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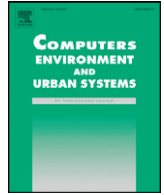
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Review

Assessing geographical effects in spatial diffusion processes: The case of euro coins



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ABSTRACT

We examine how geographical structures impact diffusion processes within a regional system. From the example of euro coin diffusion across countries, we show how the relative position and population endowment of regions impact our understanding of interregional mobility, beyond simple spatial interaction effects. The mix of coins of different origins is a complex but unique trace of the movement of individuals within a common currency area, potentially revealing a new facet of European integration. We simulate an individual-based dynamic model where agents move and exchange coins across regions. We analyse the convergence towards a homogeneous mix of coins through time for a series of different theoretical spatial systems. This sensitivity analysis demonstrates the impact of the regularity and aggregation levels, or centrality/periphery effects, on spatial diffusion dynamics. We then calibrate the model against empirical data for the regions of 5 European countries and provide estimates of mobility rates, distance decay and population attractiveness factors, affecting the diffusion of coins, hence international movements and European integration.

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1. Introduction

As emphasized by Gould (1969), the study of spatial diffusion processes should neither be reduced to the temporal dimension nor to a static analysis of their spatial patterns. It is well known that spatial processes to which a 'time arrow' is added can follow complex trajectories (Prigogine & Stengers, 1979) and encounter spatial path-dependencies (Whittle, 1954). Diffusion processes in particular are impacted by the geographic complexity of a broad variety of attractors and repellers of human movements (Gould, 1969). In economics it is also well known that cumulative processes can be spatially selective and show dependencies on initial regional conditions (Arthur, 1994; Barro & Sala-i Martin, 1992; Krugman, 1996; Petrakos, Kallioras, & Anagnostou, 2011).

Spatial interactions occur at varying geographical scales and the representation of space need to be adapted to the process under study (Rodriguez-Pose, 2011). However, the analytical tractability of geographical models has often led researchers to a simple treatment of geography, even if acknowledging that spatial simplifications cannot account for the complexity of reality (Fujita, Krugman, & Venables, 2001). New Economic Geography (NEG) models, for example, have been based on two or three regions only, or used an n -regions setting but with a simplistic geometrical structure, e.g. regions equally distributed along a line or a circle, or other symmetrical and equidistant structures (Bosker, Brakman, Garretsen, & Schramm, 2010). While these models can illustrate tendencies they encounter limits to reflect real world amplitudes (Puga & Venables, 1997). If one further recognises the dynamic and cumulative nature of processes, as in diffusion phenomena, spatial simplifications seem to impact results even more. Actually, the effect of using regular structures (lattices) or varying initial configurations is a well recognised problem in agent-based literature since Schelling (1971) and can lead to inappropriate conclusions if the geography is more asymmetric or heterogeneous (Brulhart & Torstensson, 1996; Stanilov, 2012).

In this paper we contribute an analysis of the effects of population size and relative location of places on the rapidity of the diffusion of euro coins using theoretical and empirical geographies and a dynamic disaggregated model. On January 1st 2002, European integration was pushed forward by establishing a common currency for twelve European member states. Each euro coin has on one side a common value symbol and, on the other side, a symbol representing the country where it was minted and started to circulate. Each coin can of course be used everywhere in the eurozone, whatever its production place. This specific feature of euro coins represents a fantastic opportunity to analyse mobility patterns across Europe, which are not revealed in other statistics. While coin diffusion is not a significant part of financial exchanges, observing a 'foreign' coin in a given place is an indirect signal of cross-border mobility for work, leisure, or any other business. It is well known that the mobility of people is subject to spatial interaction laws, in particular distance and agglomeration effects (Ravenstein, 1885; Roy & Thill, 2004; Stouffer, 1940). Conversely to other goods however, cash interactions with euro coins are not subject to additional transaction costs when transferred across states: the value of a coin is constant and there is no taxation at borders. It is therefore a promising tool for revealing 'pure' geographical effects on mobility.

Our analysis aims at answering the following questions: What is the effect of an increasingly complex geographical structure on the stability of the diffusion process? Is the diffusion speed between regions an independent and non-trivial geographical process, as suggested by Hagerstrand (1952)?

The remainder of the paper is organised as follows: in Section (2) we first discuss how money diffusion modelling can add to the European integration literature by emphasizing international mobility of individuals and second review existing cash diffusion models. In Section (3) we present our model and its assumptions regarding agents' mobility and the cash transaction process. In Section (4) we analyse the effects of theoretical spaces and the geography of North-West Europe on diffusion patterns and the convergence time to a steady state. In Section (5) we fit our model to empirical data and estimate convergence time in the eurozone before concluding.

