# Non-local model and global-local cracking analysis for the study of size effect

Cédric Giry, Cécile Oliver Oliver-Leblond, F. Ragueneau, Ejona Kishta

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C. Giry & C. Oliver-Leblond & F. Ragueneau
*LMT-Cachan (ENS Cachan/CNRS/UPMC/UniverSud Paris)*
*61, Avenue du Prsident Wilson, 94230 Cachan, France*

E. Kishta
*CEA,DEN,DANS,DM2S,SEMT, Laboratoire d'Etudes de Mécanique Sismique*
*CEA/Saclay F-91191 Gif-sur-Yvette, France*

ABSTRACT: The problematic of size effect for quasi-brittle materials and more particularly for concrete has been widely studied during the last decades. Several approaches have been proposed to describe and explain why the strength decreases as the size increases. Indeed, the capacity of a model to describe the size effect matters greatly when dealing with the modelling of structures.
A study of size effect is proposed in this article by considering several notched concrete beams submitted to a three points bending test. To describe the nonlinear behavior of concrete, two different nonlocal regularization methods (the original method and a method proposed recently by one of the authors) are compared to analytical model and to experimental results available in the literature in order to assess their capacity to describe size effect at the global scale as well as at the local scale. More particularly, a quantification of the crack field is obtained by using a discrete elements reanalysis method proposed recently by two of the authors. This last analysis brings an additional element to discriminate or not model capacity to describe size effect of quasi-brittle materials.

## 1 INTRODUCTION

Fracture behavior of quasi-brittle materials as concrete are commonly specimen-size and crack-size dependent. This is due to the fact that a fracture process zone with a non-negligible size in comparison with the structure dimensions develop in the media. Several theoretical approaches have been developed to describe the size effect observed in the transitional fracture behavior from small size specimen (i.e. in the order of the FPZ size) up to linear very large specimen (i.e. size of the FPZ negligible): asymptotic size effect law (from yield strength theory (YST) to linear elastic fracture mechanics (LEFM)) (Bažant 1984), multi-fractal scaling law (MFSL) (Carpinteri and Chiaia 1995) or local fracture energy concept considering the influence of boundary on the development of the FPZ (Hu and Wittmann 1992).

The quasi-brittle materials studied here show the presence of microcracks in their media. Under loading, these microcracks interact with each other, leading to nonlocal interactions. During the cracking, strain localization appears with a size and an orientation of the localized band as well as its evolution that can be directly linked to the nonlocal interactions due to microcracks. In continuous media, the microcracks are not explicitly represented. As a consequence, additional generalized constitutive equations need to be introduced in the models to take into account the nonlocal character of the propagation and coalescence of microdefects. These models replace the local internal variable by its nonlocal counterpart. For the gradient regularization method (Peerlings et al. 1996), the nonlocal internal variable fulfills a differential equation whereas for the nonlocal integral model (Pijaudier-Cabot and Bažant 1987), the nonlocal internal variable is a weighted spatial average. In addition to restoring the objectivity of the numerical modeling for strain softening behavior, these models aim at describing the behavior of quasi-brittle materials and introduce also an internal length allowing to describe size effect. Indeed, by introducing an internal length in the modeling, the size of the FPZ is explicitly defined.

(Simone et al. 2004) and (Jirásek et al. 2004) for the description of size effect, have pointed out the problematic of the description of the nonlocal interactions in the vicinity of a notch tip. To address the description of size effect for notched beams under three-point bending loading, this problem is of main concern as the FPZ develop in this area at peak load. Several attempts have been made recently to modify the description of the nonlocal interactions close to boundary (Krayani et al. 2009) and by integrating the influence of the stress state (Giry et al. 2011). This last one allows for an improved description of both the degradation in the vicinity of boundaries and the evolution of cracking at failure. This modified nonlocal integral model is used in this study and compared to the original version.

In order to get local information such as cracking, a global-local approach has been proposed (Oliver-Leblond et al. 2013). At the global scale, a non-local damage model describes the complete behaviour of the concrete structure and a reanalysis of the damaged areas observed during the loading is performed at the local scale thanks to a discrete element model. The microcrack area as well as the macrocrack propagation – with the crack opening level along it can be obtained. This information is an important aspect as it is a necessary data to perform durability analysis of concrete structures.

