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Relativistic entanglement and the Theory of Dilated Locality

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Since his famous discussions with Niels Bohr, Albert Einstein considered quantum entanglement (QE) as a spooky action at a distance, due to the violation of locality necessary for the state of the particle entangled with its companion to share at same a distance, in such a way that if a notification that is not local but instantaneous could only cover that space at a speed superior to that of light. The Theory of Relativity (TR) with Quantum Theory (QT) would face a direct consequence of this; turning QE into the bone of contention between both theories, which are no more than the pillars of Physics. Contrary to predictions, in this work we will see that QE is the meeting point of both theories, so much so, that QE could be considered as the cornerstone of the Theory of Everything (TOE). Consistent with this, the entangled particles retain certain autonomy unknown to date, and in addition, they have relativistically entangled alter egos, which will hold the effect even when the original entangled particles are extremely separated from each other. These alter-egos can be considered as black holes (with their corresponding temperature and entropy) giving rise to a wormhole. This is possible since the locality dilates according to the Lorentz factor, which is accompanied by a contraction in the effective channel and in the temporal delay to cross that channel. All this takes place while space-time is curved (hyperspace) to generate the wormhole between both black holes. In other words, QE is a local effect of infinite range so it does not outpace the speed of light, and therefore QT is a complete theory that does not clash with TR. Finally, everything we have said has direct consequences on the link between entangled particles from the point of view of quantum communications in terms of the channel and its bandwidth, latency, capacity, robustness and security.

Keywords—Black holes; EPR paradox; locality; quantum channel; quantum communication; quantum entanglement; Quantum Theory; quasi-entanglement; Theory of Relativity; wormhole.

1 Introduction

Quantum entanglement -also known as the God effect- is a physical phenomenon which takes place between two or more particles (strictly speaking, their spins) that interact after their creation in such a way that the resulting quantum state corresponds to the effect itself and not to the individual particles that make it up [1]; even when such particles are separated by an astronomical distance. So that, the resulting quantum state acts as a whole [2] with a total loss of individuality on the part of the original states. Therefore, we will refer to the entanglement as a specific case of correlation between subsystems [3]. Consequently, we can find several configurations for quantum entanglement which currently exist, in particular: GHZ state due to D. M. Greenberger, M.A. Horne and A. Zeilinger [4,5] formed by 3 or more entangled particles and the so-called configuration W which is the perfect complement of the previous one [3]. Another interesting kind of entanglement is called Hyperentanglement [6] which is a promising resource in quantum information processing with its high capacity character; defined as the entanglement in multiple degrees of freedom (DOF) of a determined quantum system, such as polarization, spatial-mode, orbit-angular-momentum, time-bin and frequency DOF of photons [7]. In the meantime, multidimensional entanglement quantum system confirms the existence of at least one dimension of 100-x-100 using spatial modes of photons [8]. On the other hand, distributed entanglement [9] is the polygamous nature present in multipartite systems with a stronger unlocalizable character [10]. Recently, two extremely important experiments have been carried out: a) entanglement between photons that have never coexisted [11], i.e., entanglement in time, not only in space, and, b) a scheme that deterministically generates wave-particle entanglement of two photons [12].

Besides, at this point, it is important to mention two outstanding aspects of entanglement: distillation, and, swapping. In the first case, we get some number of almost pure Bell pairs from N copies of an arbitrary entangled state. This transformation happens using local operations and classical communication (LOCC). That is to say, it is a powerful tool to cope with the negative influence produced by noise in quantum channels. Thanks to this previous transformation, the distillation is achieved by obtaining a smaller number of maximally entangled pairs (i.e., Bell states [1]) from less shared entangled pairs. As for swapping, it is a simple and illustrative example of Teleportation, which can be equally applied to pure and mixed states, and is considered as the state of a single subsystem of an entangled pair [1].

Moreover, quantum decoherence is the worst enemy of entanglement, and formally it is the loss of quantum coherence. In quantum mechanics, particles such as electrons also behave like waves and are described by a wavefunction. These waves can interfere, leading to the peculiar behavior of quantum

particles. As long as there is a definite phase relation between different states, the system is said to be coherent [1].

Although quantum entanglement is a key piece in Quantum Information Processing [13-15] in general and Quantum Computing [14,16-20] in particular, it is in Quantum Communications [18,19,21-23] where the most exciting challenge is presented, which consists of the following problem: if we have two entangled particles and we separate them from each other at a distance similar to that existing between Earth and Mars; the theory of quantum entanglement tells us that any local change in the state of one of the particles (either because of the quantum measurement, or by a change in the polarization of the particle or in the magnetic field in which it is immersed) causes the instantaneous change in the state of the other particle regardless of the distance between them. However, if we consider the link between these particles as a quantum channel and the change in both states as a signal, then that signal should travel at a speed greater than light to perform such a feat. This fact collides head-on with the Theory of Relativity [24] and this results in the Einstein-Podolsky-Rosen (EPR) paradox [1,25-30] and even more, with No-Communication [31] and No-Cloning [1] Theorems (two No-Go Theorems [32]). At this point, a formidable debate on the basis of four possible alternatives breaks ground:

- 1) quantum entanglement [1] is not instantaneous, therefore, it makes no contribution in relation to the communications involved in the trip to Mars, and hence, it does not collide with the Theory of Relativity [24],
- 2) quantum entanglement is instantaneous, and we do not expect it to convey information, this is the present posture (or resignation) and from which no problem is derived,
- 3) quantum entanglement is instantaneous, and we intend to carry information, then, it collides with the Theory of Relativity, which represents a very unpleasant scenario because it deepens the gap between Quantum Theory and the Theory of Relativity, and lastly,
- 4) quantum entanglement is instantaneous, and we intend to carry information, however, it does not collide with the Theory of Relativity. This is the hypothesis and the central objective of this work, whatsmore, this would be the ideal scenario, because Quantum Theory [1] would be a complete theory, being quantum entanglement an instantaneous effect, and all at the same time.

Regardless of the correct alternative, such a debate is the main pitfall of quantum communications involved in the trip to Mars.

On the other hand, in recent years, there have been significant efforts to formally link quantum entanglement with gravity in general [33-35], as well as with the entropy of black holes in particular [36-38]. This link is not trivial at all. If this were successful, it would give rise to a version of the theory of everything (TOE) [39], through which the Quantum Theory [40-43] and the Theory of Relativity [24] would coexist, neither doubting the completeness of the first (as happened from the EPR paradox [27]) nor exposing with marked discrimination the total inability of the second to explain the subatomic world; given that, definitely, the search for a formal nexus between both worlds represents the central axis of Modern Physics, and the present work.

The principle of locality [44-45] establishes that an object can only be influenced by its immediate surroundings. A local theory must necessarily include the principle of locality. This is presented as an alternative to the deeply rooted concept of instantaneous *action at a distance* [1,25-29]. Since the days of Newton, the locality has overflowed its natural frame (i.e., Classical Physics), in particular, field theories. Basically, the idea is the following: given two points A and B; if an action takes place on point A, then, the only way that this action could have influence on point B is that between them, there is a field that conveys such action. In other words, there must be a wave or particle that crosses the space between the two points so that they share that influence.

The Special Theory of Relativity [24] establishes that the limit at which the mentioned influence can travel is the speed of light. A direct consequence of this is that an action at point A cannot simultaneously and instantaneously cause the same result at point B; since, the time it takes for the influence to cross the space between both points cannot be less than $t = d/c$, where d is the distance between the points, and c is the speed of light. In fact, in 1935 Albert Einstein, Boris Podolsky and Nathan Rosen published their work on the EPR paradox [1,25-30]. In this work they theorize about the possibility of non-local behavior of quantum mechanics. Their reasoning is based on the fact that, if we make a measurement on one of the constituent particles of an entangled pair, it simultaneously and instantaneously produces the collapse of the wavefunction in both near and remote particles, incomprehensibly exceeding the speed of the light. However, this violation of locality does not allow us to use it to transmit information faster than light [29,46,47], since the collapse of the wavefunction has a probabilistic nature. In 1964, John Stewart Bell presented him a famous theorem (the *Bell inequality* [1,25]), which was violated in several experiments implying, at least apparently, that quantum mechanics does not comply with locality and realism (or local-realism [1,27]). This principle, among others, is linked with values of unmeasured magnitudes. Then, the local-realism emerges as the resultant between locality principle (establishing the speed of light as the upper limit of any cause-and-effect)

with the assumption that any physical magnitude must objectively have a real value before any possible measurement. As Albert Einstein said “I like to think that the moon is there even when I am not looking at it”.

The local realism is accepted by classical mechanics, and classical electrodynamics, but absolutely rejected by quantum mechanics. This rejection is based on the experiments carried out with remote entangled particles, in particular, that made by Aspec in 1982, which is apparently supported by the Bell's 1964 inequality theorem [1,25]: an interpretation that Einstein previously rejected (in form of a paradox). In the course of this work the reason why we say *apparently* will be unveiled. By definition, Quantum Mechanics violates either locality or realism. If an experiment shows quantum mechanics, it violates Bell's theorem. However, it is not clear if the 1982 experiment demonstrates a true violation, due to two reasons: experimental limits of the test and the complete inability to prove the sub-class of inequalities. Currently, several interpretations of quantum mechanics seem to violate various aspects of Local Realism, at least, apparently, since it can be wondered: Does the violation really happen? Is that interpretation correct? In fact, some interpretations only violate aspects of a related principle known as counter-factual definiteness [48] (CFD), i.e., it accepts the results of a measurement that was never made giving values that were never measured as valid.

The first experimental test about the Bell inequality was made by Alain Aspect in 1982 [30,49]. In such test, quantum mechanics seems to violate the inequality, so it follows that it must violate either locality or realism [1,27]. However, several scientists have noticed that these experiments contained “loopholes”, which do not allow an effective response to this question. Apparently, this problem seemed to be solved in 2015 in the experiment of Dr. Ronald Hanson at Delft University, carrying out the first loophole-free experiment [50].

However, and as we can see, the controversy continues although, at present, in a tone significantly lower than in the days of Albert Einstein, who fundamentally objected to the probabilistic nature of quantum mechanics and famously declared “I am convinced God does not play dice”. Einstein, Podolsky, and Rosen argued that “elements of reality” (in the form of hidden variables [1-3,51]) must be incorporated to quantum mechanics to explain entanglement without accepting action at a distance [1,52]. This led the authors to argue that Quantum Theory could not be considered a complete theory, although it was [27]. In other words, entanglement became the bone of contention between the two main theories of Physics. Later, Bell's theorem established that local hidden variables of certain types are impossible, and that this implies non-locality. Another famous non-local theory belongs to De Broglie–Bohm [1,2].

In a very short although extremely interesting paper [53], Prof. Alain Aspect raises doubts about what the problem is: locality or realism? Several experiments have been carried out throughout the last decades with the object of unmasking, or simply bringing out the truth in relation to this subject, including, of course, Bell's experiments. The most outstanding fact of these experiments takes place based on the so-called CHSH inequalities, which are thanks to John Clauser, Michael Horne, Abner Shimony, and Richard Holt (CHSH), who described it in a much-cited paper published in 1969 [51].

Nowadays, a series of methods for local operations and classical communication [1,3] (LOCC) complete these concepts. The first one is LOCC itself which is employed in quantum information theory, and where a local operation (i.e., a product) is carried out on part of the system. The result of that operation is communicated classically to another part where another local operation is usually performed. Distinguishing two Bell pairs [25] would constitute an example of this. Another method is the so-called NLOCC [54], or noisy LOCC, by means of which local systems in maximally mixed states only can be added; any other system must be considered in the initial state rather than in the processing stage. Lastly, the CLOCC or closed LOCC [55] method, which is a modification of the LOCC paradigm that disallows unrestricted consumption of local pure states. Horodecki *et al* [56] had previously obtained some limitations on this problem, both for the one-way and two-way CLOCC case.

Finally, and based on what we have said so far, it is clear that a reconciling theory is more necessary than a unifying one. That is the central idea of this work through the Theory of Dilated Locality developed in the next sections.

2 Results

2.1 Mutual information as a measure of correlations

For pure states, that is, states on the Bloch's sphere [13], any wavefunction

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

arises from the superposition of so-called Computational Basis States (CBS) or qubit basis states, which are located at the poles of already said sphere,

$$Spin\ up = |\uparrow\rangle = |0\rangle = North\ pole \equiv \mu_{|0\rangle}, \quad (2)$$

$$\text{Spin down} = |\downarrow\rangle = |1\rangle = \text{South pole} \equiv \mu_{|1\rangle} = -\mu_{|0\rangle}, \quad (3)$$

with $|\alpha|^2 + |\beta|^2 = 1$, such that $\alpha \wedge \beta \in \mathbb{C}$, and where $\mu_{|0\rangle} = 1$ for photons, and $\frac{1}{2}$ for electrons, being $\mu_{|0\rangle}$ and $\mu_{|1\rangle}$ the scalar versions of the *spin up* $|0\rangle$ and the *spin down* $|1\rangle$, respectively, in such a way that the complete spin is conserved, i.e., $\mu_{|0\rangle} + \mu_{|1\rangle} = 0$, similarly when a spinless particle decays in two new entangled particles with opposite spins [1,2,13]. Therefore, in this paper, the letter μ will represent spin numbers in general, exclusively, and does not necessarily mean muon. Moreover, and based on CBS, we can define another basis, which will be very useful for the rest of this paper as well as for Quantum Information in general [1], and quantum teleportation and superdense coding in particular [28,57-59], taking into account the interaction of two subsystems A and B, and that its components are pure states; and using their scalar versions too,

$$\begin{aligned} |0^A\rangle \otimes |0^B\rangle &= |0^A\rangle |0^B\rangle = |0^A, 0^B\rangle = |0^A 0^B\rangle \equiv \mu_{|00\rangle} = \mu_{|0\rangle}^2, \\ |0^A\rangle \otimes |1^B\rangle &= |0^A\rangle |1^B\rangle = |0^A, 1^B\rangle = |0^A 1^B\rangle \equiv \mu_{|01\rangle} = \mu_{|0\rangle} \mu_{|1\rangle}, \\ |1^A\rangle \otimes |0^B\rangle &= |1^A\rangle |0^B\rangle = |1^A, 0^B\rangle = |1^A 0^B\rangle \equiv \mu_{|10\rangle} = \mu_{|1\rangle} \mu_{|0\rangle}, \\ |1^A\rangle \otimes |1^B\rangle &= |1^A\rangle |1^B\rangle = |1^A, 1^B\rangle = |1^A 1^B\rangle \equiv \mu_{|11\rangle} = -\mu_{|1\rangle}^2, \end{aligned} \quad (4)$$

where \otimes is the Kronecker's product, and $\mu_{|00\rangle}$ and $\mu_{|11\rangle}$ are the scalar version of $|00\rangle$ and $|11\rangle$, respectively, with $\mu_{|00\rangle} + \mu_{|11\rangle} = 0$. The sign rules that support the equivalences and equalities of Eq.(4) will be explained at the end of this section. From Eq.(4), we are going to build the famous Bell's bases [1-3], with 2-qubit vectors the combined Hilbert space will be $H^{A \cup B} = H_2^A \otimes H_2^B$, and we dispose of the following four vectors,

$$|\Phi_{\pm}^{A \cup B}\rangle = \frac{1}{\sqrt{2}}(|0^A, 0^B\rangle \pm |1^A, 1^B\rangle), \quad |\Psi_{\pm}^{A \cup B}\rangle = \frac{1}{\sqrt{2}}(|0^A, 1^B\rangle \pm |1^A, 0^B\rangle). \quad (5)$$

At the same time, we will define the density matrix necessary to calculate the entropy for the complete set of the above-mentioned cases. We are going to begin with density matrix of the subsystems treated individually,

$$\rho^A = \rho^B = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) = \frac{1}{2}I = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (6)$$

where I is the identity matrix. While the scalar version of this operator will be

$$r^A = r^B = \frac{1}{2} \left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{\eta} \right) = \frac{1}{2} \left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{2} \right), \quad (7)$$

where η is the adjustment factor (necessary in the scalar version of the density matrix), which in the case of photons is equal to 2, since the scalar version is strongly dependent on the type of particle to which it represents. We must note that in the case of photons $r^A = r^B = \frac{1}{2}$; which is consequent with the matrix nature of Eq.(6). However, with respect to the case of the system composed of two subsystems, its density matrix will depend on whether or not these subsystems are entangled. Therefore, for the non-entangled case

$$\rho^{A \cup B} = \rho^A \otimes \rho^B, \quad (8)$$

but for the entangled case, we will have,

$$\rho^{A \cup B} \neq \rho^A \otimes \rho^B. \quad (9)$$

Although this operator will depend on each of the 3 cases that will be analyzed in the next subsection, i.e., completely independent, classically-correlated, and entangled subsystems.

Now, with respect to the entropy, we can define it in two forms: tensor and scalar. The first one,

$$S^A = S^B = -\text{tr}[\rho^A \log(\rho^A)] = -\text{tr}[\rho^B \log(\rho^B)], \quad (10)$$

then, replacing Eq.(6) into Eq.(10), yield,

$$S^A = S^B = -\text{tr}\left[\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \log\left(\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\right] = 1, \quad (11)$$

while the scalar version would be,

$$S^A = S^B = -\kappa[r^A \log(r^A)] = -\kappa[r^B \log(r^B)], \quad (12)$$

where, κ is an adjustment factor (necessary in the scalar version of the entropy). Then, replacing Eq.(7) into Eq.(12), and considering the case of photons, i.e., $\kappa = 2$ and $\mu_{|0\rangle} = -\mu_{|1\rangle} = 1$, yield,

$$S^A = S^B = -2\left[\frac{1}{2} \log\left(\frac{1}{2}\right)\right] = 1. \quad (13)$$

In the same way, for a composed system, the entropy would be,

$$S^{A \cup B} = -\text{tr}[\rho^{A \cup B} \log(\rho^{A \cup B})] = -\kappa[r^{A \cup B} \log(r^{A \cup B})]. \quad (14)$$

In this case, κ depends on the degree of correlation between both subsystems (i.e., completely independent, classically-correlated, and entangled subsystems).

