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# Integration of commodity derivative markets: Has it gone too far? 

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#### Abstract

We examine the impact of two financial crises on commodity derivative markets: the subprime crisis and the bankruptcy of Lehman Brothers. These crises are "external" to the commodity markets because they occurred in the financial sphere. Still, because commodity markets are now highly integrated with each other and with other financial markets, such events could have had an impact. In order to fully comprehend this possible impact, we rely on tools inspired by the graph theory that allow for the study of large databases. We examine the daily price fluctuations recorded in 14 derivative markets from 2000 to 2009 in three dimensions: the observation time, the space dimension - the same underlying asset can be traded simultaneously in two different places - and the maturity of the transactions. We perform an event study in which we first focus on the efficiency of the price shock's transmission to the commodity markets during the crises. Then we concentrate on whether the paths of shock transmission are modified. Finally, relying on the measure proposed by Bonacich (1987) for social networks, we focus on whether the centrality of the price system changes.


JEL Codes: E44, F15, G01, Q02, Q40
Keywords: Commodity markets, derivatives, market integration, financial crises, graph theory, minimum spanning tree, centrality.

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## 1 Introduction

In this paper, we examine the impact of two financial crises on the commodity derivative markets: the subprime crisis and the bankruptcy of Lehman Brothers. These crises are exogenous to the commodity markets because they occurred in the financial sphere. Still, such events could have propagated to the commodity markets because these markets are highly integrated with each other and with other financial markets (see Fattouh et al. (2013), Buyukşahin et al. (2010), Buyukşahin and Robe (2011), Buyukşahin and Robe (2014), Tang and Xiong (2012), Irwin and Sanders (2011)).

Specifically, in this paper, we analyze the shock transmission through the dynamic behavior of the correlations between price returns. Following Forbes and Rigobon (2002), we consider that there is transmission if market co-movements increase significantly after a shock.

In order to fully comprehend the potential impact of such crises on the commodity derivative markets, we perform an event study in which we examine price fluctuations in three dimensions: the observation time, the space dimension - the same underlying asset can be traded in two exchanges simultaneously - and the maturity of the transactions. We focus on a time window of one month (i.e., ten trading days before and after the beginning of the crises). We situate the triggering event on August 9, 2007 for the subprime crisis and on September 15, 2008 for the Lehman Brothers bankruptcy (see the Appendices A. 1 and A. 2 for more details on the chronology of the crises).
Such an analysis requires the use of high dimensional data. In this context, the tools of the graph theory have already proved to be very interesting in various fields of finance. First, they provide a way to synthesize the information contained in the data and to obtain meaningful visual representations, second because they allow for the quantification of high dimensional information (see for instance Onnela et al. (2003), Cohen-Cole et al. (2012), Lautier and Raynaud (2012)). In what follows, we rely mainly on the methodology proposed by Lautier and Raynaud (2012). These authors provide a long-term analysis of the connections between 14 derivative markets between 2000 and 2009. They give evidence of an increasing integration along the time period under scrutiny, and they show that it is a condition for systemic risk to appear. Taking advantage of the fact that between 2000 and 2009 two main financial crises occurred, we perform an event study on the same markets. This study gives us the possibility to concretely assess the potential consequences of market integration. Moreover, we introduce a new method that was initially proposed by Bonacich (1987) for social networks. This method allows us to better evaluate the organization of the graph. It gives insights into the localization of the center of the graph that, as far as systemic risk is concerned, is crucial.

Following Lautier and Raynaud (2012), the nodes of the graphs correspond to price returns: there is one node per futures contract and per maturity. The link between each pair of nodes depends on the correlations between their returns. Relying on several measures, we provide a dynamic analysis of these graphs and their behavior around the crises. We also empirically compute how exceptional these events are compared to what can be observed in the whole period.

First, in order to filter the information contained in the graphs, we use Minimum Spanning Trees -MST - (Mantegna (1999)). Because they capture the most important links between the markets, they are the most probable and the most efficient paths of price shock transmission. Taking into account the length of the MST, we can ask a first question: does the efficiency of the price shock transmission improve during crises? We then concentrate on the organization of the graph, namely the topology of the MST and ask a second question: do the paths of shock transmission change during crises and how? In order to answer these questions, several tools are used. First, we use survival ratios that indicate the number of links that change from one day to the other and give indications about large reorganizations of the graphs. Second, the allometric coefficients measure how far a tree stands from a linear or, on the contrary, a star-like organization. These two extreme configurations have radically opposite consequences from the systemic point of view: with a chain-like tree, a shock appearing at one extremity of the tree must spread through all nodes before reaching the other extremity. On the contrary, with a star-like tree, a shock arising at the center of the graph might rapidly affect all other nodes. Finally, we focus on the centrality of the price system: does it change? Does it increase? In a first approach, we simply identify the center of the price system as the most connected node. We then improve this analysis with the measure developed by Bonacich (1987): in a nutshell, instead of focusing on one single node, we take into account the whole organization of the network, that is, the number and proximity of the direct as well as the indirect neighbors of a node.

This paper is organized as follows. We first explain how to build a graph on the basis of our data. We then examine the efficiency of the shock transmission, the organization of the price system and its centrality. At each step, we compare the behavior of the price system in the whole period with what happened during the crises.

## 2 The price system

After a short description of the data used for the study, we explain the way we build price graphs.

