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## 3D-Calibration of the IMU

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**Abstract** – A new calibration method for Inertial Measurement Unit (IMU) of strapdown inertial technology was presented. IMU has been composed of accelerometers, gyroscopes and a circuit of signal processing. Normally, a rate transfer test and multi-position tests are used for IMU calibration. The new calibration method is based on whole angle rotation or finite rotation. In fact it is suggested to turn over IMU around three axes simultaneously. In order to solve the equation of calibration, it is necessary to provide an equality of a rank of basic matrix into degree of basic matrix. The results of simulated IMU data presented to demonstrate the performance of the new calibration method.

**Keywords** – *Inertial Measurement Unit; Accelerometers; Calibration; Gyroscopes.*

### I. INTRODUCTION

Inertial Measurement Unit (IMU) is a base for developing of Strapdown Inertial Navigation Systems. Each one IMU consist at least three accelerometers and three gyroscopes.

Normally, accelerometers and gyroscopes should go via autonomous testing before they will be assembled to IMU. But it is very important to determinate their features and parameters in whole IMU, because output signals of accelerometers and gyroscopes would be tied to IMU's frame. Therefore IMU calibration is an integral stage of his preparing to work or alignment of the Inertial Navigation System (INS). The calibration is a determination of IMU parameters or his errors for further using or compensation during INS's working.

Usually the IMU calibration is going on by multi-position tests [1-3] for sets of accelerometers, using a precision dividing head. A photograph of such equipment from NTUU 'KPI' laboratory is shown in Figure 1. This equipment has a setting accuracy near one second of arc, enables the sensitive axis of an accelerometer rotate with respect to the gravity vector. For calibration of a set of accelerometers it is need to measure output signals, at first to turn the them around  $ox$  axis, then to reinstall the set of accelerometers for to measure of output signals by turning around  $oy$  axis, and finally to repeat the procedure to measure of output signals by turning around  $oz$  axis. It should be noted that such technology is an artificial case. Actually the vehicle or body turns for whole angle rotation or finite rotation, which is a result of turnings around two or three axes.

For the gyroscopes calibration in IMU by rate transfer tests it is using rate table (precision turntable). A photograph of such equipment from NTUU 'KPI' is shown in Figure 2. During the transfer test the IMU is mounted on the turntable at first with one sensitive axis. For example, axis  $ox$  is parallel to the axis of rotation of the rate table and rate table is stepped through a series of angular rates  $\omega_{xi}$  starting at zero by recording data at each stage. Such test repeated for rotation around axis  $oy$  with angular rates  $\omega_{yi}$  and, finally, for rotation around axis  $oz$  with angular rates  $\omega_{zi}$ ,  $i$  – the number of tests. Need to mark that such method is required a lot of time and at real life the vehicle or body is rotated around the each one of rotation axes.

There are some methods for estimation and compensation of sensors output signal noises. Most known is Kalman filter, used in [4-7]. For estimation of noise features were used Allan variance methods and some time – Wavelet transform [8]. Also, it is well known of using of fuzzy logic techniques [9] and neural networks [10, 11].

There is the scalar method of IMU calibration used for gyroscopes and accelerometers [12, 13], based on a scalar reference input motion. For the gyroscopes in the Earth's gravitational field such scalar value is the rotation rate  $\Omega$ , and for accelerometers – the value of gravity vector  $g$ .

In this paper was suggested to use a new method of 3D-calibration. The new calibration method based on whole angle rotation. In fact it is suggested to turn over IMU around three axes simultaneously.



Fig. 1. IMU calibration with a precision dividing head.



Fig. 2. IMU MAX 21105 calibration with a two axial rate table.

## II. CALIBRATION OF IMU'S SET OF AXIAL ACCELEROMETERS

Output signals of the IMU's axial accelerometers may be expressed according to source [1]:

$$\begin{bmatrix} U_{ax} \\ U_{ay} \\ U_{az} \end{bmatrix} = \begin{bmatrix} B_{ax} \\ B_{ay} \\ B_{az} \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} n_{ax} \\ n_{ay} \\ n_{az} \end{bmatrix} \quad (1)$$

where:

$B_{ax}, B_{ay}, B_{az}$  – g-independent biases;

$a_x, a_y, a_z$  – accelerations acting along the  $x, y$  and  $z$  axes of the sensor respectively;

$n_{ax}, n_{ay}, n_{az}$  – zero-mean random biases or output measurements noises;

$Oxyz$  – body frame,  $3 \times 3$  matrix representing the g-dependent biases induced by accelerations  $a_x, a_y, a_z$ ;

$k_{11}, k_{22}, k_{33}$  – scale-factor coefficients of accelerometers, another one coefficients of the matrix are cross-coupling coefficients.

