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ROSHDI RASHED, HISTORIAN OF GREEK AND ARABIC MATHEMATICS

ATHANASE PAPADOPOULOS

*To Roshdi Rashed, on the occasion of his eightieth birthday,
with respect and admiration.*

A. P.

ABSTRACT. The Kenneth O. May Prize in history of mathematics is awarded every four years by the International Commission on the History of Mathematics (ICHM). It was awarded in 2017 to Roshdi Rashed, the Egyptian-French historian of mathematics. (See the article *Awarding of the May Prizes for 2017*, *Historia Mathematica*, Volume 44, Issue 4, November 2017, Pages 315–317). In the present article, I survey the work of Rashed on the history of Greek and Arabic mathematics. Presenting this work in some detail gives at the same time an overview of the highly important works of Greek mathematics that were transmitted to us through the Arabs (the Greek original manuscripts do not survive), as well as some of the finest pieces of Arabic mathematics came down to us.

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INTRODUCTION

Roshdi Rashed has brought about, through his work, a transformation of the landscape of our understanding of the history of Greek mathematics transmitted in Arabic, and of Arabic¹ mathematics. He discovered, edited, translated, compared and published an incredibly large number of manuscripts, shedding new light on major classical Greek texts such as Diophantus' *Arithmetica* and Apollonius' *Conics*, and bringing to the forefront of the mathematical-historical scene a number of Arabic mathematicians whose names had been forgotten despite the fact that they produced remarkable pieces of work.

Rashed is the author of nearly 40 books², covering, from a historical perspective, the fields of algebra, geometry, arithmetic, analysis, trigonometry (spherical and Euclidean), optics, astronomy, epistemology, and philosophy of mathematics. Bringing new elements in the complex relationship between these subjects has been one of his other important achievements. The extent and the diversity of his contribution manifest his very broad vision.

Most of Rashed's books concern Arabic mathematics from its golden era (IXth–XIIIth century), or Greek mathematics transmitted through Arabic translations. His writings provide new insight on the intricate relation between Greek and Arabic mathematics as well as on the impact of translation of mathematical texts on research, and vice versa. The Arabs were not mere translators, as is often thought, but there were great mathematicians among them, and their translation effort was inspired by the realization that mathematics is a science in which every generation builds on the works of the earlier ones, and where reading the old masters is a ferment for the new achievements. The translations they made were aimed at providing convenient access to the research groups to these texts. A translation and edition of difficult mathematical Greek texts by the Arabs which would be just for the purpose of learning or

¹The intense science activity produced during the period IXth–XIVth century is sometimes referred to as “Islamic science.” Rashed uses the word “Arabic,” and this is supported by three facts: First of all, the great majority of works in the period considered were written in Arabic, a language which is the only common characteristic of the various nations and populations termed as “Arabs.” The second reason is that science has not so much to do with religion. The third reason is that some – even among the very important – Arabic mathematicians of that period were not Muslims: several were Christians, and some others were Jews.

²Without counting the books edited and the translations.

teaching³ is just inconceivable. This is one of the important outcomes of Rashed's industrious work.

Rashed also wrote books on mathematics from recent times (of Descartes, Fermat, etc.) establishing some fascinating connections with Arabic mathematics, some of which are found difficult by many to accept, on account of certain stereotypical beliefs.⁴

As a general scheme, Rashed's publications on each subject he tackled include critical editions of texts originally written in Arabic – often appearing for the first time – with an extensive historical commentary, accompanied by interesting and compelling mathematical and philosophical commentaries, relating, on the one hand, Greek and Arabic mathematics, and on the other hand, later mathematics, generally from the European Renaissance period and sometimes from the modern times. This openness to and awareness of modern mathematics is another exceptional feature of Rashed's historical insight. His books are complemented by papers; he published about 100 of them.⁵

Rashed's first publication dates from 1968, and his output went constantly on the ascendant from then on. He is still very active today (at the age of 80), working and publishing on various subjects. It is simply impossible to do justice within a short report to the totality of this work, even the most important part of it. On each topic, he published several monographs, articles and critical editions. It is worth noting that Rashed's writings on each topic that he dealt with are substantial in volume and depth.

We shall limit ourselves to a general description of some of his most important publications (and making the selections was not easy), and discuss their importance and their impact. The write-up is divided according to topics as follows:

- (1) Infinitesimal mathematics from the IXth to the XIth century.
- (2) The work of Ibn al-Haytham.
- (3) Optics.
- (4) Apollonius' *Conics*.
- (5) Diophantus' *Arithmetica* and the development of Diophantine analysis.
- (6) The notion of angle.
- (7) Some other works.

On each topic, he published several monographs, articles and critical editions. It is worth noting that Rashed's writings on each topic that he dealt with are substantial in volume and depth. His approach is completely opposite to the current trend which consists of publishing quickly, even partial results. On several important topics, the result of his writing, rewriting, revision and addition over the years is a definitive and complete edition of the work.

³Or for the purpose of transmitting it to the West...

⁴The assumption that our culture, and in particular for what concerns mathematics, is based only on Greek or Western thought.

⁵Not counting the Encyclopedia articles.

1. INFINITESIMAL MATHEMATICS

One of Rashed's far-reaching achievements is his 5-volume piece of work titled *Les mathématiques infinitésimales du IXe au XIe siècle*. The French version is about 5000 pages long and it is currently being translated into English (4 volumes of the translated version appeared and the 5th is in press). This work took Rashed more than 15 years to complete (the volumes in French appeared in 1993, 1996, 2000, 2002 and 2006).

Rashed's purpose in *Les mathématiques infinitésimales* is threefold:

- (1) The edition (or re-edition) of works that led to the emergence of important mathematical communities with a profound expertise in Archimedean and Apollonian geometries which flourished in Baghdad, Damascus and other major Arabic centers of learning, starting from the IXth century.
- (2) To collect the works belonging to a broad field that Rashed designates by the generic name "infinitesimal mathematics."⁶ This word refers to a collection of methods that were used and taught, and gradually evolved into a school of thought. This includes techniques for the measurement of the surfaces or solid curves, lines, circles, parabolas, ellipses, spheres, cylinders, paraboloids, extending the work of Archimedes on this subject, the investigation of the variations of functions (trigonometric functions, or combinations of trigonometric and polynomial) for the purpose of proving inequalities or determining extreme values. It also includes research around the convexity of functions (without the terminology "convexity" and "function"), approximation techniques with a systematic use of tangents in a scheme which is in fact a geometric equivalent of taking derivatives, with applications of the methods in spherical geometry and astronomy, and in problems on isoperimetry and isoeiphany.

One notable outcome of this work that must be mentioned is the strong relation between the works of Archimedes and Apollonius.

- (3) To highlight the work of Ibn al-Haytham, one of the most brilliant scholars of the period considered, and whose name appears all along in Rashed's works. We have devoted a section to him in the present report.

Despite their triple purpose, these five volumes have a coherent and organic unity, around the theme of infinitesimal mathematics. They contain a set of fundamental texts edited and translated, many of them for the first time. The translations are accompanied by historical and mathematical commentaries in modern terms making relations with today's mathematics. Not only historians but also mathematicians will benefit from reading them. We shall now briefly review these individual volumes.

⁶The reader should note that the word "infinitesimal" is not used here in the sense of the infinitesimal calculus discovered by Leibniz and Newton. Rashed himself warns the reader about this possible confusion.

Volume I. The subtitle of this volume is *Fondateurs et commentateurs: Banū Mūsā, Ibn Qurra, Ibn Sinān, al-Khāzin, al-Qūhī, Ibn al-Samḥ, Ibn Hūd*. This volume focuses on the founding figures of the IXth and Xth century of the school referred to as the Archimedian and Apollonian mathematical school of Baghdad, the Abbaside capital which witnessed an explosion of science and philosophy, starting from the second half of the IXth century. The volume is built upon a rich collection of texts due to the three brothers Banū Mūsā and Thābit ibn Qurra, al-Khāzin, Ibrahīm ibn Sinān, al-Qūhī, Ibn al-Samḥ, and Ibn Hūd.