In the first section, the model considered to describe the nonlinear behavior of concrete is presented. In particular, the main equations of two nonlocal regularization methods are exposed. Then, in order to get local information to analyse the FPZ, an original method of reanalysis of the continuous calculation is exposed. The damage area obtained from the calculation is reanalyzed with a discrete elements model giving an explicit description of the micro-cracks and macro-crack. In the third part, the main aspects of the Bažant size effect law are recalled and the procedure to get an estimation of the FPZ size. In the last part, the models presented with both regularization methods are analyzed in the framework of size effect analysis. Notched concrete beams of different sizes submitted to a three points bending test are considered. Classical results as the global behavior (i.e. size effect on the nominal strength) are given and a more original study is performed by analyzing the description of the FPZ in comparison with experimental results from (Alam et al. 2013). A comparison between both non-local models is performed in order to assess the capacity of the model to describe size effect at the global and local scale.

## 2 DESCRIPTION OF DEGRADATION IN CONCRETE

### 2.1 *Constitutive equations*

A continuum framework is considered to describe the nonlinear behavior of concrete. The main constitutive equations are formulated within the framework of the thermodynamics for irreversible process in order to fulfill conservation and evolution principles. A model based on isotropic damage mechanics, accounting for elasticity, damage, sliding between cracked surfaces and hardenings, is used for this study ((Richard et al. 2010)). Eq.1 gives the expression of the state potential:

$$\begin{aligned}\rho\psi \;=\;& \frac{1}{2}\{\frac{\kappa}{3}((1-d)\langle\varepsilon_{kk}\rangle_+^2 - \langle-\varepsilon_{kk}\rangle_+^2) + \\ & 2(1-d)\mu\varepsilon_{ij}^D\varepsilon_{ij}^D + 2d\mu(\varepsilon_{ij}^D-\varepsilon_{ij}^\pi)(\varepsilon_{ij}^D-\varepsilon_{ij}^\pi) + \\ & \gamma\alpha_{ij}\alpha_{ij}\} + H(z),\end{aligned} \tag{1}$$

where $\rho$ is the material density, $\kappa$ and $\mu$ are the bulk and shear coefficients, respectively. $\varepsilon_{ij}$ is the second order total strain tensor, $\varepsilon_{ij}^D$ is the second order deviatoric total strain tensor and $d$ is the scalar damage variable evolving from 0 (virgin material) to 1 (failed material). $\langle A_{ij}\rangle_+$ accounts for the positive part of the tensor $A_{ij}$. $\varepsilon_{ij}^\pi$ is the second order sliding tensor, $\gamma$ is a material parameter, $\alpha_{ij}$ is the second order tensor associated to the kinematic hardening, $z$ is the internal variable related to the isotropic hardening and $H$ the consolidation function.

This potential state leads to the relation between the Cauchy stress tensor $\sigma_{ij}$ and the total strain tensor $\varepsilon_{ij}$ (Eq.2).

$$\begin{aligned}\sigma_{ij} = \frac{\partial\rho\psi}{\partial\varepsilon} = & \frac{\kappa}{3}((1-d)\langle\varepsilon_{kk}\rangle_+ - \langle-\varepsilon_{kk}\rangle_+)\delta_{ij} + \\ & 2(1-d)\mu\varepsilon_{ij}^D + 2d\mu(\varepsilon_{ij}^D-\varepsilon_{ij}^\pi),\end{aligned} \tag{2}$$

### 2.2 *Nonlocal regularization method*

A regularization method needs to be associated to the model presented in the previous section in order to keep the objectivity of the results in a numerical framework. In this work, two nonlocal regularization methods are considered and compared. The first one is the original nonlocal method (ONL) on internal variables proposed by (Pijaudier-Cabot and Bažant 1987) and the second one is a stress based nonlocal method (SBNL) proposed recently by one of the authors (Giry et al. 2011).

For the damage model considered, the regularization method replace the local damage energy released rate $\overline{Y}$ (the variable driving the damage $d$) by its nonlocal counterpart $\overline{Y}^{NL}$ according to Eq.3.

$$\overline{Y}^{NL}(\mathbf{x}) = \frac{\int_{\Omega(x)}\phi(\mathbf{x}-\mathbf{s})\overline{Y}ds}{\int_{\Omega(x)}\phi(\mathbf{x}-\mathbf{s})ds}, \tag{3}$$

$\phi(\mathbf{x}-\mathbf{s})$ is a weight function chosen as the Gaussian function (Eq.4).

$$\phi(\mathbf{x}-\mathbf{s}) = \exp\left(-\frac{4\|\mathbf{x}-\mathbf{s}\|^2}{l_c^2}\right), \tag{4}$$

where $l_c$ is the internal length of the model.