For reducing noises of output measurements we will average output signals during 30...60 seconds. Therefore in the future we will ignore of  $n_{ax}, n_{ay}, n_{az}$  value. Let's write the expression (1) for each one accelerometer and for  $i$  – position of IMU testing:

$$\begin{aligned} U_{axi} &= B_{ax} + k_{11}a_{xi} + k_{12}a_{yi} + k_{13}a_{zi}; \\ U_{ayi} &= B_{ay} + k_{21}a_{xi} + k_{22}a_{yi} + k_{23}a_{zi}; \\ U_{azi} &= B_{az} + k_{31}a_{xi} + k_{32}a_{yi} + k_{33}a_{zi}. \end{aligned}$$

We will do a set of testing measurements, which are undertaken multi-position tests for turn angles  $\alpha, \beta, \gamma$  around three axes simultaneously with exact step of turn.

Normally the precision dividing head is using for calibration of accelerometers. But in this case, when IMU is turning around three axes simultaneously or whole angle rotation or finite rotation, the three-axes turntable is used instead a typical precision dividing head. A kinematic diagram of three axial rate table is shown in Figure 3.

Modern three axial rate tables have got very precision data. For example, ACUTRONIC's AC3350-08 has a position accuracy 1,5 arc sec RSS per each one axis, command resolu-

tion 0,00001 deg and repeatability less than 1 arc sec. For angular rate the range are  $\pm 1000; \pm 500$  and  $\pm 400$  deg/sec for each one axis and command resolution is 0,0001 deg/sec per each axis. Besides, the accuracy of calibration on the rate table is depended from inclination of testing equipment under the horizon. Modern liquid levels could provide the accuracy of inclination not less 2 angular minutes.

Consider an output signal of the first accelerometer:

1-st measurement:

$$U_{ax1} = B_{ax} + k_{11}a_{x1} + k_{12}a_{y1} + k_{13}a_{z1};$$

2-nd measurement:

$$U_{ax2} = B_{ax} + k_{11}a_{x2} + k_{12}a_{y2} + k_{13}a_{z2};$$

.....  
n-st measurement:

$$U_{axn} = B_{ax} + k_{11}a_{xn} + k_{12}a_{yn} + k_{13}a_{zn}.$$

Above set of equations we can represent in matrix form:

$$\begin{bmatrix} U_{ax1} \\ U_{ax2} \\ \vdots \\ U_{axn} \end{bmatrix} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & a_{xn} & a_{yn} & a_{zn} \end{bmatrix} \begin{bmatrix} B_{ax} \\ k_{11} \\ k_{12} \\ k_{13} \end{bmatrix}.$$

It is possible to receive the same equations for other one accelerometer – for second:

$$\begin{bmatrix} U_{ay1} \\ U_{ay2} \\ \vdots \\ U_{ayn} \end{bmatrix} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & a_{xn} & a_{yn} & a_{zn} \end{bmatrix} \begin{bmatrix} B_{ay} \\ k_{21} \\ k_{22} \\ k_{23} \end{bmatrix}$$

and third:

$$\begin{bmatrix} U_{az1} \\ U_{az2} \\ \vdots \\ U_{azn} \end{bmatrix} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & a_{xn} & a_{yn} & a_{zn} \end{bmatrix} \begin{bmatrix} B_{az} \\ k_{31} \\ k_{32} \\ k_{33} \end{bmatrix}.$$

After that let's to combine the received matrix equations to single 'equation of calibration' for IMU's set of accelerometers:

$$\mathbf{U}_{a1} = \mathbf{G}_{n \times 4} \cdot \mathbf{X}_1, \quad (2)$$

where

$$\mathbf{U}_{a1} = \begin{bmatrix} U_{ax1} & U_{ay1} & U_{az1} \\ U_{ax2} & U_{ay2} & U_{az2} \\ \vdots & \vdots & \vdots \\ U_{axn} & U_{ayn} & U_{azn} \end{bmatrix}, \mathbf{G}_{n \times 4} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & a_{xn} & a_{yn} & a_{zn} \end{bmatrix},$$

$$\mathbf{X}_1 = \begin{bmatrix} B_{ax} & B_{ay} & B_{az} \\ k_{11} & k_{21} & k_{31} \\ k_{12} & k_{22} & k_{32} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}$$