2. Related work and theory

2.1. European integration and the diffusion of euro coins

European integration through the mobility of people, capital, information and ideas was one of the main purposes for the creation of the European Union. In a context of economic crisis and increased regionalism, it is important to assess the strength of territorial integration and European construction. Coin exchange practices can help the assessment of this integration by revealing cross-border mobilities not otherwise systematically reported.

Indeed the question remains of how strongly integrated is the eurozone system across regions and whether it contributes to cancelling out regional disparities within countries as well (Martin, 2001; Petrakos et al., 2011; Puga, 2002). Both Martin (2001) and Puga (2002) stress the need for further consideration of spatial proximity effects or the internal geographical structures of the eurozone in order to better understand polarisation and convergence effects, and eventually fine tune regional policy. However, more data is needed to better understand human capital, technology and knowledge diffusion across regions (Fingleton, 2004, p.397) depending on their characteristics as well as economic integration and long-run convergence (Martin, 2001). Our argument in this paper is that euro coins are an unconventional dataset but a useful marker of daily mobility, of business exchanges and eventually of territorial integration.

We are aware that using euro coins as a marker of mobility may suffer from biases. In particular, there might be differences in payment practices (use of cash, cards,...) across eurozone countries. Second, there have been differences in the way the spatio-temporal distribution of coins has been recorded in each country (Grasland, Guérin-Pace, Le Texier, & Garnier, 2012). Third, coin exchange is only an indirect and small part of economic integration, compared to investments' flows or all kinds of bank transfers between firms and households of different countries. Despite these limits, using the diffusion of euro coins to analyse integration has a series of advantages. First, the diffusion of euro coins may well reflect a deeper view of economic integration that touches individuals directly, even if cash payments are only a small part of their budget. With the understanding that economic action 'is embedded in structures of ongoing social relations' (Bathelt & Glückler, 2003; Granovetter, 1985), adopting this coin perspective actually complements the literature that is mainly limited to economic aggregates. The spatial distribution and diffusion of euro coins across Europe reflect the mobility of people within and across states via the cumulative effects of many small cash transactions.

Second, the diffusion of coins by individuals represents an important share of exchanges. For instance, in Germany, the volume of coins carried in households' wallets (estimated to 410 million euros in 2011) exceeds largely the volume of coins held by credit institution (between

120 and 370 million euros), retail outlets (130 million euros) or vending machine operators (between 30 and 100 million euros) (Deutsche Bundesbank, 2015). Coin circulation increases with the percentage of consumer goods bought in cash (Deutsche Bundesbank, 2013). According to the Bank of France (Grasland & Guerin-Pace, 2004), the movement of coins is mainly due to the mobility and purchase of people rather than to direct exchanges between shops and banks. Direct redistribution between banks is very local due to high transport costs (volume and weight of coins, and security).

Third, there is no other similar comprehensive data of cross-border and international mobility at the European level that can be associated with convergence processes. Indeed, existing mobility surveys are mainly based on information segmented by modes of transport and by the type of intra-European travellers involved (students, commuters, businessmen and women). Conversely, euro coins are carried and used by every eurozone resident in their everyday transactions. As individuals transact, they mix euro coins. This gradual 'melange' should eventually lead to a steady state where there is a homogeneous mix of coins in all regions. The more economic interactions between individuals of different regions, the more rapid should be the convergence to a homogeneous steady state. Rappaport (2004), Busnel, Bertier, and Kermarrec (2008) have shown that the economic convergence of regions is impacted by the intensity of mobility and frequency of interactions. We can hypothesise that the rapidity of reaching a coin mix steady state will be impacted similarly.

Finally, controlling for differences between regions at the beginning of a convergence process (initial state) is important when analysing economic convergence (Barro & Sala-i Martin, 1992; Petrakos et al., 2011). This is especially easy to deal with in the context of coin diffusion since the initial endowment is perfectly known. At the moment of the introduction of the euro, all coins were theoretically located in their own country of minting. The presence of a foreign coin in a territory at a later point in time indicates that it has crossed at least one border, at least once, and thus has been transported by at least one person as part of its mobility pattern.

2.2. Existing models of coin diffusion

Only few diffusion models have attempted to simulate the spatial distribution of foreign coins through time and across space from an initial, non-mixed, configuration. Each of these models makes a series of assumptions (most of the time implicit) with respect to the mobility of people and with respect to the process of exchanging coins.