Besides, in both worlds, classical and quantum-mechanical; the correlations between the systems are those established by the additional information. In the case of composite quantum systems, the mutual information $S^{A \cap B}$ is introduced to quantify that additional information, allowing us to obtain the degree of correlation between both subsystems¹,

$$S^{A \cap B} = S^A + S^B - S^{A \cup B} \geq 0. \quad (15)$$

Therefore, the entropy of the composite system $S^{A \cap B}$ indicates us that the uncertainty of a state $\rho^{A \cup B}$ is less than the two subsystems S^A and S^B added together. In other words, $S^{A \cap B}$ tells us how much more information the composite system can store compared to the individual subsystems. Besides, $S^{A \cap B}$ measures the distance between the state $\rho^{A \cup B}$ and the non-entangled state¹ $\rho^A \otimes \rho^B$.

The entropy of the subsystems S^A and S^B is equal to one in all cases ($S^A = S^B = 1$); while the entropy of the composite system $S^{A \cup B}$ will have different values for each and every of the three cases. For the sake of simplicity, the subsystems are taken to be qubits. Making use of Eq.(15), the main idea is to relate correlations and entanglement to the entropy, and in particular, to the mutual information $S^{A \cap B}$.

Then, we present the mutual information of the three types of subsystems, i.e., completely independent, classically correlated, and entangled, where the last one is critical for the development of both the alter-egos involved in the relativistic entanglement, and the Theory of Dilated Locality.

2.2 Completely independent subsystems

In this case, both subsystems have absolute and complete independence between them, i.e., $\rho^{A \cup B}$ is a Kronecker's product of density matrices like Eq.(8), as we can see in Fig.1. Therefore, there are no correlations between such subsystems.

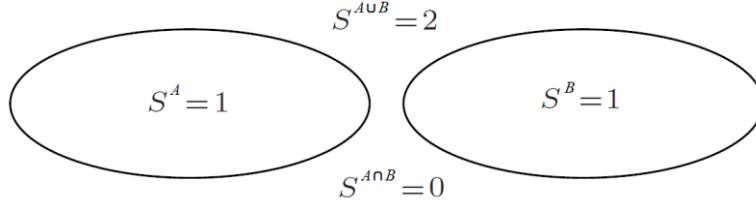


Fig. 1 Completely independent.

In the computational basis of $H^A \otimes H^B$, $\rho^{A \cup B}$ takes the following form,

$$\rho^{A \cup B} = \frac{1}{4} \left(|0^A, 0^B\rangle\langle 0^A, 0^B| + |0^A, 1^B\rangle\langle 0^A, 1^B| + |1^A, 0^B\rangle\langle 1^A, 0^B| + |1^A, 1^B\rangle\langle 1^A, 1^B| \right) = \frac{1}{4} I^{4 \times 4}, \quad (16)$$

where $I^{4 \times 4}$ is a 4x4 identity matrix. Therefore, replacing Eq.(16) into Eq.(14), $S^{A \cup B}$ will be

$$S^{A \cup B} = -\text{tr} \left\{ \frac{1}{4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \log \left(\frac{1}{4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right) \right\} = 2. \quad (17)$$

And, introducing the results of Eq.(13) and (17) into Eq.(15),

$$S^{A \cap B} = S^A + S^B - S^{A \cup B} = 1 + 1 - 2 = 0. \quad (18)$$

The result of the Eq.(18) ratifies the complete independence of the subsystems. Now, if we remember the equivalences of Eq.(4), then, the scalar version of Eq.(16) will be,

$$\begin{aligned} r^{A \cup B} &= \frac{1}{4} \left(\frac{\mu_{|00\rangle}\mu_{|00\rangle} + \mu_{|01\rangle}\mu_{|01\rangle} + \mu_{|10\rangle}\mu_{|10\rangle} + \mu_{|11\rangle}\mu_{|11\rangle}}{\eta} \right) \\ &= \frac{1}{4} \left(\frac{\mu_{|0\rangle}^2\mu_{|0\rangle}^2 + (\mu_{|0\rangle}\mu_{|1\rangle})(\mu_{|0\rangle}\mu_{|1\rangle}) + (\mu_{|1\rangle}\mu_{|0\rangle})(\mu_{|1\rangle}\mu_{|0\rangle}) + (-\mu_{|1\rangle}^2)(-\mu_{|1\rangle}^2)}{4} \right), \\ &= \frac{1}{4} \left(\frac{\mu_{|0\rangle}^4 + \mu_{|0\rangle}^2\mu_{|1\rangle}^2 + \mu_{|1\rangle}^2\mu_{|0\rangle}^2 + \mu_{|1\rangle}^4}{\eta} \right) = \left(\frac{\mu_{|0\rangle}^4 + 2\mu_{|0\rangle}^2\mu_{|1\rangle}^2 + \mu_{|1\rangle}^4}{4^2} \right) = \left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{4} \right)^2 \end{aligned} \quad (19)$$

where η will be equal to 4 for the purpose of adjusting its subsequent use into entropy, and considering that we will make the following deduction for the case of photons, i.e., $\mu_{|0\rangle}^2 = \mu_{|1\rangle}^2 = 1$, then, replacing Eq.(19) into the scalar version of Eq.(14) with $\kappa = 4$, and $S^{A \cup B} = 2$ from Eq.(17), we will have,

$$S^{A \cup B} = 2 = -\kappa \left[r^{A \cup B} \log(r^{A \cup B}) \right] = -4 \left[\left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{4} \right)^2 \log \left(\left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{4} \right)^2 \right) \right]. \quad (20)$$

Replacing $\mu_{|0\rangle}^2 = \mu_{|1\rangle}^2 = 1$ outside logarithm, and making additions and subtractions that do not alter Eq.(20),

$$\begin{aligned}
1 &= -\log\left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{4}\right) + \log\left(\frac{\mu_{|0\rangle}^2}{2}\right) - \log\left(\frac{\mu_{|0\rangle}^2}{2}\right) + \log\left(\frac{\mu_{|1\rangle}^2}{2}\right) - \log\left(\frac{\mu_{|1\rangle}^2}{2}\right), \\
&= -\log\left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{4}\right) + \log\left(\frac{\mu_{|0\rangle}^2}{2}\right) + 1 + \log\left(\frac{\mu_{|1\rangle}^2}{2}\right) + 1
\end{aligned} \tag{21}$$

and,

$$-1 = -\log\left(\frac{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}{4}\right) + \log\left(\frac{\mu_{|0\rangle}^2}{2}\right) + \log\left(\frac{\mu_{|1\rangle}^2}{2}\right) = \log\left(\frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}\right). \tag{22}$$

Finally,

$$\mu_{|\uparrow\uparrow\rangle}^2 = \mu_{|\uparrow\rangle}^2 = \frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2} = \mu_{|0\rangle}^2 // \mu_{|1\rangle}^2 = 2^{-1} = \frac{1}{2}. \tag{23}$$

where “//” is the parallel operator, and $\mu_{|\uparrow\rangle}$ is the equivalent spin for completely independent subsystems, which would have been impossible to deduce with the traditional tensor (original) analysis [1]. The graphic interpretation of Eq.(23) can be seen in Fig.2 where $\mu_{|0\rangle}^2$ in red is in position A, and $\mu_{|1\rangle}^2$ in blue is in position B, where, the pink line is parallel to the gross black line of base and perpendicular to spins arrows.

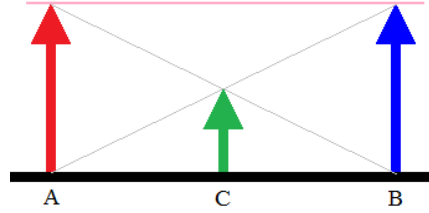


Fig. 2 Parallel operator for completely independent subsystems.

Then, as $\mu_{|\uparrow\rangle}^2 = \mu_{|0\rangle}^2 // \mu_{|1\rangle}^2$ according to Eq.(23), $\mu_{|\uparrow\rangle}^2 = 1/2$ in green is in position C, that is to say, at the intersections of two gray lines, one connects the upper part of $\mu_{|0\rangle}^2$ (in red) with the base of $\mu_{|1\rangle}^2$ (in blue) and the other connects the base $\mu_{|0\rangle}^2$ (in red) with the upper part of $\mu_{|1\rangle}^2$ (in blue). In short, this is - in itself - the geometric interpretation of the parallel operator.

In Fig.3, if we move $\mu_{|1\rangle}^2$ to position B', we leave $\mu_{|0\rangle}^2$ in its original position A, and reapply the parallel operator to this new configuration, we will see that a new $\mu_{|\uparrow\rangle}^2$ appears as a result of the two new intersected gray lines, but now in position C', which has a value of 1/2 again, i.e., similar to the previous case of Fig.2. We can see this new result in Fig.3, where, the pink lines are parallel to the gross black line of the base and perpendicular to the arrows (red, blue and green) representing the spins. In other words, the lines of pink color indicate that heights are preserved regardless of the distance between both particles (red and blue) on which the parallel operator is applied. This indicates that the parallel operator is insensitive to the distance between the two spins, that is to say, red and blue.

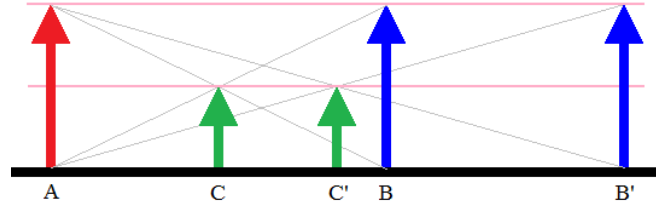


Fig. 3 Parallel operator for completely independent subsystems with an increase in separation between original spins.

Now, if we apply square root to both sides of Eq.(23), we obtain the final value of the equivalent spin $\mu_{|\uparrow\uparrow\rangle}$

$$\mu_{|\uparrow\uparrow\rangle} = \sqrt{\mu_{|\uparrow\uparrow\rangle}} = \sqrt{\mu_{|0\rangle}^2 // \mu_{|1\rangle}^2} = \sqrt{\frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|0\rangle}^2 + \mu_{|1\rangle}^2}} = \frac{\mu_{|0\rangle}}{\sqrt{1 + \left(\frac{\mu_{|0\rangle}}{\mu_{|1\rangle}}\right)^2}} = \frac{1}{\sqrt{2}}. \quad (24)$$

Finally, in the Section called Methods, we are going to analyze the position of the spin $\mu_{|\uparrow\uparrow\rangle}$.

2.3 Classically-correlated subsystems

In this case, see Fig. 4, it is possible to create correlations without obtaining entanglement for $S^{A \cap B} \neq 0$. For example, let's do this with the *separable mixture* of pure product states.

$$\rho^{A \cup B} = \frac{1}{2} (|0^A, 0^B\rangle \langle 0^A, 0^B| + |1^A, 1^B\rangle \langle 1^A, 1^B|). \quad (25)$$

As we can see, $\rho^{A \cup B}$ of Eq.(25) contains fewer terms than the case of completely independent of Eq.(16).

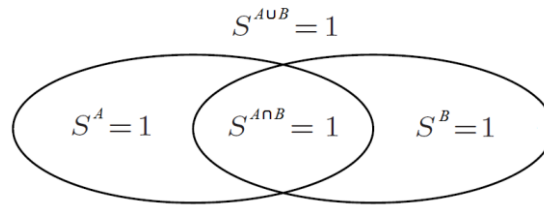


Fig. 4 Classically-correlated.

In the computational basis of $H^A \otimes H^B$, $\rho^{A \cup B}$ will be:

$$\rho^{A \cup B} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (26)$$

Therefore, replacing Eq.(26) into Eq.(14), $S^{A \cup B}$ will be

$$S^{A \cup B} = -tr \left\{ \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \log \left(\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right) \right\} = 1. \quad (27)$$

And, introducing the results of Eq.(13) and (27) into Eq.(15),

$$S^{A \cap B} = S^A + S^B - S^{A \cup B} = 1 + 1 - 1 = 1. \quad (28)$$

This result brings out a certain degree of correlation between the subsystems. Then, if we remember the equivalences of Eq.(4), the scalar version of Eq.(25) will be,

$$\begin{aligned} r^{A \cup B} &= \frac{1}{2} \left(\frac{\mu_{|00\rangle} \mu_{|00\rangle} + \mu_{|11\rangle} \mu_{|11\rangle}}{\eta} \right) = \frac{1}{2} \left(\frac{\mu_{|0\rangle}^2 \mu_{|0\rangle}^2 + (-\mu_{|1\rangle}^2)(-\mu_{|1\rangle}^2)}{4} \right), \\ &= \frac{1}{2} \left(\frac{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}{4} \right) = \frac{1}{2} \left(\frac{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}{4} \right) = \frac{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}{8} \end{aligned} \quad (29)$$

where $\eta = 4$ for the same reasons as for the completely independent case, where we also consider the use of photons, i.e., $\mu_{|0\rangle}^2 = \mu_{|1\rangle}^2 = 1$, then, replacing Eq.(29) into the scalar version of Eq.(14) with $\kappa = 2$, and

$S^{A \cup B} = 1$ from Eq.(27), we will have,

$$S^{A \cup B} = 1 = -\kappa \left[r^{A \cup B} \log(r^{A \cup B}) \right] = -2 \left[\left(\frac{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}{8} \right) \log \left(\left(\frac{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}{8} \right) \right) \right]. \quad (30)$$

Replacing $\mu_{|0\rangle}^2 = \mu_{|1\rangle}^2 = 1$ outside logarithm,

$$2 = -\log \left(\frac{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}{8} \right) = -\log(\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4) + 3. \quad (31)$$

Simplifying and making additions and subtractions that do not alter Eq.(31),

$$-1 = -\log(\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4) + \log(\mu_{|0\rangle}^4) - \log(\mu_{|0\rangle}^4) + \log(\mu_{|1\rangle}^4) - \log(\mu_{|1\rangle}^4), \quad (32)$$

being $-\log(\mu_{|0\rangle}^4) - \log(\mu_{|1\rangle}^4) = 0$, then,

$$-1 = \log \left(\frac{\mu_{|0\rangle}^4 \mu_{|1\rangle}^4}{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4} \right), \quad (33)$$

Finally,

$$\mu_{|\uparrow\uparrow\rangle}^2 = \mu_{|\uparrow\rangle}^4 = \frac{\mu_{|0\rangle}^4 \mu_{|1\rangle}^4}{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4} = \mu_{|0\rangle}^4 // \mu_{|1\rangle}^4 = 2^{-1} = \frac{1}{2}. \quad (34)$$

Anew, this would have remained hidden if not for this treatment, that is, for the scalar version. Besides, and as for the previous case, i.e., completely independent, the equivalent spin $\mu_{|\hat{n}\rangle}$ represents (in some way) the original spins involved with a similar analysis to the one made for the Fig.3, and with a similar result, i.e., in this case too, the parallel operator is completely insensitive to the distance between spins. Then, if we apply the fourth root to both sides of Eq.(34), we obtain the final value of the equivalent spin $\mu_{|\hat{n}\rangle}$

$$\mu_{|\hat{n}\rangle} = \sqrt[4]{\mu_{|\hat{n}\rangle}^2} = \sqrt[4]{\mu_{|0\rangle}^4 // \mu_{|1\rangle}^4} = \sqrt[4]{\frac{\mu_{|0\rangle}^4 \mu_{|1\rangle}^4}{\mu_{|0\rangle}^4 + \mu_{|1\rangle}^4}} = \frac{\mu_{|0\rangle}}{\sqrt[4]{1 + \left(\frac{\mu_{|0\rangle}}{\mu_{|1\rangle}}\right)^4}} = \frac{1}{\sqrt[4]{2}}. \quad (35)$$

2.4 Entangled subsystems

For this case (see Fig.5) we will take as an example one of the Bell states of Eq.(5), specifically $|\Phi_+^{A \cup B}\rangle$, which is a pure state, with $\rho^{A \cup B}$ from Eq.(36),

$$\rho^{A \cup B} = \frac{1}{4} \left(|0^A, 0^B\rangle \langle 0^A, 0^B| + |0^A, 0^B\rangle \langle 1^A, 1^B| + |1^A, 1^B\rangle \langle 0^A, 0^B| + |1^A, 1^B\rangle \langle 1^A, 1^B| \right). \quad (36)$$

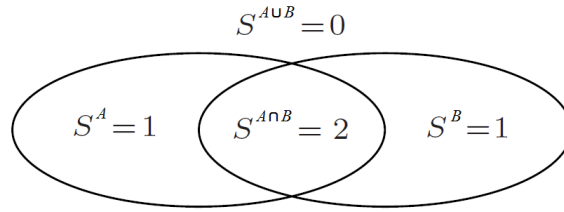


Fig. 5 Entangled subsystems.