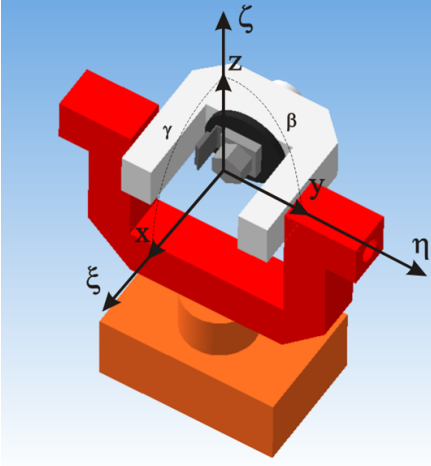


Fig. 3. IMU calibration with a three axial rate table.

We will solve the last matrix equation by least-squares method:

$$\hat{\mathbf{X}}_1 = (\mathbf{G}_{n \times 4}^T \mathbf{G}_{n \times 4})^{-1} \mathbf{G}_{n \times 4}^T \mathbf{U}_{a1}. \quad (3)$$

Here symbol  $^T$  is mean a transposed matrix.

**Sample 1.** We will consider the IMU's accelerometers with the below followings nominal parameters:

$$B_{ax} = B_{ay} = B_{az} = 2.5 \text{ V/g};$$

$$k_{11} = k_{22} = k_{33} = 1.0 \text{ V/g};$$

$$k_{12} = 0.01 \text{ V/g}, k_{13} = -0.01 \text{ V/g}, k_{21} = -0.01 \text{ V/g},$$

$$k_{23} = 0.01 \text{ V/g}, k_{31} = 0.01 \text{ V/g}, k_{32} = -0.01 \text{ V/g},$$

Body's turns with angles  $\alpha, \beta, \gamma$  which will change from 0 to 400 degrees with the same step of 10 degrees (the number of positions is 40).

The calculated values of output signals (2) for matrix  $\mathbf{G}_{n \times 4}$  are shown on Fig. 4.

The above output signals should be used for calculation of (3).

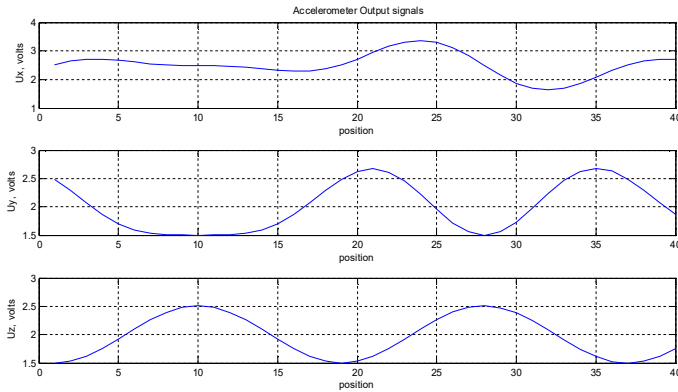


Fig. 4. Averaging output signals of axial accelerometers.

After calculations according to (3), we will have:

$$\hat{\mathbf{X}}_1 = \begin{bmatrix} 2.5 & 2.5 & 2.5 \\ 1.0 & -0.01 & 0.009999 \\ 0.01 & 1.0 & -0.009999 \\ -0.009999 & 0.01 & 1.0 \end{bmatrix}.$$

Thus, as a result of measuring (2) and calculations (3) it is succeeded to get estimations of the biases  $\hat{\mathbf{B}}_a$ , scale factors and cross-coupling coefficients – elements of matrix  $3 \times 3$ .

Consider that the above method can be used for the IMU's set of pendulum accelerometers.

### III. CALIBRATION OF IMU'S SET OF PENDULUM ACCELEROMETERS

Output signals of the IMU's set of pendulum accelerometers may be expressed according to source [1]:

$$\begin{bmatrix} U_{ax} \\ U_{ay} \\ U_{az} \end{bmatrix} = \begin{bmatrix} B_{ax} \\ B_{ay} \\ B_{az} \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \cdot \begin{bmatrix} a_x a_y \\ a_y a_z \\ a_x a_z \end{bmatrix} + \begin{bmatrix} n_{ax} \\ n_{ay} \\ n_{az} \end{bmatrix}. \quad (4)$$