Stoyan (2002), Stoyan, Stoyan, and Doge (2004), Seitz, Stoyan, and Toter (2009, 2012) first attempted to replicate observed distributions of the different origins of coins across countries (especially the Netherlands and Germany). Their very aim was to characterise the dynamics of the diffusion process in order to eventually use these dynamics for more general spatio-temporal processes such as epidemics. Their models are based on Markov chains with transition probabilities defined at the country scale. They assume two states (national or foreign) for each coin (no heterogeneity of coins values¹), and rely on a dynamic stochasticity by assuming a random walk-Brownian movement of coins. They further assume that the structure of each wallet and monetary transactions repeat through time using given proportional constants. These models are aggregate approaches in the sense that the mobility patterns of individuals are not explicit. In addition, they are non-spatial models in the sense that the geographical organisation of places is ignored.

Dragulescu and Yakovenko (2000) used a non-aggregate, agent-based, simulation model with successive and random monetary transactions between agents in a closed system. They show that the wealth distribution between agents reaches a thermodynamic equilibrium

with Boltzmann-Gibbs distribution. Chakraborti and Germano (2010) use a similar approach but introduce different saving strategies by categories of agents. Although they contribute knowledge on the complexity of the system, these models do not integrate heterogeneity in space, again ignoring geographical impacts on exchange behaviour.

Brockmann and Theis (2008), Brockmann (2010) showed that classic Brownian movements were not a good solution to capture the mobility of individuals, as revealed by the circulation of 1 dollar bills in the US. They simulated the circulation of money with the help of Levy-flights, thus describing random individual movements as a combination of a few long distance deviations to trajectories and many small distance deviations to trajectories. This approach is based on the assumption that the different modes of transport allow people to move at very different geographical scales within a short period of time. This diffusion process is a step further in accounting for geographical effects, but still assumes an isotropic spatial system. Proximity interactions with neighbouring units may vary from one spatial unit to another because of shape, size or intensity of opportunities. Moreover the two-way interactions between two units can be unbalanced for similar reasons.

From empirical analyses in France, Grasland (2009), Guerin-Pace and Le Texier (to be published) showed that the socio-economic characteristics of individuals impact on the probability to own an 'internationalised' wallet and that this effect varies across space. Their findings stress the importance of socio-spatial heterogeneities in the distribution of foreign coins in addition to standard size and distance effects, i.e. those explaining why there is a higher probability to find German coins rather than Luxembourg coins in France (size effect) or why we have a higher probability to find Belgium rather than Portuguese coins in the Netherlands (distance effect). Adding to the socio-economic effects, Berroir, Grasland, Guérin-Pace, and Hamez (2005) showed barrier effects of languages by comparing the diffusion of French coins in the French-speaking and Flemish-speaking regions of Belgium.

In sum, dynamic models have considered a rough geography so far, while empirical analyses were non-temporal, but geographically and socially detailed. In this paper we attempt further integration by proposing a dynamic and geographically rich model. Moreover we test for its empirical validity. We extend the two-region model proposed by Le Texier and Caruso (in press) who have compared different dynamic models, including aggregate and individual agent-based approaches. Both individual-based and aggregate simulation models lead to a steady-state. The analytical resolution of the aggregate model with continuous time also concludes for a convergence to a perfect mix of coins and shows the convergence depends only on the amount of coins produced in each country. Any deviation is therefore likely to derive from either the geographical structure, which is what we test here, or from parameters varying across space or time, which can only leave for future research and finding relevant historical data.

The aggregate models of Le Texier and Caruso (in press) predict a perfect mix of coins around 2020 while their individual-based simulation counterparts delays the mix until 2060. In both cases, the comparison with empirical data for France revealed an unexpected slow-down of the speed of the mixing process after 2008, which might be understood as a result of the economic crisis, slowing down short-term inter-regional mobility such as tourism in Europe (as reported by Smeral, 2009). In this paper we focus on understanding the effects of geographical structures far beyond a 2-regions model by gradually increasing the complexity of the geographical setting and by applying the model to the NW part of the eurozone.

3. The model

Our model is dynamic and disaggregated in order to represent transactions between individuals across time. This methodological choice builds on comparative tests made by Le Texier and Caruso (in press). Here, conversely to this previous work however, we use more than two territorial units and consequently redefine the destination choice

¹ Euro coins exist with following values: 2 or 1 euro, or 50, 20, 10, 5, 2 and 1 cent. As shown by Grasland et al. (2012), the speed of diffusion varies with these values.

of individuals and the cash transaction process. We first describe how our geographical inputs are gradually complexified (Section 3.1) and then the components of the model (Section 3.2).