From Banū Mūsā, we have an important text from their *Book of the measurement of plane and spherical figures*. It deals with areas, volumes, means, trisection of angles, constructions with mechanical instruments, and the use of conics. The treatise is written in the tradition of Archimedes, including new developments. Thābit ibn Qurra, who belongs to the same school, besides being a mathematician was fluent in Greek. He translated into Arabic a considerable number of Greek texts, including major ones such as Archimedes' *The sphere and the cylinder*, Books V to VII of Apollonius' *Conics* and the *Arithmetical introduction* of Nicomachus of Gerasa. He also revised several translations made by others, like one of Euclid's *Elements* and Ptolemy's *Almagest*. Thābit ibn Qurra is a typical example of a translator who was a great mathematician. This brings us to another important idea that recurs in Rashed's work, the relation between translation and research: the impact of translation on mathematical research, and the role of research as a catalyst for translation. Translations of mathematical texts were often sponsored by the Arabic rulers, because of their need in research communities. Several translations were done by mathematicians, rather than linguists, and are more of redactions, rather than translations. Ibn Qurra's treatises that are edited in this volume concern computations of areas bounded by conic sections and volumes of solids of revolution, using approximation methods of an infinitesimal nature, as well as results involving geometric transformations. The volume also contains a critical edition of his three treatises *On the measurement of the parabola*, *On the measurement of paraboloids*, and *On the section of the cylinder and its lateral surface*. Among the important notions contained in these books, we mention the notion of motion, introduced by Thābit ibn Qurra in his treatise on the section of the cylinder. The work of Ibn Qurra was greatly influential on Ibn al-Haytham and Sharaf-al-dīn al-Ṭūsī, whose work we shall mention below.

The other texts edited in Volume I include Ibn Sinān's analysis and synthesis and the measure of the parabola, Al-Khāzin's application of the theory of conics to the solution of 3rd-degree equations, to problems of integer Diophantine analysis and to geometric solutions of isoperimetric problems, his *Commentary on the first book of Ptolemy's Almagest* dealing with plane isometric figures and solid figures with equal surface areas, Al-Qūhī's two treatises on the determination of the volume of the paraboloid, a revision by Rashed of a French translation by Tony Lévy of the Hebrew version of Ibn al-Samḥ's fragment on the cylinder and its plane sections, as well as the edition

of passages from Ibn Hūd's *Mathematical Encyclopedia* concerning his measurement of the parabola and his treatment of the isoperimetric problem. The richness and breadth of the texts, most of them published for the first time, is admirable. It should be clear to anyone reading this list that this volume gives a broad overview of a prolific and coordinated activity involving a number of mathematicians with related interests.

Volume II. The subtitle of this volume is *Ibn al-Haytham*. It constitutes a comprehensive exploration of infinitesimal mathematics of the XIth century through the legacy of this author, with a critical edition of nine of his papers, eight of which are translated and published for the first time. The papers are classified in three groups:

(1) The quadratures of lunes and circles, including Ibn al-Haytham's *Treatise on the lunes*, the *Quadrature of the circle* and the *Exhaustive treatise on the figures of the lunes*.

(2) Volumes of paraboloids and of the sphere, and the method of exhaustion, including Ibn al-Haytham's *On the measure of the paraboloid*, *On the measure of the sphere* and *On the division of two different quantities mentioned in Euclid's Elements X,1*.

(3) Isoperimetric problems, isoeperimetric problems, and the theory of the solid angle, including his *On the sphere which is the largest of the solid figures having equal surface areas and on the circle which is the largest of the plane figures having equal perimeters*, *On extraction of the square root* and *On the extraction of the cubic root*.

These works of Ibn al-Haytham are amazingly interesting, in terms of level of difficulty of the problems addressed and the novelty of the ideas involved. The treatises on root extraction contain an algorithm which Ibn al-Haytham tries to justify, differing in this respect from his contemporaries; the algorithm leads to that of Ruffini-Horner. Relations with the work of Euler are highlighted by Rashed. Volume II also contains a commentary on a list of 96 treatises attributed to Ibn al-Haytham. Another important contribution of Rashed in this volume is also the sorting out of several confusions around the authorship of Ibn al-Haytham. The works presented show that Ibn al-Haytham is a worthy successor of the Greek geometers (Archimedes, Hypocrates of Chios, etc.), but we also learn that he spent a substantial amount of his time *copying* mathematical texts (once a year he used to copy Euclid's *Elements*), showing the importance he associated to transmitting these texts to future generations.

Volume III. The subtitle of this volume is *Ibn al-Haytham. Théorie des coniques, constructions géométriques et géométrie pratique*. The texts edited are mostly by Ibn al-Haytham. The first one is his *Completion of Apollonius's Conics*. (The reader may recall that the VIIIth book of Apollonius *Conics* (third century BCE) was already lost in the IVth century.) Ibn al-Haytham tried to complete it by writing the present treatise. This is not the first time this text is edited, but most interesting is Rashed's discussion on the meaning and goal of Ibn al-Haytham's completion. The discussion is based on a profound knowledge of the totality of Ibn al-Haytham's extant works

as well as all the existing texts of Apollonius' works. This makes this edition of Ibn al-Haytham's treatise a definitive one. Another text by Ibn al-Haytham, edited in this book, is a correction of a lemma by Banū Mūsā on the *Conics*. We thus arrive at one of the recurrent themes of Rashed, that the work on the conics is closely related to infinitesimal mathematics. The "geometric constructions" in the title of this volume are also related to the conics. These constructions were developed by the Greeks to solve problems on solid geometry: points are found as intersections of conics, etc. Again, Rashed's introduction and his mathematical commentary on this subject are extremely interesting, and they place all the previously existing editions on the subject in a coherent context. The volume contains editions of several texts on problems of geometrical constructions. The last chapter concerns "practical geometry" or "the art of measuring," where three other treatises of Ibn al-Haytham are edited, translated and commented.

All these texts, together with Ibn al-Haytham's comments and Rashed's own comments, have strong connections with the first two Volumes of Rashed's series, whose aim is the restitution of the Arabic Archimedean and infinitesimal tradition.

The volume also contains an appendix with a collection of 15 exceptional texts on the construction of the regular heptagon by Archimedes, by Thābit ibn Qurra, Abū al-Jūd, al-Sijzī, al-Qūhī, al-Ṣāghānī, al-Shannī, Naṣr ibn 'Abd Allāh, Ibn Yūnus and an anonymous author. A second appendix contains texts of Sinān ibn al-Faṭḥ's on *Optical measurements*. The relation of optics with infinitesimal mathematics is through the conics. This is also an original idea of Rashed.

Volume IV. The subtitle of this volume is *Ibn al-Haytham. Méthodes géométriques, transformations ponctuelles et philosophie des mathématiques*. It contains critical editions, translations, and an extensive commentary of a series of treatises of Ibn al-Haytham; among them is his *Properties of circles*, a study based on the theory on similarity properties. According to Rashed, this is the first time in the history of mathematics where homothety is treated as a point transformation. The claim is reasonable. It is based on a theorem contained in this manuscript, asserting that lines drawn through the point of contact of two circles map figures on one of the circles onto similar figures of the other circle. The idea of similarity (although the word is not used) is inherent in this statement and in its proof.

The volume contains the critical edition and analysis of several other works of Ibn al-Haytham, including his *Analysis and Synthesis* as a mathematical method of invention, which deals with geometry as well as with the theory of numbers, equations, astronomy, and music. In this treatise, Ibn al-Haytham deals with the use of analysis (in the ancient Greek sense of this word) in geometry, arithmetic, and astronomy, with examples.

Another important text edited by Rashed in Volume IV is Ibn al-Haytham's *Knowns*, which is a complement to Euclid's *Data* when rigid motions are accepted in geometry.