In order to take into account a modification of the nonlocal interactions close to free boundaries and an evolution of these interactions during the progressive degradation of the material, the stress based nonlocal method introduced a coefficient of influence $\rho$ expressed as a function of the stress state at the location of the redistributing point. The internal length is written (Eq.5):

$$l_c(\mathbf{x}, \sigma(\mathbf{s})) = \rho(\mathbf{x}, \sigma(\mathbf{s})).l_{c_0}, \tag{5}$$

# 3 CRACKING ANALYSIS OF A CONTINUOUS MEDIA

In this section the global/local method used in a reanalysis of the continuous calculation is described. This approach allows to give local information relative to cracking (e.g. location, opening...) by reanalyzing the damaged areas of the structure studied in a continuous framework (Oliver-Leblond et al. 2013). Fig.1 gives the process of analysis of the method. At

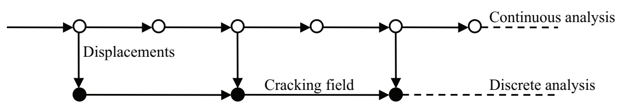



Figure 1: Global/Local sequential analysis.

the global scale, the nonlinear behavior of the structure is described thanks to the model presented in Section 2. The obtained damage pattern is studied and several Regions of Interest (ROIs) are defined corresponding to distinct areas of damage. Then, the loading steps for the extraction of crack features are determined and will correspond to the steps of reanalysis. For those loading steps, boundary conditions are extracted from the continuous displacement field and applied on the non-free surfaces of the ROIs  namely the ones which cut the whole domain. The natural way to transfer the displacement field from the global scale to the local scale is to use the shape functions of the finite elements used for the global computation. Then, the displacement $u_L(x_L)$ applied at a local node $x_L$ of a non-free surface of a ROI is directly obtained with Eq.6.

$$u_L(x_L) = \sum_{j=1}^{N_{FE}} N_j u_L(x_L) u_j, \tag{6}$$

where $N_j$ are the shape functions of the finite element model, $u_j$ is the displacement vector computed at the global scale and $N_{FE}$ is the number of finite element nodes. The cracking pattern obtained at the previous step will be retrieved at the current step of the local reanalysis in order to follow the crack propagation accurately. The discrete computation of the chosen ROIs can be parallelised.

## 3.1 *Local model*

The discrete model used at the local scale has been proposed by (Delaplace 2005) and offers a reliable description of concrete behaviour for tensile loadings. The material is described as an assembly of polyhedral particles linked by Euler-Bernoulli beams. The quasi-brittle behaviour of the material is obtained through a brittle behaviour for the beams. The breaking threshold $P_{ab}$ of $a-b$, the beam linking the particles $a$ and $b$, not only depends on the beam extension $\varepsilon_{ab}$ but also on the rotations of the two particles $\theta_a$ and $\theta_b$ (Eq.7).

$$P_{ab} = \left( \frac{\varepsilon_{ab}}{\varepsilon^{cr}}, \frac{|\theta_b - \theta_a|}{\theta^{cr}} \right) > 1, \tag{7}$$

The six parameters of the beam $a-b$ need to be calibrated. First, the length and the area are imposed by the geometry. Then, the inertia and the elastic modulus are identified in order to retrieve the elastic behaviour of the global computation. Finally, the calibration of the breaking thresholds of the beam $\varepsilon^{cr}$ and $\theta^{cr}$ allows us to fit the peak and post-peak behaviour of the global model. The calibration is performed on an independent case study (Oliver-Leblond et al. 2013). Our study focuses on a fine description of crack pattern and on the measurement of the crack opening. The crack pattern is defined as the common side of the particles initially linked by the breaking beams. The opening of the crack is computed by considering the relative displacement $u_b - u_a$ of the unlinked particles $a$ and $b$. The measure of the opening between those particles $e_{ab}$ is projected on the normal $n_{ab}$ of the local discontinuity (Eq.8).

$$e_{ab} = \langle (u_b - u_a).n_{ab} \rangle_+, \tag{8}$$

## 3.2 *Quantification of cracks properties*

In order to get information on cracking from the discrete element reanalysis, additional tools have been considered. The micro cracked domain obtained from the discrete element reanalysis is described in graph theory (Fig.2).

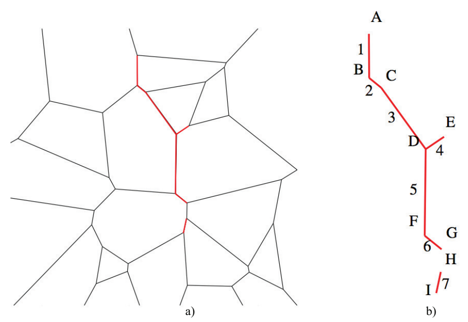



Figure 2: Discrete elements description of the media with micro cracks a) Representation of microcracks in graph theory b).

