organized and random velocity increment,  $\Delta \tilde{u}_i = \tilde{u}_i^+ - \tilde{u}_i$  and  $\Delta u'_i = u'_i{}^+ - u'_i$  respectively :

$$\begin{aligned} & \frac{\partial \Delta \tilde{u}_i}{\partial t} + \Delta \left( \bar{U}_j \frac{\partial \tilde{u}_i}{\partial x_j} \right) + \Delta \left( \tilde{u}_j \frac{\partial \bar{U}_i}{\partial x_j} \right) + \Delta \left( \frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j - \bar{\tilde{u}_i \tilde{u}_j}) \right) \\ & + \Delta \left( \frac{\partial}{\partial x_j} (\langle u'_i u'_j \rangle - \bar{u'_i u'_j}) \right) = - \left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \Delta \tilde{p} + \nu \left( \frac{\partial^2}{\partial x_j \partial x_j} + \frac{\partial^2}{\partial x_j^+ \partial x_j^+} \right) \Delta \tilde{u}_i \end{aligned} \quad (3)$$

$$\begin{aligned} & \frac{\partial \Delta u'_i}{\partial t} + \Delta \left( \bar{U}_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left( \tilde{u}_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left( u'_j \frac{\partial \bar{U}_i}{\partial x_j} \right) + \Delta \left( u'_j \frac{\partial \tilde{u}_i}{\partial x_j} \right) \\ & + \Delta \left( \frac{\partial}{\partial x_j} (u'_i u'_j - \langle u'_i u'_j \rangle) \right) = - \left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \Delta p' + \nu \left( \frac{\partial^2}{\partial x_j \partial x_j} + \frac{\partial^2}{\partial x_j^+ \partial x_j^+} \right) \Delta u'_i \end{aligned} \quad (4)$$

At this stage, the statistics at the two points  $\vec{x}$  and  $\vec{x}^+$  are considered as independent variables in a similar manner than Hill (2002) or Danaïla *et al.* (2004). Then, considering the gradient with respect to the mid-point  $\vec{X} = \frac{1}{2} (\vec{x} + \vec{x}^+)$  (Hill (2002), Danaïla *et al.* (2004)) :

$$\frac{\partial}{\partial x_j} = -\frac{\partial}{\partial r_j} + \frac{1}{2} \frac{\partial}{\partial X_j}, \quad \frac{\partial}{\partial x_j^+} = \frac{\partial}{\partial r_j} + \frac{1}{2} \frac{\partial}{\partial X_j} \quad (5)$$

multiplying (3) and (4) by  $2\Delta\tilde{u}_i$  and  $2\Delta u'_i$  respectively, applying a phase followed by a time averaging, and noting that :

$$\langle \Delta u'_j \Delta u'_i \rangle = \langle \Delta u_i \Delta u_j \rangle - \Delta\tilde{u}_i \Delta\tilde{u}_j \quad (6)$$

$$\overline{\langle \Delta u_j \Delta q^2 \rangle} = \overline{\Delta\tilde{u}_j \Delta\tilde{q}^2} + \overline{\Delta\tilde{u}_j \langle \Delta q'^2 \rangle} + \overline{\langle \Delta u'_j \Delta q'^2 \rangle} + 2\Delta\tilde{u}_i \overline{\langle \Delta u'_j \Delta u'_i \rangle} \quad (7)$$

we obtain the energy budgets for the organized and the random motion :