### 2.1 Data

For the empirical study, we examine 14 futures markets corresponding to three different sectors of activity: 6 energy markets that comprise 2 markets each of crude oil, natural gas and petroleum products; 4 agricultural markets (wheat, corn, soy oil and soy bean) and 4 financial assets (Mini S\&P500 index, gold, USD/EUR exchange rate, and 3-month Eurodollar interest rate). We selected the contracts that were characterized by the largest transaction volumes over a long time period, thanks to the Futures Industry Association's monthly volume reports. We used Datastream in order to collect settlement prices on a daily basis.
In the absence of reliable spot data for most commodity markets, we approximated all spot prices with the nearest futures prices. Such an approximation is very common in finance. We also rearranged the futures prices in order to reconstitute the daily term structures, i.e., the relationships linking, at a specific date, several futures contracts with different delivery dates. We removed some maturities from the database because the price curves were shorter at the beginning of the period. The number of contract maturities indeed usually rises on a derivative market; the growth in the transaction volumes of existing contracts results in the introduction of new delivery dates. Finally, when performing spatial and 3D analyses, we used the longest common time period for all of the underlying assets, from 2000/01/04 to 2009/08/12. Once these selections have been carried out, our database still contains more than 655,000 prices, that comprise 220 time series in the 3D analysis and a subset of 14 in the spatial one.

Table 1 summarizes the main characteristics of our database.

### 2.2 Building the graphs

Our graphs are built on the basis of the correlations between the price returns. We use this measure in order to capture the synchronous price movements in the system. To obtain a graph, these correlations are transformed into distances.

Correlations of price returns The first step towards the analysis of market integration is the computation of the synchronous correlation coefficients $\rho_{i j}(t)$ of the price returns, defined as follows:

$$
\begin{equation*}
\rho_{i j}(t)=\frac{\left\langle r_{i} r_{j}\right\rangle-\left\langle r_{i}\right\rangle\left\langle r_{j}\right\rangle}{\sqrt{\left(\left\langle r_{i}^{2}\right\rangle-\left\langle r_{i}\right\rangle^{2}\right)\left(\left\langle r_{j}^{2}\right\rangle-\left\langle r_{j}\right\rangle^{2}\right)}}, \tag{1}
\end{equation*}
$$

In the spatial dimension, $i$ and $j$ stand for the nearby futures contracts of a pair of assets (crude oil or corn for example), whereas they stand for pairs of delivery dates in the maturity dimension. Both are present in the 3D analysis with the 220 time

| Underlying asset | Exchange-Zone | Maturities | \# contracts |
| :--- | :--- | :--- | :--- |
| Light crude oil | CME-US | up to 84 | 33 |
| Brent crude | ICE-EU | up to 18 | 17 |
| Heating oil | CME-US | up to 18 | 18 |
| Gasoil | ICE-EU | up to 12 | 12 |
| Natural gas (US) | CME-US | up to 36 | 36 |
| Natural gas (Eu) | ICE-EU | up to 9 | 9 |
| Wheat | CME-US | up to 15 | 6 |
| Soy bean | CME-US | up to 14 | 7 |
| Soy oil | CME-US | up to 15 | 15 |
| Corn | CME-US | up to 25 | 4 |
| Eurodollar | CME-US | up to 120 | 40 |
| Gold | CME-US | up to 60 | 17 |
| USD/EUR Exchange rate | CME-US | up to 12 | 4 |
| Mini S\&P500 | CME-US | up to 6 | 2 |

Table 1: Characteristics of the collected data: nature of the underlying asset, trading location (CME stands for Chicago Mercantile Exchange, ICE for Inter Continental Exchange, US for United States and EU for Europe), longest maturity traded (in months), number of contracts (this number is added just after the name of the underlying asset in the figures).
series. The daily logarithm price differential stands for the price returns $r_{i}$, with $r_{i}=$ $\left(\ln F_{i}(t)-\ln F_{i}(t-\Delta t)\right) / \Delta t$, where $F_{i}(t)$ is the price of the futures contract $i$ at date $t$. The time interval is $\Delta t$ and $\langle$.$\rangle denotes the statistical average performed over time, for$ the trading days of the study period.
For a given time period and a given set of data, we thus compute the matrix $\mathbf{C}$ of $N \times N$ correlation coefficients, for all of the pairs $i j$. $\mathbf{C}$ is symmetric with $\rho_{i j}(t)$ equal to one when $i=j$. Thus, it is characterized by $N(N-1) / 2$ coefficients.

Performing dynamic studies on the basis of rolling windows requires the choice of a proper window length. On the one hand, we want it to represent typical economic periods (one semester, one year, five years...) and to be as short as possible in order to give evidence of sudden changes. On the other hand, we are confronted with a technical constraint: in order to ensure representative results, the number of observations has to be larger than the number of nodes. Having to deal with 220 series of price returns (i.e., 220 nodes), we thus use a rolling window of one year (252 trading days). We do the same in the spatial dimension for comparison purposes. As robustness checks, we also perform computations with 2-year windows, as illustrated in the appendix B.1. Further, we use rolling windows situated before the observation date. So when we look at what
happens on August 9, 2007, the information used is situated one year before that event. Fortunately, because the two crises are separated by more than one year, there is no overlap between them.