In this treatise, Ibn al-Haytham begins with a lengthy philosophical discussion of the concept “known,” which according to him has a wider meaning than “constructible” by Euclidean or other means. He then presents 13 theorems on loci and he concludes with about 30 propositions that are in the style of the Euclid’s *Data*. Other treatises dealing with geometrical constructions are also edited: the construction of a triangle whose base, circumference and area are given; of a triangle in which the sum of the distances of any point in the interior to the three sides is given, etc. These problems in Euclidean geometry might seem easy but in fact they are not. Euler wrote several papers dealing with similar problems.

Volume IV also contains a critical edition of the treatise *On Space* (or *On Place*) by Ibn al-Haytham, where this author explains his opposition to Aristotle’s view expressed in his *Physics*. Ibn al-Haytham defines place in a mathematical way as the imagined space filled by a body, and he argues against the Aristotelian definition of place as the surrounding surface.

The last part of Volume IV is an appendix giving texts by Thābit ibn Qurra and al-Sijzī on Analysis as a method of geometric invention, by al-Sijzī, and earlier translations and/or compilations of Greek authors, followed by texts by Ibn Hūd which show that Ibn al-Haytham’s work was probably known in the West already during his lifetime, and by the edition of the critique of an Aristotelian philosopher, al-Baghdādī, of Ibn al-Haytham’s work on space, as well as a positive note of the philosopher and theologian al-Rāzī on the same topic.

Again, one needs the stature of Rashed to be able to make the connections between the various topics and to make out of these manuscripts an organized and consistent volume.

Volume V. The subtitle of this volume is *Ibn al-Haytham. Astronomie, géométrie sphérique et trigonométrie*. The volume contains important works by Ibn al-Haytham on geometry and astronomy which are published for the first time. The most important by its scope and content is his *Configuration of the motions of the seven wandering stars*. The “stars” that are referred to are the sun, the moon, and the five known planets. The treatise was intended to be the sum of its author’s knowledge on astronomy, and its scope is comparable to the famous *Book on optics* by the same author, which we shall mention below. The *Configuration* was originally in three books, but only the first one, whose subject is mathematical astronomy and which contains the author’s planetary theory, survives. This book has two parts: the mathematical propositions and the planetary theory. The first part consists of a set of propositions on plane and spherical geometry, written in the style of Theodosius’ *Spherics* and Ptolemy’s *Almagest*, but often going beyond them in terms of depth and difficulty. Some of these propositions provide inequalities for ratios of distances between points on a great circle on the sphere and the lengths of the altitudes when these points are projected on a great circle, in terms of the angle it makes with the first one. The propositions are

of increasing complexity. Although they are aimed for their use in astronomy, the results are also interesting from the purely mathematical point of view. The proofs are geometric. Such proofs, without the help of modern differential calculus, require ingenious constructions. The second part of the book, with its deep mathematical basis, is a study of the apparent motion of the planets (including the sun and the moon) from the point of view of geometric kinematics, freed from any cosmological and metaphysical considerations. Indeed, Ibn al-Haytham is not interested in the *causes* of these motions, but only in their mathematical description, in space and time. In this sense, our author is an eye-opener towards modern science. Ibn al-Haytham also differentiates himself from Ptolemy by rejecting some of his technical hypotheses, concerning the uniform rotation of the sphere about axes which are not its diameter, which contradict the observations that he collected. For this reason, he proposed a system which replaces Ptolemy's celestial circles by other configurations which avoid Ptolemy's contradictions and are at the basis of a completely novel geometric celestial kinematics. Ibn al-Haytham's description of the motion of a planet culminates in his proof that during its daily motion the height of a planet above the horizon reaches exactly one maximum value and exactly twice a minimum value. The other works of Ibn al-Haytham edited by Rashed in Volume V include *On the variety that appears in the heights of the wandering stars*, *On the hour lines*, *On horizontal sundials* and *On compasses for large circles*.

The quality of Rashed's edition of *Mathématiques infinitésimales* is extremely high. Rashed also provides a remarkable mathematical commentary and a faithful historical overview of the texts presented. Because of the richness of the Arabic texts presented, together with the extraordinary usefulness of the information, explanations and commentaries, these five volumes now occupy a key place in our mathematical and historical literature.

2. IBN AL-HAYTHAM

Rashed investigated extensively the work of Ibn al-Haytham. From his publications, mathematicians will agree on the fact that a large number of writings of Ibn al-Haytham are of a high level of difficulty. He often tries to solve open problems, either formulated by him or by his predecessors. In his commentaries on Euclid and Ptolemy, he is extremely critical; he points out weaknesses and mistakes.

We already talked about Ibn al-Haytham's work on infinitesimal or Archimedean mathematics, on geometrical constructions and on astronomy. We know that his work on astronomy includes twenty-five treatises, which constitute about one third of his works. Ibn al-Haytham wrote on other mathematical topics, including some major works on optics, and he also wrote on physics and philosophy of science. Rashed published a corpus of papers on these other works.

One of Ibn al-Haytham's works on arithmetic is considered in Rashed's paper *Ibn al-Haytham et les nombres parfaits* (Hist. Math. 16, 1989, no. 4, 343-352). It concerns Ibn al-Haytham's work on perfect numbers. Euclid proved that if $2^p - 1$ is a prime, then $2^{p-1}(2^p - 1)$ is a perfect number (Proposition 36 of Book IX of the *Elements*). Ibn al-Haytham claimed that the converse is true and he described his ideas for a proof. Descartes, in a letter to Mersenne, dated November 15, 1638, claimed that he had a proof of this converse under the additional condition that the perfect number is even. Proofs of these claims were eventually provided by Euler in 1747, in his paper *De numeris amicibilibus*. One cannot be completely sure of whether Ibn al-Haytham had a proof of his claim or not, but nevertheless, it is remarkable that a proof of the claim he made (even as a conjecture) was published about eight centuries after him.

Rashed's publications on Ibn al-Haytham also include *Ibn al-Haytham's construction of the regular heptagon* (J. Hist. Arabic Sci. 3, 1979, no. 2, 309-387), *Ibn al-Haytham and the measurement of the paraboloid* (J. Hist. Arabic Sci. 5, 1981, no. 1-2, 262-191), and *Ibn al-Haytham et le théorème de Wilson* (Arch. Hist. Exact Sci. 22, 1980, 305-321). This brings us to optics, one of the favourite fields of Ibn al-Haytham. Rashed published several books and papers which constitute an invaluable reference corpus on Ibn al-Haytham's work on optics. It is not possible to analyse his work here, but we mention his *Discours de la lumière d'Ibn al-Haytham* (Revue d'histoire des sciences et de leurs applications, 21, 1968, no. 3 197-224), which is his first published article, containing his critical translation of the text, as well as his *Optique géométrique et doctrine optique chez Ibn al-Haytham* (Arch. Hist. Exact Sci. 6, 1970, 271-298), and *Le modèle de la sphère transparente et l'explication de l'arc-en-ciel : Ibn al-Haytham, al-Fārisī* (Rev. Hist. Sci. Appl. 23, 1970, 109-140). The first reference includes a translation and a commentary of Ibn al-Haytham's *Treatise on light*, a work considered a supplement to his famous *Book on Optics*.⁷ It appears from Rashed's publications on Ibn al-Haytham that the latter reshaped the foundations of optics, as he did for astronomy.

On the philosophy of Ibn al-Haytham, we mention Rashed's *Philosophie des mathématiques d'Ibn al-Haytham. I. L'analyse et la synthèse. II. Les connus*. Mélanges 20, 1991, 31-231 and 21, 1993, 87-275.