In this case [1], $S^{A \cup B} = 0$, then, introducing this value and the results of Eq.(13) into Eq.(15),

$$S^{A \cap B} = S^A + S^B - S^{A \cup B} = 1 + 1 - 0 = 2. \quad (37)$$

Therefore, we will go directly to the calculation of $r^{A \cup B}$,

$$\begin{aligned} r^{A \cup B} &= \frac{1}{4} \left(\frac{\mu_{|00\rangle} \mu_{|00\rangle} + \mu_{|00\rangle} \mu_{|11\rangle} + \mu_{|11\rangle} \mu_{|00\rangle} + \mu_{|11\rangle} \mu_{|11\rangle}}{\eta} \right) \\ &= 4 \left(\frac{\mu_{|0\rangle}^2 \mu_{|0\rangle}^2 + \mu_{|0\rangle}^2 (-\mu_{|1\rangle}^2) + (-\mu_{|1\rangle}^2) \mu_{|0\rangle}^2 + (-\mu_{|1\rangle}^2) (-\mu_{|1\rangle}^2)}{4} \right), \\ &= \frac{1}{4} \left(\frac{\mu_{|0\rangle}^4 - 2\mu_{|0\rangle}^2 \mu_{|1\rangle}^2 + \mu_{|1\rangle}^4}{4} \right) = \left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right)^2 \end{aligned} \quad (38)$$

where $\eta = 4$ for the same reasons that apply to the previous cases, where we also consider the use of photons, i.e., $\mu_{|0\rangle}^2 = \mu_{|1\rangle}^2 = 1$, then, replacing Eq.(38) into the scalar version of Eq.(14) with $\kappa = 4$, we will have,

$$\begin{aligned}
S^{A \cup B} = 0 &= -\kappa \left[r^{A \cup B} \log(r^{A \cup B}) \right] = -4 \left[\left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right)^2 \log \left(\left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right)^2 \right) \right] \\
&= -\frac{1}{2} \left[\left(\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2 \right)^2 \log \left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right) \right]
\end{aligned} \tag{39}$$

Clearing appropriately,

$$\frac{0}{\frac{1}{2}(\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2)^2} = \infty = -\log \left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right). \tag{40}$$

That is, on the left side of Eq.(40), denominator tends to zero with more power than the numerator. In fact, the right side verified it, because, $-\log(0) = \infty$. Now, doing additions and subtractions that do not alter Eq.(40)

$$\begin{aligned}
\infty &= -\log \left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right) + \log \left(\frac{\mu_{|0\rangle}^2}{2} \right) - \log \left(\frac{\mu_{|0\rangle}^2}{2} \right) + \log \left(\frac{\mu_{|1\rangle}^2}{2} \right) - \log \left(\frac{\mu_{|1\rangle}^2}{2} \right) \\
&= -\log \left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right) + \log \left(\frac{\mu_{|0\rangle}^2}{2} \right) + 1 + \log \left(\frac{\mu_{|1\rangle}^2}{2} \right) + 1
\end{aligned} \tag{41}$$

Then,

$$\begin{aligned}
\infty - 2 &= \infty = -\log \left(\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4} \right) + \log \left(\frac{\mu_{|0\rangle}^2}{2} \right) + \log \left(\frac{\mu_{|1\rangle}^2}{2} \right) \\
&= \log \left(\frac{\frac{\mu_{|0\rangle}^2}{2} \frac{\mu_{|1\rangle}^2}{2}}{\frac{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}{4}} \right) = \log \left(\frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2} \right)
\end{aligned} \tag{42}$$

Finally,

$$\mu_{|\uparrow\uparrow\rangle} = \mu_{|\uparrow\rangle}^2 = \frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2} = \mu_{|0\rangle}^2 // \mu_{|1\rangle}^2 = +\infty \quad (\text{actually, } \pm\infty, \text{ see Eq.44 and Fig.6}) \tag{43}$$

Equation (43) represents one of a pair of super-spins, and this result justifies in itself the whole paper and in a few words tells us that the parts are infinitely superior to the whole, then, another super-spin will be,

$$\mu_{|\downarrow\downarrow\rangle} = -\mu_{|\downarrow\rangle}^2 = -\infty, \tag{44}$$

therefore, $\mu_{|\uparrow\uparrow\rangle} + \mu_{|\downarrow\downarrow\rangle} = 0$. The attribute of these last two equations was hidden with the traditional treatment [1] and through them, we will try to explain the exceptional attributes of entanglement. Clearly, the value of these spins infinitely exceeds the value of the original spins. The traditional analysis [1] based on vector or tensor notation is unable to detect this scenario, in fact, the expressions based on scalar notation (Equations 38 and 39) are key, which leads to the equation as it could have never been possible with the original version, i.e. Eq.(36) and $S^{A \cup B}$ based on it. On the other hand, Fig.6 shows us the geometric representation of Eq.(43) with $\mu_{|00\rangle}$ (in red) in position A and $\mu_{|11\rangle}$ (in blue) in position B.

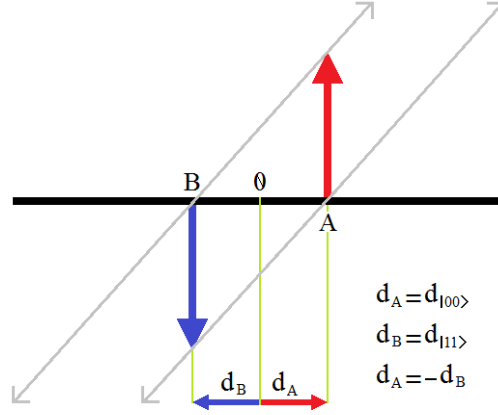


Fig.6 Parallel operator for entangled subsystems.

Figure 6 shows that $\mu_{|\psi\rangle}$ and $\mu_{|\uparrow\uparrow\rangle}$ are in the positions $-\infty$ and $+\infty$, respectively, i.e., to both ends of the figure, namely, where both gray lines intersect. Therefore, it is as if the positions were also entangled,

$$d_{|\uparrow\uparrow\rangle} = d_A + \frac{\mu_A d_{BA}}{\mu_A + \mu_B} = d_{|00\rangle} + \frac{\mu_{|00\rangle}(d_{|11\rangle} - d_{|00\rangle})}{\mu_{|00\rangle} + \mu_{|11\rangle}} = +\infty, \quad (45)$$

$$d_{|\psi\rangle} = d_B + \frac{\mu_B d_{AB}}{\mu_A + \mu_B} = d_{|11\rangle} + \frac{\mu_{|11\rangle}(d_{|00\rangle} - d_{|11\rangle})}{\mu_{|00\rangle} + \mu_{|11\rangle}} = -\infty, \quad (46)$$

Equations (45) and (46) will be analyzed in depth in the section called Methods. They keep the same relationship as in the case of spins, i.e., $d_{|\psi\rangle} + d_{|\uparrow\uparrow\rangle} = 0$.

In Fig.7, if we move $\mu_{|00\rangle}$ from position A to position A', then, we will have $\mu_{|\uparrow\uparrow\rangle} = +\infty$ and $\mu_{|\psi\rangle} = -\infty$ again. Besides, and as in the case of completely independent subsystems, Fig.7 shows that $\mu_{|\uparrow\uparrow\rangle}$ and $\mu_{|\psi\rangle}$ are absolutely insensitive to the distance between original spins, i.e., if $\mu_{|00\rangle}$ does not change neither in modulus nor in its orientation (polarization), however much it shifts, such displacement does not alter the final result; in this case, $\mu_{|\psi\rangle} = -\infty$ and $\mu_{|\uparrow\uparrow\rangle} = +\infty$. This confirms the success of the parallel operator since it attempts to model the entanglement, and as we know -from several laboratory experiences- it does not seem to care about the distance between the entangled particles.

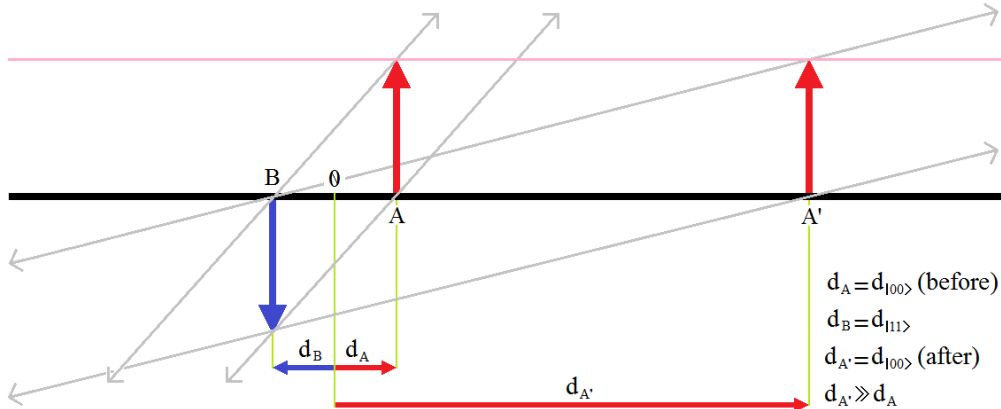


Fig.7 Parallel operator for entangled subsystems with an increase in the inter-spin separation.

Finally, if we apply square root to both sides of Eq.(45), we obtain the final value of the equivalent spin

$$\mu_{|\uparrow\rangle} = \sqrt{\frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2}} = \frac{|\mu_{|1\rangle}|}{\sqrt{1 - \left(\frac{\mu_{|1\rangle}}{\mu_{|0\rangle}}\right)^2}} = \mu_{|0\rangle} \gamma = +\infty, \quad (47)$$

with $\mu_{|\downarrow\rangle} = \mu_{|1\rangle} \gamma = -\infty$, $|\mu_{|1\rangle}| = \mu_{|0\rangle} = -\mu_{|1\rangle}$, and, $\mu_{|\uparrow\rangle} + \mu_{|\downarrow\rangle} = 0$, and where,

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{\mu_{|1\rangle}}{\mu_{|0\rangle}}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{\mu_{|0\rangle}}{\mu_{|1\rangle}}\right)^2}}, \quad (48)$$

is the Lorentz factor when the speed $v = c$ (speed of light). This is the first testimony of the relativistic nature of spins involved in quantum entanglement. In fact, both spins are relativistic. These are the alter-egos. The same situation applies with the locations of these alter-egos,

$$d_{|\uparrow\uparrow\rangle} = d_{|\uparrow\rangle}^2 = d_A + \gamma^2 d_{BA} = d_{|00\rangle} + \gamma^2 (d_{|11\rangle} - d_{|00\rangle}) = +\infty \quad (49)$$

$$d_{|\downarrow\downarrow\rangle} = -d_{|\downarrow\rangle}^2 = d_B + \gamma^2 d_{AB} = d_{|11\rangle} + \gamma^2 (d_{|00\rangle} - d_{|11\rangle}) = -\infty \quad (50)$$

Equations (49) and (50) will also be analyzed in depth in the section called Methods. This tells us that both, the spins resulting from the entanglement and their locations, suffer a dilation according to the Theory of Relativity [24]. On the other hand, the original spins and their alter-egos form a spatial homothecy between them, where the alter-egos are the shadows of the original spins projected to infinite with the same morphology but with infinite magnitude.

2.5 Spatiotemporal analysis

Figures 6 and 7 show us the case analyzed so far, where the coordinates of point A are (d_A, t_A) , while those of point B are (d_B, t_B) , this means, same time and different space. However, the model allows an entanglement between spins that have never coexisted [11]. See Fig.8. In this figure, it can also be seen (as in the case of spatial separation) that the model is insensitive to inter-spin temporal separation. In this case, we have same space and a different time (A in the future and B in the past). Finally, Figures 9 and 10 show us the spin A in the present in the position d_A , while spin B is in the position d_B , in the past in Fig.9 and in the future in Fig.10.

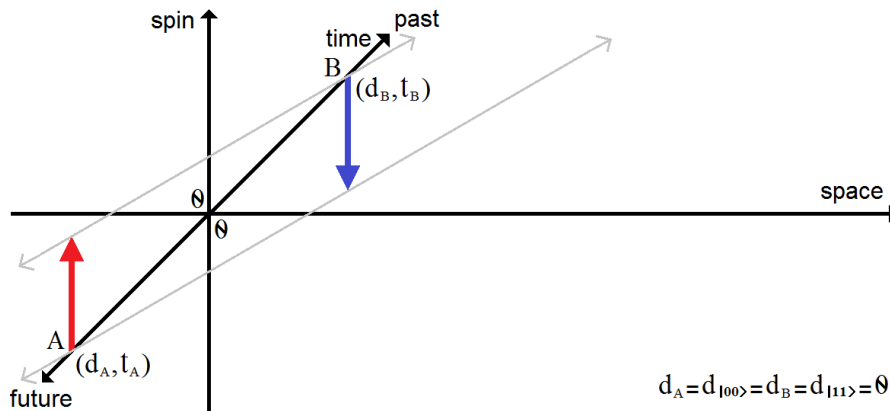


Fig.8 Entanglement between particles that have never coexisted.

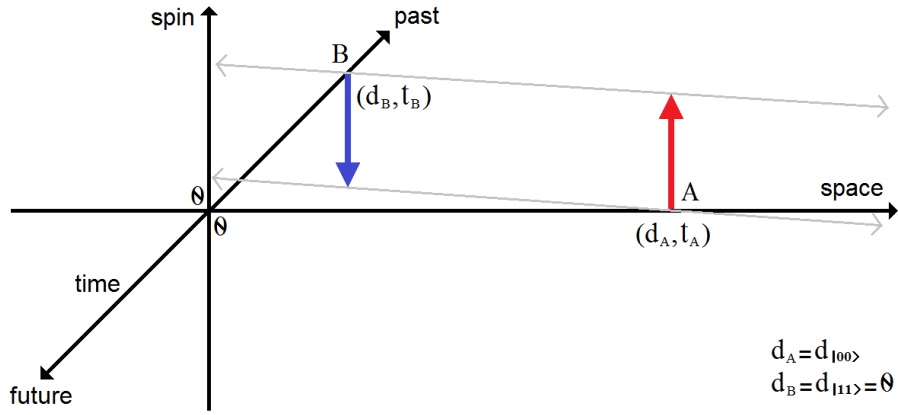


Fig.9 Particle A in the present in position d_A , and particle B in the past in position d_B .

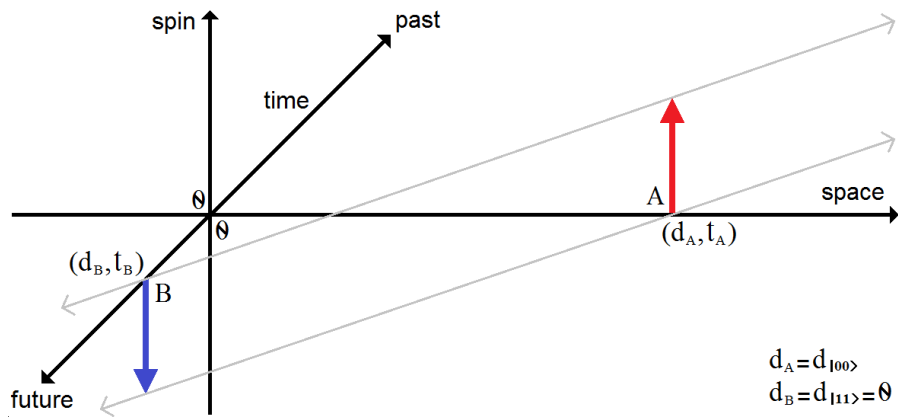


Fig.10 Particle A in the present in position d_A , and particle B in the future in position d_B .

2.6 Final thoughts on this section

From the core of Eq.(39), i.e., from the scalar version of $S^{A \cup B}$, we obtained the alter-egos of Equations (43 and 44). With the original tensor notation (no scalar) we would have never deduced the alter-egos because, with the tensor notation, values, signs, forms, and orientations are blurred; which is fulminant when working with spins. In other words, this notation is inimical to the spins that are the spice of Physics, so the tensor notation is like a bull in a china shop.

Besides, Table 1 shows us some attributes of the parallel operator “//” employed in Eq.(23, 34 and 43) which will be taken into account later.

Table 1. Some attributes of the parallel operator “//”.					
//	0	x	∞	$-x$	$-\infty$
0	0	0	0	0	0
x	0	$x/2$	x	$\pm\infty$	x
∞	0	x	∞	$-x$	$\pm\infty$
$-x$	0	$\pm\infty$	$-x$	$-x/2$	$-x$
$-\infty$	0	x	$\pm\infty$	$-x$	$-\infty$

Where x is a scalar and generic value.

On the other hand, the model for the representation of the entanglement based on the parallel operator “//” predicts strange configurations not tested in the laboratory to date. That is the case of three spins oriented according to a single axis z , that we can see in Fig.11. The spin numbers of these particles are (1,

1, -1/2). These spins do not comply with the principle of conservation of spin because $1+1-1/2 \neq 0$. The first two (μ_A and μ_B) are completely independent (red and blue in Fig.11); however, their equivalent spin (in green) obtained after the application of Eq.(23) is μ_C , which is entangled with another spin in position D, i.e., μ_D (in yellow). The result (according to the model) is a perfect entanglement between μ_C and μ_D , as long as the spins in C and D are equal and opposite, and both parallel lines (of light blue color) that link both spins through the parallel operator intersect at $+\infty$ and $-\infty$, to both sides of the Fig.11 and to infinite distances. The literature about entanglement curiously does not mention anything about it, maybe because it does not exist in reality

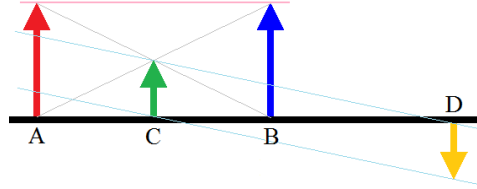


Fig.11 Two completely independent spins (μ_A and μ_B) are entangled with a third spin μ_D .

Additionally, the same model also predicts the existence of quasi-entanglement when we have two opposite spins oriented respect to the same axis but with different spin numbers, see Fig.12.

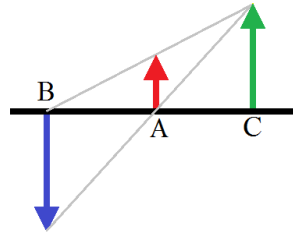


Fig.12 Quasi-entanglement of two opposite spins (μ_A in red and μ_B in blue) oriented respect to the same axis but with different spin numbers, produces the resulting spin μ_C in green.