From correlations to distances In order to use the tools of the graph theory, we need to introduce a metric. The correlation coefficient $\rho_{i j}$ cannot be used as a distance $d_{i j}$ between $i$ and $j$ because it does not fulfill the three axioms that define a metric (Fréchet (1906), p30):

- $d_{i j}=0$ if and only if $i=j$,
- $d_{i j}=d_{j i}$
- $d_{i j} \leq d_{i k}+d_{k j}$

However, a metric $d_{i j}$ can be extracted from the correlation coefficients through a nonlinear transformation. This Euclidean distance is defined as follows ${ }^{1}$ :

$$
\begin{equation*}
d_{i j}(t)=\sqrt{2\left(1-\rho_{i j}(t)\right)} . \tag{2}
\end{equation*}
$$

A distance matrix $\mathbf{D}$ is thus extracted from each correlation matrix $\mathbf{C}$ (at each date t ) according to the Equation (2). The matrices $\mathbf{C}$ and $\mathbf{D}$ are both $N \times N$ dimensional. While the coefficients $\rho_{i j}(t)$ can be positive for the correlated returns or negative for the anti-correlated returns, the distance $d_{i j}(t)$ is always positive. The distance matrix corresponds to a fully connected graph; it represents all the possible connections in the price system.

## 3 The efficiency of the shock transmission

Considering the dimensionality of our price system and the number of nodes in our graph, it is very difficult to visualize. We thus resort to a filtering technique which is especially suited to our context: the Minimum Spanning Tree (MST).

### 3.1 The minimum spanning tree

In order to understand the organizing principles of a system through its representation as a graph, the latter needs to be spanned. However, there are a lot of paths that span a graph. For a weighted graph like ours, the MST divulges the most relevant connections of each element of the system and it reduces the information space from $N(N-1) / 2$ to $N-1$.

[^0]The MST is the path spanning all the nodes of the graph without any loop. It has less weight than any other tree and is unique. The distance $d_{i j}(t)$ is more than just an Euclidean metric; it is an ultrametric that satisfies the triangular inequality: $d_{i j}(t) \leq$ $\max \left\{d_{i k}(t) ; d_{k j}(t)\right\}$.
When the graph is weighted with distances, the latter corresponding to the correlations between the price returns, the MST is especially useful for the study of systemic risk. In an analogy with signal transmission, the ultrametric provides the shortest path between all of the nodes, that is, the path where the signal suffers the least losses and travels the fastest. We interpret this feature as the efficiency (in speed and in accuracy) in the transmission of the signal. Furthermore, if a price shock is assimilated to a signal and if transmission is appreciated through the analysis of the dynamic behavior of the correlations between the price returns, then the MST "can be assimilated into the shortest and most probable path for the propagation of price shocks" (Lautier and Raynaud (2012)).

The visualization of the trees (which are plotted with the software Graphviz) addresses the meaningfulness of the taxonomy that emerges from the system. Because we are considering the links between markets and/or delivery dates belonging to the MST, if a link between two markets or maturities does not appear in the tree, it only means that this link does not correspond to a minimal distance. Note also that, in such an analysis, the results depend on the nature and the number of markets chosen for the study.

Figure 1 presents the MST obtained on the basis of our price system for the spatial dimension and over the whole period. It is scaled: the closest nodes correspond to the most correlated price series. Three sectors can be identified: energy is in the top left-hand. It gathers American as well as European markets and is situated between agriculture (on the right) and financial assets (at the bottom).
The link between the energy and agricultural products passes through soy oil. This is interesting because soy oil can be used for fuel. The link between commodities and financial assets passes through gold, which is also meaningful, because gold can be seen as a commodity as well as a reserve of value. The only surprise comes from the Mini S\&P500 that is more correlated to soy oil than to financial assets. This connection between the Mini S\&P500 and agricultural markets could be interpreted as evidence of the financialization of the commodity markets. However, in a dynamic analysis, this connection is very unstable. At least two reasons could explain such a result: first, Buyukşahin et al. (2014) find that the correlations between grains and equities fluctuate a lot; and second, compared to all other contracts taken into account, the Mini S\&P500 is the least actively traded.
At first glance (if we accept that counting the number of links allows for the identification of the center of the graph) the most connected node is the one corresponding to Brent


Figure 1: Scaled MST in the spatial dimension, 2000-2009.
crude oil, which makes it - a priori - the best candidate for the transmission of price fluctuations in the tree (actually, the same could be said for American crude oil - Light Crude - because the distance between these products is very short). Last but not least, the energy sector seems the most integrated, as the distances between the nodes are short. Such a star-like organization leads to specific conclusions regarding systemic risk. A price movement appearing in the energy markets, situated at the heart of the price system, will have more impact than a fluctuation affecting the peripheral markets such as interest rates or wheat. This configuration explains why we consider the subprime and the Lehman Brothers crises as exogenous events in this study.
The 3D MST comprises 220 time series (nodes). Depicted by Figure 2, it is less easy to read (this is why we removed the captions in the nodes), but it can be interpreted through the prism of the spatial tree. The same topology prevails, except that adding the maturities introduces linear branches in each market (with the noticeable exception of American natural gas). Moreover, this scaled representation shows that some markets are more integrated than others: clusters of maturities can be seen, at the center of the graph, for the energy sector (except for the two natural gas markets). Strong integration
can also be observed in the financial branch; this is especially true for the Eurodollar contract after the eighth maturity.
Because these topologies are very stable over time (Lautier and Raynaud (2012)), we use them as references in the remainder of this study.