When the set of  $n$ -measurements will be done and after averaging of output signals during 30...60 seconds we will have a new matrix 'equation of calibration':

$$\mathbf{U}_{a2} = \mathbf{G}_{n \times 7} \cdot \mathbf{X}_2, \quad (5)$$

where

$$\mathbf{U}_{a2} = \begin{bmatrix} U_{ax1} & U_{ay1} & U_{az1} \\ U_{ax2} & U_{ay2} & U_{az2} \\ \vdots & \vdots & \vdots \\ U_{axn} & U_{ayn} & U_{azn} \end{bmatrix},$$

$$\mathbf{X}_2 = \begin{bmatrix} B_{ax} & k_{11} & k_{12} & k_{13} & l_{11} & l_{12} & l_{13} \\ B_{ay} & k_{21} & k_{22} & k_{23} & l_{21} & l_{22} & l_{23} \\ B_{az} & k_{31} & k_{32} & k_{33} & l_{31} & l_{32} & l_{33} \end{bmatrix}^T,$$

$$\mathbf{G}_{n \times 7} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} & a_{x1}a_{y1} & a_{y1}a_{z1} & a_{x1}a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} & a_{x2}a_{y2} & a_{y2}a_{z2} & a_{x2}a_{z2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & a_{xn} & a_{yn} & a_{zn} & a_{xn}a_{yn} & a_{yn}a_{zn} & a_{xn}a_{zn} \end{bmatrix}.$$

Solving the last matrix equation (5) by least-squares method:

$$\hat{\mathbf{X}}_2 = (\mathbf{G}_{n \times 7}^T \mathbf{G}_{n \times 7})^{-1} \mathbf{G}_{n \times 7}^T \mathbf{U}_{a2}. \quad (6)$$

**Sample 2.** We will consider the IMU's pendulum accelerometers with the below followings nominal parameters:

$$B_{ax} = B_{ay} = B_{az} = 2.5 \text{ V/g};$$

$$k_{11} = k_{22} = k_{33} = 1.0 \text{ V/g};$$

$$k_{12} = 0.01 \text{ V/g}, k_{13} = -0.01 \text{ V/g}, k_{21} = -0.01 \text{ V/g},$$

$$k_{23} = 0.01 \text{ V/g}, k_{31} = 0.01 \text{ V/g}, k_{32} = -0.01 \text{ V/g},$$

$$l_{11} = l_{22} = l_{33} = -0.001 \text{ V/g}^2;$$

$$l_{12} = l_{13} = l_{21} = l_{23} = l_{31} = l_{32} = 0.001 \text{ V/g}^2;$$

Body's turns for angles  $\alpha, \beta, \gamma$  will change from 0 to 400 degrees with the same step of 10 degrees (the number of positions is 40).

After calculations according to (6), we will have:



$$\hat{\mathbf{X}}_2 = \begin{bmatrix} 2.5 & 2.5 & 2.5 \\ 1 & -0.01 & 0.01 \\ 0.01 & 1 & -0.01 \\ -0.01 & 0.01 & 1 \\ -0.001 & 0.001 & 0.001 \\ 0.001 & -0.001 & 0.001 \\ 0.001 & 0.001 & -0.001 \end{bmatrix}.$$

Thus, as a result of measuring (5) and calculations (6) it is succeeded to get estimations of the biases  $\hat{\mathbf{B}}_a$  and elements of matrixes  $3 \times 3$ .

#### IV. CALIBRATION OF IMU'S SET OF GYROSCOPES

Output signals of the IMU's gyroscopes may be expressed according to source [1]:

$$\begin{bmatrix} U_{\omega x} \\ U_{\omega y} \\ U_{\omega z} \end{bmatrix} = \begin{bmatrix} B_{\omega x}^* \\ B_{\omega y}^* \\ B_{\omega z}^* \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} n_{\omega x} \\ n_{\omega y} \\ n_{\omega z} \end{bmatrix}. \quad (7)$$

Here  $B_{\omega x}^*$ ,  $B_{\omega y}^*$ ,  $B_{\omega z}^*$  - biases of each one gyroscopes, which may be depends from  $g$  and  $g^2$  - drifts,  $\omega_x$ ,  $\omega_y$ ,  $\omega_z$  - turn rates, acting along the  $x$ ,  $y$  and  $z$  axes of the sensor respectively,  $n_{\omega x}$ ,  $n_{\omega y}$ ,  $n_{\omega z}$  - zero-mean random biases or output measurements noises,  $3 \times 3$  matrix representing the biases induced by  $\omega_x$ ,  $\omega_y$ ,  $\omega_z$ ,  $n_{11}$ ,  $n_{22}$ ,  $n_{33}$  - scale-factor coefficients of gyroscopes, another one coefficients of the matrix are cross-coupling coefficients.