Rashed, in his extensive historical research on Ibn al-Haytham, cleared out a confusion in the literature between two persons, our author, al-Ḥasan ibn al-Haytham, and another one, Muḥammad Ibn al-Haytham. The confusion started with an ancient author, Ibn Abī Uṣaybi'a. This is discussed at length in Volume II of *Mathématiques infinitésimales*. In particular, a treatise titled *On the configuration of the universe*

⁷The year 2015 has been declared the "year of light" by the UNESCO. One of the main reasons was to celebrate Ibn al-Haytham's *Book on optics* (*Kitāb al-Manāẓir*) that was written 1000 years before. An international conference on Ibn al-Haytham's work on optics took place at the UNESCO headquarters in Paris, on September 14-15, 2015. The title of "father of modern optics," which Ibn al-Haytham already carried, was confirmed during that conference.

written in the purely Ptolemaic style was traditionally attributed to al-Ḥasan ibn al-Haytham but in reality it belongs to Muḥammad Ibn al-Haytham.

Distinguishing between the two mathematicians not only corrects a historical error, but it also makes the works of Ibn al-Haytham more coherent.

3. OPTICS

In the Arab world, a long tradition of research in optics started in the IXth century, conducted by Qusṭā ibn Lūqā, al-Kindī and their successors. Since Greek antiquity, there were several competing theories on optics, among them intromission doctrine, described in Aristotle's *De Anima*, the emission doctrine supported by Euclid, Ptolemy and other geometers (and described by Aristotle in his *Meteorology*), and the Stoico-Galenic theory of pneuma. Optics, in the hand of the Arabs, who were the heirs of the Greeks, became a subject within both fields of geometry and physics. Qusṭā ibn Lūqā and al-Kindī were supporters of the so-called visual ray theory, that is, vision is the result of a radiant power emitted from the eye, acquiring physical reality and describing straight lines in the air. They were most critical of the emission doctrine of Euclid and Ptolemy because it was not compatible with the laws of perspective. The field of optics attained a high degree of maturity in the XIth century, thanks to the work of Ibn al-Haytham, with Ibn Sahl's dioptrical research, and with Kamāl al-Dīn al-Fārisī's quantitative research. These works superseded everything that was done in the West on the subject, where nothing equivalent was discovered until the epoch of Kepler.

In 1993, Rashed published an important book, in French, on optics, *Géométrie et Dioptrique au Xe Siècle : Ibn Sahl, al-Qūhī et Ibn al-Haytham*, (Paris, Les Belles Lettres), then, in 2005, a revised version in English, *Geometry and Dioptrics in Classical Islam* (London, Al-Furqān Islamic Heritage Foundation). The book constitutes a new milestone in the history of geometrical optics, in particular dioptrics, and it is closely related to the geometry of conics. It comes after a series of articles on the subject. Faithful to his way of always improving and expanding his work, the number of pages of Rashed's English version (1176 pages) is almost the double of that of the French one. The result is a breakthrough in the history of geometrical optics, especially that of dioptrics. The work also deals with the mathematical theory of conics. Indeed, it is one of Rashed's mottos that optics and the geometry of conics, for what concerns Greek and Arabic mathematics, cannot be separated. In this book, Rashed gives a broad overview of the subject, starting with the works of the founders, including Thābit ibn Qurra, al-Sijzī and others. With Ibn Sahl and his *Treatise on Burning Instruments* (986) a new series of questions arose: to understand the way light is refracted in the air and traverses instruments, setting fire with mirrors and lenses, with a light source at finite or infinite distance (that is, the rays that reach the instrument either issue from a point or are parallel). Ibn Sahl also completely reformulated the problem of burning

mirrors (devices that cause burning by focusing the rays of the sun) and lenses. His work, *On burning instruments*, is naturally divided into four parts, according to the various ways of setting fire : (1) by reflection with parallel rays (parabolic mirror); (2) by reflection with rays issuing from a single point (ellipsoidal mirror); (3) by refraction with parallel rays (plane-convex lens); (4) by refraction with rays issuing from a single point (bi-convex lens). Knowing this, it is not surprising that this research heavily relies on the geometry of conic sections. It is at that epoch that dioptrics was born as a new field. We can find in Ibn Sahl's work the so-called Snell law on the refraction of light.⁸ Rashed writes (pp. 62-63): "The presence of this formula in Ibn Sahl's work in the Xth century not only overturns our image of history, it also leads us to formulate differently the problem of the successive rediscoveries of this law. In other words, to the names of Snell, Harriot, and Descartes we must henceforth add that of Ibn Sahl." The experimental research of Ibn al-Haytham and al-Fārisī is then presented and contrasted with the previous theoretical researches. Rashed's book contains critical editions and translations of Ibn Sahl's *Treatise on Burning Instruments and proof that the celestial sphere is not of extreme transparency*, of the extremely important *Seventh Book of Optics* of Ibn al-Haytham and his *Treatise on the Burning Sphere*, with the redaction by al-Fārisī. The book also contains a chapter titled *On Conic Sections and their Applications*. It concerns the work done by the Arabs between the IXth and the XIIth centuries on geometrical constructions in the tradition of Euclid and Apollonius using conic sections. Rashed describes these scholars as "Hellenistic Arab mathematicians" (p. 295). This chapter is followed by another one, titled: *A Tradition of Research: Continuous Drawing of Conic Curves and the Perfect Compass*, where the mechanical applications are highlighted. In this work on conics, a foundational issue is discussed, concerning "measurable" and "non-measurable" curves, that is, curves which are subject or not to the theory of proportions. Rashed relates this with Descartes' and Leibniz's classification of curves into geometrical, mechanical, algebraic, and transcendent.⁹ Again, a collection of extremely interesting manuscripts on this subject are edited.

⁸Snell's refraction law (also known as the Snell-Descartes law) describes the change in direction of a light ray at the interface between two different media by the formula $\frac{\sin i}{\sin r} = \text{constant}$. The law is attributed to the Dutch astronomer Willebrord Snellius (1580–1626). Rashed had already pointed out in a paper published in 1990 that Ibn Sahl formulated accurately this law in the Xth century; cf. R. Rashed, "A pioneer in anacastics: Ibn Sahl on burning mirrors and lenses," *Isis* 81, 1990, no. 3, 461-491.

⁹It is important to note that this subject of mechanical curves was also at the heart of some XXth century mathematics; cf. Thurston's conjecture on the construction of curves by mechanical linkages, solved by Mnev.

Talking about conics, and for the sake of being more extensive, it is natural to discuss projections, because the subjects are closely related and because the same mathematicians (al-Qūhī, Ibn Sahl, etc.) worked on both subjects. This is why Rashed's book includes a chapter titled *Conical and Cylindrical Projections, and Astrolabes*.

The profound feeling which stems from Rashed's work on this book is again that the so-called scientific revolution that took place in seventeenth-century Europe has to be re-evaluated in terms of the Middle-Ages contribution of the Arabs.

One has also to mention another book on optics published by Rashed in 1997, *L'optique et la catoptrique d'al-Kindī* (Brill, Leiden, 790 p.), which is the first volume of an edition of the collected works of al-Kindī (Œuvres philosophiques et scientifiques d'al-Kindī, vol. I). This volume contains the complete works of al-Kindī on optics. We mention the titles of these works, because they indicate the diversity of the subjects discussed: *The rectification of the errors and the difficulties due to Euclid in his book called "The Optics"; On the solar rays; On the magnitudes of figures immersed in water; Fragment on a concave mirror whose arc is the third of its circle; On the causes of the diversities of perspective and on the geometric demonstrations that we ought to give them.* The book also contains the *editio princeps* of two texts on burning mirrors and catoptrics by Qustā ibn Lūqā and Ibn 'Īsā, written after the works of al-Kindī on optics.

There are other works of Rashed on the history of optics, and we shall consider them below in the section devoted to the *Conics*.