Using the Bellman-Ford algorithm in this framework, an identification of the macro-crack in the domain reanalyzed is obtained. In order to choose the right crack path at a crossroad, the crack opening associated to each discrete element is used as a weight.

## 4 SIZE EFFECT ANALYSIS

### 4.1 *Size effect law*

Quasi-brittle materials such as concrete shows a dependency of their nominal strength on structure size. A typical test to analyse size effect is a 3-points bending test on notched beams of different sizes. In this configuration, a large notch is developed before reaching the maximum load and the size effect on the nominal strength is mainly energetic with a small influence of the material heterogeneity (Bažant and Xi 1991). The evolution of the nominal strength $\sigma_N$ in function of a characteristically size of the beam $D$ can be approximately be described by the size effect law in Eq.9 (Bažant 1984).

$$\sigma_N = Bf'_t \frac{1}{\sqrt{1+\frac{D}{D_0}}}, \tag{9}$$

with $D_0$ and $Bf'_t$ expressed as (Eq.10):

$$D_0 = c_f \frac{g'(\alpha_0)}{g(\alpha_0)} \quad Bf'_t = \sqrt{\frac{EG_f}{c_f g'(\alpha_0)}}, \tag{10}$$

with $\alpha_0 (= a_0/D)$ the ratio between the notch size and the height of the beam, $c_f$ the effective length of fracture process zone for an infinite media, $G_f$ the fracture energy and $g$ is a dimensionless energy release function of equivalent LEFM characterizing the specimen geometry (12).

### 4.2 *Fracture process zone length*

(Bažant and Kazemi 1990) propose from the size effect law presented in the previous subsection a method to estimate the fracture process zone length $c$. This value corresponds to the length of the area at the crack tip in which microcracking or void formation takes place. It can be estimated by solving Eq.11.

$$c = c_f \frac{D}{D+D_0} \frac{g'(\alpha_0)}{g(\alpha_0)} \frac{g(\alpha)}{g'(\alpha)}, \tag{11}$$

where $c_f$ is the value identified in Eq.10 corresponding to $\lim_{D\to\infty} c = c_f$. $\alpha$ is the ratio between $a$ (=$a_0 + c$) and $D$ the characteristical size of the structure. The function $g$ considered in this study is an approximation derived by (Pastor et al. 1995) (Eq.12).

$$\begin{aligned} g(\alpha) &= k_{\frac{S}{D}}(\alpha)^2 \\ k_{\frac{S}{D}}(\alpha) &= \sqrt{\alpha} \frac{p_{\frac{S}{D}}(\alpha)}{(1+2\alpha)(1-\alpha)} \\ p_{\frac{S}{D}}(\alpha) &= p_\infty(\alpha) + \frac{4D}{S}[p_4(\alpha) - p_\infty(\alpha)], \end{aligned} \tag{12}$$

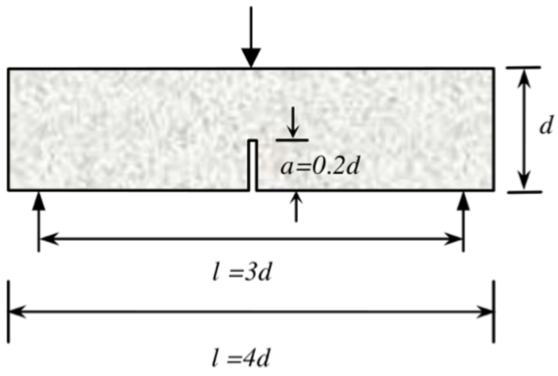



Figure 3: Geometry of the notched beams (from Alam et al.,2013).

with
$p_4(\alpha) = 1.9 - \alpha[-0.089 + 0.603(1-\alpha) - 0.441(1-\alpha)^2 + 1.223(1-\alpha)^3]$
and
$p_\infty(\alpha) = 1.989 - \alpha(1-\alpha)[0.448 - 0.458(1-\alpha) + 1.226(1-\alpha)^2]$

## 5 THREE-POINT BENDING TEST

### 5.1 *Presentation of the experiments and model parameters identification*

The experimental campaign on three-point bending test performed by (Alam et al. 2013) are used in the present work in order to assess the capacity of the models presented to capture cracking process in the framework of size effect. Three sizes for the notched beams have been considered with a constant depth b (=10cm) and the following characteristical dimension $d$ per size (Fig.3): $D_1$ = 10 cm, $D_2$ = 20 cm and $D_3$ = 40 cm.