$$\begin{aligned} & \overline{\frac{\partial}{\partial t} \Delta\tilde{q}^2} + \overline{U_j \frac{\partial \Delta\tilde{q}^2}{\partial x_j}} + \frac{1}{2} \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \left[ \overline{(\tilde{u}_j + \tilde{u}_j^+) \Delta\tilde{q}^2} + 2 \overline{\langle (u'_j + u'_j^+) \Delta u'_i \rangle \Delta\tilde{u}_i} \right] \\ & + 2 \overline{\Delta\tilde{u}_i \Delta\tilde{u}_j \frac{\partial \overline{U}_i}{\partial x_j}} - \overline{\langle (u'_j + u'_j^+) \Delta u'_i \rangle} \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \Delta\tilde{u}_i + 2 \left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \overline{\Delta\tilde{u}_i \Delta\tilde{p}} \\ & \quad + \frac{\partial}{\partial r_j} \overline{\Delta\tilde{u}_j \Delta\tilde{q}^2} + 2\Delta\tilde{u}_i \frac{\partial}{\partial r_j} \overline{\langle \Delta u'_i \Delta u'_j \rangle} \\ & \nu \left[ - \left( 2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \overline{\Delta\tilde{q}^2} - 4 \left( \frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial \tilde{u}_j}{\partial x_i} + \frac{\partial \tilde{u}_i^+}{\partial x_j^+} \frac{\partial \tilde{u}_j^+}{\partial x_i^+} \right) \right] + 2(\tilde{\epsilon} + \tilde{\epsilon}^+) = 0 \quad (8) \end{aligned}$$

$$\begin{aligned} & \overline{\frac{\partial}{\partial t} \Delta q'^2} + \overline{U_j \frac{\partial \Delta q'^2}{\partial x_j}} + \frac{1}{2} \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \left[ \overline{(u'_j + u'_j^+) \Delta q'^2} + \overline{(\tilde{u}_j + \tilde{u}_j^+) \langle \Delta q'^2 \rangle} \right] \\ & + 2 \overline{\Delta u'_i \Delta u'_j \frac{\partial \overline{U}_i}{\partial x_j}} + \overline{\langle (u'_j + u'_j^+) \Delta u'_i \rangle} \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^+} \right) \Delta\tilde{u}_i + 2 \left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \overline{\Delta u'_i \Delta p'} \\ & \quad + \frac{\partial}{\partial r_j} \left( \overline{\langle \Delta u_j \Delta q^2 \rangle} - \overline{\Delta\tilde{u}_j \Delta\tilde{q}^2} \right) - 2\Delta\tilde{u}_i \frac{\partial}{\partial r_j} \overline{\langle \Delta u'_i \Delta u'_j \rangle} \\ & \nu \left[ - \left( 2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \overline{\Delta q'^2} - 4 \left( \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} + \frac{\partial u'_i^+}{\partial x_j^+} \frac{\partial u'_j^+}{\partial x_i^+} \right) \right] + 2(\epsilon' + \epsilon'^+) = 0 \quad (9) \end{aligned}$$

where  $\overline{\Delta\tilde{q}^2} = \overline{\Delta\tilde{u}_i \Delta\tilde{u}_i}$  and  $\overline{\Delta q'^2} = \overline{\Delta u'_i \Delta u'_i}$  (double indices indicate the summation) is the turbulent kinetic energy at a given scale of the organized and random motion respectively.

$\tilde{\epsilon} = \frac{\nu}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)^2$  and  $\epsilon' = \frac{\nu}{2} \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)^2$  is the mean dissipation of the coherent and the random motion.

For the sake of simplicity, (8) and (9) can be formally written as :

$$\mathcal{A}_{cm} + \mathcal{D}_{cc} + \mathcal{D}_{rc}^1 + \mathcal{P}_{cm} + \mathcal{P}_{rc} + \mathcal{D}_{cp} + \mathcal{T}_c + \mathcal{F}_c + \mathcal{V}_c + 2(\tilde{\epsilon} + \tilde{\epsilon}^+) = 0 \quad (10)$$

$$\mathcal{A}_{rm} + \mathcal{D}_{rr} + \mathcal{D}_{rc}^2 + \mathcal{P}_{rm} - \mathcal{P}_{rc} + \mathcal{D}_{rp} + \mathcal{T} - \mathcal{T}_c - \mathcal{F}_c + \mathcal{V}_r + 2(\epsilon' + \epsilon'^+) = 0 \quad (11)$$

where  $\mathcal{A}$ ,  $\mathcal{D}$ ,  $\mathcal{P}$ ,  $\mathcal{T}$ ,  $\mathcal{F}$  et  $\mathcal{V}$  denote respectively the advection, diffusion, production, transfer, forcing and viscous terms. The subscript  $m$ ,  $c$ ,  $r$  correspond to the mean, coherent and random motion, and  $\mathcal{D}_p$  indicate the pressure diffusion.