3.1. Geographical complexity

Our objective is to explore the effects of the geographical structure on coin diffusion, especially as it is defined by its topological organisation and population endowment hierarchy. We first apply the model to a controlled abstract environment of three regions/countries, where we vary the topology of the geographical structure (linear or circular systems) and the relative population endowments of the regions. Second, we apply the model to Christaller-Loeschian landscapes. We finally apply the model to a subset of the eurozone regional system to understand the effect of this specific geographical structure, and test the capacity of our model to reproduce empirically observed patterns of euro coins' diffusion.

3.1.1. Circular vs. linear systems

Even in the relatively simple case of three regions, the hierarchy levels and the choice between a circular or linear topology result in an important combination of border and size effects (is the biggest region close to a mid-size region or a small one, at the periphery or in the centre of the system? etc.) and, presumably, of potential outputs. Fig. 1 illustrates these topologies and change in population for two hierarchical levels.

Possibilities with n regions and n hierarchical levels cannot be enumerated, so we stick to comparing the outputs of the model in circular and linear spaces with one and two hierarchical levels.

3.1.2. Christaller-Loeschian systems

Next, we apply the model to a fictitious central place system of regularly positioned regions. There have been many criticisms of early location theory models, such as Christaller (1933), Loesch (1940), nevertheless they appear to be still relevant when modelling the evolution of geographical systems (Fujita & Krugman, 2004, p.147). Theoretical travel behaviour studies for example, especially trip-chaining, have been developed in connection with central place theory and the formulation of consumer demand in space by Loesch (1940) (see e.g. Thill & Thomas, 1987). In our case we are interested in the location of regions, their size and distances rather than market areas, and these can be accounted for by a Christaller-Loeschian landscape.

We see the spatial structure of a central place system as one way to achieve more heterogeneity (compared to lines or circles) in a controlled experimental environment with a regular spacing between places. Although it has not been used much as an input to simulation, this system has the advantage of combining a certain degree of regularity with heterogeneity across hierarchical levels, thus offering a reasoned way to vary the attractive endowments and agglomeration

effects across places (second nature in the sense of Krugman (1993), Ioannides and Overman (2004)) and permitting cumulative or self-reinforcing effects through time (third nature). One could, of course, also easily add first nature heterogeneity into such a system, e.g. natural attributes, which is particularly appealing in the context of coin exchanges since tourism (mountain, sea) is an important local catalyst for the diffusion of coins (Grasland, 2009).

We use a set of regions, hierarchically defined according to three levels and located within three different countries. The structure follows an administrative principle in the sense of Loesch (1940) (i.e. the scale parameter is equal to 7, meaning that each central place at the center of an hexagon over-arches six secondary places). This structure prevents regions to be located exactly on borders, and the same internal structure is made within the different countries. We have 111 regions in total: 3 large (capital-cities), 18 middle-sized and 90 small regions. To strictly distinguish topological effects from the combined effects of topology and population, we use a Christaller-Loeschian system where only positions are used and then a Christaller-Loeschian system where population differences are also introduced (see Fig. 2)

3.1.3. Subset of the eurozone regional system

Regularities and symmetries in population size and accessibility within a central place system may lead to under or overestimations of diffusion speed compared to the real eurozone. We therefore also apply the model within a geographical system representing the regions of Belgium, France, Germany, Luxembourg and the Netherlands—all countries for which empirical data have been collected. In order to avoid strong discrepancies in surface and have comparable regional levels, different administrative levels (NUTS) have been selected in each country. The population of the considered regions is taken from European Commission (2014) and represents 171 million people in total. For computational reasons we divide population by 1000 to obtain the agents for our simulations. Regional numbers are summarised in Table 1 and mapped onto Fig. 3. Euclidean distances between centroids are used for assessing the proximity of regions (D_{ij} in Eq. (4)).

3.2. Model components

3.2.1. Geography and agents

3.2.1.1. Geographical system. Our system consists of a set of regions i, j, \dots, k endowed with total populations P_i, P_j, \dots, P_k . Each region belongs to a country I, J, \dots, K . Each country I, J, \dots, K produces a certain number of coins with a specific symbol: F_i, F_j, \dots, F_k . For simplification and in line with the European case, we assume that the total amount of coins produced by a country is directly proportional to its population. Each agent receives the same amount of coins at the beginning of the simulation (which we normalised to one coin per inhabitant without loss of generality). We aim at having a constant number of coins among agents so as to focus on the diffusion process and thus their purses or wallets are supposed to be independent of national richness or economic characteristics of individuals. Empirically there is no evidence that more mobile or less mobile agents have different wallets size (Le Texier, 2014). It is therefore a realistic assumption.