A Levenberg-Marquardt optimization algorithm has been applied to get a set of parameters for both nonlocal method (Le Bellégo et al. 2003) by minimizing the functional $\Im(\mathbf{p})$ defined in Eq.13.

$$\Im(\mathbf{p}) = \sqrt{\sum_{i=1}^{n} \left( \frac{C^i_{num}(\mathbf{p}) - C^i_{exp}(\mathbf{p})}{C^i_{exp}(\mathbf{p})} \right)^2}, \tag{13}$$

with $n$ the number of measurements during the loading, $\mathbf{p}$ the vector of the parameters to optimize, $C^i_{num}(\mathbf{p})$ the i$^{th}$ point of the global behavior from the numerical calculation and $C^i_{exp}(\mathbf{p})$ the i$^{th}$ point of the global behavior from the experiment.

The model parameters identified are summarized in Tab.1.

In order to identify the model parameters associated to the discrete element reanalysis, another mechanical test has been performed with a continuous modeling using both nonlocal methods and with the discrete elements. By minimizing the error between the discrete elements analysis and the finite element analysis, the parameters given in Tab.2 have been obtained. A Weibull distribution is considered for the elements. $k$ corresponds to the shape parameter, $\lambda$ to the scale factor and $\varepsilon$ to the cutoff frequency (Tab. 2).

Table 1: Model parameters obtained from the identification method performed on the medium beam.

| Parameters | Original nonlocal | | Stress based nonlocal | |
|---|---|---|---|---|
| $E$ | 32 | GPa | 32 | GPa |
| $\nu$ | 0.21 | - | 0.21 | - |
| $f_t$ | 3.5 | MPa | 3.5 | MPa |
| $A_{Dir}$ | $3.7\times 10^{-3}$ | $J^{-1}m^3$ | $3.0\times 10^{-3}$ | $J^{-1}m^3$ |
| $A_{Ind}$ | $3.5 \times 10^{-4}$ | $J^{-1}m^3$ | $3.5\times 10^{-4}$ | $J^{-1}m^3$ |
| $\gamma$ | $5.0 \times 10^6$ | Pa | $5.3 \times 10^6$ | Pa |
| $a$ | $7.0 \times 10^7$ | $Pa^{-1}$ | $7.0 \times 10^7$ | $Pa^{-1}$ |
| $l_c\ \gamma$ | 12 | mm | 12.5 | mm |

Table 2: Model parameters for the discrete element reanalysis.

| Parameters | Original nonlocal | | Stress based nonlocal | |
|---|---|---|---|---|
| $l_p$ | 2 | mm | 2 | mm |
| $E$ | 40.5 | GPa | 40.5 | GPa |
| $\alpha$ | 0.83 | - | 0.83 | - |
| $\lambda_{\varepsilon_{cr}}$ | $2.9\times 10^{-4}$ | - | $3.2\times 10^{-4}$ | - |
| $\lambda_{\theta_{cr}}$ | $4 \times 10^{-3}$ | - | $4\times 10^{-3}$ | - |
| $k$ | 0.75 | - | 0.88 | - |
| $\varepsilon_{min}^{cr}$ | $1.5\times 10^{-5}$ | - | $1.2 \times 10^{-5}$ | - |

## 5.2 *Global analysis of size effect*

The results obtained for the experimental average evolution of the force applied versus the crack mouth opening displacement (CMOD) at the notch (average over 3 tests per size) (Alam et al. 2013) and for the numerical test with both nonlocal methods are given in Fig.4 .

From the average nominal strength of each size one can identify the parameter of the size effect law (Eq.9) : $B.f_t' = 3.03$ MPa and $D_0 = 420.5$ mm. Fig.5 gives the results obtained for the nominal strength for the experimental, the numerical results and the size effect law.

One can see from Fig.5 that both numerical models give a good description of the global behavior. By identifying the parameters on one size (beam $D_2$), both models are capable of reproducing size effect on nominal strength for the two other beam sizes.