(8) and (9) are the general formulations of the scale by scale budgets which account for the coherent motion in which each term depends on the separation vector  $\vec{r}$ . For homogeneous flows and in the limit to large separations, the scale-by-scale budgets (8) and (9) are fully consistent

with the one point energy budget provided by Reynolds & Hussain (1972) (Eq(3.2b) et (3.2c) p.266 of Reynolds & Hussain (1972)). By means of DNS, each term can be evaluated without any other assumption. (8) and (9) will therefore constitute the general fundamental basis to unravel the physics of the coherent and random fields interaction.

As different from Danaïla *et al.* (2004), some additional terms appear. Particularly, the terms  $\mathcal{P}_{rc}$ ,  $\mathcal{T}_c$  and  $\mathcal{F}_c$ , identified as the production of random fluctuations by the coherent motion, the coherent kinetic energy transfer and the forcing associated by the presence of a coherent motion are emphasized. These terms are both present in the two equation (8) and (9) but their sign differ from one to the other. This means that which represents a loss of energy for the organized motion (8) constitutes a gain for the random motion (9). Furthermore we put light on the transport of random statistical quantities by the organized motion  $\mathcal{D}_{rc}^1$  and  $\mathcal{D}_{rc}^2$ .

## 2.2. Simplifications in the locally homogeneous and isotropic context

We now turn our attention on the derivation of (8) and (9) in a locally homogeneous and isotropic context. This assumption yields to considerable simplifications as regards to analytical treatments and experimental confrontations (Danaïla *et al.* (1999), Danaïla *et al.* (2001)). Furthermore, some comparisons with some previous isotropic considerations (Antonia *et al.* (1997), Danaïla *et al.* (2004), Yaglom (1949)) are needed at this stage.

Considering first a locally homogeneous turbulent flow, the viscous term simplifies :

$$\nu \left[ - \left( 2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \overline{\Delta q'^2} - 4 \left( \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i}} + \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i}} \right) \right] = 2\nu \frac{\partial^2}{\partial r_j^2} \overline{\Delta q'^2} \quad (12)$$

since  $\overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i}} = 0$  and  $\frac{\partial^2}{\partial X_j^2} = 0$  as shown by Hill (1997). The same simplification holds for the coherent motion. Then, in the context local isotropy, the divergence and the Laplacian operator are expressed as :

$$\frac{\partial}{\partial r_j} = \frac{2}{r} + \frac{\partial}{\partial r}, \quad \frac{\partial^2}{\partial r_j^2} = \left( \frac{2}{r} + \frac{\partial}{\partial r} \right) \frac{\partial}{\partial r} \quad (13)$$

Using (13), multiplying (8) and (9) by  $r^2 = r_j r_j$ , integrating over  $r$  and dividing by  $r^2$ , yields :

$$\begin{aligned} & \frac{1}{r^2} \int_0^r s^2 (\mathcal{A}_{cm} + \mathcal{D}_{cc} + \mathcal{D}_{rc}^1 + \mathcal{P}_{cm} - \mathcal{P}_{rc} + \mathcal{D}_{cp}) ds \\ & + \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds - 2\nu \frac{\partial}{\partial r} \overline{\Delta \tilde{q}^2} + \frac{4}{3} \tilde{\epsilon} r = 0 \end{aligned} \quad (14)$$

$$\begin{aligned} & \frac{1}{r^2} \int_0^r s^2 (\mathcal{A}_{rm} + \mathcal{D}_{rr} + \mathcal{D}_{rc}^2 + \mathcal{P}_{rm} + \mathcal{P}_{rc} + \mathcal{D}_{rp}) ds \\ & + \overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle} - \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} - \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds - 2\nu \frac{\partial}{\partial r} \overline{\Delta q'^2} + \frac{4}{3} \epsilon' r = 0 \end{aligned} \quad (15)$$

(14) and (15) are the scale-by-scale energy budgets of the organized and random motion respectively in a locally homogeneous and isotropic context.  $s$  is a dummy variable and the subscript  $\parallel$  denotes the direction parallel to direction of the separation vector. From experiments for which the spatial separation is inferred from the Taylor hypothesis, this direction coincides with the direction of the mean flow.