4. THE CONICS

Apollonius' *Conics* (3rd-2nd c. BCE) is one of the three or four greatest treatises of Greek mathematics that have reached us. It was originally in eight books, but the last one had already disappeared at the epoch of Pappus (IVth century A.D.). We already mentioned that Ibn al-Haytham tried to reconstruct it, and we talked about Rashed's edition on this subject. We mentioned the *Conics* in relation to optics, and we have noted how much Rashed emphasizes the fact that the two fields cannot be separated. We shall present briefly Rashed's comprehensive edition of this major work by Apollonius and we shall explain the importance of this edition. Let us note of the side consequences which is of major importance of this work on the *Conics*, even if it may be considered as a side result. This edition gives an example of the fact that Arabic translations that reached us of a Greek mathematical work can be much superior to Greek manuscripts of the same work which also survived and which are anterior to the Arabic ones. This fact appears clearly from Rashed's historical analysis contained in his edition and in the articles he published on this subject. The main reason is that the Greek manuscripts of Books I to IV of Apollonius' *Conics*, dating back to the Byzantine mathematician Eutocius of Ascalon¹⁰ (VIth century) is

¹⁰Eutocius was one of the last heads of the Neoplatonic School of Athens, which was founded around the year 400 A.D. and which lasted until the VIth century. One of the famous teachers in that school

a revision in which, according to his own account, he “simplified” the text by omitting the proofs of the difficult propositions. The reason is that Eutocius wanted the work to be more accessible to his students. The Arabic translation which survives of the same books and which is obviously posterior to Eutocius’ version, is much closer to the Greek original and is based on manuscripts which are older than the ones which Eutocius used. It permits the reconstruction of several propositions and proofs that are missing in Eutocius’ edition.

Eutocius’ edition of Books I to IV was edited several times and translated into Latin since the Renaissance. The well known editions are those of Commandino (Bologna, 1566), Halley (Oxford, 1710) and Heiberg (Leipzig, 1891–1893). Books V to VII, which are the most interesting from the mathematical point of view (because of the difficulty of the propositions they contain), survive only in the Arabic translation we mentioned. This translation was done in Baghdad in the IXth century, under the supervision of the Banū Mūsā. The translation comprises the totality of the first seven books. One remarkable aspect of the history of the edition of the *Conics* is that the various editors of the first four books did not find it useful to translate the Arabic versions, considering, like an axiom, that the Greek version is more valuable than the Arabic.

Rashed’s critical edition of the Arabic manuscripts, published by de Gruyter in 5 volumes (more than 2500 pages),¹¹ is the result of 20 years of hard work, searching the world for all the extant manuscripts, collecting them, working on the translation and the critical edition, and writing the history and the mathematical commentary. The result is a definitive edition. Rashed’s commentary is based on a thorough analysis of Eutocius’ edition and the Arabic one and the differences between them. The differences concern the number of propositions, their content, the proofs and the figures. The Arabic version exists in seven different manuscripts, among which four contain the totality of Books I to VII. One of the manuscripts, preserved at Istanbul, is copied by the hand of Ibn al-Haytham (XIth century). In his edition, Rashed also discusses epistemological and philosophical questions that arise from the text of the *Conics*. They

was the philosopher and mathematician Proclus, who taught there between the years 438 and 485. The school was closed in 529, together with other philosophical schools, after Christianity became the official religion of the Roman Empire. Besides his edition of Books I to IV of Apollonius *Conics*, Eutocius wrote a *Commentary* on these books, and commentaries on three works of Archimedes: *On the sphere and the cylinder*, *On the quadrature of the circle*, and the *Two books on equilibrium*.

¹¹Apollonius : Les Coniques, tome 1.1 : Livre I, commentaire historique et mathématique, édition et traduction du texte arabe, de Gruyter, 2008, 666 p.; Apollonius : Les Coniques, tome 2.2 : Livre IV, commentaire historique et mathématique, édition et traduction du texte arabe, de Gruyter, 2009, 319 p.; Apollonius : Les Coniques, tome 3 : Livre V, commentaire historique et mathématique, édition et traduction du texte arabe, de Gruyter, 2008, 550 p.; Apollonius : Les Coniques, tome 4 : Livres VI et VII, commentaire historique et mathématique, édition et traduction du texte arabe, Scientia Graeco-Arabica, vol. 1.4, de Gruyter, 2009, 572 p.; Apollonius : Les Coniques, tome 2.1 : Livres II et III, commentaire historique et mathématique, édition et traduction du texte arabe, de Gruyter, 2010, 682 p.

concern the notion of “equality” and “similarity” of conic sections, of “transformation” and the (multiple) use of the notion of motion. It is to be noted that all these notions were thoroughly discussed by the later Arabic mathematicians, and Rashed bases his discussion on a profound knowledge of these manuscripts. The fact that the published Arabic manuscripts are translations of Greek texts that are prior to the existing Greek editions on which the previous Western translations are based, makes Rashed’s edition much more faithful to the initial text of Apollonius, even with regard to Books I to IV which were already known. It provides us a better understanding of the original work of Apollonius.

In conclusion, we have, thanks to the conscientious work of Rashed, a new, comprehensive, and coherent text of the *Conics*. His historical commentary includes, besides the comparison between the Greek and the Arabic texts of the first books, an analysis and an explanation of the divergence of the various sources. The result constitutes one of the works of Rashed which transformed the subject on which he devoted years of study and research.

Besides the edition of Apollonius’ *Conics*, Rashed published a rich collection of articles and books related to this subject. To begin with, one may mention his publication with H el ene Bellosta of the critical edition of Apollonius’ book on the cutting of a ratio, published in 2009.¹² It contains a whole theory of analysis-synthesis. This work is in the background of the *Conics*. Rashed’s articles around the *Conics* include *Lire les anciens textes math ematiques : le cinqui eme livre des Coniques d’Apollonius*, *Bollettino di storia delle scienze matematiche*, vol. XXVII, fasc. 2, 2007, 265-288, *Qu’est-ce que les Coniques d’Apollonius ?*, in *Les Courbes :  tudes sur l’histoire d’un concept*, a book by Rashed and Crozet, Paris, Blanchard, 2013, *L’asymptote : Apollonius et ses lecteurs*, *Bollettino di storia delle scienze matematiche*, vol. XXX, fasc. 2, 2010, 223-254, *Al-Sijz i et Ma monide: Commentaire math ematique et philosophique de la proposition II-14 des coniques d’Apollonius*, *Arch. Int. Hist. Sci.* 37, 1987, no.119, 263-315, and there are others. The last article mentioned is most important, because it highlights the strong relationship between mathematics and philosophy in Arabic science. The proposition in question says that the asymptote and the hyperbola approach each other indefinitely, the distance between them becoming smaller than any given segment, without ever meeting. From the purely mathematical point of view, the implications of this statement are enormous, and there are relations with the foundations of hyperbolic geometry. The relation between the work of al-Sijz i and that of the Jewish Medieval scholar Maimonides is also clearly established in Rashed’s article. One should also mention his article *Le concept de tangente dans les coniques d’Apollonius*, in *Kosmos und Zahl. Beitr age zur Mathematik und Astronomiegeschichte, zu Alexander von Humboldt und Leibniz*, Berlin, 2008, 361-371. It appears from all these works

¹²Apollonius de Perge, La section des droites selon des rapports, commentaire historique et math ematique,  dition et traduction du texte arabe par Roshdi Rashed et H el ene Bellosta, *Scientia Graeco-Arabica*, vol. 2, Berlin / New York: de Gruyter, 2009, 493 p.

that Apollonius was the first who studied tangents (or support lines of convex arcs) for a whole class of curves, the conics, using harmonic division as a main tool.

We emphasize the fact that Rashed's work on the *Conics* is inseparable from some his other works. One idea that clearly manifests itself in his mathematical commentaries is that without understanding the conics, it is not possible to fully understand the rest of Greek mathematics, including geometry, algebra and optics. In fact, Rashed himself arrived at the conics through his work on algebra and optics. He highlighted the relations between the conics and algebra in his book *Les catoptriciens grecs. I. Les miroirs ardents*, Collection des Universités de France, Les Belles Lettres Paris, 450 p. (2000). This book contains critical editions of the Arabic translations of almost all the Greek texts on burning mirrors. These texts include in particular the treatises on elliptical and parabolic burning mirrors by Diocles,¹³ by an unknown Greek mathematician with the arabicized name of Dtrūms, and by the Byzantine scholar (VIth century) Anthemius of Tralles.¹⁴ The second treatise, which is lost in Greek, starts with a detailed study of propositions of Apollonius' *Conics* and of others which are relevant to the foci of parabolas and paraboloids, together with ruler and compass constructions. There are close relations between catoptrics and the mathematical theory of conics.