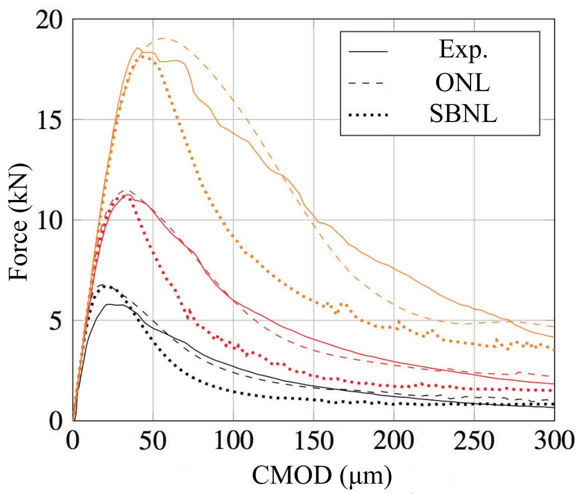



Figure 4: Evolution of the force versus the CMOD for the three size ($D_1$, $D_2$ and $D_3$).

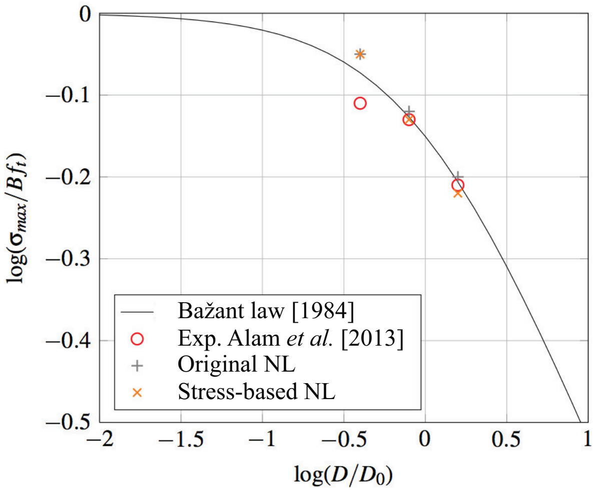



Figure 5: Logarithmic diagram of the size effect law for the three beam sizes.

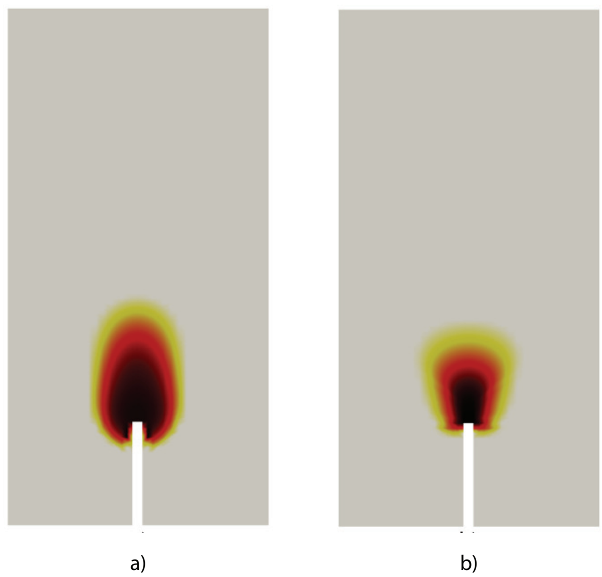



Figure 6: Damage field at the peak load for the original nonlocal method a) and the stress based nonlocal method b).

## 5.3 *Crack analysis at peak load*

Fig.6 gives the damage field observed at peak load for both nonlocal methods.

The difference observed for the damaged area at peak load between both nonlocal models can be analyzed with Fig. 7 and 8. One can see on these figures the nonlocal quantities redistributed in a notched plate under tension. The redistributed quantities for a point at notch tip, a point back to the tip and a point in front of the tip are given.

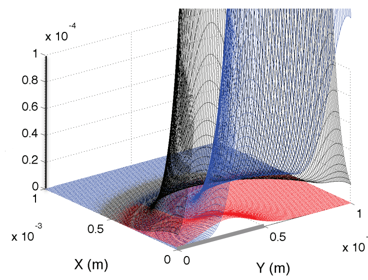



Figure 7: Nonlocal quantities redistributed in the tip area (Original nonlocal model).

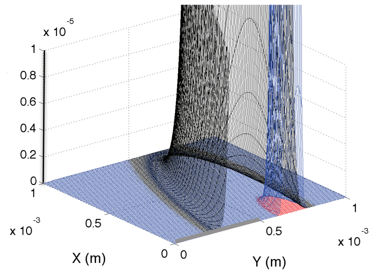



Figure 8: Nonlocal quantities redistributed in the tip area (Stress based nonlocal model).