The first line of both (14) and (15) represents the energy contribution of largest scales (Danaila *et al.* (2004)). The main difference with the classical Yaglom's equation (second line of (15)) consists in some extra terms due to the presence of the coherent motion. The effective energy transfer of the random velocity component is explicit and is thus constituted of the total energy transfer  $\overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle}$  (coherent + random) from which is subtracted the coherent energy transfer  $\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2}$  and the forcing term  $\frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2} \langle \Delta u'_{\parallel} \Delta u'_i \rangle ds$ .

### 3. Results in the wake of a circular cylinder

#### 3.1. Measurements

The analytical considerations previously mentioned are now used to assess the essential physics and particularly the dynamical nature associated with the presence of the organized motion. Some previously reported hot wire measurements made in Newcastle (Australia) by Zhou *et al.* (2003) under the supervision of R.A Antonia are thus reinterpreted in terms of two-points statistics. The measurements were conducted in a open circuit wind tunnel with a square working section of  $0.35 \times 0.35m$  and  $2.4m$  long. The cylinder of diameter  $d = 12.7mm$  is placed horizontally to generate the wake flow. The three vorticity components are measured simultaneously by means of a four-X-wire probe (see Zhou *et al.* (2003)). The downstream location investigated here is  $40d$ , sufficiently far from the energy injection to expect the local isotropy to be verified and close enough to accurately extract the organized motion. The free stream velocity  $U_0$  is  $3m.s^{-1}$  corresponding to a Reynolds number based on the cylinder diameter and the upstream velocity of  $Re_d = 2525$  and a turbulent Reynolds number based on a typical fluctuation and the Taylor micro-scale of 70 on the wake centerline. For the estimates of velocity derivatives, the separations  $\Delta x \approx \Delta y \approx \Delta z$  are set equal to  $6\eta$ . This spatial resolution leads to an attenuation of velocity derivatives which were corrected using the spectral method proposed by Zhu & Antonia (1996). The mean dissipation rate is estimated using the isotropic relation  $\epsilon = 15\nu \overline{\left(\frac{\partial u}{\partial x}\right)^2}$  in which  $\overline{\left(\frac{\partial u}{\partial x}\right)^2}$  were corrected following the procedure of Zhu & Antonia (1996). The spatial separation  $\Delta x$  is calculated by means of the Taylor hypothesis,  $\Delta x = -U_c \Delta t$ , where  $U_c$  is the average convection velocity of vortices. This velocity is defined as the mean velocity at the vortices center location and is equal to  $U_c = 0.92U_0$  at  $40d$  downstream the cylinder (Zhou *et al.* (2003)).