Consider a calibration when IMU turns around three axes simultaneously.

When the set of  $n$ -measurements will be done with turn rates  $\omega_{xi}$ ,  $\omega_{yi}$ ,  $\omega_{zi}$  and after averaging of output signals during 30...60 seconds we will have a new matrix 'equation of calibration':

$$\mathbf{U}_{\omega 3} = \mathbf{\omega}_{n \times 4} \cdot \mathbf{X}_3, \quad (8)$$

where

$$\mathbf{U}_{\omega 3} = \begin{bmatrix} U_{\omega x1} & U_{\omega y1} & U_{\omega z1} \\ U_{\omega x2} & U_{\omega y2} & U_{\omega z2} \\ \vdots & \vdots & \vdots \\ U_{\omega xn} & U_{\omega yn} & U_{\omega zn} \end{bmatrix}, \mathbf{X}_3 = \begin{bmatrix} B_{\omega x}^* & B_{\omega y}^* & B_{\omega z}^* \\ n_{11} & n_{21} & n_{31} \\ n_{12} & n_{22} & n_{32} \\ n_{13} & n_{23} & n_{33} \end{bmatrix},$$

$$\mathbf{\omega}_{n \times 4} = \begin{bmatrix} 1 & \omega_{x1} & \omega_{y1} & \omega_{z1} \\ 1 & \omega_{x2} & \omega_{y2} & \omega_{z2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \omega_{xn} & \omega_{yn} & \omega_{zn} \end{bmatrix}.$$

On the face of it the structure of the equation (8) is very similar to equation (2). The difference is that matrix  $\mathbf{\omega}_{n \times 4}$  consists from turn rates  $\omega_{xi}$ ,  $\omega_{yi}$ ,  $\omega_{zi}$  and matrix  $\mathbf{G}_{n \times 4}$  - from accelerations  $a_{xi}$ ,  $a_{yi}$ ,  $a_{zi}$ . However if we have ordered turn series without any limitations for calibration of IMU's set of accelerometers, otherwise we should provide a below following rule:

An equation of calibration will have a decision if the rank of coefficient matrix  $\mathbf{\omega}_{n \times m}$  is equal to number of columns of the same matrix or by other words  $\text{rank } \mathbf{\omega}_{n \times m} = m$ .

The above rule is based on Kronecker-Capelli theorem [14].

This condition could be provided, for example due nonlinear depending of last columns of matrix  $\mathbf{\omega}_{n \times 4}$ .

Solving the last matrix equation (8) by least-squares method:

$$\hat{\mathbf{X}}_3 = (\mathbf{\omega}_{n \times 4}^T \mathbf{\omega}_{n \times 4})^{-1} \mathbf{\omega}_{n \times 4}^T \mathbf{U}_{\omega 3}. \quad (9)$$

**Sample 3.** Consider the IMU's gyroscopes with the below followings nominal parameters:

$$\begin{aligned} B_{\omega x}^* &= B_{\omega y}^* = B_{\omega z}^* = 2.0 \text{ V}; \\ n_{11} &= 0.01 \text{ V/(d/s)}; \quad n_{12} = 0.01 \text{ V/(d/s)}; \quad n_{13} = -0.02 \text{ V/(d/s)}; \\ n_{21} &= -0.01 \text{ V/(d/s)}; \quad n_{22} = 0.1 \text{ V/(d/s)}; \quad n_{23} = 0.01 \text{ V/(d/s)}; \\ n_{31} &= 0.03 \text{ V/(d/s)}; \quad n_{32} = -0.01 \text{ V/(d/s)}; \quad n_{33} = 0.1 \text{ V/(d/s)}; \end{aligned}$$

To provide the rank of matrix  $\mathbf{\omega}_{n \times 4}$  to order of matrix:

$$\text{rank } \mathbf{\omega}_{n \times 4} = 4,$$

we will arrange the projections of angular rate of turntable in below following series:

$$\omega_{xi} = \omega_i, \quad \omega_{yi} = \omega_i^{1/2}, \quad \omega_{zi} = \omega_i^{1/3}.$$

We will change the angular rate  $\omega_i$  from 0 to 100 degrees/sec with the same step of 10 degrees/sec. The number of tests is 10.