One can see on Fig. 7 and 8 that by considering an isotropic influence for the original nonlocal model (i.e. the internal length of the model is a scalar) an overestimation of the redistributed quantities is obtained. As a consequence, points located back to the notch tip have nonlocal quantities that overpass the threshold for damage initiation. In contrary, by introducing a influence factor for the nonlocal redistribution, points back to the notch tip are no more influenced by the points at notch tip.

Tab.3 gives a comparison for the fracture process zone length at the peak obtained from LEFM (size effect law) and for numerical simulations with both nonlocal models for the three beam sizes.

Table 3: Fracture process zone length $c$.

| Beam | Size effect law | ONL | SBNL |
|---|---|---|---|
| $D_1$ (mm) | 15.1 | 23 | 22 |
| $D_2$ (mm) | 26.1 | 47 | 32 |
| $D_3$ (mm) | 33.9 | 78 | 42 |

One can get an estimation of the fracture process zone length $c$ from the displacement jump along the height of the beam obtained at the peak load and measured with Digital Image Correlation (Alam et al. 2013). An approximate value of 30 mm is obtained for $c$ for the beam $D_2$.

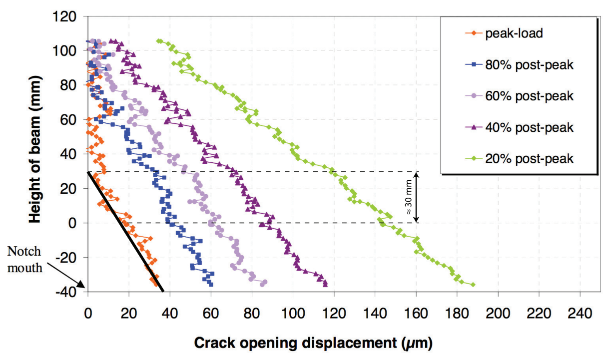



Figure 9: Displacement jump along the height of the beam at different loading stages.

For the beam $D_2$, by solving Eq. 11, a value of 26 mm is obtained for $c$. Considering the errors inherent to the procedure used to estimate $c$ from the experimental results (precision of the measure, contribution of the elastic field...), a good correlation is observed between the experiment and the analytical estimation of $c$.

A comparison between the crack field obtained from the discrete reanalysis of the continuous calculation is performed. One can see on Fig. 10 the crack field obtained from the discrete element reanalysis and for both nonlocal regularization method. By ap-

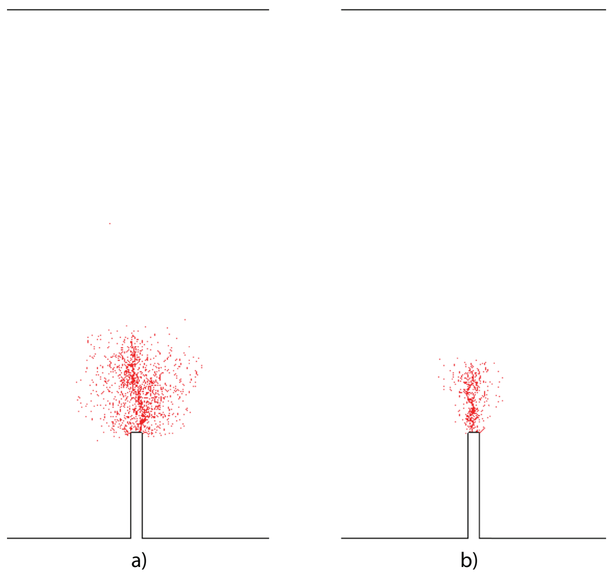



Figure 10: Crack field at the peak load for the original nonlocal method a) and the stress based nonlocal method b).

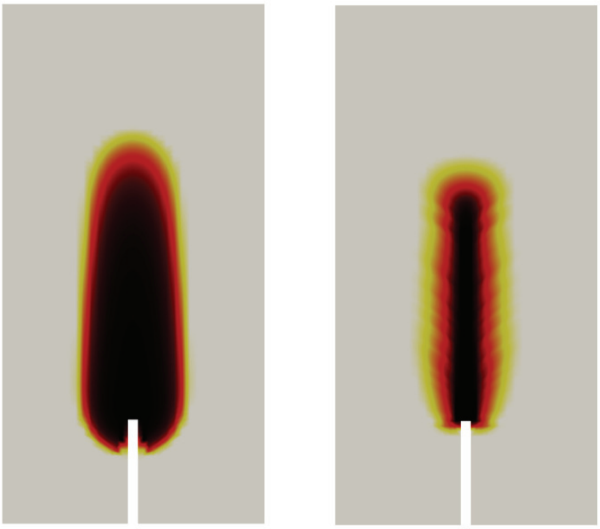
plying the method to quantify the crack properties exposed in 3.2, one can get an estimation of $c$ from the discrete reanalysis. A value of 47 mm is obtained for the original nonlocal method and a value of 32 mm is obtained for the stress based nonlocal method. The overestimation of the damaged area observed for the original nonlocal method leads to an overestimation of $c$ compared to the value obtained analytically and experimentally.