With i a particular region and n the number of regions in the country I , we denote by F_i^t the total number of coins produced by I found in region i at time t and by $\sum_{i=1}^n F_i^t$ the total number of coins from country I available in the system at time t . F_i^t describing the total number of coins available in region i at time t , the initial conditions are such that $\forall i \in I$:

$$F_i^{t_0} = \sum_{i=1}^n F_i^{t_0} \tag{1}$$

(similarly for J, \dots, K) meaning that national coins are distributed only

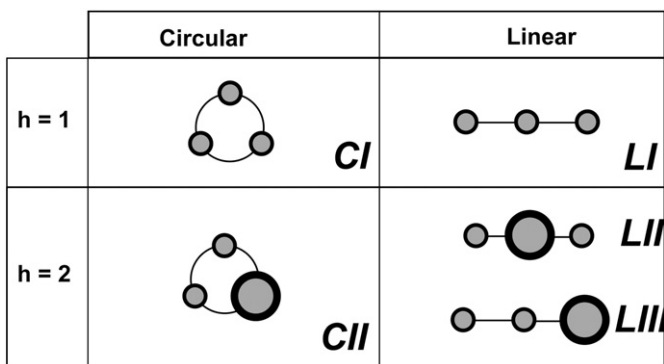


Fig. 1. Linear and Circular systems: combination of 3 hierarchy levels (h) with three regions.

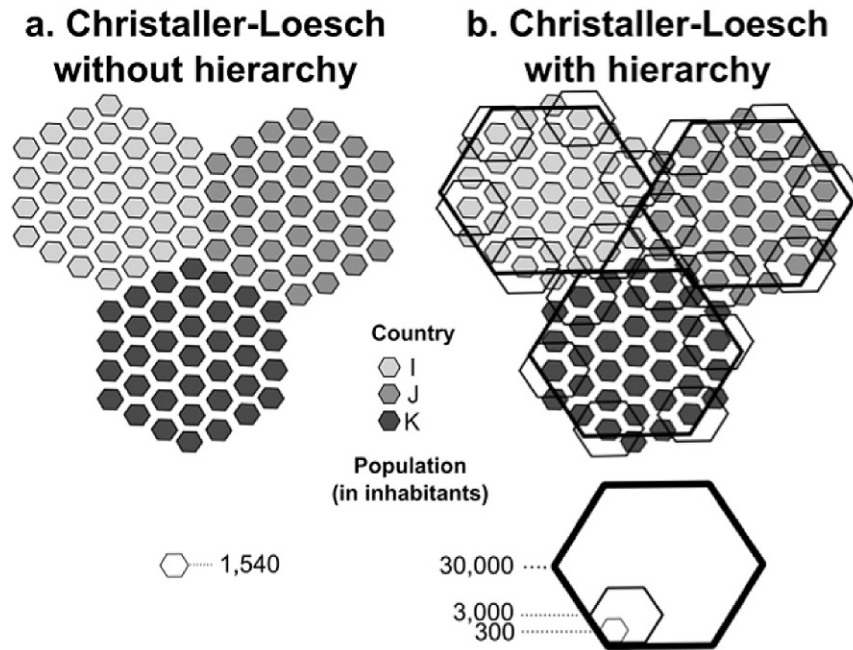


Fig. 2. Christaller-Loesch geographical systems.

among their own total population at the beginning of the system, which depicts the eurozone at the time euro currency entered circulation (January 1, 2002).

The whole system is closed in the sense that there is neither additional production nor loss of coins (no export):

$$F_i^t = F_i^{t+1} \quad (2)$$

and similarly for F_j, \dots, F_k .

3.2.1.2. Agents. The population for each region, P_i , is exogenously fixed. Within each regional population a given subset is mobile, M_i , and the rest is immobile ($P_i - M_i$). We assume that only mobile people take coins outside the regional boundaries as part of their commuting and business trips. The immobile agents participate in transactions among themselves and with mobile agents, although they do not move out of their home region. We denote by μ the mobility rate of the population and apply it homogeneously across regions. The mobile population in each place is thus:

$$M_i = \mu P_i \quad (3)$$

One can interpret μ in terms of professional skills since we consider work mobility only. This hypothesis is in line with geographical economics literature, where the interregional mobility of workers is mainly analysed using a high and low skills dichotomy (see for instance Pas (1984); Pissarides and Wadsworth (1989); Korsu and Wenglenski (2010)). Further, following Beaverstock's (2004) analysis of intercity trajectories of managerial elites and Taylor, Catalano, and Walker's

(2002) observation of intercity flows between service firms, we see that white collar business trips are a major driving force of transnational flows between cities.