Solving the equation of calibration according to (9) by least-squares method we will have

$$\hat{\mathbf{X}}_3 = \begin{bmatrix} 2.0 & 2.0 & 2.0 \\ 1.0 & -0.01 & 0.03 \\ 0.01 & 1.0 & -0.01 \\ -0.02 & 0.01 & 1.0 \end{bmatrix}.$$

Thus, as a result of measuring (8) and calculations (9) it is succeeded to get estimations of the gyro's biases and elements of the matrix  $3 \times 3$ .

##### A. Extended model.

Output signals of the IMU's gyroscopes for extended model may be expressed according to source [1]:

$$\begin{bmatrix} U_{\omega x} \\ U_{\omega y} \\ U_{\omega z} \end{bmatrix} = \begin{bmatrix} B_{\omega x}^* \\ B_{\omega y}^* \\ B_{\omega z}^* \end{bmatrix} + \mathbf{M}_{1\omega} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \mathbf{M}_{2\omega} \cdot \begin{bmatrix} \omega_x \omega_y \\ \omega_y \omega_z \\ \omega_x \omega_z \end{bmatrix} + \begin{bmatrix} n_{\omega x} \\ n_{\omega y} \\ n_{\omega z} \end{bmatrix}. \quad (10)$$

Here

$$\mathbf{M}_{1\omega} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}, \quad \mathbf{M}_{2\omega} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}.$$

Still consider the calibration when IMU turns around three axes simultaneously.

When the set of  $n$ -measurements will be done with turn rates  $\omega_{xi}$ ,  $\omega_{yi}$ ,  $\omega_{zi}$  and after averaging of output signals during 30...60 seconds we will have a new matrix 'equation of calibration':

$$\mathbf{U}_{\omega 4} = \boldsymbol{\omega}_{n \times 7} \cdot \mathbf{X}_4, \quad (11)$$

where

$$\mathbf{U}_{\omega 4} = \begin{bmatrix} U_{\omega x1} & U_{\omega y1} & U_{\omega z1} \\ U_{\omega x2} & U_{\omega y2} & U_{\omega z2} \\ \vdots & \vdots & \vdots \\ U_{\omega xn} & U_{\omega yn} & U_{\omega zn} \end{bmatrix}, \mathbf{X}_4 = \begin{bmatrix} \mathbf{B}_{\omega}^T \\ \mathbf{M}_{1\omega}^T \\ \mathbf{M}_{2\omega}^T \end{bmatrix},$$

$$\mathbf{B}_{\omega}^T = [\mathbf{B}_{\omega x}^* \quad \mathbf{B}_{\omega y}^* \quad \mathbf{B}_{\omega z}^*];$$

$$\boldsymbol{\omega}_{n \times 7} = \begin{bmatrix} 1 & \omega_{x1} & \omega_{y1} & \omega_{z1} & \omega_{x1}\omega_{y1} & \omega_{y1}\omega_{z1} & \omega_{x1}\omega_{z1} \\ 1 & \omega_{x2} & \omega_{y2} & \omega_{z2} & \omega_{x2}\omega_{y2} & \omega_{y2}\omega_{z2} & \omega_{x2}\omega_{z2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \omega_{xn} & \omega_{yn} & \omega_{zn} & \omega_{xn}\omega_{yn} & \omega_{yn}\omega_{zn} & \omega_{xn}\omega_{zn} \end{bmatrix}.$$

Solving the last matrix equation (11) by least-squares method:

$$\hat{\mathbf{X}}_4 = (\boldsymbol{\omega}_{n \times 7}^T \boldsymbol{\omega}_{n \times 7})^{-1} \boldsymbol{\omega}_{n \times 7}^T \mathbf{U}_{\omega 4}. \quad (12)$$

**Sample 4.** Consider the IMU's gyroscopes with the below followings nominal parameters:

$$\begin{aligned} B_{\omega x}^* &= B_{\omega y}^* = B_{\omega z}^* = 2.0V; \\ n_{11} &= 0.01 \text{ V/(d/s)}; \quad n_{12} = 0.01 \text{ V/(d/s)}; \quad n_{13} = -0.02 \text{ V/(d/s)}; \\ n_{21} &= -0.01 \text{ V/(d/s)}; \quad n_{22} = 0.1 \text{ V/(d/s)}; \quad n_{23} = 0.01 \text{ V/(d/s)}; \\ n_{31} &= 0.03 \text{ V/(d/s)}; \quad n_{32} = -0.01 \text{ V/(d/s)}; \quad n_{33} = 0.1 \text{ V/(d/s)}; \\ p_{11} &= p_{22} = p_{33} = -0.001 \text{ V/(d/s)}^2; \\ p_{12} &= p_{13} = p_{21} = p_{23} = p_{31} = p_{32} = 0.001 \text{ V/(d/s)}^2; \end{aligned}$$