As we can see in Fig.12, the resulting spin of quasi-entangled subsystems is on the same side and has the same sign as the spin of the smaller modulus. That is to say, by the mere fact that μ_A and μ_B have a different module, i.e., $|\mu_A| \neq |\mu_B|$, we obtain, $|\mu_C| \neq \infty$. We are going to analyze the position of the spin μ_C in the Section called Methods. Besides, and as we were able to see in the cases of completely independent, classically-correlated, and entangled spins; the results of the application of the parallel operator “//” to quasi-entangled subsystems is completely insensitive to the distance between original spins, that is to say, to the inter-spin distance. See Fig.13.

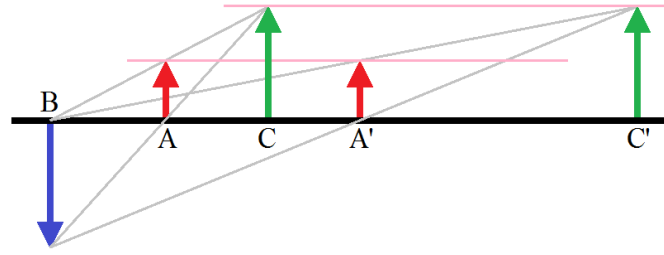


Fig.13 When spin μ_A is displaced from position A to position A', the result of the quasi-entanglement gives us a new resulting spin μ_C in a new position C'. This tells us that just as in the case of entanglement, here also, the inter-spin distances are entangled and it will have much-smaller-than-infinity module.

Table 2 compares two attributes (quantity and size) of the resulting spin after applying the parallel operator “//” to the four viewed cases, depending on the degree of correlation between both subsystems, i.e., completely independent, classically-correlated, entangled, and quasi-entangled. As we can see in Table 2, in the case of entangled subsystems only two spins are generated, i.e., both alter-egos have infinitely large module, whereas, in the other cases, a single spin is generated and it has finite module.

Table 2. Attributes (number and size) of the spins resulting from applying the parallel operator “//” according to the degree of correlation of the subsystems.				
attribute	independent	correlated	entangled	quasi-entangled
number	1	1	2	1
size	finite	finite	infinite	finite
In the case of quasi-entangled subsystems, the resulting spin is on the same side and has the same sign as the spin of smaller module, as shown in Figures 12 and 13.				

But at this point, a question automatically arises: What does each resulting spin represent physically for each of the 4 cases? In all mentioned cases, except entanglement, the resulting spin is a kind of figurehead or representative of the population of spins involved, and without being in the least bound to some kind of correlation between the spins involved. In particular, we are referring to these cases: completely independent, quasi-entangled of Figures 12 and 13, and the curious case of Fig.11. This is the typical case of some diatomic molecules, where if classical physics were applicable, the nuclei would have energy [62],

$$E_{classical} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}kx^2 \quad (51)$$

where m_1 and m_2 are the masses of the nuclei, p_1 and p_2 are the magnitudes of its momenta, k is a constant and x denotes displacement or distance. In the *center-of-mass* frame, we can set $p_1 = p_2 = p$ and, by introducing the reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = m_1 // m_2, \quad (52)$$

we obtain

$$E_{classical} = \frac{p^2}{2\mu} + \frac{1}{2}kx^2 \quad (53)$$

However, in the case of entanglement, the resulting spins have a philosophically and physically deeper (ontological) meaning. As we will see in the following section both alter-egos, are the ones that support the effect; since they generate a locality aisle (that involves all the mentioned spins, i.e., originals and alter-egos,

which will be developed later on). It is them that will give rise to all the extraordinary attributes of entanglement.

On the other hand, it is important to make clear the sign rules for the scalar spins of Eq.(4), with

$$\text{sign}(\mu_{|0\rangle}) > 0 \quad (54a)$$

$$\text{sign}(\mu_{|1\rangle}) < 0 \quad (54b)$$

$$\mu_{|xy\rangle} = \mu_{|x\rangle} \mu_{|y\rangle} \quad (55a)$$

$$-\mu_{|xy\rangle} = -\mu_{|x\rangle} \mu_{|y\rangle} \quad (55b)$$

where $\text{sign}(\bullet)$ means *sign of* (\bullet), and x and y can only be 0 or 1. Such rules are

Rule #1: $\forall \mu_{|xy\rangle}$

if $x = 0$,

$$\text{sign}(\mu_{|x\rangle}) > 0$$

else

$$\text{sign}(\mu_{|x\rangle}) < 0$$

$$\text{sign}(\mu_{|y\rangle}) = -\text{sign}(\mu_{|y\rangle})$$

end if

Rule #2: $\forall \mu_{|xy\rangle}$

$$\text{sign}(\mu_{|xy\rangle}) = -\text{sign}(\mu_{|\bar{y}\bar{x}\rangle})$$

where \bar{x} and \bar{y} mean inverse of x and y (e.g., if $x = 0$, then $\bar{x} = 1$, and vice-versa).

Rule #3: $\forall -\mu_{|xy\rangle}$

$$\text{sign}(-\mu_{|xy\rangle}) = -\text{sign}(-\mu_{|\bar{y}\bar{x}\rangle})$$

Therefore,

$$\mu_{|00\rangle} = \mu_{|0\rangle}^2 \quad (56a)$$

$$\mu_{|11\rangle} = -\mu_{|1\rangle}^2 \quad (56b)$$

$$\mu_{|00\rangle} + \mu_{|11\rangle} = 0 \quad (57a)$$

$$\mu_{|01\rangle} + \mu_{|10\rangle} = 0 \quad (57b)$$

In this way, the rules of the signs necessary for their use in the deduction of Equations (23, 34 and 43) are completed.

Finally, in today's Physics, we employ the following syllogism:

Major premiss:

The Theory of Relativity [24] says that any instantaneous phenomenon can only be local since it cannot have a speed that outpaces the speed of light [29,46,47,60]; and if there is an instantaneous nonlocal phenomenon, this can't transport information.

Minor premiss:

In Quantum Theory [4,26,42], the works of Bell [4,5,25,30,44,48-50], CHSH [51], Aspect [30], Zeller [58], and many others, establish that the entanglement is a nonlocal effect, like another phenomenon predicted by the Theory of Relativity, the ER bridge [61] (by Einstein and Rosen) or also called wormhole between two black holes.

Therefore:

There is no instantaneous transmission of information thanks to the entanglement or through the interior of a wormhole.

However, and from what has been seen so far in this work, it is logical to ask ourselves: what is the impact of this work on both theories and the apparent crack that separates them? As we have said before, the truth is that a reconciling and binding theory is needed to close the gap between both pillar theories of Physics and at the same time to test the earlier syllogism; in fact, this work will show that there is a problem of interpretation with the results of the experiments cited in the *minor premise*, not having had at their time the information provided by this work. This is what we will try to do throughout the following sections.

3 Discussion

3.1 Dilated Locality

Equations (43 and 47) tell us about two new spins (alter-egos) that arise as a consequence of the quantum entanglement, whose values are infinite in the module and are at infinite distances from the center of origin (at opposite ends of the universe). From the previous analysis, we can't deduce anything about the respective eventual masses of both spins. In fact, there are two possibilities:

- their masses are null, i.e., $m_{|\uparrow\uparrow\rangle} = m_{|\downarrow\downarrow\rangle} = 0$, and
- their masses (finite or infinite) have opposite signs, in such a way that $m_{|\uparrow\uparrow\rangle} + m_{|\downarrow\downarrow\rangle} = 0$.

From here to the rest of this section, we will work with the first ponder idea. This possibility is very interesting because if an infinite massless spin could bend the space-time, a large number of pairs of massless spins (alter-egos) could be responsible for the expansion of the universe in all directions. This fact would constitute a revolution in Physics, since, as we know from the Theory of Relativity [63-69] a big mass curves the space-time diverting the path of light beam composed by massless photons, but there is still no evidence that space-time is curved by a massless spin, even if this one was infinite. If the above is correct, we could conjecture that a pair of such extravagantly immense spins (even if they do not have mass) could divert the trajectory of elements with and without mass, or attract them directly. Besides, in a section called Methods, we are going to analyze the second one, i.e., masses with opposite signs.

On the other hand, in the literature there are numerous works about particles with infinite spin [70-75] (which, of course, have not yet been discovered, if such particles exist) as well as massless infinite spin [74,76-84] particles (with identical considerations than the previous ones regarding their eventual existence), which can also be found under the name of massless continuous spin [72,85-89], although of course, all these works are mere theoretical speculations. We must not forget the contribution of the Vasiliev Higher-Spin theory [90-95], which is a minimal extension of gravity with (massless, Gauge) fields of spin $s > 2$. It is a higher-derivative completion of gravity that has good chances of being a consistent theory of quantum gravity (without super-symmetries and extra dimensions). The idea of a relativistic spin [96-104] was not born with the present work, and as a consequence of this, a relativistic entanglement [105-117] comes to light. Besides, in recent years numerous works have appeared linking entanglement with the Theory of Relativity [118-128] directly or indirectly, some of which pose an entanglement between black holes (giving rise to an ER bridge or wormhole, even if temporarily) and others instead of the relativistic consequences of entanglement. The above concerns can be channeled through an exhaustive analysis of Fig.7, which is the key to the whole effect, given that this figure shows the immunity of entanglement to the distance between the original spins to give the same alter-egos with their same locations, i.e., at infinity. In other words, even if we modify at will the separation between $\mu_{|00\rangle}$ and $\mu_{|11\rangle}$, the distance between $\mu_{|\uparrow\uparrow\rangle}$ and $\mu_{|\downarrow\downarrow\rangle}$ will always be the same (based on Equations 49 and 50 and considering for simplicity $d_{|11\rangle} + d_{|00\rangle} = 0$), and it is represented in Fig.14, as

$$\begin{aligned}
d_{|\uparrow\uparrow\rangle} - d_{|\downarrow\downarrow\rangle} &= d_{|00\rangle} + \gamma^2(d_{|11\rangle} - d_{|00\rangle}) - (d_{|11\rangle} + \gamma^2(d_{|00\rangle} - d_{|11\rangle})) \\
&= d_{|00\rangle} + \gamma^2(d_{|11\rangle} - d_{|00\rangle}) - (-d_{|00\rangle} - \gamma^2(d_{|11\rangle} - d_{|00\rangle})) \\
&= d_{|00\rangle} + \gamma^2(d_{|11\rangle} - d_{|00\rangle}) + (d_{|00\rangle} + \gamma^2(d_{|11\rangle} - d_{|00\rangle})) \\
&= 2(d_{|00\rangle} + \gamma^2(d_{|11\rangle} - d_{|00\rangle})) = 2d_{|\uparrow\uparrow\rangle} = 2\infty = \infty
\end{aligned} \tag{58}$$

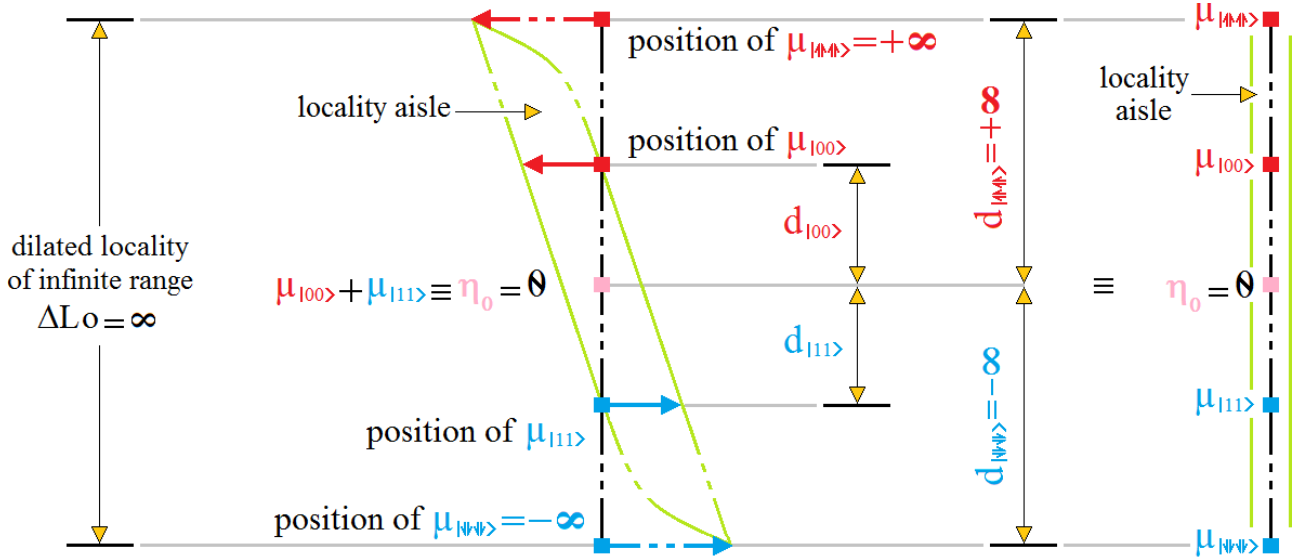


Fig.14 On the left, the dilatation of the locality with an infinite range, in the middle, the locality aisle between two parallel green lines (which is curved in the vicinity of the alter-egos since the graph is not in scale), and, on the right, a simplified version of the graphic, where η_0 means neutral meson which sometimes decays into two oppositely charged muons, and it represents the spin baseline.

This follows from Eq.(49) where $d_{|\uparrow\rangle} = +\infty$ and $d_{|\downarrow\rangle} = -\infty$, with $d_{|\uparrow\uparrow\rangle} = d_{|\uparrow\rangle}^2$ and $d_{|\downarrow\downarrow\rangle} = -d_{|\downarrow\rangle}^2$, which represents a case identical to the rule of the signs of the spins seen above. In the middle of Fig.14, we can see the original spins and their alter-egos linked by two green parallel lines which intersect exactly in the alter-egos. These lines have been drawn curved in the vicinity of the alter-egos since it is impossible to graph infinite values in such figure. As we can see, both green lines define among themselves a locality aisle whose range is infinite and is determined by Eq.(58). Let's test that statement with a simple mental experiment (*Gedankenexperiment* - in german) based on Fig.7:

- if the distance between $\mu_{|\uparrow\uparrow\rangle}$ and $\mu_{|\downarrow\downarrow\rangle}$ is so small that the experiment may be considered local, then, the application of Eq.(43) will give the same alter-egos (i.e., $\pm\infty$) and the Eq.(45 and 46) will give the same locations (i.e., $\pm\infty$), now,
- if we start to separate $\mu_{|\uparrow\uparrow\rangle}$ and $\mu_{|\downarrow\downarrow\rangle}$ until the experiment may be considered non local by our knowledge of the works of Bell [4,5,25,30,44,48-50], CHSH [51], Aspec [30], Zellinger [58], and many others, then, the application of Eq.(43) will give the same alter-egos (i.e., $\pm\infty$) and the Eq.(45 and 46) will give the same locations (i.e., $\pm\infty$) again.

At this point, there is an apparent contradiction given that in one case the experiment is local, and in the other case it is non-local, however, the effect is the same with identical results. How is it possible? If the effect is the same, then both times it is local or both times it is not local. But the same effect can't be disguised as different things according to the occasion. It is clear that something happens with the locality, as well as the fact that the alter-egos are under the same area of the effect caused by the original spins when they are entangled according to Eq.(43). On the other hand, if in the first case the original spins are so close

that the measurement instruments of the experiment are overlapped, that is, they collide, while in the second case the original spins may be as far apart as the alter-egos and even though being part of the same area of influence of the effect (which involves -of course- the above mentioned alter-egos), the only explanation is that in both cases the experiment is local with a significant influence on the locality, which translates into a dilation of it infinitely until sheltering, containing or covering the alter-egos. That is to say, based on Eq.(58), the locality is stretched like a chewing gum. Then, this dilation of the locality can be represented by the following equation

$$\Delta Lo = d_{|\uparrow\uparrow\rangle} - d_{|\downarrow\downarrow\rangle} = 2\left(d_{|00\rangle} + \gamma^2(d_{|11\rangle} - d_{|00\rangle})\right) = 2d_{|\uparrow\uparrow\rangle} = 2\infty = \infty. \quad (59)$$

ΔLo is represented to the left of Fig.14. Besides, to the right of this figure, we have an equivalent simplification of the central scheme which will help us to better understand the concepts developed in relation to Fig.15. Finally, η_0 means neutral meson which sometimes decays into two oppositely charged muons, and it represents the spin baseline, and it is simply a reference which establishes the value of spin equal to zero.

The graph to the right of Fig.14 is the same graph to the left of Fig.15, and this -in turn- is equivalent to the graph on its right, which interprets the alter-egos as two massless black holes [129-133] (massless, at least for the moment). The alter-egos are entangled with each other forming an EPR bridge or wormhole (in yellow) which can be seen in the third and fourth graph (from the left to the right) of Fig.15. In these graphs we can see that the locality aisle becomes the hyperspace (in green) when bending according to the Theory of Relativity [63-69] and the entanglement of the two massless black holes (alter-egos) forms a particular case of wormhole, which has already been vastly treated in the literature although with a slightly different approach [36,38,126,134-136], particularly in the works of Susskind [36,126] and Maldacena [36,37].