Figure 2: Scaled MST in 3D, 2000-2009.

### 3.2 How does the length of the MST behave?

We first explain how this measure can be obtained and how it behaves on the whole sample. We then study it around crises.

### 3.2.1 The measure

The normalized length of the tree can be defined as the average of the lengths of the edges belonging to the MST:

$$
\begin{equation*}
\mathcal{L}(t)=\frac{1}{N-1} \sum_{(i, j) \in M S T} d_{i j}(t), \tag{3}
\end{equation*}
$$

where $t$ denotes the date of the construction of the tree and $N-1$ is the number of edges in the MST. The length of a tree is higher when the distances increase and consequently when correlations are low. Thus, the more the length diminishes, the more integrated the system is.


Figure 3: Normalized tree's length in the spatial dimension, 2000-2009.

Figure 3 represents the dynamic behavior of the normalized length of the MST in the spatial dimension over the whole period under consideration. The general pattern is that the length decreases, which reflects the increasing integration of the system. Thus the most efficient transmission path for price fluctuations becomes shorter as time goes by. This finding is consistent with e.g., Silvennoinen and Thorp (2013) and Tang and Xiong (2012). A more in-depth examination of the graph also shows some very important moves at specific dates, one of them being around the Lehman Brothers bankruptcy.

### 3.2.2 The length of the trees around the crises

A first appraisal of the importance of the crises consists in measuring whether the changes in the length of the MST that occurred around the events were tail events or not.
We compute the empirical distribution of the length variations over the whole sample and examine the probability of the occurrence of fluctuations situated above (for increases) or below (for decreases) those observed around the crises. At 5\%, the changes recorded on August 16, 2007 (five trading days after the beginning of the subprime crisis) and on September 12, 2008 (one trading day before the bankruptcy of Lehman Brothers) are in the tail of the distribution, both in the spatial dimension and in 3D. In the spatial dimension only, we can add August 14, 2007, and in 3D only September 17, 2008. These last


Figure 4: Normalized tree's length in the spatial dimension and in 3D for each event.
two events and the one recorded on September 12, 2008 have a probability of occurrence that is close to $1 \%$. Consequently, compared to what was observed between 2000 and 2009, the two crises have generated exceptional changes in the length of the MST.

A recurrent result in finance is the observation of an increase in the price correlations just after a crisis (see, e.g., Chakraborti et al. (2003) for an analysis of the equity market around Black Monday on October 19, 1987, Buyukşahin and Robe (2014) and Tang and Xiong (2012) for commodity-equity markets, or Pesaran and Pick (2007) for a review of several studies on these topics). Figures 4 (a) to 4 (d), which represent the evolution of the length of the trees on a 1-month time window around the crises under consideration both in the spatial dimension and in 3D, do not exhibit such behavior. On the contrary, in three cases (subsets (a), (b) and (c)) out of four, we find an increase in the length of the MST. For the subprime crisis, the peak appears on August 15, 2008, four trading days after the beginning of the crisis. For the Lehman Brothers bankruptcy, the change in the behavior of the tree arrives before the event, between September 11 and 12 of 2008. These dates correspond to the period when the difficulties encountered by the bank became public knowledge (see the Appendices A. 1 and A.2).
However, this increase in the global length of the MST comes with a decrease in certain subsets of the trees. This is especially the case for the Eurodollar market around the Lehman Brothers bankruptcy as shown by the scaled MST on Figure 5, where there clearly is a shrink in the trees around the two crises. Such a result is reasonable: first because
the 3-month interest rate is a pure financial asset and second because what we observe here is a branch of the tree where only the maturity dimension is taken into account. As mentioned by Lautier and Raynaud (2012), under the pressure of arbitrage operations, the markets are more integrated in the maturity dimension than in the spatial one.

The analysis of the length of the trees shows that, even if our price system becomes more and more integrated between 2000 and 2009, these two crises, born in the financial sphere, did not harm the commodity markets as a whole. This conclusion is consistent with the findings of Buyukşahin and Robe (2011) who observed that the links between the equity index and the energy futures is weaker in times of crises or of Corsetti et al. (2005) who find that correlations decreases in some episodes of crisis. As expected, these crises had an impact on the financial sphere: there is a local increase in the integration of the futures contracts written on the financial assets. However, as far as commodity markets are concerned, they became temporarily less connected with the financial assets.

## 4 The organization of the tree

Measuring the length of the MST does not give the possibility to ask whether or not the paths for shock transmission change during the crises. In order to answer this question, the graph theory provides several tools: first the survival ratios and second the allometric coefficients.