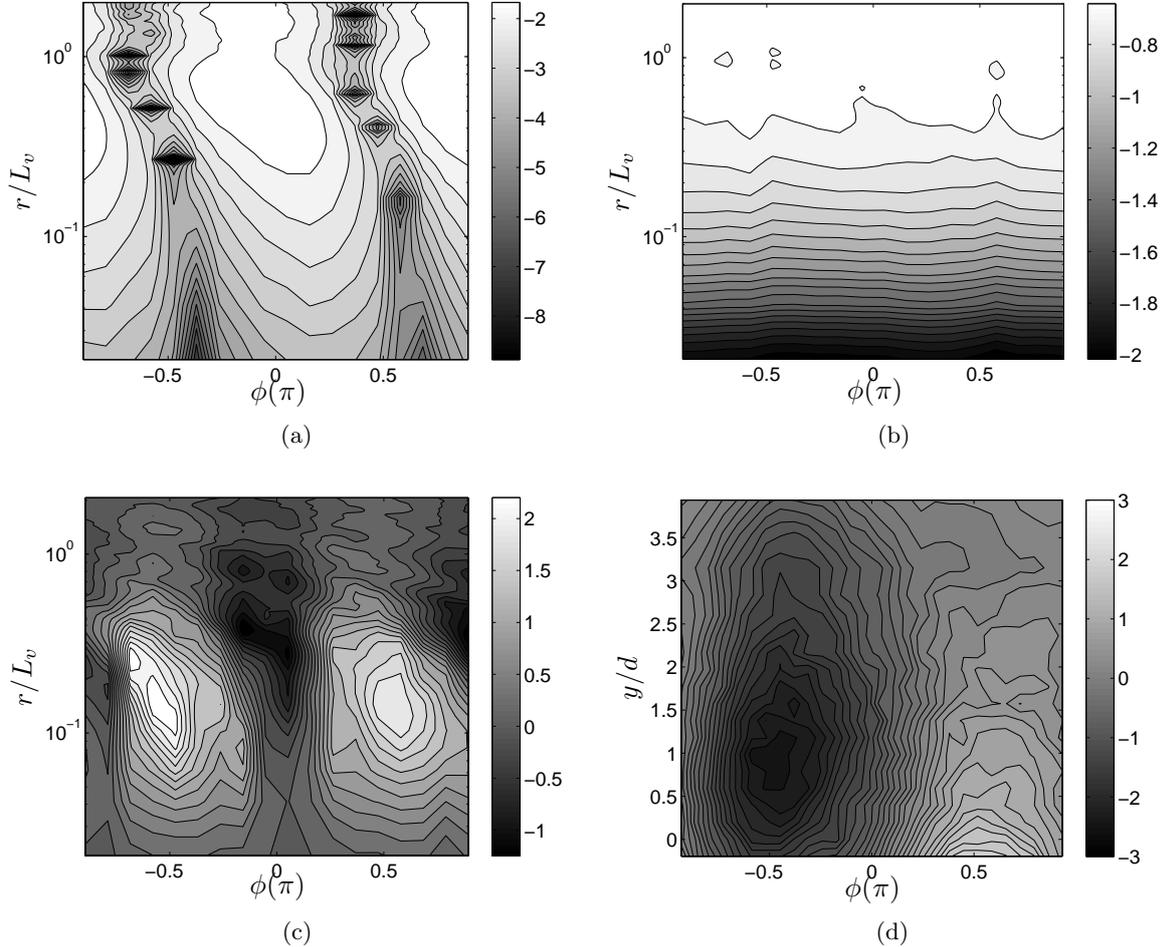
To calculate phase averaged statistics, the transverse velocity signal  $v$  is bandpass filtered at frequency centred on Strouhal frequency. The Hilbert transform  $h$  of the filtered signal  $v_f$  is calculated and the phase is inferred from  $\phi(t) = A \tan\left(\frac{h(t)}{v_f(t)}\right)$ . Finally, the phase is divided into 32 segments and phase averaged statistics are calculated for each class. We dispose of about 750 integral time-scales for each segment providing a correct convergence of statistics.

#### 3.2. Phase-scale second- and third-order structure functions

The essential difference compared to the classical energy budget is the phase averaging operation which allows us to assess the temporal dynamic associated with the coherent motion one step beyond the time averaging. Second- and third-order functions are as usual function of  $r$ , and also of the phase  $\phi$  of the organized wave, before being time-integrated.

Fig1(a) and 1(b) report the phase dynamics of the scale-by-scale distribution of the turbulent kinetic energy of the organized and random motion respectively on the wake center-line. In Fig1(a), one can note a strong temporal periodicity of period  $\phi = \pi$  on the kinetic energy distribution of the organized motion. This periodicity is also observable on the  $r$  axis when the organized motion is highly intense (not shown). This reveals the spatio-temporal periodicity of the Von Kármán street. Concerning the energy distribution of random fluctuations (Fig1(b)),

the influence of the organized motion is less perceptible. Its shape in scale  $r$  is very similar to that of the 'classical' time integrated second-order structure function.