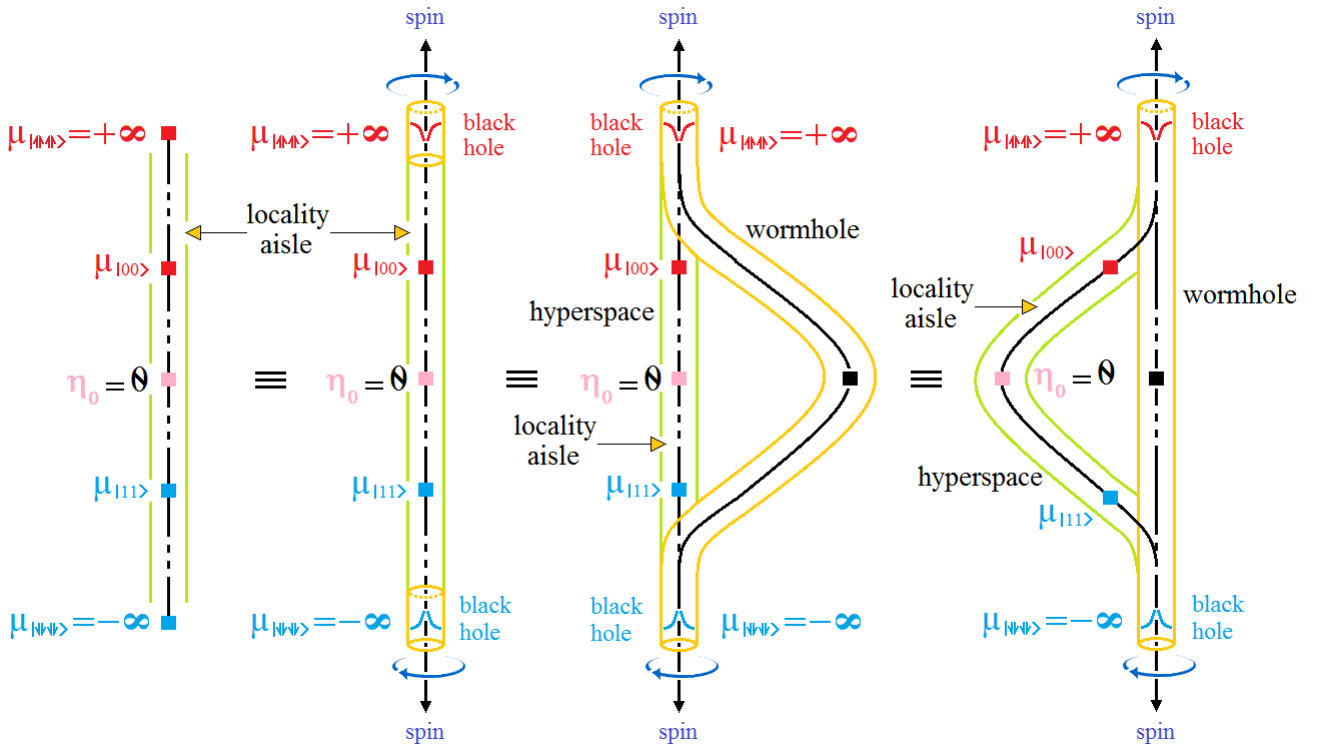


Fig.15 From bipartite entanglement to wormhole where the locality aisle curves according to the Theory of Relativity forming the hyperspace.

Obviously, the alter-egos are entangled with each other, in fact, as entangled as the original spins to each other in Eq.(43). Then and taking into account everything seen so far

$$\begin{aligned}
\mu_{|\hat{n}\hat{n}\rangle_2} &= \mu_{|\hat{n}\hat{n}\rangle} // \mu_{|\psi\psi\rangle} = \frac{\mu_{|\hat{n}\hat{n}\rangle} \mu_{|\psi\psi\rangle}}{\mu_{|\hat{n}\hat{n}\rangle} + \mu_{|\psi\psi\rangle}} = \frac{\mu_{|\hat{n}\rangle}^2 \left(-\mu_{|\psi\rangle}^2 \right)}{\mu_{|\hat{n}\rangle}^2 + \left(-\mu_{|\psi\rangle}^2 \right)} = \frac{\mu_{|\hat{n}\rangle}^2}{1 - \left(\frac{\mu_{|\hat{n}\rangle}}{\mu_{|\psi\rangle}} \right)^2} \\
&= \frac{\mu_{|0\rangle}^2 \gamma^2}{1 - \left(\frac{\mu_{|0\rangle} \gamma}{\mu_{|1\rangle} \gamma} \right)^2} = \frac{\mu_{|0\rangle}^2 \gamma^2}{1 - \left(\frac{\mu_{|0\rangle}}{\mu_{|1\rangle}} \right)^2} = \mu_{|0\rangle}^2 \gamma^4 = \mu_{|\hat{n}\rangle}^2 \gamma^2
\end{aligned} \tag{60}$$

where, $\mu_{|\hat{n}\hat{n}\rangle_2}$ means second level entanglement, with,

$$\mu_{|\hat{n}\rangle_2} = \sqrt{\mu_{|\hat{n}\hat{n}\rangle} // \mu_{|\psi\psi\rangle}} = \mu_{|0\rangle} \gamma^2 = \mu_{|\hat{n}\rangle} \gamma \tag{61}$$

Therefore, Eq.(47, 49, 58 and 61) constitute an approximation to the Theory of Everything [39], e.g., in the case of Eq.(61) we can see that this equation is a perfect balance between a spin (which is the fifth essence of Quantum Theory) and the Lorentz's factor (which is a central tool in the Theory of Relativity); i.e., entanglement generates relativistic spins and hence relativistic entanglement. The negative complement of Eq.(61) will be

$$\mu_{|\psi\rangle_2} = \mu_{|\psi\rangle} \gamma = - \left| \mu_{|\psi\rangle} \right| \gamma \tag{62}$$

Thus, Eq.(61) and its negative complement (i.e., Eq.62) represent an ER bridge or wormhole, where the spin of Eq.(47), i.e., $\mu_{|\hat{n}\rangle} = \mu_{|0\rangle} \gamma$ and its negative complement $\mu_{|\psi\rangle} = \mu_{|1\rangle} \gamma$ are the massless black holes of Fig.15, which when entangled generate as a result the spins of the Eq.(61 and 62). That is to say, the effect is being chained, and each time more pronounced, getting alter-egos from alter-egos. It is clear at this point the main role of the alter-egos in the entanglement and an eventual quantum communication through a wormhole. To do justice to the importance of alter-egos in the entanglement we propose the following mental experiment (*Gedankenexperiment* - in german) based on Fig.7 again. Let's define the following rates spin/distance-from-center

$$\mu'_{|00\rangle} = \frac{\mu_{|00\rangle} \times [\text{units of distance}]}{d_{|00\rangle}} \quad \text{and} \quad \mu'_{|11\rangle} = \frac{\mu_{|11\rangle} \times [\text{units of distance}]}{d_{|11\rangle}} \tag{63}$$

where $\mu'_{|00\rangle}$ and $\mu'_{|11\rangle}$ don't have physical units (like spins). Then, if we start with both spins relatively close (and this statement can be as arbitrary as we want), and we separate them, both ratios of Eq.(63) start deteriorating (i.e., they are approaching zero), therefore, and according to Fig.7 and Eq.(43) the alter-egos remain intact in magnitude and position, while the ratios of Eq.(63) disappear, which clearly indicates that the alter-egos sustain the effect, that is, the entanglement, even more so than the original spins themselves, which give existence to the alter-egos. Evidence of this can be obtained by replacing the alter-egos and their positions in Eq.(63); in this case, the ratios will always be equal to 1, regardless of the position occupied by the original spins, i.e., these new ratios do not degrade with the positions of the original spins.

On the other hand, the graph on the right of Fig.15 (i.e., the last one) is equivalent to the graph on the left of Fig.16 (i.e. the first one), with a pronounced curvature of hyperspace (or locality aisle).

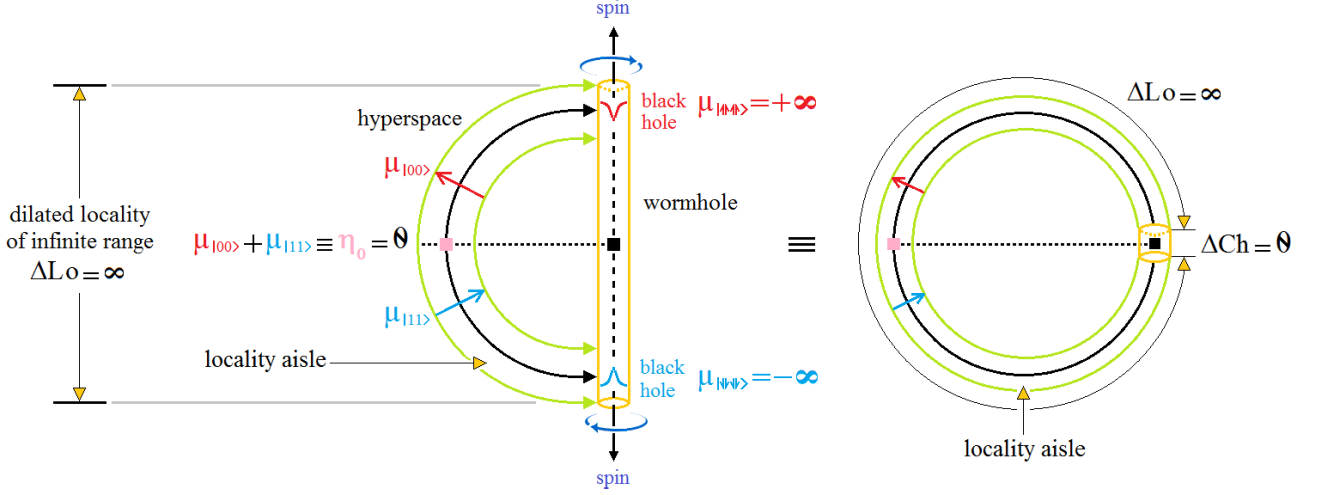


Fig.16 Graph on the left is a continuity of Fig.15, with a very pronounced curvature of hyperspace (locality aisle) according to Equations (60 and 61). The graph on the right shows us the complete curvature of hyperspace to the point of provoking a super-compression (or overshinking) on the channel (ER bridge or wormhole). This last graphic is the platform to analyze the virtues of entanglement in communications.

Equations (60 and 61) tell us that as the alter-egos are entangled, then the curvature of space-time becomes extravagantly exaggerated to the point of completely closing in a perfect circle (infinite stretching of the locality aisle) crushing the ER bridge or wormhole to a null trajectory or channel length with dramatic consequences. The most relevant of these consequences consists of the appearance of a trade-off between the range of the locality ΔL_o (see Eq.59) and the length of the channel (ER bridge or wormhole) ΔCh , both in meters

$$\Delta L_o \Delta Ch \geq \frac{l_p^2}{4\pi} \quad (64)$$

where $l_p = 1.616229(38) \times 10^{-35}$ is the Planck length in meters. Equations (64) will be deduced formally in a next section called Method, and it is the *entanglement's uncertainty principle*. The most outstanding thing to highlight at this moment is that, as the locality stretches to infinity, the channel is compressed (shrunk) to zero. This phenomenon is of vital importance when we later deduce the attributes of entanglement from the point of view of quantum communications, i.e., bandwidth, latency and channel capacity.

3.2 Final thoughts on this section

Although Eq.(59) shows a dilation of the locality directly proportional to γ^2 instead of γ , from the point of view of an eventual communication between both alter-egos, everything is local, i.e., the link behaves operatively as a point. In fact, and as we have seen, the channel has null length. This last characteristic has surprising implications, since the main argument by which we cannot transmit useful information thanks to a link based on the entanglement, is that -given its instantaneousness- any transmission at such a distance d , that if divided by a time $t = 0$ would result in a velocity greater than the speed of light, i.e., $v > c$. This would automatically imply that the information traveled faster than c , which results in an inconceivable concept and it is in clear opposition to what is established by the Theory of Relativity. That is, what is questioned is not the instantaneousness (null latency) of entanglement but the fact that it could overcome the speed of light to transmit information. However, and based on what is seen here in this work, particularly if we look at the graph on the right of Fig.16, and relate it to Eq.(64) we can carry out another mental experiment (*Gedankenexperiment* - in german). Imagine a turtle (*Dermochelys coriacea*) crossing the length channel $\Delta Ch = 0$ at a speed

$$v_{turtle} = .09 \left[\frac{meter}{sec} \right] \ll c \approx 300,000,000 \left[\frac{meter}{sec} \right]. \quad (65)$$

How long would it take the turtle to cross that channel?

$$Latency = \frac{\Delta Ch}{v_{turtle}} = \frac{0[meter]}{.09[meter/sec]} = 0 sec. \quad (66)$$

That is to say, and to exemplify, the trip of a turtle through a channel (ER bridge or wormhole) of null length is instantaneous even when the speed used by the turtle is much less than c . In this way, the entanglement retains its main attribute without contradicting the Theory of Relativity, because it is a local effect of infinite range, and as we have already said, it is also the key to the TOE [39], given that if we rewrite Eq.(5) based on the original spins

$$\begin{aligned} |\Phi_{\pm}^{A \cup B}\rangle &= \frac{1}{\sqrt{2}} \left(|\mu_{|0\rangle}^A, \mu_{|0\rangle}^B \rangle \pm |\mu_{|1\rangle}^A, \mu_{|1\rangle}^B \rangle \right) = \frac{1}{\sqrt{2}} \left(|\mu_{|00\rangle} \rangle \pm |\mu_{|11\rangle} \rangle \right), \\ |\Psi_{\pm}^{A \cup B}\rangle &= \frac{1}{\sqrt{2}} \left(|\mu_{|0\rangle}^A, \mu_{|1\rangle}^B \rangle \pm |\mu_{|1\rangle}^A, \mu_{|0\rangle}^B \rangle \right) = \frac{1}{\sqrt{2}} \left(|\mu_{|01\rangle} \rangle \pm |\mu_{|10\rangle} \rangle \right) \end{aligned} \quad (67)$$

we can define symbolically the entanglement between both black holes of Fig.16 based on the Eq.(67)

$$\begin{aligned} |\Phi_{\pm}^{black\ holes}\rangle &= \frac{1}{\sqrt{2}} \left(|\mu_{|\uparrow\uparrow\rangle} \rangle \pm |\mu_{|\downarrow\downarrow\rangle} \rangle \right) = \frac{1}{\sqrt{2}} \left(|\mu_{|00\rangle} \rangle \pm |\mu_{|11\rangle} \rangle \right) \gamma^2 = |\Phi_{\pm}^{A \cup B}\rangle \gamma^2 \\ |\Psi_{\pm}^{black\ holes}\rangle &= \frac{1}{\sqrt{2}} \left(|\mu_{|\uparrow\downarrow\rangle} \rangle \pm |\mu_{|\downarrow\uparrow\rangle} \rangle \right) = \frac{1}{\sqrt{2}} \left(|\mu_{|01\rangle} \rangle \pm |\mu_{|10\rangle} \rangle \right) \gamma^2 = |\Psi_{\pm}^{A \cup B}\rangle \gamma^2 \end{aligned} \quad (68)$$

Then, Eq.(68) represents the essence of the TOE, where its last terms have on the left the original Bell's bases, which constitute a main role in Quantum Theory and, on the right the Lorentz factor as its counterpart in the Relativity Theory. In fact, Eq.(68) is the relativistic version of the Bell's bases.

Thanks to the Theory of Dilated Locality there is no confrontation between the Theory of Relativity and Quantum Theory because of the entanglement. On the contrary, there is a union since the Theory of Relativity assists entanglement with a super curvature of space-time and so prevents the entanglement from violating the Theory of Relativity, that is to say, it is the Theory of Relativity itself that collaborates with entanglement so as not to be violated.

Besides, if the channel is reduced to a point (that is, no channel) and the information is completely shared, it is clear that the following principle is applied: *What You Put is What You Get* (WYPiWYG). Therefore, the bandwidth of a link via entanglement is infinite, and as there is no channel then there is no noise, at least, there is no noise due to the channel, therefore, with infinite bandwidth and without noise, we will have a channel capacity which is also infinite. And finally, the cherry on the cake: it is impossible to attack, intercept or hack a channel that does not exist, i.e., thanks to the entanglement we can do without the compression and the encryption of the data that is being manipulated. Actually and given that the channel is reduced to a point, instead of talking about transmitting information we prefer to talk about sharing information at that point, it is even as if there were no signal. The arguments mentioned above and the following are those that avoid violating the No-Communications Theorem [31,137-141] to a transmission of information through a link based on entanglement, thus, we will start with a simple analysis about this important theorem. The No-Communication Theorem works with some matrices called Kraus matrices which satisfy certain conditions and their means –for example– that Alice's measurement apparatus does not influence Bob's subsystem, i.e., it starts from the non-locality established by Bell's theorem, CHSH and the apparent experimental verifications of such non-locality due to Aspec, among others. We say “apparent experimental verifications” because such experiments were consistent with a preconception which is completely displaced in this work. Then, the mentioned Kraus matrices continue being manipulated until they reach a scalar on which the theorem says: *from this scalar, we can argue that, statistically, Bob cannot discern the difference between what Alice did and a random measurement (or if she just did something)*. That is, everything begins from the assumption that non-locality is the only reality around entanglement; but as we have seen throughout this work; the entanglement is a local phenomenon of infinite range, however, it does not violate the No-Communication Theorem. Besides, entanglement does not need a transmission at a speed $v > c$ to share information instantaneously. As we have previously seen, in relation to the Eq.(63) it is the alter-egos that sustain the effect and, in fact, they are the only ones that communicate, what's more, they share information given that they are in the same point, because the channel has zero length, i.e., they communicate without doing so. Therefore, the No-Communication Theorem does not apply to alter-egos, which are the only ones that manipulate information without violating the Theory of Relativity. In other words, in relation to entanglement we can draw two interesting conclusions:

- it is a local phenomenon of infinite range,
- the original spins do not communicate but they do it through their representatives, i.e., their alter-egos, and
- in every experiment related to the entanglement, the measured entangled pairs are those of Eq.(5) also known as singlet states, Bell bases, or the bases of Eq.(5), and they are based on the original spins of Eq.(4), nevertheless, alter-egos are could never be measured (for obvious reasons). However, if such thing as measuring an alter-ego was possible, in fact, if we measured one, we would measure the other because they share the same space (point = channel).

Therefore, there is not the slightest inkling of limitation or impediment to the instantaneous sharing of useful information via a link based on entanglement. At least until the entanglement is destroyed, for example, through a quantum measurement.