The relation between conics and optics is also the subject of Rashed's article *Les constructions géométriques entre géométrie et algèbre: l'Épître d'Abū al-Jūd à al-Bīrūnī*, *Arabic Sciences and Philosophy*, 20, 2001, no. 1, 1-51. Abū al-Jūd is a mathematician of the Xth century who contributed most to the development of geometric constructions of the problems of solids and super-solids, and at the same time to the question of solving cubic and bi-quadratic equations with the aid of conics. His work is significant in terms of the relations it established between algebra and geometry.

Another important application of conics is number theory (the use of geometry in the solution of polynomial equation), and it is addressed in Rashed's book *Œuvre mathématique d'al-Sijzī*. Vol. I. Géométrie des coniques et théorie des nombres au Xe siècle (Peeters, Leuven, 2004. 541 p.). This book is also the occasion to present the work of the Persian mathematician al-Sijzī, a prolific geometer (with more than 50 treatises to his credit) who is an excellent representative of the Xth century. Rashed, together with his former student Pascal Crozet, devoted more than 15 years to rehabilitate his forgotten work, in another volume. His work on conics is also analyzed

¹³Diocles was a mathematician who was the first to prove the focus-directrix property of the parabola. He was a contemporary of Apollonius and in his work on the burning mirrors he uses several properties contained in the *Conics*.

¹⁴Anthemius of Tralles was a mathematician, engineer and architect. He is the inventor a large number of machines. Together with Isidore of Miletus, he was responsible for the reconstruction of Agia Sofia under Justinian Ist after the earlier church at the same location burnt.

in the paper *Al-Qūhī et al-Sijzī : Sur le compas parfait et le tracé continu des sections coniques*, Arabic Sci. Philos. 13, 2003, 9-43, which presents his completely new approach to the problem of the classification of curves.

One also has to mention Rashed's paper with Neugebauer, *Sur une construction du miroir parabolique par Abū al-Wafā' al-Būzjānī*, Arabic Science and Philosophy 9, 1999, no.2, 261-277, containing the critical edition of al-Būzjānī's paper where he explains a method of construction of a parabolic mirror.

In conclusion, the work of Rashed on the *Conics* consists of

- his remarkable edition of Apollonius' *Conics*;
- his edition of the works of the Arabs on conics;
- his research leading to the strong relation between conics and algebra, geometry, optics and astronomy.

5. DIOPHANTUS' *Arithmetica*

At the same time as the *Conics*, Rashed worked on another major Greek work, Diophantus' *Arithmetica*, a book on number theory, which had a tremendous influence on Fermat, Euler, and on all modern and contemporary mathematics. It gave rise to Diophantine analysis, a field which was thoroughly developed by the Arabs, and reconstructing this development is one of Rashed's major contributions to the field, in addition to his critical edition of the texts.

Diophantus' *Arithmetica* consisted originally of thirteen books. Only six of them survived in Greek, and they were edited and translated into Latin, starting from the XVIth century. The other books that reached us are in Arabic.

Rashed dedicated several decades of hard work on this subject. His first book was published in 1975, and the last one in 2013. The Arabic manuscripts he discovered and published constitute an invaluable source for the reconstruction of Diophantus' original work. He learned first the existence of four books that were lost in Greek, from a manuscript of al-Karajī who made a summary of them. Thus, instead of six, we have now ten books of the *Arithmetica*. As a matter of fact, there were probably seven books in Arabic, and not only four.

There are still open historical questions and conjectures in this domain. Rashed's publications on this subject include several books: *The art of algebra in Diophantus* (Arabic), Cairo, National Library, 253 p. (1975) ; *Diophante : Les Arithmétiques, Livre IV, vol. 3*, Collection des Universités de France, Paris, Les Belles Lettres, 487 p. (1984); *Diophante : Les Arithmétiques, Livres V, VI, VII, vol 4*, Collection des Universités de France, Paris, Les Belles Lettres, 451 p. (1984); and *Les Arithmétiques de Diophante : Lecture historique et mathématique* (with Christian Houzel), de Gruyter, 629 p. (2013). Besides these books, one has to mention Rashed's *Abū Kāmil: Algèbre et analyse diophantienne*, de Gruyter, 818 p. (2012), and his *Histoire de l'analyse diophantienne classique : D'Abū Kāmil à Fermat*, de Gruyter 349 p. (2013). The first book contains

the *editio princeps* and the translation of the book on algebra of Abū Kāmil and of another book he wrote, *On the volatiles*. Abū Kāmil is an Egyptian mathematician, from the IXth-Xth century, who worked in the lineage of al-Khwārizmi, the founder of the field of algebra. In the preamble of his book, *Kitāb al-jabr wa-al-muqābala*, Abū Kāmil writes that his aim is to comment on the book of al-Khwārizmi that carries the same title. He emphasizes the fact that his famous predecessor not only invented the field of algebra, but also provided a model for a treatise on this subject, containing, in this order, chapters on equations, algebraic calculus, study of problems that can be solved using equations, solutions of geometric problems using algebra, etc. In fact, Abū Kāmil writes a book in the footsteps of his predecessor, but in which several ideas, methods and proofs are further developed and made more explicit, and several algorithms for equation solving are provided with proofs. Part of the program consists in providing the theory of equations with a geometric solid basis. Furthermore, in applying the techniques of algebra to indeterminate problems, Abū Kāmil sets the bases of a new field which became known as rational indeterminate analysis, also known nowadays as rational Diophantine analysis. As Rashed formulates it, Abū Kāmil paved the way to another field, that of “arithmetization of algebra,” which was born almost one century after him, by his successor, al-Karajī. The latter, as well as his own successor, al Samaw’al, and much later, Fibonacci, borrowed a great number of problems to Abū Kāmil.

This brings us to the second book, *Histoire de l’analyse diophantienne classique : D’Abū Kāmil à Fermat*. The subject is Diophantine analysis and its development by the Arabs and the later Western mathematicians. Rashed succeeds, in this book, to recover the Arabic tradition of research in Diophantine analysis. The first part concerns the works of Abū Kāmil, Al-Karajī, Al-Khāzin, Al-Sijzī, Abū al-Jūd, Ibn al-Haytham and Ibn al-Khawwām. These mathematicians addressed new problems on integer solutions of equations that were motivated by their reading of Diophantus. They made lists of all these problems, some of which remained open for centuries. These include, for example, the lists made by Ibn al-Khawwām that crossed the centuries. Kamāl al-Dīn al-Fārisī (died in 1319) reproduced some of its problems in his treatise on algebra. The Arabs worked on the classification of equations, according to the number of solutions (finite or infinite), they conceived the notion of impossible problems (the question of the non-existence of a solution). We also find there the first attempts to prove the unsolvability of the so-called Fermat equation $x^4 + y^4 = z^2$. In relation with Diophantine analysis and especially the research problems proposed by al-Khawwām and other Arabic mathematicians, Rashed insisted on seeing there the notion of “impossible and inaccessible Diophantine problems,” a notion which posed tremendous logical and philosophical questions.

The field of integer Diophantine analysis was already on a firm basis by in the first half of the Xth century, thanks to the works of mathematicians like al-Khujandī and al-Khāzin. We also learn from Rashed that the edification of this field of Diophantine

analysis was necessitated by the development of algebra since al-Khwārizmī, around the year 830.

The second part of the book concerns the Western developments, by Fibonacci, Rafael Bombelli, Simon Stevin, François Viète, Bachet de Méziriac and Fermat. Describing, in his commentaries, the modern developments, using modern terms, in this book and in others, is one of Rashed's invaluable achievements which contributes making history a living science. It is important to note that this does not lead to any anachronism, because the language used in the mathematical commentary is independent and does not affect the precise archaic language which Rashed uses in his translations of the ancient texts.