To provide  $\text{rank } \boldsymbol{\omega}_{n \times 7} = 7$ , we will arrange the projections of angular rate of turntable in below following series:

$$\omega_{xi} = \omega_i, \quad \omega_{yi} = e^{-\omega_i}, \quad \omega_{zi} = \omega_i^2.$$

We will change the angular rate  $\omega_i$  from 0 to 10 degrees/sec with the same step of 1 degree/sec. The number of tests is 10.

Solving the equation of calibration according to (12) by least-squares method we will have

$$\hat{\mathbf{X}}_4 = \begin{bmatrix} 2 & 2 & 2 \\ 0.01 & -0.01 & 0.03 \\ 0.01 & 0.1 & -0.01 \\ -0.02 & 0.01 & 0.1 \\ -0.001 & 0.001 & 0.001 \\ 0.001 & 0.001 & -0.001 \\ 0.001 & -0.001 & 0.001 \end{bmatrix}.$$

Thus, as a result of measuring (11) and calculations (12) it is succeeded to get estimations of the gyro's biases and elements of the matrixes  $\mathbf{M}_{1\omega}$ ,  $\mathbf{M}_{2\omega}$ .

However, it should be noted that output signal's models (7) and (10) are inherent to optical sensors like ring laser and fiber optic gyroscopes.

## V. AN ERROR ESTIMATION OF 3D-CALIBRATION

For considering an error of estimation, we can rewrite expressions (3), (6), (9) and (12) as following bellow:

$$\hat{\mathbf{X}} = \mathbf{B} \cdot \mathbf{U}, \quad (13)$$

where, for example, for axial accelerometers from item II, we will have  $n \times 4$ -matrix

$$\mathbf{B} = (\mathbf{G}_{n \times 4}^T \mathbf{G}_{n \times 4})^{-1} \mathbf{G}_{n \times 4}^T.$$

For the axial accelerometers, the expression (13) will have such view

$$\begin{bmatrix} B_{ax} & B_{ay} & B_{az} \\ k_{11} & k_{21} & k_{31} \\ k_{12} & k_{22} & k_{32} \\ k_{13} & k_{23} & k_{33} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ b_{31} & b_{32} & \dots & b_{3n} \\ b_{41} & b_{42} & \dots & b_{4n} \end{bmatrix} \begin{bmatrix} U_{ax1} & U_{ay1} & U_{az1} \\ U_{ax2} & U_{ay2} & U_{az2} \\ \vdots & \vdots & \vdots \\ U_{axn} & U_{ayn} & U_{azn} \end{bmatrix}$$

According to last expression, we can receive expression for each one seeking parameter as a function of components  $n \times 4$ -matrix and measured output signals:

$$B_{ax} = F_{Bx}(b_{1i}, U_{xi}),$$

$$B_{ay} = F_{By}(b_{1i}, U_{yi}),$$

$$B_{az} = F_{Bz}(b_{1i}, U_{zi}).$$

To estimate an error of biases we should calculate

$$\delta B_{ax} = \sum_{i=1}^n \frac{\partial F_{Bx}}{\partial b_{1i}} \delta b_{1i} + \sum_{i=1}^n \frac{\partial F_{Bx}}{\partial U_{xi}} \delta U_{xi},$$

$$\delta B_{ay} = \sum_{i=1}^n \frac{\partial F_{By}}{\partial b_{1i}} \delta b_{1i} + \sum_{i=1}^n \frac{\partial F_{By}}{\partial U_{yi}} \delta U_{yi},$$

$$\delta B_{az} = \sum_{i=1}^n \frac{\partial F_{Bz}}{\partial b_{1i}} \delta b_{1i} + \sum_{i=1}^n \frac{\partial F_{Bz}}{\partial U_{zi}} \delta U_{zi}.$$

Here  $\delta b_{1i}$ ,  $\delta U_{xi}$  - errors of components 1<sup>st</sup>-line  $n \times 4$ -matrix and errors of measured output signals.

Similar expressions could be received for scale-factor coefficients of accelerometers and cross-coupling coefficients of  $3 \times 3$ -matrix from (1).