3.2.2. Actions

Once the geographical setting and the agents are defined and initiated, the simulation runs into 6 sub-steps within each time step t : (1) the mobile agents move to destination (Mobility); (2) they meet other agents at destination (Meetings); (3) exchange coins with paired agent (Exchange); (4) go back home (Return); (5) meet other agents at home (Meetings); and (6) finally exchange coins with paired agent at home (Exchange). This procedure is depicted in Fig. 4 and the actions detailed below.

3.2.2.1. Mobility. Destination choices follow a simple gravitational model, expressed in a probabilistic manner. According to Huff's (1964) destination model, we determine the probability for each region j to

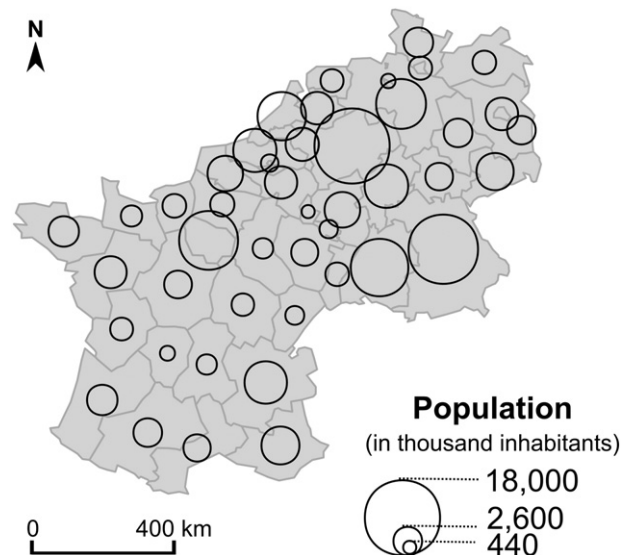


Fig. 3. Agents population in the selected eurozone regions.

Table 1
Levels and population range for the selected eurozone countries.

Country	NUTS Level	Number of units	Population ($\times 1000$)				
			Tot.	Min.	Max.	Mean	SD
France	2	21	58927	710	11055	2806	2253
Belgium	1	3	10560	971	5959	3427	2495
Germany	1	16	85408	660	18027	5338	5152
Luxembourg	0	1	442	-	-	-	-
Netherlands	1	4	16042	1678	7472	4011	2454

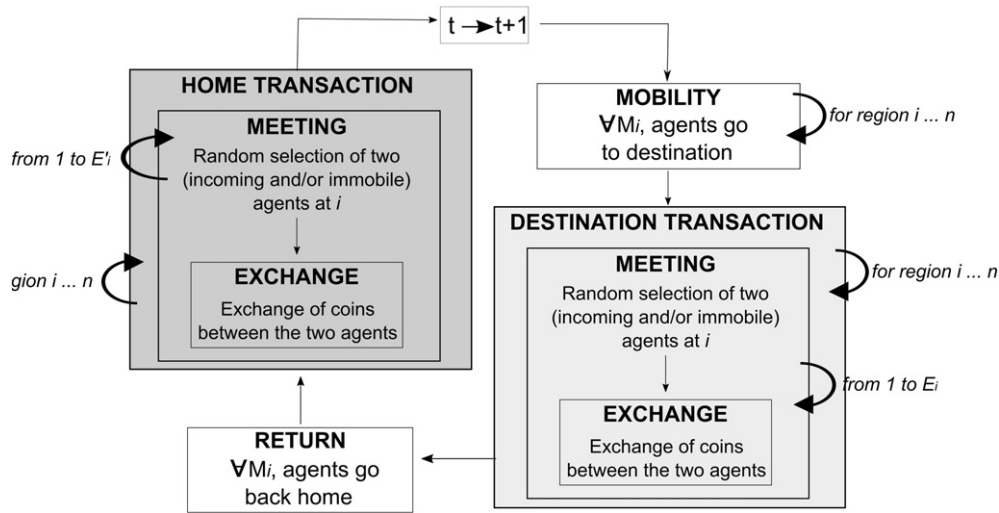


Fig. 4. Flow chart of model actions.

be visited by an agent from i as:

$$p_{i,j} = \frac{D_{i,j}^{-\alpha} P_j^{\beta}}{\sum_{k=1}^n D_{i,k}^{-\alpha} P_k^{\beta}} \quad (4)$$

where:

- p_{ij} is the probability for people living in i to go in j ²
- $D_{i,j}$ is the distance between regions i and j
- $\alpha > 0$ is a distance decay parameter
- P_j is the population of region j
- $\beta > 0$ is a population weighting parameter
- n is the total number of regions forming the geographical system

At each time step t , all mobile agents from all regions choose a destination according to that probability. We readily see from this equation that when $\beta = 0$, population attractiveness does not play a role and that the model will then behave similarly to Levy-flights model with a power distance function.

