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Qingyuan Hu, Franz Chouly, Andreas Zilian, Gengdong Cheng, Stéphane Bordas. Generalized local B-bar method for locking phenomenon in Reissner-Mindlin shell and skew-symmetric Nitsche method for boundary conditions imposing and patch coupling in IGA. 7th GACM Colloquium on Computational Mechanics for Young Scientists from Academia and Industry, Oct 2017, Stuttgart, Germany. hal-01654498

HAL Id: hal-01654498

<https://hal.science/hal-01654498>

Submitted on 4 Dec 2017

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Generalized local B-bar method for locking phenomenon in Reissner-Mindlin shell and skew-symmetric Nitsche method for boundary conditions imposing and patch coupling in IGA

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Micro Abstract

We adopt the generalized local B-bar method to deal with the locking phenomenon in Reissner-Mindlin shell. The local element-wise projection saves computational effort, and projected basis functions of different orders are used to achieve good accuracy. The skew-symmetric Nitsche method is introduced for boundary conditions imposing and patch coupling. It has an advantage of unconditional stability wrt the stable parameter, i.e. parameter-free.

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Introduction

It is possible to construct Kirchhoff shells [10] thanks to the \mathcal{C}^1 -continuity of NURBS basis functions in isogeometric analysis (IGA). However due to the fact that Kirchhoff shells can only simulate thin structures, it is worthwhile to investigate locking in Reissner-Mindlin shells [5] to address both thick and thin structures. The \bar{B} method was introduced into IGA in [6]. Thanks to contributions about the local \bar{B} concept [2], the computational expense is decreased for a given accuracy level. We proposed the generalized local \bar{B} method for Timoshenko rods [8] and achieved excellent locking alleviation. Rich of this experience we extended the generalized local \bar{B} method for Reissner-Mindlin shells [9].

Nitsche's method plays an important role in IGA, especially in applications of boundary conditions imposing and multi-patch coupling [11, 12]. However numerical stability of (symmetric) Nitsche's method relies upon the appropriate choice of Nitsche's parameter, which can be determined by solving a generalized eigenvalue problem along the interface. Here we introduce the skew-symmetric Nitsche method [4, 7], which is unconditionally stable with respect to the stabilisation parameter, i.e. parameter-free.

1 The generalized local \bar{B} method

1.1 Classical \bar{B} formulation

We construct lower order B-spline basis functions \bar{N} and by the help of which we build a new strain in the discretized form

$$\bar{\epsilon}^h = \sum_{\bar{A}=1}^{\bar{n}\bar{m}} \bar{N}_{\bar{A}} \bar{\epsilon}^{\bar{A}^h}. \quad (1)$$

We then perform the L_2 projection on the physical domain as

$$\int_{\Omega} \bar{N}_{\bar{B}} (\boldsymbol{\varepsilon}^h - \bar{\boldsymbol{\varepsilon}}^h) d\Omega = 0, \quad \bar{B} = 1, \dots, \bar{n}\bar{m}. \quad (2)$$

By solving the above equations we obtain the unknowns $\bar{\boldsymbol{\varepsilon}}^{\bar{A}^h}$.

Assuming the bilinear term in the weak form as

$$b(\mathbf{U}, \mathbf{U}^*) = \int_{\Omega} \boldsymbol{\varepsilon}(\mathbf{U}^*)^T \mathbf{D}_g \boldsymbol{\varepsilon}(\mathbf{U}) d\Omega, \quad (3)$$

we use the new strain, instead of the locking strain to formulate the bilinear term

$$\bar{b}(\mathbf{U}, \mathbf{U}^*) = \int_{\Omega} \bar{\boldsymbol{\varepsilon}}^{*\top} \mathbf{D} \bar{\boldsymbol{\varepsilon}} d\Omega. \quad (4)$$

1.2 The generalized local \bar{B} formulation

The local type of \bar{B} formulation constructs the new strain locally as

$$\bar{\boldsymbol{\varepsilon}}_e^h = \sum_{\bar{A}=1}^{(\bar{p}+1)(\bar{q}+1)} \bar{N}_{\bar{A}} \bar{\boldsymbol{\varepsilon}}_e^{\bar{A}^h}, \quad (5)$$

and projects the locking strain locally as

$$\int_{\Omega_e} \bar{N}_{\bar{B}} (\boldsymbol{\varepsilon}^h - \bar{\boldsymbol{\varepsilon}}^h) d\Omega = 0, \quad \bar{B} = 1, \dots, (\bar{p}+1)(\bar{q}+1). \quad (6)$$

The generalized \bar{B} formulation allows to use various orders of $\bar{N}_{\bar{A}}$ for different elements.