### 4.1 The survival ratios

This measure $\left(S_{R}\right)$ indicates the fraction of links that survives, in the MST, between two consecutive trading days (Chakraborti et al. (2003)):

$$
\begin{equation*}
S_{R}(t)=\frac{1}{N-1}|E(t) \cap E(t-1)| \tag{4}
\end{equation*}
$$

In this equation, $E(t)$ refers to the set of the tree edges at date $t, \cap$ is the intersection operator and $|$.$| gives the number of elements contained in the set. Due to the finite$ number of links, the ratios take discrete values.
The use of this measure naturally raises the same question as before: how exceptional are the values of the survival ratios observed around the crises? As before, we evaluate the probability of the occurrence of high reconfigurations in the graph. We find that only the changes recorded on September 18 and 19 of 2008 (the $17^{\text {th }}$ is close) are below the $5 \%$ probability of occurrence in the spatial dimension. In 3D, only September 17 and 24 of 2008 appear below the $5 \%$ threshold. According to these figures, the subprime crisis shows nothing specific: even if, as shown by the length of the MST, the trees locally shrink in the financial sphere on this occasion, the path of the price shock transmission


Figure 5: Scaled MST in the maturity dimension, Eurodollar market. Subset (a): 2000-2009; subset (b): 1-month time window including the subprime crisis; subset (c): 1-month time window including the Lehman Brothers bankruptcy.


Figure 6: Survival ratios in the spatial dimension, 2000-2009.
remains the same.
This result is confirmed by Figure 6. The figure shows first that under normal circum-
stances, the topology of the trees is very stable between two dates, in the spatial dimension as well as in 3D: most of the time, between 2000 and 2009, more than $85 \%$ of the links remain unchanged from one day to the next. Second, nothing special happens around the subprime crisis. This is far less obvious for the Lehman Brothers bankruptcy. In this case, the most important reorganizations appear in the spatial dimension, where more than $30 \%$ of the graph is reorganized.

Finally, while some fluctuations of the survival ratios might be due to real changes in the behavior of the system, it is worth noting that others might simply be due to noise. This is why a deeper analysis is needed. We will perform it through the use of allometric coefficients.

### 4.2 The allometric coefficients

The computation of the allometric coefficients of a MST quantifies where this tree stands between two asymptotic topologies: star-like trees and chain-like trees. These two topologies have very different implications for systemic risk.
The first model of the allometric scaling on a spanning tree was developed by Banavar et al. (1999). In their method, the first step consists in assigning a value $A_{i}$ equal to 1 to each node $i$. Then the root (also called the central node) of the graph must be identified. In what follows, the root is determined with Bonacich's measure defined in section 5. As a robustness check, we perform the same tests with a root identified as the node with the highest number of links. The results remain qualitatively the same and are available on request
Starting from the root, the second step of the method consists in updating the coefficients $A_{i}$ and in assigning the coefficients $B_{i}$ of each node $i$ as follows:

$$
\begin{equation*}
A_{i}=\sum_{j} A_{j}+1 \text { and } B_{i}=\sum_{j} B_{j}+A_{i}, \tag{5}
\end{equation*}
$$

where $j$ stands for all of the nodes connected to $i$ in the MST. The allometric scaling relation is defined as the relationship between $A_{i}$ and $B_{i}$ :

$$
\begin{equation*}
B \sim A^{\eta} \tag{6}
\end{equation*}
$$

where $\eta$ is the allometric exponent. It represents the degree or complexity of the tree and stands between two extreme values: $1^{+}$for star-like trees (Figure 7) and $2^{-}$for chain-like trees (Figure 8).

A MST belonging to the first or to the second structure will not have the same implications in terms of shock transmission. One way to explain such an interpretation is to rely once again on the analogy with the transmission of a signal in a network. Let


Figure 7: Star-like structure


Figure 8: Chain-like structure
us assume that a signal is transmitted in each network represented by Figures 7 and 8 . In each case, the signal is transmitted from node $S$ at time $t$ and there is some latency in the transmission. In the star-like tree, all of the others nodes (A, B, C, D and E) will receive the transmission simultaneously at time $t+1$. Comparatively, in the chain-like tree, the first receiver is node A, the second is node B, etc. In such a topology with $N$ nodes, it takes $N-1$ time periods (i.e., five in the Figure 8) before reaching the end of the network. Meanwhile, if there is noise in the transmission channel, the signal will suffer some losses. In our case, where the distances in the networks stand for correlations between price returns, a price shock emerging at node $S$ will spread more efficiently if the structure of the tree is star-like, because it will more quickly reach all of the other nodes. It is thus crucial to correctly identify the center of the graph.
Relying on the allometric coefficients, Lautier and Raynaud (2012) show that: i) the MST are almost linear in the maturity dimension of most markets, ii) they stand right in the middle of the two extreme configurations in the spatial dimension at 1.5, and iii) the allometric coefficients are around 1.75 in the 3D case. Around the crises, as shown by Figure 9, the levels of the allometric coefficients remain the same. Moreover, their variations are not exceptional at 5\% except those recorded in 3D on September 2, 2008 and on September 29, 2008, around the bankruptcy of Lehman Brothers.

## 5 Examining the centrality of the graphs

When studying systemic risk, it is important to correctly detect the center of the trees. For regulatory authorities, such nodes can be assimilated to regions of higher fragility. Even though we examine exogenous events in this study, the question of centrality remains crucial. What if these events create shocks that reach the center of the graph? They would then spread rapidly to all of the other markets, as noted in the above sub-


Figure 9: Allometric coefficients, in the spatial dimension and in 3D, for each event.
section.
The most common way to identify the center of a graph is to assess the degree (i.e., the number of links) of each node in the trees. However, such an analysis might be insufficient: first because it does not take into consideration the distances between the nodes, and second because it only accounts for the direct neighbors of a node. It could be interesting, on the contrary, to be mindful of the overall configuration of the graph. In what follows, we first give an example of an analysis based on degree only: we focus on the evolution of the trees in the spatial dimension around the Lehman Brothers bankruptcy. As noted on the basis of the survival ratios, there is indeed an important reconfiguration of the graph on this occasion. Then, we propose the use of a new measure of centrality that was introduced by Bonacich in 1987 for social networks and recently used by Bloch and Quérou (2013). To the best of our knowledge, this is the first time that such a method has been employed in finance.