On the same subject, Rashed also published the articles *Les travaux perdus de Diophante, I*, *Revue d'Histoire des Sciences*, 27, 1974, no. 1, 97-122, *Les travaux perdus de Diophante, II*, *Revue d'Histoire des Sciences*, 28, 1975, no. 1, 3-30 and *Notes sur la version arabe des trois premiers livres des Arithmétiques de Diophante, et sur le problème 1.39*, *Historia Scientiarum*, 4, 1994, no. 1, 39-46.

The work by Rashed on Diophantus' *Arithmetica* and Diophantine analysis in the Arab world are impressive new testimonies of the genius, at the same time of Diophantus, and of the Arabic mathematicians.

6. ANGLES

The book *Angles et grandeur : d'Euclide à Kamāl al-Dīn al-Fārisī* (de Gruyter, 2015), is a recent achievement by Rashed. This is a fundamental work, 706 pages long, extremely rich in historical, mathematical and philosophical content. One can admire the seek for clarity and precision in the mathematical language. Efforts were made in that direction by the various authors quoted. The subject is *Angle as a magnitude*, based on Arabic manuscripts dating from the IXth century until the XIVth. The questions addressed are at the heart of mathematics and they had tremendous repercussions in the later philosophic-mathematical literature and a real impact in the development of geometry. The book contains critical editions and analyses of Greek writings translated into Arabic on the one hand, and original contributions of the Arabic mathematicians on the other hand. The sum of all these texts presented, most of them published here for the first time, together with the commentaries by Rashed, constitute a major contribution to our mathematical, historical and philosophical literature. The fascinating comments by Rashed show that the questions discussed are among the most fundamental for what concerns classical Greek mathematics and its continuation by the Arabs. Some of the authors whose texts are presented are well known in the later Latin world: Ibn al-Haytham (Alhazen), Ibn Sīnā (Avicenna), Naṣīr al-Dīn al-Ṭūsī, but there are also others: al-Nayrīzī, al-Anṭākī, Ibn Hūd, al-Sijzī, al-Samaw'al, al-Fārisī, al-Qūshjī, al-Abharī and al-Shīrāzī. All of them were valuable mathematicians.

It is in trying to resolve difficulties that appear in the *Elements* that Ibn al-Haytham wrote his two famous treatises, the *Explanation of the postulates of the book of Euclid* and the *Book on the solution of doubts relative to the book of Euclid on the Elements and the explanation of its notions*. In these works, Ibn al-Haytham created a new geometry where the notions of angle and of superposition are primitive elements. Ibn al-Haytham applied his theory of infinitesimals to the setting of angles, in particular for the comparison between a contact angle and a rectilinear angle, two “magnitudes” that do not satisfy proposition X.1 of the *Elements*. For this comparison, Ibn al-Haytham introduced two sequences, an increasing one and a decreasing one bounding the first one from below and from above (cf. p. 101 of *Angles et grandeur*). Ibn al-Haytham’s research on angle is also part of his contribution on isoperimetry and isepiphany, and it is a continuation and an outcome of the work of the Arabic mathematicians of the two centuries that preceded him. The work of Ibn al-Haytham was continued by several Arabic mathematicians, including Naṣīr al-Dīn al-Ṭūsī in his commentary on Archimedes’ *Sphere and cylinder* in which he addresses the question of the comparability of lines and curves, and of curvilinear angles. Naṣīr al-Dīn used in particular a notion of “rolling onto each other,” while he compared lengths of curves (cf. p. 469ff.). Rashed’s book also contains a critical edition of a text by the algebraist al-Samaw’al ibn Yaḥyā al-Maghribī (d. 1175) titled *Epistle on the angle of contact*, in which this author gives an explanation of the non-homogeneity and the non-comparability of figures, based on the example of the angle of contact. Together with Kamāl al-Dīn al-Fārisī (d. 1319) al-Samaw’al is one of the successors of Ibn al-Haytham who continued his research on the angle of contact.

In Rashed’s book the philosophical character of the research on angle by Ibn al-Haytham, Ibn Sīnā (980–1037), the well-known physician and philosopher known in the Western world under the Latinized name Avicenna, and Abū Sahl al-Masīḥī, an erudite who was familiar with Greek science and literature, is represented in a masterly way.

The book also contains a chapter on solid angles and their comparison. Al Sijzī considered solid angles that are not bounded by planes. In the XIth century, Ibn al-Haytham developed the theory of solid angles in his research on isoperimetry and equal surface areas. He was motivated by the problem of approximating the volume of the sphere by volumes of convex polyhedra, in his infinitesimal approach to the sphere. In his works, solid angles are subject to the usual operations that apply to Archimedean magnitudes and to the theory of proportions. He used not only the work of Archimedes on the sphere, but also conical projections, and spherical geometry. One may recall here that in the Western world and after the Hellenistic period, research on solid angles started only after the XVIIth century, in works of Descartes, followed by Euler, de Gua, Legendre, Cauchy and others.

The general question is how to compare magnitudes, taking into account the Aristotelean injunction of involving motion. The author reviews questions related to equality, superposition, congruence, similarity, etc. in the works of Euclid, Apollonius, Proclus and their Arabic successors. This chapter contains critical editions of fragments of the redaction of Archimedes' book *On the sphere and on the cylinder* by Naṣīr al-Dīn al-Ṭūsī and a *Treatise on rolling motion and on the relation between the rectilinear and the curve* by Al-Shīrāzī (second part of the XIIIth century).

7. MENELAUS' *Spherics*

More recently, Rashed, together in collaboration with Athanase Papadopoulos, They published a critical edition of Menelaus' *Spherics* from the Arabic manuscripts, with English translation and historical and mathematical commentaries: *Menelaus' Spherics: Early Translation and al-Māhānī/ al-Harawī's Version* De Gruyter, 2017, 877 p.

From the purely mathematical point of view, Menelaus' *Spherics* is the most important treatise on spherical geometry ever written. The author flourished in the 1st-2nd cent. A. D. The work is a systematic treatise on spherical geometry, comparable to Euclid's *Elements*, containing 91 propositions of increasing difficulty (some of them very difficult to prove). The methods used in the proofs of the propositions are constructive and amazingly intrinsic to the sphere, considering this surface in itself, without any reference to the ambient space.

From the historical point of view, this piece of work is highly representative of the contribution of the Arabs to Greek mathematics: The Greek manuscript is lost, and before it was lost, it was translated into Arabic. What survives today is a set of Arabic translations, commentaries and rectifications, by various Arabic authors, including leading mathematicians, starting from ninth century, including al-Māhānī, al-Harawī, Ibn 'Irāq, Naṣīr al-Dīn al-Ṭūsī, Ibn Hūd, Ibn al-Haytham, al-Bīrūnī, Ibn Abī Jarrāda and others.

Before completing the of this authoritative complete edition of the *Spherics*, Rashed and Papadopoulos published two papers on Menelaus' Proposition III.5 (in the numbering of Ibn 'Irāq ; that is, Proposition 71 in the version of al-Māhānī/ al-Harawī) : *On Menelaus' Spherics III.5 in Arabic mathematics, I and II: Arabic Science and Philosophy* vol. 24 (2014), pp. 1–68 and vol. 25 (2015), pp. 1–32. These two papers alone show the amount of work dedicated, along several centuries, to a single proposition by Menelaus, by some of the leading Arabic mathematicians.

8. SOME OTHER WORKS

The book *Classical mathematics from Al-Khwārizmī to Descartes* is another extensive (748 pages) book, published in 2015 (Routledge/Taylor & Francis), a new edition of a book in French which appeared in 2011 (Hermann, Paris). The period involved is

from 800 to 1650. Besides algebra, Rashed considers in this book combinatorics, number theory, geometry, astronomy, and optics, and the stress is on the logical relation and comparison between Western and Arabic mathematics. In particular, he shows with convincing arguments that the classification of curves, started by the Greeks (Plato, Pappus, Proclus, etc.) and leading to Descartes passes necessarily by the works of al-Sijzī and al-Khayyām.