With what has been said so far, it should be clear that Entanglement is the meeting point and not the cause of disagreement between the two main pillars of Physics, that is, it is the cornerstone or instrument that ends up amalgamating everything. This is extremely positive since a better understanding of entanglement could lead us to convert a measurement into a message, or rather, part of it.

It is also clear the impact of this better understanding of entanglement in quantum communications in general, and in a possible interplanetary communication in particular, such as for the much-mentioned trip to Mars. Besides, thanks to the present work three types of the communication system are revealed:

- classic (via an electromagnetic transmission), for disambiguation in a quantum information transmission, e.g., Teleportation [1-3,58],
- quantum (via optical fiber), for a classic information transmission, e.g., Superdense Coding [28, 57-59], and
- quantum (via entanglement), which will give rise to a new communication system with links that do not have obstacles that interrupt it, neither fading nor vanishing. Everything said, with the real possibility of giving rise to new protocols for Teleportation, which allow a disambiguation without the use of a classic channel. This would allow a completely instantaneous Teleportation without violating the Theory of Relativity.

On the other hand, and since the entanglement is "monogamous" and taking into account that $\Delta Ch = 0$, the security of the link should be completely preserved. However, the impact on Teleportation and Superdense coding goes much further, since under the parallel operator of Eq.(43) we could think about protocols considering not only an exclusive wavefunction representative of entanglement (as we know from the literature on this subject [1-3]) but on the contrary, recognizing more individuality to the original spins involved in that eventual protocol, that is to say, a new protocol where Alice and Bob do not lose their individualities. Moreover, and already in the terrain of Figures 15 and 16, a possible teleportation based on the entanglement of both black holes could be thought as a distribution of an entangled pair of the type of Eq. (68) between Alice and Bob, instead of sharing pairs of the type of Eq.(67). This opens up a whole new world of possibilities to Teleportations of another type, which to date are mere speculations.

Furthermore, from the Eq.(43), it is clear to understand why there is no entanglement in the classical world and the reason is that in the world of the macroscopic (whether natural or man-made) there are not two perfectly equal things in such a way that with opposite sign they can be entangled by some means and reproduce an effect as extraordinary as the case of the entanglement for elementary spins, which (and unlike any classic counterparts) if they can be perfectly equal, they cancel the denominator of the Eq.(43) and give infinite alter-egos, with everything which that entails. That is to say, everything is reduced to the Eq.(43), being both spins perfectly equal in that the denominator of the equation is annulled.

For the option chosen in this section, that is, that the alter-egos are massless black-holes, we conclude that they do not emit Hawking's radiation [38,142,143], precisely because such radiation depends on the mass of the black hole.

Although we have taken the decision to completely strip this paper of all complicated symbolic logic so that the physical effect and its attributes survive above everything else, several questions remain to be answered:

- What attributes of the original spins are transferred to alter-egos? Since these spins are one of the so many attributes of the particle to which the spin belongs, e.g., if the original spins are photons (bosons) with mass = 0, electric charge = 0, mean lifetime: stable, among others, all this seems to be transferred from the original to the alter-egos, except the spin which in photons is 1, and it is precisely this one that is reached by the relativistic dilation becoming ∞ , and
- What kind of black holes are the alter-egos? We know that they are massless infinite spins, but what else? What about their linear momentum, charge, etc?

These questions will remain for an upcoming work, however, in the next section, we will delve into the relationship between alter-egos and black holes in order to verify the attributes of entanglement for quantum communications.

4 Methods

In this section, we will develop the necessary method for the verification of the concepts conceived in the previous sections, in particular, regarding the main attributes of entanglement in quantum communications, i.e., bandwidth and latency. This method is called Quantum Spectral Analysis-Frequency at Time (QSA-FaT) [144,145]. Also, we will demonstrate some theorems related to the projection of the mentioned attributes in communications. Besides, we will develop a new deduction of the alter-egos but this time from the Hamiltonian of entanglement. Finally, we will expose the possibility that the pair of alter-egos has equal masses and of opposite sign.

4.1 Prolegomena to Quantum Spectral Analysis-Frequency at Time

A unitary operator U can transform a quantum state into another one, with, $U: H \rightarrow H$ on a Hilbert space H , where U will be a unitary operator if it satisfies the condition: $U^\dagger U = U U^\dagger = I$. Besides, $(\bullet)^\dagger$ is the adjoint of (\bullet) , and I is the identity matrix. Such condition is required to preserve inner products, inasmuch as, we can transform $|\chi\rangle$ and $|\psi\rangle$ to $U|\chi\rangle$ and $U|\psi\rangle$, thus $\langle\chi|U^\dagger U|\psi\rangle = \langle\chi|\psi\rangle$. In particular, unitary operators preserve lengths:

$$\langle\psi|U^\dagger U|\psi\rangle = \langle\psi|\psi\rangle = \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\alpha|^2 + |\beta|^2 = 1, \quad (69)$$

Besides, the unitary operator satisfies the following differential equation known as the Schrödinger equation [13,16,17]:

$$\frac{d}{dt} U(t + \Delta t, t) = \frac{-i \hat{H}}{\hbar} U(t + \Delta t, t) \quad (70)$$

where \hat{H} represents the Hamiltonian matrix of the Schrödinger equation, while $i = \sqrt{-1}$, and \hbar is the reduced Planck constant, i.e., $\hbar = h/2\pi$. Multiplying both sides of Eq.(70) by $|\psi(t)\rangle$ and setting

$$|\psi(t + \Delta t)\rangle = U(t + \Delta t, t) |\psi(t)\rangle \quad (71)$$

Being $U(t + \Delta t, t) = U(t + \Delta t - t) = U(\Delta t)$ a unitary transform (operator and matrix), yields

$$\frac{d}{dt} |\psi(t)\rangle = \frac{-i \hat{H}}{\hbar} |\psi(t)\rangle \quad (72)$$

The Hamiltonian operator represents the total energy of the system and controls the evolution process.

In most of the cases, the Hamiltonian is formed by kinetic and potential energy. However, if the particle is stationary thus the kinetic energy is cancelled, leaving only the potential energy which will be the only one that will be linked to external forces applied to this particle. Thus the control of the external forces is at the same time the control of the evolution of the states of the system [1,2,13,16,17,146,147]. For example, in the case of bosons (in particular, photons), they possess integer spin (i.e., $\mu = 1$), we would have a momentum,

$$\sigma \cdot P = \begin{pmatrix} P_z & P_x - iP_y \\ P_x + iP_y & -P_z \end{pmatrix}, \quad (73)$$

being $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ the Pauli's matrices, that is to say,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (74)$$

while the spin will be,

$$S = \hbar \sigma = \hbar \mu \sigma = \hbar \mu (\sigma_x, \sigma_y, \sigma_z). \quad (75)$$

Then, the Hamiltonian takes the following form,

$$H = \frac{c S \cdot P}{\hbar} = \frac{c \mu \hbar \sigma \cdot P}{\hbar} = \frac{c \mu \hbar}{\hbar} \begin{pmatrix} P_z & P_x - iP_y \\ P_x + iP_y & -P_z \end{pmatrix} = \hbar \mu \Omega \quad (76)$$

being c the speed of light, Ω will result in this case,

$$\Omega = \frac{c}{\hbar} \begin{pmatrix} P_z & P_x - iP_y \\ P_x + iP_y & -P_z \end{pmatrix} \quad (77)$$

At this point, if we consider a polarization of spin regarding the z-axis exclusively, thus,

$$\Omega_z = \frac{c}{\hbar} \begin{pmatrix} P_z & 0 \\ 0 & -P_z \end{pmatrix} = \frac{c}{\hbar} P_z \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{c}{\hbar} P_z \sigma_z \quad (78)$$

with a Hamiltonian,

$$H = \hbar \mu \Omega_z = \hbar \mu \frac{c}{\hbar} P_z \sigma_z = \hbar \mu \omega \sigma_z \quad (79)$$

where ω is the angular frequency, which is the same in all directions since we will consider Ω as spatially isotropic and homogeneous.

Finally, solving Eq.(72) depending on the Hamiltonian of Eq.(79), we will have the solution to Schrödinger equation based on the exponential matrix of the Hamiltonian matrix, i.e.,

$$|\psi(t + \Delta t)\rangle = e^{\frac{-i \hat{H} \Delta t}{\hbar}} |\psi(t)\rangle \quad (\text{if Hamiltonian is not time-dependent}) \quad (80)$$

$$|\psi(t + \Delta t)\rangle = e^{\frac{-i}{\hbar} \int_t^{t+\Delta t} \hat{H} dt} |\psi(t)\rangle \quad (\text{if Hamiltonian is time dependent}) \quad (81)$$

Discrete versions of Equations (80) and (81) for a time-dependent (or not) Hamiltonian, being k the discrete time. With,

$$|\psi_{k+\Delta k}\rangle = e^{\frac{-i \hat{H} \Delta k}{\hbar}} |\psi_k\rangle = e^{-i \mu \omega_k \sigma_z \Delta k} |\psi_k\rangle \quad (\text{if Hamiltonian is not time-dependent}) \quad (82)$$

$$|\psi_{k+\Delta k}\rangle = e^{\frac{-i}{\hbar} \sum_k^{k+\Delta k} \hat{H}_k} |\psi_k\rangle = e^{-i \mu \sigma_z \sum_k^{k+\Delta k} \omega_k} |\psi_k\rangle \quad (\text{if Hamiltonian is time-dependent}) \quad (83)$$

$$|\psi_{k+1}\rangle = e^{-i \mu \sigma_z \sum_{i=1}^{k+1} \omega_i} |\psi_0\rangle \quad (\text{with } \Delta k = 1, \text{ and starting from initial state }^{13} |\psi_0\rangle) \quad (84)$$

Moreover, replacing Eq.(79) into Eq.(80) and (81), we will have another main equation for this paper,

$$|\psi(t + \Delta t)\rangle = e^{-i \mu \omega(t) \sigma_z \Delta t} |\psi(t)\rangle, \quad (85)$$

$$|\psi(t + \Delta t)\rangle = e^{-i \mu \sigma_z \int_t^{t+\Delta t} \omega(t) dt} |\psi(t)\rangle. \quad (86)$$

Finally, and considering an incremental approximation of Eq.(72) as well as in its discrete version, and considering the proper replacements of Eq.(79), both versions of Schrödinger's equation will take the following form respectively,

$$\frac{|\Delta\psi(t+\Delta t)\rangle}{\Delta t} = -i\mu\omega(t)\sigma_z|\psi(t+\Delta t)\rangle \quad (87)$$

and

$$|\Delta\psi_{k+\Delta k}\rangle = \frac{|\psi_{k+\Delta k+1} - \psi_{k+\Delta k-1}\rangle}{2} = -i\mu\omega_k\sigma_z|\psi_{k+\Delta k}\rangle \quad (88)$$

4.2 Quantum Spectral Analysis-Frequency at Time (QSA-FaT)

This tool plays a main role in the development of this section, besides, it is so important when it is applied in signal analysis, in particular for the practical calculation of the bandwidth of any type of signal in a much more direct way than the Fourier theory [148-150]. In fact, A *quantum time-dependent spectrum analysis*, or simply, quantum spectral analysis: frequency at time (QSA-FaT) complements and completes the Fourier theory, especially its maximum exponent, i.e., the fast Fourier transform (FFT) [151-154]. For all the above, QSA-FaT constitutes a practical temporal-spectral bridge^{144,145}. Finally, QSA-FaT is a metric, which assesses the impact of the flanks of a signal on its frequency spectrum at each instant, which is not taken into account by Fourier theory and even less in real time. This is the reason why they must both work together.

Next, we are going to deduce this operator in its continuous and discrete forms for a quantum state. There are several versions of QSA-FaT [144,145], in this case, we will deduce this operator in its continuous and discrete versions of Equations (87) and (88), respectively. Therefore, if we multiply by $\langle\psi|$ both sides of Eq.(87), we will have,

$$\langle\psi(t)|\frac{\Delta\psi(t)}{\Delta t}\rangle = -i\mu\omega(t)\langle\psi(t)|\sigma_z|\psi(t)\rangle \quad (89)$$

then,

$$\Delta\omega(t) = \mu\omega(t) = i\frac{1}{\langle\psi(t)|\sigma_z|\psi(t)\rangle}\langle\psi(t)|\frac{\Delta\psi(t)}{\Delta t}\rangle \quad (90)$$

Now, if we multiply both sides of Eq.(88) by $\langle\psi_k|$, we will have,

$$\frac{\langle\psi_k|\psi_{k+1} - \psi_{k-1}\rangle}{2} = -i\mu\omega_k\langle\psi_k|\sigma_z|\psi_k\rangle \quad (91)$$

then,

$$\Delta\omega_k = \mu\omega_k = i\frac{\langle\psi_k|\psi_{k+1} - \psi_{k-1}\rangle}{2\langle\psi_k|\sigma_z|\psi_k\rangle} = i\frac{(\langle\psi_k|\psi_{k+1}\rangle - \langle\psi_k|\psi_{k-1}\rangle)}{2\langle\psi_k|\sigma_z|\psi_k\rangle} \quad (92)$$

That is to say, we are going to have a $\Delta\omega$ at each instant of the signal (continuous or discrete, classical or quantum). On the other hand, a very interesting attribute of this operator is that it is not affected by the quantum measurement problem, because its output is a classical scalar, i.e., it can be measured with complete accuracy. In fact, the operator $\Delta\omega$ arises from a hybrid algorithm with quantum and classical parts, as we can see in Fig.17 where a single fine line represents 1 or N qubits, while a single thick line represents 1 or N classical bits. Moreover, the quantum part of the operator $\Delta\omega$ must respect the concept of reversibility because it is closely related to energy consumption, and consequently to the Landauer's Principle¹³, for this reason, $|\psi_k\rangle$ also appears on the way out. Thus,

Quantum part:

$$a_k = \langle \Psi_k | \Psi_{k+1} \rangle, \quad b_k = \langle \Psi_k | \Psi_{k-1} \rangle, \quad c_k = \langle \Psi_k | \sigma_z | \Psi_k \rangle \quad (93)$$

Classical part:

$$\Delta\omega_k = \mu \omega_k = i \frac{(a_k - b_k)}{2c_k} \quad (94)$$

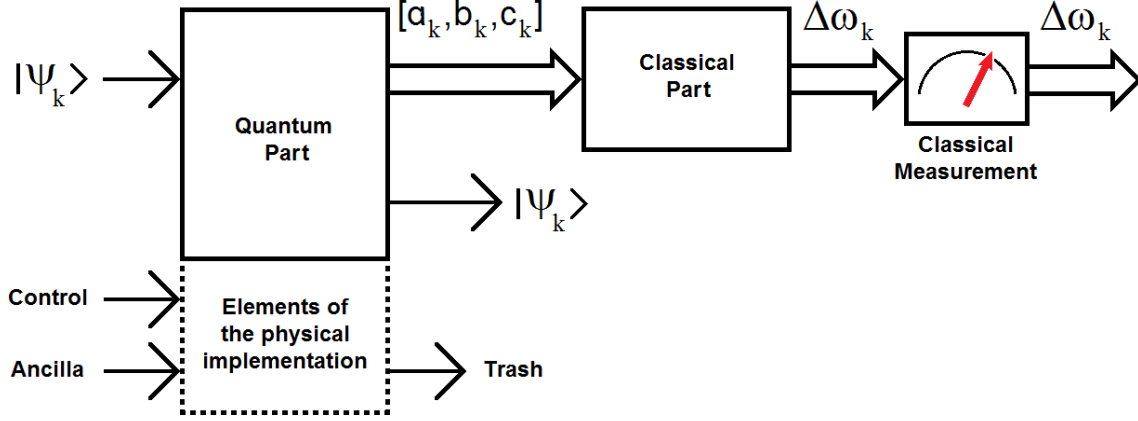


Fig. 17 A hybrid algorithm with quantum and classical parts.