### 5.1 The degree of the nodes

The scaled MST in the spatial dimension at the Lehman Brothers bankruptcy is depicted by Figure 10. If we compare this tree with the one computed for the whole period as illustrated by Figure 1 (as shown in subsection 4.1 the MST is very stable; the tree computed on the whole period can thus be taken as a reference) then we can see some
changes: the Mini S\&P500 is not linked to soy oil anymore, but now to wheat; the UK natural gas is not directly connected to the energy sector anymore; and, more importantly, gold now stands at the center of the graph. From an economic point of view, such a result is very reasonable. In a situation where high uncertainty affects the whole financial system, we indeed expect investors to consider gold as a reserve of value. Yet the story is not so simple.


Figure 10: Scaled MST in the spatial dimension at Lehman Brothers bankruptcy (September 15, 2008).

### 5.2 The Katz-Bonacich centrality measure

We first present the method and its advantages. Then we use it for the event study.

### 5.2.1 The method

The Katz-Bonacich centrality measure aims at taking into consideration the whole configuration of a graph, that is, the direct as well as the indirect neighbors of a node. Looking only at the direct neighbors, as done when one relies on the degree, might be insufficient as illustrated by Figures 11: the node labelled "A" (or B or C) exhibits the highest degree (four in this case). However, the " S " node is obviously the most central one.

The measure proposed by Bonacich (1987) is an extension of the one developed by Katz (1953). This author was the first to pay attention to the indirect neighbors of a node. In addition, the measure developed by Bonacich (1987) gives the possibility of taking into account the "negative" relations, i.e. the fact that, if the value of a node increases, then its neighbors' value decreases.
The centrality vector, which gives one value per node, is computed as follows:

$$
c(\alpha, \beta)=\alpha(\mathbf{I}-\beta \mathbf{R})^{-1} \mathbf{R} \mathbf{1}
$$



Figure 11: Connectivity (A, B, or C) versus centrality (S).
where $\mathbf{I}$ is the identity square matrix, $\mathbf{R}$ is the matrix of the weights of the graph, and $\mathbf{1}$ a vector of 1 s . The coefficient $\alpha$ is a scale factor. According to Bonacich, the coefficient $\beta$ can be interpreted in different ways: "the degree to which an individual's status is a function of the statuses of those to whom he or she is connected" or "a radius within which the researcher wishes to assess centrality". Note also that the centrality values are sensitive to both the weights of the graph and its topology. Since these values take into account infinitely far neighbors, a small change in the topology of the graph can result in large changes in the centrality values.
The use of this relationship matrix requires first a measure of similarity: the quantities in $\mathbf{R}$ must be such that, the higher the $\beta$, the easier the transmission. A second requirement is that all $R_{i j}$ are positive. Third the $R_{i i}$ must be equal to zero. To fulfill the first requirement, we use the correlation matrix for $\mathbf{R}$. More precisely, because we are interested in the central node of the MST, we consider the prices correlations in the MST, and we compute $\mathbf{R}$ as follows:

$$
R_{i j}(t)=C_{i j}(t) * E_{i j}^{M S T}(t),
$$

where $\mathbf{C}(t)$ is the correlation matrix and $\mathbf{E}^{M S T}(t)$ is the edge matrix of the tree; $E_{i j}^{M S T}(t)$ equals to one if there is a link between $i$ and $j$ in the MST and zero otherwise. This matrix is symmetric, with $N-1$ ones.
The use of the filtered correlation matrix for $\mathbf{R}$ simplifies the application of the method developed by Bonacich. This matrix can be directly identified to $\mathbf{R}$, because it fits all of the requirements. Moreover, such a choice leads to more precise results, because it allows for taking into account the specific value of each link instead of averaging them into a $\beta$ coefficient (which we thus drop).

### 5.2.2 Empirical results

For comparison purposes, it is interesting to go back to the scaled MST in the spatial dimension commented on in section 3 and represented by Figure 1. When relying on the degrees of the nodes, the root of the tree corresponds to crude oil. However, taking into account the overall organization of the tree leads to a conclusion that is more nuanced. Table 2 presents the results of the method when it is applied in the spatial dimension between 2000 and 2009. Relying on the centrality measures leads to putting more emphasis on both heating oil and crude oil; the heating oil is ranked first. Moreover, a dynamic analysis shows that, especially after August 17, 2005, the agricultural markets play a more important role. This result calls for further analysis, but it is probably due to the introduction of the rules regarding bioethanol in the United States in 2005. Second, half of the markets under consideration in the spatial analysis never reach a centrality value above 1: this is true for the 3-month eurodollar, the USD/EUR exchange rate, the Mini S\&P500 index, gold, gasoil and for the US and UK natural gases. These markets thus have a centrality that is unusually low and are hence less important.

| Market | Centrality measure | Rank |
| ---: | ---: | ---: |
| Heating Oil | 1.148228 | 1 |
| Brent | 1.108484 | 2 |
| Light Crude | 0.856703 | 3 |
| Gasoil | 0.591487 | 4 |
| US Natural Gas | 0.364067 | 5 |
| Gold | 0.231502 | 6 |
| USD/EUR Exchange Rate | 0.036973 | 7 |
| UK Natural gas | 0.034241 | 8 |
| Eurodollar | -0.00875 | 9 |
| Mini S\&P500 | -0.189855 | 10 |
| Wheat | -1.144788 | 11 |
| Soy Oil | -1.159204 | 12 |
| Corn | -1.890017 | 13 |
| Soy Bean | -1.979338 | 14 |

Table 2: Bonacich's centrality measure in the spatial dimension, 2000-2009.