If we will have results of  $m$ -tests ( $m \neq n$ ), it is possible to calculate a root-mean-square error for each one bias:

$$\sigma_{B_{ax}} = \sqrt{\frac{1}{m-1} \sum_{j=1}^m (\delta B_{axj})^2};$$

$$\sigma_{B_{ay}} = \sqrt{\frac{1}{m-1} \sum_{j=1}^m (\delta B_{ayj})^2};$$

$$\sigma_{B_{az}} = \sqrt{\frac{1}{m-1} \sum_{j=1}^m (\delta B_{azj})^2}.$$

Therefore, errors of 3D-Calibration are depended from errors of measured output signals, from an accuracy of angular positions of precision rate table for calibration of accelerometers and an accuracy of angular rate of testing equipment for calibration of gyroscopes. Also, the accuracy of 3D-Calibration is

depended from inclination of testing equipment under the horizon.

## CONCLUSIONS

A new calibration method for IMU of strapdown inertial technology was suggested. The new calibration method is based on whole angle rotation. In fact it is suggested to turn over Inertial Measurement Unit around three axes simultaneously. In order to solve the equation of calibration, it is necessary to provide an equality of a rank of basic matrix to degree of basic matrix. It is received the general error estimation of 3D-Calibration. The results of simulated IMU data are presented to demonstrate the performance of the new calibration method.

## REFERENCES

- [1] Titterton D. H. and Weston J. L. Strapdown Inertial Navigation Technology. - IEE Radar, Sonar, Navigation and Avionics Series 17, 2004. – pp. 558.
- [2] Nebot E., Durrant-Whyte H., Initial Calibration and Alignment of Low Cost Inertial Navigation Units for Land Vehicle Applications. – *Journal of Robotics Systems*, Vol. 16, No. 2, February 1999, pp. 81-92.
- [3] Artese G., Trecroci A. Calibration of a low cost MEMS INS sensor for an integrated navigation system. – *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*. Vol. XXXVII. Part B5. Beijing 2008, pp. 877-882.
- [4] Grewal M. S., Henderson, V. D., Miysako R. S. Application of Kalman filtering to the calibration and alignment of inertial navigation systems. *IEEE Transactions on Automatic Control*, 1991, vol. 36, pp. 3-13.
- [5] Hide C., Moore T., Smith M. Adaptive Kalman filtering for low-cost INS/GPS. *The Journal of Navigation*, 2003, vol. 56, pp. 143-152.
- [6] Nikbakht N., Mazlom M., and Khayatian A. Evaluation of solid-state accelerometer for positioning of vehicle. *Proceeding of the IEEE International Conference on Industrial Technology*, Hong Kong, 2005, pp. 729-733.
- [7] Pang G., Liu, H. Evaluation of a low-cost MEMS accelerometer for distance measurement. *The Journal of Intelligent and Robotic Systems*, 2001, vol. 30, pp. 249-265.
- [8] Sheng-Chih Shen, Chia-Jung Chen, Hsin-Jung Huang. A new calibration method for low cost MEMS inertial sensor module. *Journal of Marine Science and Technology*, 2010, vol. 18, No. 6, pp. 819-824.
- [9] Gaysse, J. A low cost absolute position calculation system. *Proceeding of SICE-ICASE International Joint Conference*, Busan, Korea, 2006, pp. 5658-5661.
- [10] El-Rabbany A., El-Diasty, M. An efficient neural modal for de-noising of MEMS-based inertial data. *The Journal of Navigation*, 2004, vol. 57, pp. 407-415.
- [11] Wang H., Tian W. Modeling the random drift of micro-machined gyroscope with neural network. *Neural Processing Letters*, 2005, vol. 22, pp. 235-247.
- [12] Avrutov V. On Scalar Calibration of an Inertial Measurement Unit / V. Avrutov, S. Golovach, T. Mazepa // 19-th St. Petersburg International Conference on Integrated Navigation Systems. St. Petersburg, Russia, 2012. - State Research Center (CSRI) Elektropribor, 2012, pp.117-121.
- [13] Avrutov V. Scalar Diagnostics of the Inertial Measurement Unit// *I.J. Intelligent Systems and Applications*, 2015, vol.11, pp.1-9.
- [14] Bugrov Y.S., Nikolsky S.M. Higher mathematics. Elements of linear algebra and analytic geometry. – Moscow: 'Nauka', 1984. – pp.192 (*In Russian*).