3.2.2.2. Meetings. We assume that once the agents have travelled to a particular region, mobile agents have the possibility to meet the immobile agents of this region, as well as the other mobile agents who have travelled in. They then have the possibility to make a transaction at the place of destination. After all transactions are completed, all mobile agents go back home and make a second meeting and transaction with the home population (immobile and mobile). Agents therefore exchange coins twice during a single time iteration.

We simulate a sequence of exchange meetings between pairs of individuals present at the same place at a given time. For the first transaction at each destination place, pairs of individuals are randomly picked from a list containing the immobile people of the destination place and the mobile agents incoming from the other regions. For the second transaction, the list is made of the population of the region, i.e. the immobile agents plus the mobile ones (back from their trip).

To keep the model as simple as possible, it is considered that on average each individual can take part in an exchange once per meeting sequence (i.e. twice per time step). Obviously the likelihood of exchanging coins also depends on the duration of a visit or its purpose, but we cannot control for that here. Since in the model every transaction involves two individuals, we fix the number of exchanges at destination E and

at home E' to half of the population present in a place, i.e.:

$$E_i = \frac{[(1-m_i)P_i + m_j P_j + \dots + m_k P_k]}{2} \quad (5)$$

for the first round at destination, and:

$$E'_i = \frac{P_i}{2} \quad (6)$$

for the second round at home.

Finally, we assume that the pairing of individuals is made from a random drawing with replacement. This reflects the idea that (1) all exchanges are sequential; (2) each agent can make several or no exchange independently of its origin; and (3) at each destination place, the probability to exchange with a local or a visitor is equal. The latter sounds reasonable although we could also imagine—at the expense of handling more parameters—a system where visitors concentrate in some parts of a city and have more interactions among themselves.

3.2.2.3. Exchange. Our focus is set on the mixing of coins from different origins as a proxy for European integration at regional level. Conversely to econophysics (see for instance Dragulescu and Yakovenko (2000)) we are not interested in the amount of money an agent gets from transactions. We therefore consider for simplification that an exchange consists of averaging the wallets of the paired individuals at each meeting. Since all individuals have the same number of coins (unitary) at the beginning of the simulation, we can keep this as a constant through time. Le Texier and Caruso (in press) have explored other exchange strategies and have shown how they impact on the speed of the mixing process.

Suppose a pair of individuals $a = (1,2)$, $f_{i,a}$ the share of I coins in the wallet of the individual a , and τ referring to a particular exchange moment within a given time, we have:

$$f_{i,1}^{\tau} = f_{i,2}^{\tau} = (f_{i,1}^{\tau-1} + f_{i,2}^{\tau-1})/2 \quad (7)$$

describing the update of a wallet at each transaction based on the characteristics of the wallets involved in the meeting prior to the transaction. This is similar for the other types J, K, \dots of coins.

The mix of coins in a given region i after completion of a full time step is measured when every mobile agent is back home:

$$F_{i,i}^t = \frac{\sum_{a=1}^{P_i} f_{i,a}^t}{P_i}, \forall a \in i \quad (8)$$

² As internal mobility is taken into account, a probability p_{ij} has been defined too.

with $F_{i,t}^I$ the share of I coins in region i at time t (and similarly for J, \dots, K coins).

3.2.2.4. Perfect mix and steady state. Our goal is to find how the mix of coins changes with time and when it reaches a steady state. Le Texier and Caruso (in press) have demonstrated analytically, with continuous and discrete time, that the steady state exists in the case of two regions and that it corresponds to a perfect mix. The perfect mix is defined as a situation where the proportion of the different origins of coins within each region corresponds to the share of the different origins of coins in the total production. We do not know ex-ante whether a steady state will be reached and whether it will correspond to perfect mix in the case of complex spatial configurations. In this paper, the existence and conditions of a perfect mix of coins are investigated in the cases of multiple regions and varying topological structures.