The results associated with the centrality measures around the crises are depicted, for the spatial dimension, in Appendices B.2.1 and B.2.2. Once again, the subprime crisis does not affect the organization of the trees, whereas the Lehman Brothers bankruptcy has an impact (mostly temporary, though). Around this event, the ranking of the nodes
puts light crude oil first, gold second and heating oil third.
In 3D, the most central nodes are about the same as in the spatial dimension. Due to the large number of nodes (220), we cannot display the tables in this case but the results are available on request. As before, we do not find many changes around the subprime crisis and many more around the Lehman Brothers bankruptcy.
Finally, the most interesting phenomena appear in the maturity dimension around the Lehman Brothers bankruptcy. There are some changes in the direction of certain propagation paths. The most illustrative example of such behavior is that of light crude on September 10, 2008: before that date, many short-term maturities of light crude oil are among the most central nodes of the tree (they are situated above the rank of 20 according to the centrality measure), while most of the long-term maturities are among the least central (below the rank 200). From one day to the next, however, there is an inversion: the least central nodes become the most central ones (they even reach the rank of one) while the previously most central ones go as low as rank 220. Finally, things revert back to the initial state.

## 6 Conclusion

For a decade, commodity derivative markets have been experiencing a process of financialization due to managers seeking the diversification of their portfolios and to the arrival of new actors. This phenomenon has raised questions and worries about the eventuality of meaningless links, from an economic point of view, between commodities and more traditional financial markets like bonds and stocks. These fears have been largely confirmed by the acknowledgment of a growing integration between commodity markets as well as between commodities and other financial assets. One could wonder to what extent a shock originating from financial markets could propagate to commodities and strongly impact them. Investigating such a question is the purpose of this paper.
To this aim we examine the impact, on commodity markets of two recent financial crises: the Subprime crisis and the Lehman Brothers bankruptcy. Using the insightful tools of the graph theory, on the basis of several measures, we show that those shocks did not affect the commodity markets as hard as one might have expected.

## References

Banavar, J. R., Maritan, A., and Rinaldo, A. 1999. Size and form in efficient transportation networks. Nature 399:130-132.

Bloch, F. and Quérou, N. 2013. Pricing in social networks. Games and Economic Behavior.

Bonacich, P. 1987. Power and centrality: A family of measures. American Journal of Sociology 92:1170-1182.

Brunnermeier, M. K. 2009. Deciphering the liquidity and credit crunch 2007-2008. Journal of Economic Perspectives 23:77-100.

Buyukşahin, B., Haigh, M. S., And Robe, M. A. 2010. Commodities and equities: ever a "market of one"? Journal of Alternative Investment pp. 76-95.

Buyukşahin, B. and Robe, M. A. 2011. Does Paper Oil Matter? Energy Markets' Financialization and Equity-Commodity Co-Movements. Working Paper, American University, (Revised) July .

Buyukşahin, B. and Robe, M. A. 2014. Speculators, Commodities and Cross-market Linkages. Journal of International Money and Finance 42:38-70.

Buyukşahin, B., Robe, M. A., and Bruno, V. G. 2014. The financialization of food? Working Paper .

Chakraborti, A., Kaski, K., Kertész, J., and Onnela, J.-P. 2003. Dynamic asset trees and black monday. Physica A 324:247-252.

Cohen-Cole, E., Kirilenko, A., Patacchini, E., Fouque, J., and Langsam, J. 2012. Strategic interactions on financial networks for the analysis of systemic risk. Handbook on Systemic Risk p. 306.

Corsetti, G., Pericoli, M., and Sbracia, M. 2005. 'Some contagion, some interdependence': More pitfalls in tests of financial contagion. Journal of International Money and Finance 24:1177-1199.

Fattouh, B., Killian, L., and Mahadeva, L. 2013. The role of speculation in oil markets: What have we learned so far? The Energy Journal 34.

Forbes, K. J. and Rigobon, R. 2002. No contagion, only interdependence: measuring stock market comovements. The journal of finance 57:2223-2261.

Fréchet, M. 1906. Sur quelques points du calcul fonctionnel. Rendiconti del Circolo Mathematico di Palermo 22:1-74.

Irwin, S. H. and Sanders, D. R. 2011. Index funds, financialization, and commodity futures markets. Applied Economic Perspectives and Policy 33:1-31.

Katz, L. 1953. A new status index derived from sociometric analysis. Psychometrika 18:39-43.

Lautier, D. and Raynaud, F. 2012. Systemic risk in energy derivative markets: A graph-theory analysis. The Energy Journal 33:215-239.

Mantegna, R. N. 1999. Hierarchical structure in financial markets. The European Physical Journal B 11:193-197.

