

$$\begin{aligned}
\eta_2^{\mathbb{Z}} \left(\begin{array}{c} d \quad c \\ \vdots \\ a \quad b \end{array} \right) &= a \otimes \begin{array}{c} d \quad c \\ \vdots \\ \perp \end{array} b + b \otimes \begin{array}{c} d \quad c \\ \vdots \\ \perp \end{array} a + c \otimes \begin{array}{c} b \quad a \\ \vdots \\ \perp \end{array} d + d \otimes \begin{array}{c} b \quad a \\ \vdots \\ \perp \end{array} c \\
&= a \otimes [[d, c], b] + b \otimes [a, [d, c]] + c \otimes [[b, a], d] + d \otimes [c, [b, a]].
\end{aligned}$$