1.3 Numerical examples

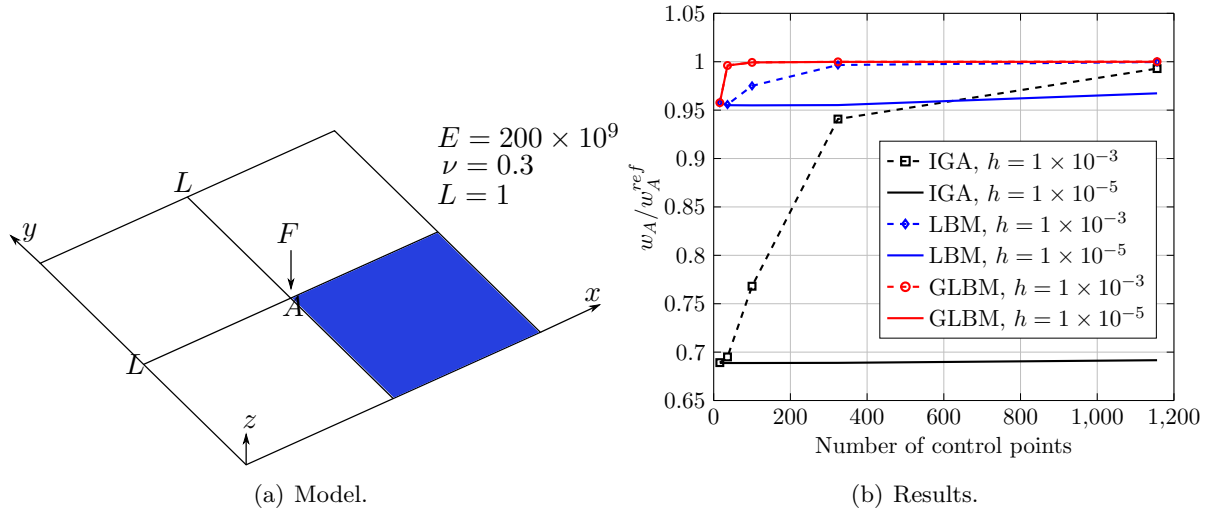


Figure 1. Square thin plate under concentrated load, only 1/4 of the plate (blue area) is modeled. Normalized center displacement w_A with various thickness h is plotted. LBM stands for local \bar{B} , GLBM stands for the generalized local \bar{B} . IGA of order 2 suffers from locking. GLBM achieves good accuracy.

Consider a square plate with simply supported edges and subjected to a point load F at the center, as shown in Figure 1. Denote its thickness by h . This problem is computed using degenerated Reissner-Mindlin plate/shell elements. Results show that for thickness $h = 10^{-3}$ IGA gets inaccurate results for coarse meshes but the results tend to improve upon refinement. However, IGA suffers from shear locking when $h = 10^{-5}$, indicating that the elements lock. GLBM provides good results when $h = 10^{-5}$. Thus, the proposed formulation alleviates shear locking and yields accurate results even for thin plates with coarse meshes.

2 The skew-symmetric Nitsche method

2.1 Formulation

Denote the $\llbracket \mathbf{u} \rrbracket$ and $\langle \boldsymbol{\sigma} \mathbf{N} \rangle$ as displacement jump and average stress resultant along the interface. For boundary conditions imposing the jump and average operators writes

$$\begin{aligned}\llbracket \mathbf{u} \rrbracket &:= \mathbf{u} - \bar{\mathbf{u}}, \\ \langle \boldsymbol{\sigma} \mathbf{N} \rangle &:= \boldsymbol{\sigma} \mathbf{N},\end{aligned}\tag{7}$$

where $\bar{\mathbf{u}}$ denotes the boundary conditions. For patch coupling the jump and average operators are defined as

$$\begin{aligned}\llbracket \mathbf{u} \rrbracket &:= \mathbf{u}^1 - \mathbf{u}^2, \\ \langle \boldsymbol{\sigma} \mathbf{N} \rangle &:= \frac{1}{2}(\boldsymbol{\sigma}^1 \mathbf{N} + \boldsymbol{\sigma}^2 \mathbf{N}),\end{aligned}\tag{8}$$

in which the superscripts 1, 2 are used to mark the corresponding variables of the partitioned domain.