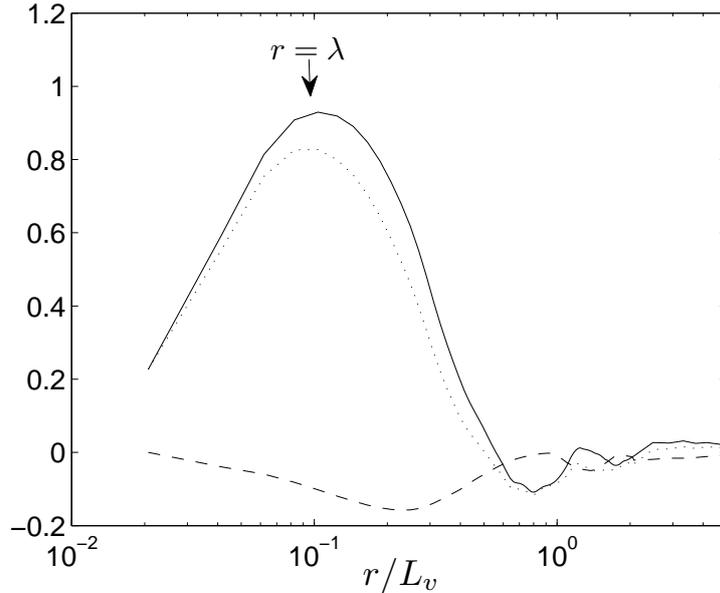
**Figure 1.** (a)  $\log_{10}(\Delta \tilde{q}^2)$  function of  $r$  and  $\phi$  on the wake center line at  $x = 40d$ ,  $r$  is normalized by  $L_v = 4.2d$ , the streamwise distance between two consecutive vortices. (b)  $\log_{10}(\langle \Delta q^2 \rangle)$  function of  $r$  and  $\phi$  on the wake center line at  $x = 40d$ . (c) Transfer term  $-\langle \Delta u_{\parallel} \Delta q^2 \rangle / \epsilon r$  function of  $r$  and  $\phi$  on the wake center line at  $x = 40d$ . (d) Phase averaging of the spanwise vorticity component  $\tilde{\omega}_z$  normalized by  $d/U_0$  in the plan  $(\phi, y)$ .

The non linear transfer term  $-\langle \Delta u_{\parallel} \Delta q^2 \rangle$  divided by  $\epsilon r$  is displayed on Fig1(c). The temporal periodicity is strongly discernible, and reveal two maximum at a phase location  $\phi = \pm\pi/2$  and a scale  $r \approx L_v/10 \approx \lambda$ , where  $\lambda$  is the Taylor micro-scale. Furthermore, at  $\phi = 0$  this term reveal some negative values.

In Fig1(d), is calculated the phase averaged spanwise component of the vorticity field. The Von Kármán street is thus magnified, and reveal a negative sign vortex centred at  $\phi = -\pi/2, y = 0.75d$ . Its partner of positive sign, not visible on the figure, is located at  $\phi = \pi/2, y = -0.75d$ . Therefore, the two maximum previously emphasized on the non linear transfer term coincide with the vortex center. It is therefore evidence that the organized motion induce a strong temporal dynamic on the transfer of kinetic energy at a scale  $r$ .

### 3.3. Scale-by-scale budget

We now turn our attention on time averaged structure functions. In Fig2, are displayed the total non linear transfer  $-\langle \overline{\Delta u_{\parallel} \Delta q^2} \rangle$ , the additional coherent transfer and forcing due to the coherent motion  $\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$  and the effective transfer inferred from their difference, in function of the separation  $r/L_v$ .



**Figure 2.** Non linear transfer term divided by  $\epsilon r$ . — total transfer term  $-\langle \overline{\Delta u_{\parallel} \Delta q^2} \rangle$ , - - - coherent transfer and forcing term  $\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$ , ..... difference  $-\langle \overline{\Delta u_{\parallel} \Delta q^2} \rangle + \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds$