Finally, for all mentioned cases, that is to say, continuous or discrete, classical or quantum signals, the bandwidth BW will result from the difference between the maximum and the minimum frequency of such signal,

$$BW = f_{\max} - f_{\min} = \frac{1}{2\pi} (\Delta\omega_{\max} - \Delta\omega_{\min}) \quad (95)$$

A practical example will make things clearer. This is the case of the application of QSA-FaT to a classical signal. There are several versions and ways to apply QSA-FaT to a classical signal [144,145]. For any signal (in general) it is necessary to equalize it and calibrate it in quadrature with FFT [144,145], but for a pure tone this procedure is unnecessary, so we can access the result in a much simpler way. Therefore, the direct classical continuous version of Equations (90) and (92) will be of the form,

$$\omega(t) = \frac{\eta}{s(t)} \frac{ds(t)}{dt}, \quad (96)$$

where $s(t)$ is the signal, and η is an adjustment factor. While the discrete version will be,

$$\omega_k = \frac{\eta}{s_k} \frac{(s_{k+1} - s_{k-1})}{2}. \quad (97)$$

The problem with Equations (96) and (97) consists in the indeterminacy of $\Delta\omega$ when the signal is null at that instant. Then, we will use a modified version of the signal called *baseline-less* (BLL) which consists of,

$$\omega(t) = \frac{1}{s_{BLL}} \frac{ds(t)}{dt}, \quad (98)$$

with $\eta = 1$, where,

$$s_{BLL} = \frac{s_{\max} - s_{\min}}{2}, \quad (99)$$

then, ω , f_{max} and f_{min} will be

$$\omega(t) = \frac{1}{\left(\frac{s_{max} - s_{min}}{2}\right)} \frac{ds(t)}{dt}, \quad (100)$$

$$f_{max} = \frac{1}{2\pi} \frac{1}{\left(\frac{s_{max} - s_{min}}{2}\right)} \left(\frac{ds(t)}{dt}\right)_{max} = \frac{(ds(t)/dt)_{max}}{\pi(s_{max} - s_{min})}, \quad (101)$$

$$f_{min} = \frac{1}{2\pi} \frac{1}{\left(\frac{s_{max} - s_{min}}{2}\right)} \left(\frac{ds(t)}{dt}\right)_{min} = \frac{(ds(t)/dt)_{min}}{\pi(s_{max} - s_{min})}, \quad (102)$$

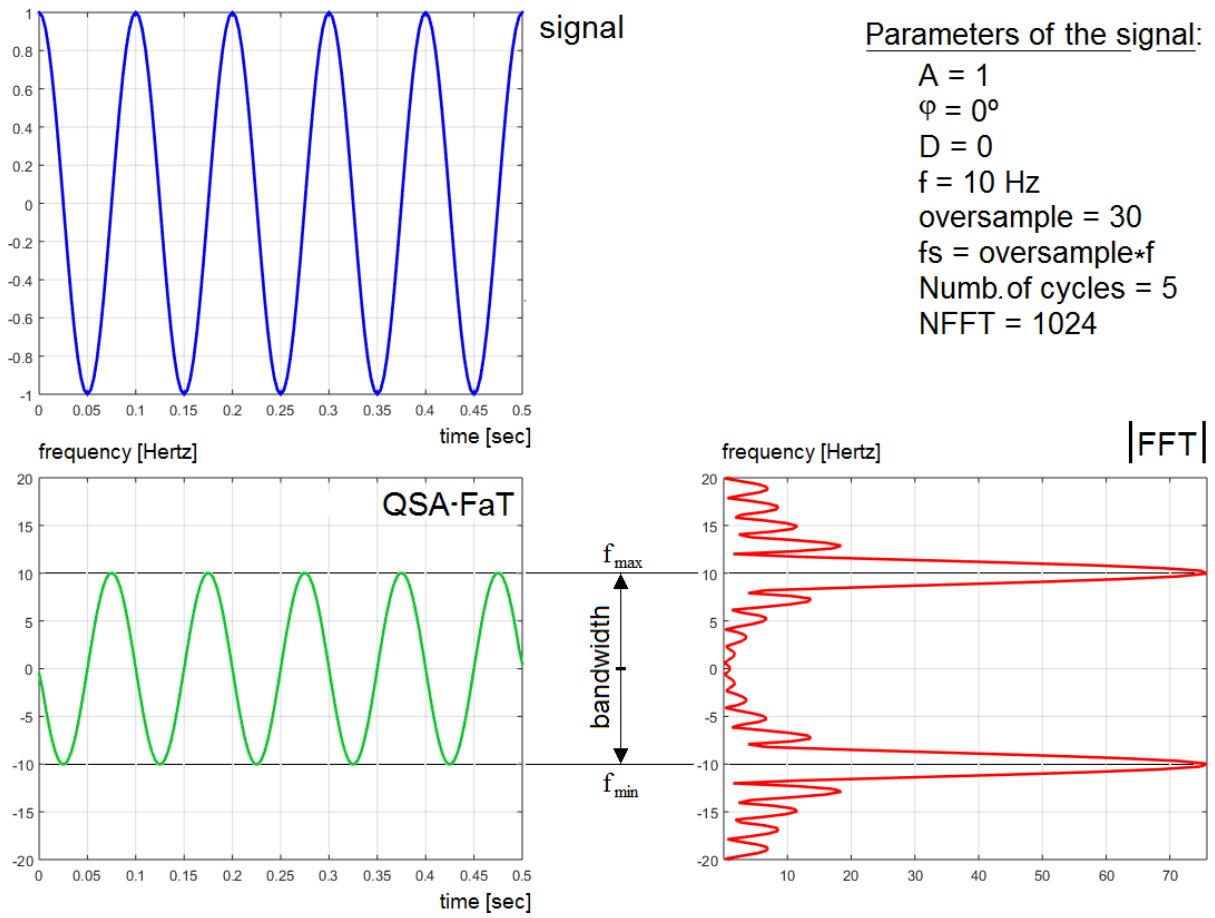


Fig. 18 Example of signal (in blue), QSA-FaT (in green), and |FFT| (in red) of cosine.

Now, if we consider a signal like Fig.18 (in blue),

$$s(t) = A \cos(\omega t + \varphi) + B, \quad (103)$$

where A is the amplitude, φ is the phase, and B is the baseline, with,

$$\frac{ds(t)}{dt} = -A \omega \sin(\omega t + \varphi), \quad (104)$$

then,

$$s_{\max} = A + B, \quad s_{\min} = -A + B. \quad (105)$$

Now, replacing Equations (104) and (105) into (100), we will have,

$$\omega(t) = \frac{1}{\left(\frac{(A+B) - (-A+B)}{2}\right)} (-A \omega \sin(\omega t + \varphi)) = -\omega \sin(\omega t + \varphi) \quad (106)$$

in green in Fig.18, then,

$$f_{\max} = \frac{\omega}{2\pi} = \frac{2\pi f}{2\pi} = f, \quad f_{\min} = \frac{-\omega}{2\pi} = \frac{-2\pi f}{2\pi} = -f. \quad (107)$$

Then, replacing Eq.(107) into (95), we will have,

$$BW = f_{\max} - f_{\min} = f - (-f) = 2f. \quad (108)$$

This result can be seen in the lower part of Fig.18 between QSA-FaT and |FFT|, which is the total aperture of QSA-FaT (in green) and at the same time, the distance between the peaks of |FFT| (in red).

Now, we are going to calculate the spectral analysis of one of the original spins thanks to the operator QSA-FaT. Therefore, if we resort to Eq.(87) but for one of the bases of Eq.(4), e.g., $|00\rangle$

$$\frac{d|00\rangle}{dt} = (-i \mu_{|00\rangle} \omega) [\sigma_z \cdot \oplus \sigma_z] |00\rangle \quad (109)$$

we're going to need to use a new operator " $\cdot \oplus$ " (which is easy to generalize) on the Pauli matrix σ_z of Eq.(74), this new operator is the only substantial difference between the Equations (87) and (109), and accounts for the dimensional difference between the two equations. So that, if

$$\begin{aligned} A &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ and } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \text{ therefore} \\ A \cdot \oplus B &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \oplus \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + b_{11} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + b_{12} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + b_{21} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + b_{22} \end{bmatrix} \\ &= \begin{bmatrix} \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{11} \\ a_{21} + b_{11} & a_{22} + b_{11} \end{bmatrix} & \begin{bmatrix} a_{11} + b_{12} & a_{12} + b_{12} \\ a_{21} + b_{12} & a_{22} + b_{12} \end{bmatrix} \\ \begin{bmatrix} a_{11} + b_{21} & a_{12} + b_{21} \\ a_{21} + b_{21} & a_{22} + b_{21} \end{bmatrix} & \begin{bmatrix} a_{11} + b_{22} & a_{12} + b_{22} \\ a_{21} + b_{22} & a_{22} + b_{22} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{11} & a_{11} + b_{12} & a_{12} + b_{12} \\ a_{21} + b_{11} & a_{22} + b_{11} & a_{21} + b_{12} & a_{22} + b_{12} \\ a_{11} + b_{21} & a_{12} + b_{21} & a_{11} + b_{22} & a_{12} + b_{22} \\ a_{21} + b_{21} & a_{22} + b_{21} & a_{21} + b_{22} & a_{22} + b_{22} \end{bmatrix} \end{aligned} \quad (110)$$

Now, applying the new operator on the Pauli matrices

$$\begin{aligned}
\sigma_{z^*} \oplus \sigma_z &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \oplus \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + 1 & \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + 0 \\ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + 0 & \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - 1 \end{bmatrix}, \\
&= \begin{bmatrix} \begin{bmatrix} 1+1 & 0+1 \\ 0+1 & -1+1 \end{bmatrix} & \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} \\ \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} & \begin{bmatrix} 1-1 & 0-1 \\ 0-1 & -1-1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & -1 & -2 \end{bmatrix},
\end{aligned} \tag{111}$$

Then, if we multiply both sides of the Eq.(109) by $\langle 00|$,

$$\langle 00| \frac{d00}{dt} \rangle = (-i \mu_{|00\rangle} \omega) \langle 00| \sigma_{z^*} \oplus \sigma_z | 00 \rangle. \tag{112}$$

Then,

$$\Delta \omega_{\max} = \Delta \omega_{|00\rangle} = \mu_{|00\rangle} \omega = \mu_{|0\rangle}^2 \omega = \omega = \frac{i \langle 00| \frac{d00}{dt} \rangle}{\langle 00| \sigma_{z^*} \oplus \sigma_z | 00 \rangle}. \tag{113}$$

That is, Equations (92) and (113) coincide in their form, and independently of the term of the extreme right of the Eq.(113), it is clear that the spectral analysis for its counterpart will be,

$$\Delta \omega_{\min} = \Delta \omega_{|11\rangle} = \mu_{|11\rangle} \omega = -\mu_{|1\rangle}^2 \omega = -\omega. \tag{114}$$

Then, the bandwidth of the original entangled spins will be,

$$BW_{original} = \frac{1}{2\pi} (\Delta \omega_{\max} - \Delta \omega_{\min}) = \frac{1}{2\pi} (\omega - (-\omega)) = \frac{2\omega}{2\pi} = \frac{\omega}{\pi} < \infty. \tag{115}$$

That is to say, the bandwidth of the link between the original spins is finite. Now, what happens with the bandwidth of the link between the alter-egos? Well, replacing the alter-egos in the Eq.(113),

$$\begin{aligned}
\Delta \omega_{\max} &= \Delta \omega_{|\uparrow\uparrow\rangle} = \mu_{|\uparrow\uparrow\rangle} \omega \\
\Delta \omega_{\min} &= \Delta \omega_{|\downarrow\downarrow\rangle} = \mu_{|\downarrow\downarrow\rangle} \omega
\end{aligned} \tag{116}$$

Then, the bandwidth will be

$$BW_{alter-egos} = \frac{1}{2\pi} (\Delta \omega_{|\uparrow\uparrow\rangle} - \Delta \omega_{|\downarrow\downarrow\rangle}) = \frac{1}{2\pi} (\infty - (-\infty)) = \frac{2\infty}{2\pi} = \frac{\infty}{\pi} = \infty = BW_{original} \gamma^2. \tag{117}$$

This result should not be surprising given the exceptional performance of the quantum entanglement and therefore of a quantum channel. But again we can appreciate that are the alter-egos that sustain not only the effect but also the exceptional attributes of that effect.

Now, if we consider that $|\uparrow\uparrow\rangle = \left| \frac{\Delta \uparrow\uparrow}{\Delta t} \right\rangle$ is a good approximation of $|\uparrow\uparrow\rangle = \left| \frac{d \uparrow\uparrow}{dt} \right\rangle$, because $|\uparrow\uparrow\rangle$ is a stationary wavefunction, then

$$\Delta\omega \Delta t = \mu_{|\uparrow\uparrow\rangle} \omega \Delta t = \frac{i \langle \uparrow\uparrow | \Delta \uparrow\uparrow \rangle}{\langle \uparrow\uparrow | \sigma_z \oplus \sigma_z | \uparrow\uparrow \rangle}. \quad (118)$$

Equation (118) represents a trade-off, such that, if we consider the division by 2 of the derivative, and we take the module on the right side of the equality, then, the trade-off becomes

$$\Delta\omega \Delta t = \frac{1}{2} \left| \frac{i \langle \uparrow\uparrow | \Delta \uparrow\uparrow \rangle}{\langle \uparrow\uparrow | \sigma_z \oplus \sigma_z | \uparrow\uparrow \rangle} \right| = \frac{1}{2} |i| \left| \frac{\langle \uparrow\uparrow | \Delta \uparrow\uparrow \rangle}{\langle \uparrow\uparrow | \sigma_z \oplus \sigma_z | \uparrow\uparrow \rangle} \right| = \frac{1}{2} \left\{ (i)^* (i) \right\}^{1/2} = \frac{1}{2} \left\{ (-i)(i) \right\}^{1/2} \geq \frac{1}{2}, \quad (119)$$

which represents the famous Heisenberg Uncertainty Principle^{1-3,13}, and it is the same for the original spins and for the alter-egos,

$$\Delta\omega_{alter-egos} \Delta t_{alter-egos} = \left(\Delta\omega_{originals} \gamma^2 \right) \left(\frac{\Delta t_{originals}}{\gamma^2} \right) = \Delta\omega_{originals} \Delta t_{originals} \geq \frac{1}{2}. \quad (120)$$

4.3 Entanglement and black holes

Next, we will try to verify the connection between Equations (64) and (119), since and as we can see, both equations represent a trade-off between their two respective components, i.e., if in Eq.(64) $\Delta L_o \rightarrow \infty$, then, $\Delta Ch \rightarrow 0$, and if in Eq.(119) $\Delta\omega \rightarrow \infty$, then, $\Delta t \rightarrow 0$. We will begin by demonstrating three theorems, which are key to this mission.

Perfect Quantum Channel Theorem (PQChT):

Theorem: A quantum channel with infinite bandwidth and then null time latency is a perfect quantum channel.

Proof: From Eq.(119), we know that if $\Delta\omega = \infty$, thus $\Delta t = 0$. Now, replacing $\Delta t = 0$ and $t = t_0$ into Eq.(71), we will have,

$$|\psi(t_0 + 0)\rangle = |\psi(t_0)\rangle = U(t_0 + 0, t_0) |\psi(t_0)\rangle = U(t_0 + 0 - t_0) |\psi(t_0)\rangle = U(0) |\psi(t_0)\rangle \quad (121)$$

It is easy to deduce that the only equality that admits Eq.(121) is

$$U(0) = I \quad (122)$$

where I is the identity matrix. Consequently, Eq.(122) will be,

$$|\psi(t_0)\rangle = |\psi(t_0)\rangle \quad (123)$$

Therefore, we will obtain a perfect quantum channel U under these conditions. ■

No-Quantum Channel Theorem (NQChT):

Theorem: Every perfect quantum channel has null effective Euclidean dimensions.

Proof: If we do,

$$\Delta\omega = \frac{2\pi \Delta L_o C}{l_p^2} \quad (124)$$

and

$$\Delta t = \frac{\Delta Ch}{C} \quad (125)$$

where ΔL_o is the range or locality of the effect (i.e., quantum entanglement), $l_p = 1.616229(38) \times 10^{-35}$ is the Planck length, and ΔCh is the channel length (the three in meters), and C is the speed of light, then, replacing Equations (124) and (125) into (119), we will have,

$$\Delta\omega \Delta t = \left(\frac{2\pi \Delta L_o C}{l_p^2} \right) \left(\frac{\Delta Ch}{C} \right) = 2\pi \left(\frac{\Delta L_o}{l_p} \right) \left(\frac{\Delta Ch}{l_p} \right) \geq 1/2 \quad (126)$$

with

$$2\pi \Delta l_o \Delta c_h \geq 1/2 \quad (127)$$

and then

$$\Delta l_o \Delta c_h \geq 1/4\pi \quad (128)$$

being Δl_o the range of effect (unitless) and Δc_h the channel length (unitless). Clearly, Eq.(128) holds the trade-off between Δl_o and Δc_h (like between $\Delta\omega$ and Δt), so, if $\Delta l_o \rightarrow \infty$, then, $\Delta c_h \rightarrow 0$. ■

This theorem does not tell us that there is no quantum channel but -under these conditions- there is a quantum channel of null dimension, that is, the quantum channel is reduced to a point. Besides, from Eq.(126) we can rearrange terms so that thanks to it we can deduce Eq.(64).

Moreover, in the two theorems developed up to this point we could have considered (in relation to the graph on the right of Fig.16) only the upper semicircle (or north arch) above the dotted line, that is, $(\Delta L_o/2)(\Delta Ch/2)$ instead of $\Delta L_o \Delta Ch$ (see Fig.19), although for these two theorems the final result does not change at all.

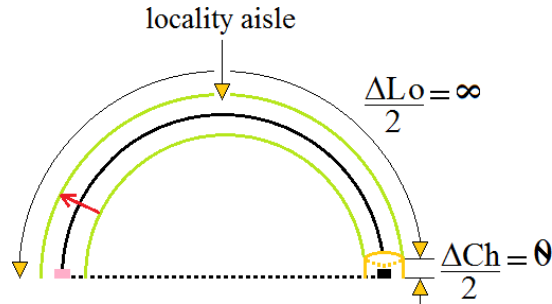


Fig.19 North arch of Fig.16.

However, for the following theorem, we will take Fig.19 into account. Besides, for the next theorem we will consider the remaining option with respect to the masses of the black holes of a bipartite entanglement, that is, both black holes have mass (finite or infinite) but of opposite sign. In particular, the following theorem exclusively involves the north arch, which has positive mass (finite or infinite), as we can see in Fig.19.

North Arch Theorem (NAT):

Theorem: The north arch of a bipartite quantum entanglement behaves like a black hole.

Proof: Let's start from Eq.(126), considering only the northern arch, and regrouping elements, we will have,

$$\Delta\omega \Delta t = 2 \frac{\left(\pi \Delta L_o/2 \Delta Ch/2 \right)}{l_p^2} \geq 1/2 \quad (129)$$

where the term inside parentheses represents the lateral area of the cylinder of Fig.20, in fact, A is the area of the event horizon, and the width of the channel is equal to its length since it is a point. Then,

$$\frac{A}{l_p^2} \geq \frac{1}{4} \quad (130)$$

where the lateral area $A = \pi \frac{\Delta L_o}{2} \frac{\Delta Ch}{2}$, being the diameter of the cylinder equivalent to $\frac{\Delta Ch}{2}$, and its long equal to $\frac{\Delta L_o}{2}$, see Fig.20.