### 5.4 *Crack analysis at failure*

During the softening behavior of the beam, one can observe the propagation of the fracture process zone. Fig.11 gives the damage field at 50% of the peak load for both nonlocal methods. The damage field observed for the original nonlocal model tends to be more spread than the one with stress based nonlocal model. Furthermore, the damage field with the original model tends to go back to the notch tip. This difference is due to the introduction of a stress state factor to describe the nonlocal interactions between points in the media. When a macrocrack goes through the media, the principal stress values tend to 0 leading to decreasing of the nonlocal quantities redistributed in its close neighborhood for the stress based nonlocal method. As a consequence, there is no damage diffusion at failure for this last method compared to the original one.

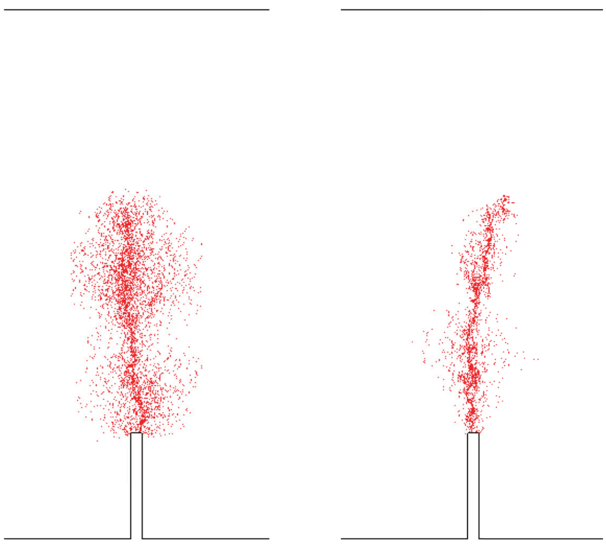
Fig.11 gives the crack field at 50% of the peak load for both nonlocal methods.

One can see on Fig.12 that the crack field tends to be similar for both nonlocal models. These observations can see some similarities with the ones made by



a) b)

Figure 11: Damage field at 50% of the peak load for the original nonlocal method a) and the stress based nonlocal method b).



a) b)

Figure 12: Crack field at 50% of the peak load for the original nonlocal method a) and the stress based nonlocal method b).

(Jirásek et al. 2004). Indeed, they observed that the better way to describe size effect on fracture energy for notched beams in an original nonlocal continuous framework was to replace the explicit description of the notch by a fully damage area. It confirms the fact that the original nonlocal model failed to catch local phenomena as the description of the fracture process zone close to a notch tip for size effect analysis whereas they are only small differences during the propagation of a fracture process zone in a continuous media (e.g. evolution of the damage area in front of a fully damage band representing a notch). For the presentation associated to this article, some local results as the crack opening along the crack will be also provide and compared to the experimental ones.

## 6 CONCLUSIONS

An analysis of size effect for quasi-brittle materials has been presented in this paper. Numerical simulations of three-point bending tests have been performed using a damage model with two different nonlocal regularization methods. It has been shown that both methods achieved to catch size effect at the global scale (i.e. size effect on nominal strength) in comparison with experimental results and values given by a theoretical approach of size effect (Bažant size effect law). In this framework, a cracking analysis has also been performed by considering a reanalysis of the continuous calculation with a discrete elements modeling. It has been observed that when local quantities as cracking are considered, the original nonlocal method tends to overestimate the size of the fracture process zone at the peak load. Indeed, the original nonlocal model considers isotropic nonlocal interactions close to the notch tip leading to a bad description of the development of damage in this area. By introducing a stress state factor, a better estimation of the fracture process zone length is obtained in comparison with experimental results with DIC and with the results from the theoretical approach. This error tends to decrease during the softening behavior of the structure with the propagation regime of the fracture process zone. The kinematic conditions at the boundaries of the area reanalyzed by discrete elements approach seems to be less sensitive to the fracture process zone evolution during the progressive failure of the beam than to the fracture process zone initiation in the neighborhood of the notch tip even if some damage diffusion can be observed with the original nonlocal model.

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