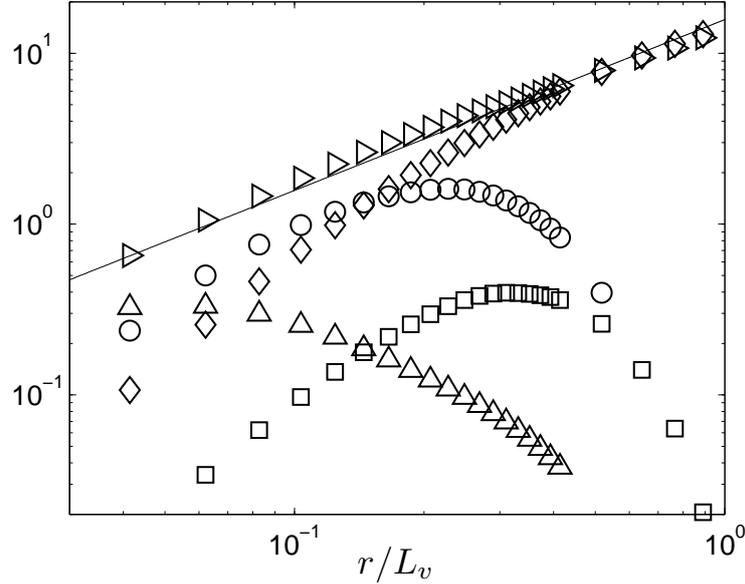
For weakly turbulent flows the non linear transfer term is less than  $\frac{4}{3}\epsilon r$  because of the cross over between viscous and large scale effects (Danaila *et al.* (1999), Danaila *et al.* (2004)). Here,  $-\langle \overline{\Delta u_{\parallel} \Delta q^2} \rangle / \epsilon r \approx 0.93$ . The additional energy transfer associated with the coherent motion is negative, its value is quite small but not negligible. Its contribution is non zero for all separation with a maximum contribution located at about  $2\lambda$ . Finally, the effective transfer of the random motion is quite smaller than the total transfer of about 12% at the maximum location.

On the wake centerline, the isotropic scale-by-scale budget of the random motion is :

$$-\frac{1}{r^2} \int_0^r s^2 \mathcal{A}_{rm} ds - \langle \overline{\Delta u_{\parallel} \Delta q^2} \rangle + \overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} + \frac{2}{r^2} \int_0^r \overline{\Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle} ds + 2\nu \frac{\partial}{\partial r} \overline{\Delta q'^2} = \frac{4}{3} \epsilon' r \quad (16)$$

which means that in the limit of large scales, the advection term is almost only compensated by dissipation of energy. All terms of the scale-by-scale budget (16) are shown on Fig3.

The balance of the right- and left-hand side of (16) is relatively well verified for all scales. The degree of accuracy of the experimental validation of (16) is as precise as the local isotropy is achieved. Therefore, the experimental investigation in the cylinder intermediate wake supports the analytical considerations provided in Section 1.



**Figure 3.** Term in equation (16) divided by  $\epsilon'$ . — :  $\frac{4}{3}r$ ,  $\diamond$  :  $-\frac{1}{r^2} \int_0^r s^2 \mathcal{A}_{rm} ds$ ,  $\circ$  :  $-\overline{\langle \Delta u_{\parallel} \Delta q^2 \rangle}$ ,  $\square$  :  $-\overline{\Delta \tilde{u}_{\parallel} \Delta \tilde{q}^2} - \frac{2}{r^2} \int_0^r \Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_i \rangle ds$ ,  $\triangle$  :  $2\nu \frac{\partial}{\partial r} \Delta q^2$ ,  $\triangleright$  Left hand side of (16)

#### 4. Concluding remarks

The scale-by-scale budgets of the kinetic energy of the organized and random field are derived in a general and isotropic formulation. It emphasized some additional diffusion, production, transport terms as well as some extra energy transfer and forcing associated with the presence of the coherent motion. These considerations explicitly evidences that the effective energy transfer of the random motion is constituted by the total energy transfer from which it is subtracted the coherent energy transfer and the forcing by the organized structures. The hot wire measurements in the cylinder wake are then employed to emphasized the temporal dynamic associated with the presence of the organized wave. It is shown that the energy transfer is clearly influenced by the coherent motion, revealing two maximum located in the vortex center. The time-integrated scale-by-scale budget (16) is also well supported by experimental calculations.

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