Onnela, J.-P., Chakraborti, A., Kaski, K., Kertesz, J., and Kanto, A. 2003. Dynamics of market correlations: Taxonomy and portfolio analysis. Physical Review E 68:056110.

Pesaran, M. H. and Pick, A. 2007. Econometric issues in the analysis of contagion. Journal of Economic Dynamics and Control 31:1245-1277.

Silvennoinen, A. and Thorp, S. 2013. Financialization, crisis and commodity correlation dynamics. Journal of International Financial Markets, Institutions and Money 24:42-65.

Tang, K. and Xiong, W. 2012. Index Investing and the Financialization of Commodities. Financial Analysts Journal 68:54-74.

## A Timelines around the events

## A. 1 Some important events around the Subprime crisis

Based on Brunnermeier (2009), News feeds, Wikipedia

| Trading date | Calendar <br> date | Events |
| :---: | :---: | :---: |
| S-10 | 2007-07-26 | - Home sales declined and largest home builder reported loss |
| S-7 | 2007-07-31 | - American Home Mortgage Investment Corp. faces difficulties |
| S-6 to S | $\begin{aligned} & 2007-08-01 \\ & 2007-08-09 \end{aligned}$ | - Quantitative hedge funds suffered losses that trigger margin calls, fire sales, and correlation across strategies |
| S-6 | 2007-08-01 | - US Crude oil prices reach a new high due to declining stocks and decreased output |
| S-4 | 2007-08-03 | - Officials state that the housing crisis should not spread |
| S-3 | 2007-08-06 | - America Home Mortgage Investment Corp. goes bankrupt |
| S | 2007-08-09 | - BNP Paribas froze redemption of 3 of its investment funds due to inability to value structured products <br> - Triggered the first illiquidity wave on the interbank market and support from Central Banks |
| S+1 | 2007-08-10 | - Decreases propagate to Asian markets, triggering support from Central Banks |
| $\mathrm{S}+2$ to | 2007-08-13 | - Central Banks increase their support and lower rates |
| S+8 | 2007-08-21 |  |

Table 3: Important events around the Subprime crisis (S denotes the date of the trigger of the crisis, on August $9^{\text {th }}, 2008$ )

## A. 2 Some important events around Lehman Brothers bankruptcy

Based on Brunnermeier (2009), News feeds, Wikipedia

| Trading <br> date | Calendar <br> date | Events |
| :---: | :---: | :---: |
| L-6 | 2008-09-05 | US Government's plan to bail out Fannie Mae and Freddie Mac leaks |
| L-3 | 2008-09-10 | OPEC will cut oil production by 500,000 barrels a day Announcement of the worst losses of Lehman |
| L-1 | 2008-09-12 | The Federal Reserve tries to find buyers for Lehman and warns CME of a potential default |
| L | 2008-09-15 | Lehman files for bankruptcy in the morning, because of lack of buyers and of bail out <br> Merrill Lynch is sold to Bank of America |
| L+1 | 2008-09-16 | AIG is bailed out |
| L+2 | 2008-09-17 | Russia helps its biggest banks |
| L+3 | 2008-09-18 | Russia extends help <br> Lloyds TSB purchases HBOS, largely exposed to subprime mortgages |
| L+4 | 2008-09-19 | The Troubled Asset Relief Program leaks <br> US Treasury guarantees money market mutual funds up to $\$ 50$ billion <br> Nigerian oil production is cut by 280,000 barrels per day and a pipeline of Royal Dutch Shell was destroyed |
| L+5 | 2008-09-22 | G7 commits to protect the financial system |
| L+9 | 2008-09-26 | The Federal Deposit Insurance Corporation seizes Washington Mutual to sell it to JPMorgan Chase |

Table 4: Important events around Lehman Brothers bankruptcy (L denotes the date of Lehman Brothers default, on September 15, 2008)

## B Additional results

## B. 1 Robustness checks

This section of the appendix is devoted to a sensitivity analysis. It provides the results obtained around the two events, with the different measures used in the analysis (length of the MST, survival ratios and allometric coefficients) when the rolling window is extended to two years instead of one year. The comparison shows that overall, the behavior remains qualitatively the same. As expected, compared with the 1-year rolling window, the twoyear window has a smoothing effect.


Figure 12: Normalized length in the spatial dimension and in 3D, around the events


Figure 13: Survival ratios in the spatial dimension and in 3D, around the events


Figure 14: Allometric coefficients in the spatial dimension and in 3D, around the events
B. 2 Evolution of the markets rankings by centrality, around the events and sector by sector
B.2.1 Ranking by centrality measure in the spatial dimension, around the Subprime crisis (August 9, 2007)


Figure 15: Evolution of the ranks in the spatial dimension around the Subprime crisis. Subset (a) is for agricultural markets, subset (b) is for only 4 energy markets (for readability) and subset (c) is for financial markets.
B.2.2 Rankings by centrality measure in the spatial dimension, around the Lehman Brothers bankruptcy (September 15, 2008)


Figure 16: Evolution of the ranks in the spatial dimension, around the Lehman Brothers bankruptcy. Subset (a) is for agricultural markets, subset (b) is for only 4 energy markets (for readability) and subset (c) is for financial markets.


[^0]:    ${ }^{1}$ Taking the square of $\rho_{i j}(t)$ has no impact on the results (computations are available on request)