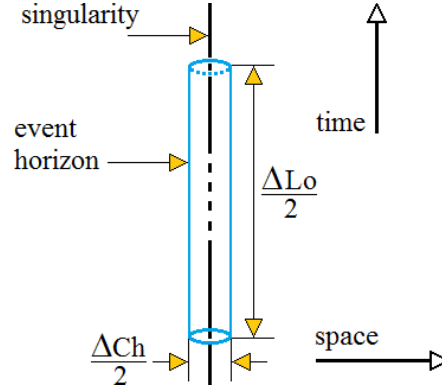


Fig.20 A cylinder contains the singularity of the entanglement black-hole whose lateral area contributes to the entropy of the effect, while such cylinder is its event horizon.

Now, if we divide by 4 and multiply by the Boltzmann's constant $k_B \approx 1.38064852(79) \times 10^{-23} \text{ J/K}$ both sides of inequation (130), we will have the entropy of the entanglement black-hole,

$$S_{BH} = \frac{k_B A}{4 l_p^2} \geq \frac{k_B}{16}. \quad (131)$$

The central term of Eq.(131) represents the Bekenstein–Hawking [14,38] formula for the entropy of a black hole. The subscript *BH* indiscriminately refers to "black hole" or "Bekenstein–Hawking". Now, if $l_p = \sqrt{\frac{G_N \hbar}{C^3}}$, where, G_N is Newton's gravitational constant, and if we replace it into Eq.(131), we will have,

$$S_{BH} = \frac{k_B A}{4 \left(\frac{G_N \hbar}{C^3} \right)} = \left(\frac{k_B C}{\hbar} \right) \left(\frac{A C^2}{4 G_N} \right) \geq \frac{k_B}{16}. \quad (132)$$

On the other hand, the Hawking temperature [155] for a black-hole is

$$T_H = \frac{\hbar \kappa}{2\pi k_B C} = \frac{\kappa}{2\pi \left(\frac{k_B C}{\hbar} \right)}, \quad (133)$$

where κ is the acceleration due to gravity at the horizon of black-hole. Then,

$$\left(\frac{k_B C}{\hbar} \right) = \frac{\kappa}{2\pi T_H}. \quad (134)$$

Now, replacing Eq.(134) into (132), we will have,

$$S_{BH} = \left(\frac{\kappa}{2\pi T_H} \right) \left(\frac{AC^2}{4G_N} \right) \geq \frac{k_B}{16}, \quad (135)$$

then,

$$T_H \leq \frac{2\kappa AC^2}{\pi k_B G_N}. \quad \blacksquare \quad (136)$$

It is evident that the north arch of a bipartite quantum entanglement behaves like a black hole because of the strict values that its temperature, $T_H = \frac{2\kappa AC^2}{\pi k_B G_N}$, and its entropy, $S_{BH} = \frac{k_B}{16}$ reach. It is a very particular black

hole, but definitively, a black hole. However, a question immediately arises: What happens with the southern arch? Obviously, it is another black hole, but, how does it interact with the northern arch? It is clear that one is the alter-ego of the other, and being both in full contact, more precisely, one over the other, if the positive mass radiates, the other one will receive that radiation until both masses are annulled. If it does not happen instantly because the masses of the alter-egos are equal (finite or not) and of opposite sign, then, both black holes end up being massless black holes. The question that remains is whether in this case, the extinction of the radiation eliminates the entanglement, or if it is simply a scenario similar to the original case of massless black holes.

4.4 Deduction of parallel operator via a Hamiltonian analysis

We know from the literature [157] that the Hamiltonian of the entanglement has a form like the following

$$H = H^A \otimes I^B + I^A \otimes H^B. \quad (137)$$

But we also know¹ that a more consistent and complete model for entanglement is the following:

$$H^{AB} = H^A \otimes I^B + I^A \otimes H^B + H_{\text{int}}^{AB}, \quad (138)$$

where the interaction between both subsystems S^A and S^B is described by the Hamiltonian $H_{\text{int}}^{AB} \neq 0$ so that each individual subsystem is an *open* quantum system. The Hamiltonian as a whole, and under these circumstances, takes the form of Eq.(138). However, and considering two fundamental aspects:

- a greater approximation between the treatment based on entropy and that based on the Hamiltonian, and
 - the possibility of working with a Hamiltonian model notably more simplified and at the same time closer to entanglement from the physical point of view, which is used with remarkable success¹⁵⁸,
- is that we understand that the best option is to use the following Hamiltonian,

$$\begin{aligned} H^{A \cup B} &= H^A + H^B - H^{A \cap B} \\ &= H^A + H^B - \frac{1}{H^{//}} H^A H^B \end{aligned} \quad (139)$$

where,

$$H^A = \mu_{|00\rangle} \hbar \omega \sigma_z \quad (140a)$$

$$H^B = \mu_{|11\rangle} \hbar \omega \sigma_z \quad (140b)$$

$$H^{//} = \mu_{|\uparrow\uparrow\rangle} \hbar \omega \sigma_z \quad (140c)$$

$$H^{A \cup B} = (\mu_{|00\rangle} + \mu_{|11\rangle}) \hbar \omega \sigma_z = 0 \quad (140d)$$

Therefore, replacing Eq.(140) into (139) so that a pair of matrices σ_z cancel each other before being replaced in $H^{A \cap B}$, we will have,

$$0 = \mu_{|00\rangle} \hbar \omega \sigma_z + \mu_{|11\rangle} \hbar \omega \sigma_z - \frac{1}{\mu_{|\uparrow\uparrow\rangle} \hbar \omega} \mu_{|00\rangle} \hbar \omega \mu_{|11\rangle} \hbar \omega \sigma_z. \quad (141)$$

Thus,

$$\mu_{|\uparrow\uparrow\rangle} = \frac{\mu_{|00\rangle} \mu_{|11\rangle}}{\mu_{|00\rangle} + \mu_{|11\rangle}} = \frac{\mu_{|0\rangle}^2 (-\mu_{|1\rangle}^2)}{\mu_{|0\rangle}^2 - \mu_{|1\rangle}^2} = \frac{\mu_{|0\rangle}^2 \mu_{|1\rangle}^2}{\mu_{|1\rangle}^2 - \mu_{|0\rangle}^2} = \mu_{|\uparrow\rangle}^2, \quad (142)$$

which coincides completely with Eq.(43). In fact, there are two more methods that converge to the same result and that have been omitted in this paper due to space issues and because they are considered unnecessary. Such coincidences clearly indicate the consistency in the genesis of alter-egos.

4.5 Analysis of the positions of equivalent spins

In the case of two hypothetical and completely independent spins, we will use Eq.(23) for the calculation of the equivalent spin μ_C , and Eq.(45) for its position d_C

$$\mu_C = \frac{\mu_A \mu_B}{\mu_A + \mu_B} = \frac{12 \times 4}{12 + 4} = 3,$$

$$d_C = d_A + \frac{\mu_A d_{BA}}{\mu_A + \mu_B} = 5 \text{ cm} + \frac{12 \times (15 \text{ cm} - 5 \text{ cm})}{12 + 4} = 12.5 \text{ cm},$$

where $\mu_A = 12$, $\mu_B = 4$, $d_A = 5 \text{ cm}$, and $d_B = 15 \text{ cm}$. Besides, and although Fig.21 is not in scale, the proportions are preserved. In that figure, we can see that $d_{BA} = d_B - d_A$.

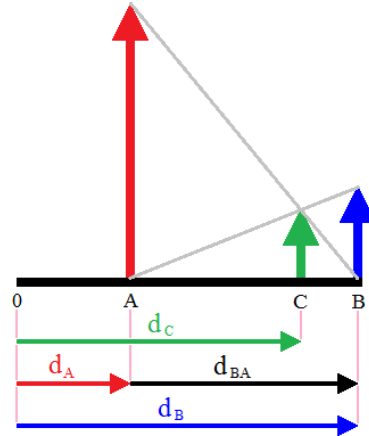


Fig.21 Sketch for the case of two hypothetical and completely independent spins.

We will apply the same equations of the previous case to the case of two hypothetical quasi-entangled spins.

$$\mu_C = \frac{\mu_A \mu_B}{\mu_A + \mu_B} = \frac{-4 \times 8}{-4 + 8} = -8,$$

$$d_C = d_A + \frac{\mu_A d_{BA}}{\mu_A + \mu_B} = -4 \text{ cm} + \frac{-4 \times (8 \text{ cm} - (-4 \text{ cm}))}{-4 + 8} = -16 \text{ cm},$$

where $\mu_A = -4$, $\mu_B = 8$, $d_A = -4 \text{ cm}$, and $d_B = 8 \text{ cm}$. On the other hand, Fig.22 is not in scale, although, the proportions are preserved. However, it is the entangled case by far the most interesting one. Because of this we resort to Eq.(45) and (46). Making the convenient replacements in them, we will obtain the following:

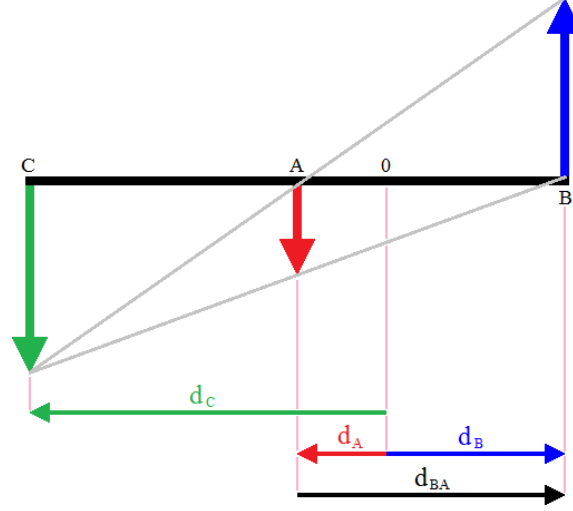


Fig.22 Sketch for the case of two hypothetical quasi-entangled spins.

$$\begin{aligned}
 d_{|\uparrow\uparrow\rangle} &= d_{|00\rangle} + \frac{\mu_{|00\rangle}(d_{|11\rangle} - d_{|00\rangle})}{\mu_{|00\rangle} + \mu_{|11\rangle}} = d_{|00\rangle} + \frac{\mu_{|00\rangle}(d_{|11\rangle} - d_{|00\rangle})}{\mu_{|00\rangle} + \mu_{|11\rangle}} \frac{\mu_{|11\rangle}}{\mu_{|11\rangle}} \\
 &= d_{|00\rangle} - \frac{\mu_{|\uparrow\uparrow\rangle}(d_{|11\rangle} - d_{|00\rangle})}{\mu_{|11\rangle}} = d_{|00\rangle} + \frac{\mu_{|11\rangle} \gamma^2 (d_{|11\rangle} - d_{|00\rangle})}{\mu_{|11\rangle}} = d_{|00\rangle} + \gamma^2 (d_{|11\rangle} - d_{|00\rangle}) = +\infty
 \end{aligned} \tag{143}$$

while,

$$\begin{aligned}
 d_{|\downarrow\downarrow\rangle} &= d_{|11\rangle} + \frac{\mu_{|11\rangle}(d_{|00\rangle} - d_{|11\rangle})}{\mu_{|00\rangle} + \mu_{|11\rangle}} = d_{|11\rangle} + \frac{\mu_{|11\rangle}(d_{|00\rangle} - d_{|11\rangle})}{\mu_{|00\rangle} + \mu_{|11\rangle}} \frac{\mu_{|00\rangle}}{\mu_{|00\rangle}} \\
 &= d_{|11\rangle} - \frac{\mu_{|\downarrow\downarrow\rangle}(d_{|00\rangle} - d_{|11\rangle})}{\mu_{|00\rangle}} = d_{|11\rangle} + \frac{\mu_{|00\rangle} \gamma^2 (d_{|00\rangle} - d_{|11\rangle})}{\mu_{|00\rangle}} = d_{|11\rangle} + \gamma^2 (d_{|00\rangle} - d_{|11\rangle}) = -\infty
 \end{aligned} \tag{144}$$

We can clearly see that Equations (143) and (144) are identical to Equations (49) and (50).

4.6 Final comments

As we could see up to here, the entanglement bandwidth is $BW = \infty$ (i.e., it is unlimited) and the length of the quantum link based on entanglement is null, and that, as a direct consequence of this: the system interchanges information in a robust way (i.e., the link is immune to noise) and could not be attacked (intercepted by an indiscreet third part). Besides, we must remember that a bipartite entanglement is a monogamous process [1-3]. However, what is the capacity of a quantum channel based on quantum entanglement? If we resort to a classical approach to the problem, e.g., the Shannon's Channel Capacity Theorem, then, we will have that the Shannon-Hartley Theorem¹⁵⁶ states that,

$$C = BW \log_2 \left(1 + \frac{S}{N} \right) \text{ [bits/sec]} \tag{145}$$

with these elements: C is the channel capacity, BW is the bandwidth in Hertz, S is the signal power and N is the noise power, $N_0 BW$ with $N_0/2$ is the two-sided noise Probability Density Function (PSD), and S/N is the ratio in watt/watt, not decibels. Then, C rises according to the increase of the available BW and the increase/improvement in S/N . Besides, Eq.(145) seems to tell us that as the BW increases the capacity should increase proportionally. But this does not happen because an increase in the bandwidth, also increases the noise power $N = N_0 BW$ giving:

$$C = BW \log_2 \left(1 + \frac{S}{N} \right) = BW \log_2 \left(1 + \frac{S}{N_0 BW} \right) \quad (146)$$

$$= \frac{S}{N_0} \frac{N_0 BW}{S} \log_2 \left(1 + \frac{S}{N_0 BW} \right) = \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 BW} \right)^{\frac{N_0 BW}{S}}$$

Now, if $BW \rightarrow \infty$ then $\left(\frac{S}{N_0 BW} \right) \rightarrow 0$. On the other hand, the expression $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$. This means that as $BW \rightarrow \infty$, $\left(\frac{S}{N_0 BW} \right) \rightarrow 0$, and $\left(1 + \frac{S}{N_0 BW} \right)^{\frac{N_0 BW}{S}} \rightarrow e$. The channel capacity, therefore, goes to:

$$\lim_{BW \rightarrow \infty} C = \lim_{BW \rightarrow \infty} \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 BW} \right)^{\frac{N_0 BW}{S}} = \frac{S}{N_0} \log_2 e = 1.44 \frac{S}{N_0} \quad (147)$$

If we don't have the channel, then, we don't have channel noise, therefore, when $BW \rightarrow \infty$ and $N_0 \rightarrow 0$, the channel capacity is also infinite. This was the last attribute of entanglement which was still to be explored in a context of absolute interest for quantum communications.

On the other hand, we rescue a particular equation, which constitutes a bridge between the Theory of Relativity and the Quantum Theory, and which constitutes a fundamental tool of the TOE,

$$\mu_{|\uparrow\uparrow\rangle} = \mu_{|00\rangle} \gamma^2, \quad \mu_{|\uparrow\downarrow\rangle} = \mu_{|01\rangle} \gamma^2, \quad \mu_{|\downarrow\uparrow\rangle} = \mu_{|10\rangle} \gamma^2, \quad \mu_{|\downarrow\downarrow\rangle} = \mu_{|11\rangle} \gamma^2. \quad (148)$$

In fact, we can consider a simplification like this,

$$\mu_{|XY\rangle} = \mu_{|xy\rangle} \gamma^2, \quad (149)$$

where, X and Y are the first and the second subscript of the alter-egos, respectively, which can be \uparrow or \downarrow , while, x and y are the first and the second subscript of the original spins, respectively, which can be 0 or 1. In this way, we arrive at a unified effect equation.

Finally, there is a large territory for the exploration of different possibilities, namely:

- If we consider the case of alter-egos with masses different from zero and with opposite sign, and supposing the case in which the masses did not cancel each other; would the alter-egos repel each other? In Eq.(140) we can see that the sign of the energy is conditioned by the sign of the corresponding spin. Therefore, if one of the components of Eq.(140) has negative energy, we can suppose that this will have negative mass, given that $E = m c^2$.
- In the previous case, can we consider an alter-ego constituted of matter and another one by anti-matter? Would they annihilate each other?
- Could we picture a scenario where one black hole attracts and another one repels?
- What is the relationship of the previous points with dark matter?
- Could a massless black hole with an infinite spin radiate foreign matter (borrowed from its environment)?
- Could a massless spin (especially if that spin is infinite) bend space-time?
- What is the relationship between alter-egos and an eventual pair of the black-hole/white-hole?
- What is the projection of this work in N-partite entanglement?
- Is the entanglement of a spin with itself possible? That is to say, if the self-entanglement possible? See Fig.23.

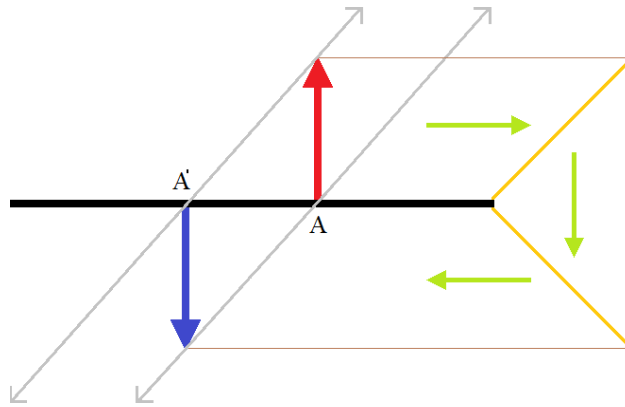


Fig.23 Symbolic sketch for a self-entanglement.

- What is the incidence of decoherence in the analysis seen in this paper?
- Can we think in a new Physics where the spin has more prominence than the mass when curving space-time?
- The entanglement is presented as a link system that does not allow blocking to its effect, which is understandable given that it is impossible to block a channel that does not exist, then: Can we think of a communications system based on entanglement without antenna or transmission power? What's more, from the point of view of quantum communications: Can we think of Teleportation protocols that do not need to be disambiguated thanks to a classic channel?

These are just some of the questions that remain to be answered in the future.

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