On the development of algebra, one has to mention the book edited by Rashed and Vahabzadeh, *Al-Khayyām mathématicien*, Paris: Blanchard 428 p. (1999), a complete critical edition of the mathematical works of al-Khayyām, with extensive introductions, commentaries, and translations; this edition supersedes all the previous editions on this famous algebraist.

Rashed's contribution on the history of algebra includes his edition of works in the volume *Entre arithmétique et algèbre. Recherches sur l'histoire des mathématiques arabes* (Paris, Les Belles Lettres, (1984) 323 p.), a reprint of eleven articles on the history of algebra and number theory during the Arabic period.

One of the ideas of Rashed, expressed in his paper *Fermat and algebraic geometry*, *Historia Sci.* (2) 11, 2001, no. 1, 1-23, is that algebraic geometry in its primitive sense, the study of equations through the geometry of the curves they represent, may be traced back to the meeting of al-Khwārizmī's Algebra, Apollonius' Conics, and Euclid's Data. It is based on these three topics that 'Umar al-Khayyām (1048–1131) wrote his *Algebra*, containing the classification of the cubic equations and their solution using the intersecting conic sections. Half a century later, Sharaf al-Dīn al-Tūsī gave the discipline of algebra an analytical direction. This duality is studied and commented upon by Rashed and it is discussed in relation with the comparison between the approaches of Fermat and Descartes.

One should also mention the book *Thābit ibn Qurra: Science and Philosophy in Ninth-Century Baghdad*, edited by Rashed (de Gruyter, 2009) and the article by Rashed and Houzel on Thābit's theory of parallel lines (*Arabic Sciences and Philosophy*, vol. 15, 2005, 9-55). The latter contains a new commented edition of two pamphlets of Ibn Qurra on the theory of parallels, titled *If we draw two straight lines making angles smaller than two right, then they meet* and *On the proof of the famous postulate of Euclid*. Ibn Qurra, like all the great geometers from the period of Euclid and until the beginning of the nineteenth century, tried to prove that Euclid's fifth postulate (the one that concerns parallel lines) follows from the other postulates, and these two pamphlets are the result of his research on the subject. His "proof" in the first work is based on the assumption that if two non-intersecting lines diverge on one side, then they converge on the other side, and in the second work, it is based on the existence of equidistant lines. We know that the two assumptions are equivalent to the parallel postulate. The two pamphlets by Ibn Qurra contain interesting developments. The first pamphlet includes, for the first time, a proof of the fact that equidistance is a symmetric relation, that is, if all the points on a given straight line are at the same

distance from a second straight line, then all the points on the second straight line are at the same distance from the first straight line. In his investigation of equidistance, Ibn Qurra uses the notion of rectilinear translation. As a matter of fact, motion is used as a geometrical primitive element in the treatise of Thābit, and we also find there the use of Archimedes' axiom and the so-called Pasch axiom. We note that the notion of motion was already used as an essential element in the work of Thābit's teacher al-Ḥasan ibn Mūsā. We already mentioned that Thābit belonged to the IXth century Baghdad school of the Banū Mūsā, and he became, after their death, the head of the school they had founded. The second pamphlet of Thābit ibn Qurra contains an examination of the two classes of quadrilaterals, the trirectangular quadrilaterals that were later given the name *Ibn al-Haytham-Lambert quadrilaterals* and the isosceles birectangular quadrilaterals that were later given the name *Khayyām-Saccheri quadrilaterals*. Thābit ibn Qurra was prominent in the Archimedean tradition of infinitesimal geometry, in Euclidean geometry and Euclidean arithmetic, in spherical geometry, in statics, in astronomy, in philosophy, and in other fields. We know that Thābit ibn Qurra wrote between thirty and forty dissertations on astronomy, among which eight reached us. They were edited, translated and commented by Rashed's student Régis Morelon (*Thābit ibn Qurra : Œuvres d'astronomie*, Collection Sciences et philosophies arabes, Paris, Les Belles Lettres, 1987).

9. BY WAY OF A CONCLUSION

Roshdi Rashed is a historian of mathematics with an immense curiosity for all that concerns science, ancient and modern, and with a notable ability for grasping the importance, the subtlety and the complexity of the mathematical notions and results contained in the manuscripts he researches into. His work is colossal, in terms of volume, depth and variety. He possesses a broad knowledge in mathematics and history, an unusual ability to convey the important Arabic mathematical texts and to comment on them, combined with a deep insight in the foundations of mathematics and the interrelations between the different fields of science. His familiarity with Arabic manuscripts and all the ways of writing in Arabic language, together with his knowledge of the primary and secondary sources of Arabic texts translated from the Greek and his incomparable intuition of what is important put him at the forefront of research in the history of Greek and Arabic mathematics. Rashed untiringly discovers, translates, edits and comments Arabic manuscripts, but above all he helps us understanding their meaning and their authors' original aim. His writings render an incomparable service to the scientific community. They show in particular that a lot more must be done in the field of History of Arabic mathematics.

Besides the interest of his work from the point of view of history, mathematicians will find a treasure of new ideas in reading these original pieces of works. It is also important to note that Rashed has a remarkable style, and his papers and books are

extremely well written, with great pedagogical finesse, sometimes with a discrete irony and with the amount of skepticism that goes with a scientific mind.

Rashed's contribution to the history of mathematics is immense and will be acknowledged by the community of historians and mathematicians.

10. A SELECTION OF EDITIONS OF GREEK/ARABIC TEXTS PUBLISHED BY RASHED

- (1) R. Rashed, Diophante: Les Arithmétiques, Livre IV, vol. 3, Les Belles Lettres, 1984, 487 p.
- (2) R. Rashed, Diophante: Les Arithmétiques, Livres V, VI, VII, vol 4, Les Belles Lettres, 1984, 451 p.
- (3) R. Rashed and Ch. Houzel, Les Arithmétiques de Diophante: Lecture historique et mathématique, Walter de Gruyter, 2013, ix-629 p.
- (4) R. Rashed, Apollonius: Les Coniques, tome 1: Livre I, commentaire historique et mathématique Walter de Gruyter, 2008, xv-666 p.
- (5) R. Rashed, Apollonius: Les Coniques, tome 3: Livre V, commentaire historique et mathématique Walter de Gruyter, 2008, xv-550 p.
- (6) R. Rashed, Apollonius: Les Coniques, tome 2: Livre IV, commentaire historique et mathématique Walter de Gruyter, 2009, xii-319 p.
- (7) Rashed, Apollonius: Les Coniques, tome 4: Livres VI et VII commentaire historique et mathématique, Walter de Gruyter, 2009, xi-572 p.
- (8) R. Rashed, Les Catoptriciens grecs. I: Les miroirs ardents, édition, traduction et commentaire, Collection des Universités de France, Paris, 2000.
- (9) R. Rashed, The Transmission of Greek Heritage into Arabic, in Classical Mathematics from al-Khwārizmī to Descartes, Culture and Civilization in the Middle East, London, 2014, pp. 19–56.
- (10) R. Rashed, Greek into Arabic: Transmission and Translation, in J. E. Montgomery (ed.), Arabic Theology, Arabic Philosophy. From the Many to the One: Essays in Celebration of Richard M. Frank, Orientalia Lovaniensia Analecta 152, Leuven/Paris, 2006, pp. 157–196.
- (11) R. Rashed, Al-Kindī's Commentary on Archimedes' The Measurement of the Circle, Arabic Sciences and Philosophy, vol. 3.1, 1993, pp. 7–53.
- (12) R. Rashed and A. Papadopoulos, On Menelaus' *Spherics* III.5 in Arabic mathematics, I and II: Arabic Science and Philosophy vol. 24 (2014), pp. 1–68 and vol. 25 (2015), pp. 1–32.
- (13) R. Rashed and A. Papadopoulos, Menelaus' Spherics: Early Translation and al-Māhānī/al-Harawī's Version, edition, translation and commentary, De Gruyter, 2017, x-877 p.

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