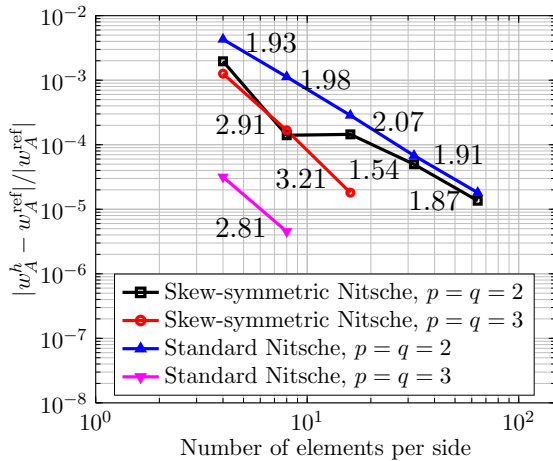
We introduce the Nitsche's formulation as

$$a(\mathbf{u}, \mathbf{v}) - \int_{\Gamma} \langle \boldsymbol{\sigma}(\mathbf{u}) \mathbf{N} \rangle \llbracket \mathbf{v} \rrbracket d\Gamma - \theta \int_{\Gamma} \llbracket \mathbf{u} \rrbracket \langle \boldsymbol{\sigma}(\mathbf{v}) \mathbf{N} \rangle d\Gamma + \alpha \int_{\Gamma} \llbracket \mathbf{u} \rrbracket \llbracket \mathbf{v} \rrbracket d\Gamma = L(\mathbf{v}),\tag{9}$$

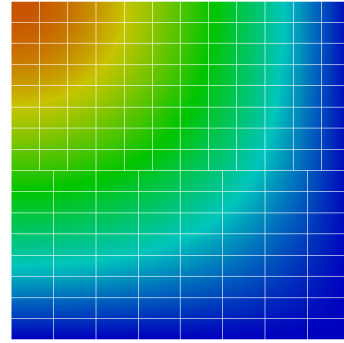
where the parameter α is the stabilization parameter, and the value of θ can be chosen to obtain various symmetry properties:

- For $\theta = 1$, the standard symmetric Nitsche's method is obtained. In order to make the bilinear form coercive a suitable choice for α is necessary [1];
- For $\theta = -1$, the skew-symmetric Nitsche method is obtained. The stability parameter can be chosen as $\alpha = 0$, resulting in a parameter-free formulation [3].

2.2 Numerical examples



(a) Symmetric boundary conditions imposing.



(b) Patch coupling.

Figure 2. Left figure: results for the 1/4 Kirchhoff plate, in which the symmetric rotational boundary conditions are imposed by Nitsche's method. Right figure: patch coupling of thick plate modeled by two patches of non-matching meshes.

By using work-conjugate pairs, such as displacement-force or rotation-moment, the Nitsche's method is suitable for applying not only Dirichlet displacement boundary conditions but also the rotational boundary conditions, as shown in the left figure. The right figure shows that Nitsche's coupling is effective for gluing displacement fields from two non-matching meshes.

Conclusions

The generalized local \bar{B} method was developed to tackle locking in Reissner-Mindlin shells. The local \bar{B} method saves computational time by projecting the standard strains locally (element-wise), and the generalized \bar{B} method introduces flexibility and achieves good accuracy.

The skew-symmetric Nitsche method was used to apply boundary conditions and to couple multi-patches. This formulation is free of stability parameters and preserves good accuracy and convergence performance.

Acknowledgements

Qingyuan Hu is funded by China Scholarship Council. Stéphane Bordas thanks the financial support of the European Research Council Starting Independent Research Grant (ERC Stg grant agreement No. 279578) entitled ‘Towards real time multiscale simulation of cutting in non-linear materials with applications to surgical simulation and computer guided surgery.’ Stéphane Bordas is also grateful for the support of the Fonds National de la Recherche Luxembourg FNRS-FNR grant INTER/FNRS/15/11019432/EnLightenIt/Bordas. Franz Chouly thanks Région Bourgogne Franche-Comté for partial funding (“Convention Région 2015C-4991. Modèles mathématiques et méthodes numériques pour l’élasticité non-linéaire”).

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