We denote by t_i^* the time at which the perfect mix is obtained in region i and by t^* the time at which all regions have converged, i.e. $t^* = \max_i(t_i^*)$. We simulate a number of iterations such that there is no further change in the proportion of coins in all regions or until an external computation limit is reached. We then verify whether the converged time, if any, corresponds to the perfect mix t^* . The time lapse between two time steps is undetermined –not interpreted– in the model experiments presented in Section 4 since it is a theoretical exercise. Obviously, in reality it should be related to the number of trips and exchanges people are actually doing. Here it is assumed to be proportional to population and uniform across agents, which is simple but a reasonable assumption. In Section 5, a calibration procedure is undertaken in order to fit our model to empirical observations of the mix of coins within the eurozone while interpreting t in number of months since the introduction of the euro.

4. Simulations

4.1. Parameters

Apart from the total population, which we equal in all theoretical spaces to the total number of agents as of the selected eurozone (171,000 agents), there are three parameters to be exogenously defined in order to run the model: the distance decay parameter α , the population size effects β affecting the destination choice (Eq. (4)) and μ , the share of mobile people in a region (Eq. (3)).

α and β are fixed at $\alpha = 1$ and $\beta = 1$ since—at this stage—we do not benefit from any accurate estimation of their effects. Therefore, we assume for simplicity that the relationships of distance and population size on mobility are linear.

We fix the mobility rate $\mu = 0.1$ from the French National Survey 2006, where we find that managerial elite and academic professionals represent about 10% of the population, which we interpret as the share of high skilled workers, i.e. our mobile population. Another way to obtain a reasonable parameter is to observe cross-border mobility, though this information is rare and likely to vary by purpose and socio-economic group. According to Spierings and Van Der Velde (2008), cross-border shopping mobility between Germany and the Netherlands would concern 0.6% of the shopping trips, which can be rounded at one cross-border shopping trip per year and per inhabitant. Since our model includes a broader set of mobilities, setting $\mu = 0.1$ is a rather reasonable—though minimal—assumption.

These parameters are not satisfactory for prediction, especially because they do not relate to the measurement units of population and time in the model. However they provide a reasonable weighting of effects and a base for showing numerically the impacts of varying geographical structures. We provide a calibration of α , β and μ against empirical data later in Section 5.

Table 2
Measures of time to convergence towards perfect mix per geographical system.

System	Regions	Population	Number of neighbors	Convergence t_i^*	
Circular with equal populations (CI)	Region 1	57,000	2	139	
	Region 2	57,000	2	139	
	Region 3	57,000	2	139	
Circular with different populations (CII)	Region 1	42,750	2	121	
	Region 2	85,500	2	122	
	Region 3	42,750	2	122	
Linear with equal populations (LI)	Region 1	57,000	1	160	
	Region 2	57,000	2	120	
	Region 3	57,000	1	160	
Linear with different populations (LII)	Region 1	42,750	1	112	
	Region 2	85,500	2	113	
	Region 3	42,750	1	112	
Linear with different populations (LIII) bis	Region 1	42,750	1	157	
	Region 2	42,750	2	124	
	Region 3	85,500	1	144	
Christaller-Loesch without hierarchy	111 regions	1,40	–	[231; 280]	
	Christaller-Loesch with hierarchy	Level 1 (3 regions)	30,000	–	[295; 299]
		Level 2 (18 regions)	3,00	–	[270; 327]
	Level 3 (90 regions)	300	–	[262; 359]	
North-West of eurozone	45 regions	171,100	–	[192; 332]	

4.2. Convergence speed and geographical structure

Simulations have been programmed with Scala (<http://www.scalalang.org/>) run on a multi-core processor. The time for a simulation ranges between 2 min and 10 min. We did 100 replications given the presence of a random component so that robust but computationally feasible results could be obtained. Our simulations show that all regions converge to a perfect mix steady state whatever the geographical system and distribution of population. However the time at which each region reaches the perfect mix and the overall steady state then depends on the relative location and population size of places (Table 2 and Fig. 5).

4.2.1. Circular vs. linear cases

As expected, when no population heterogeneity is introduced in the circular system, there are no disparities in convergence time. Indeed, there is no incentive to move to one region or another. More interestingly, when there are population differences in the system, i.e. one region is larger than the two others, we expect that this larger region converges more quickly. But this is not the case because the largest region actually constitutes an opportunity for the mobile agents from every region to meet, thus homogenizing the mixing process across all regions.

Turning to the linear case, we can test whether a peripheral/centrality effect affects this conclusion. In a linear system with the largest region at the center, we expect now that convergence will favour the larger region which would benefit both from its population mass and its centrality. Again however, this is not what we find. The same effect is at stake and cancels out population and centrality differences. Conversely, when the leading effect of the largest region is removed, as is the case with the homogeneous linear system, the central region converges more quickly. Centrality seems to become the main driver only if there is homogeneity in space.

In order to have a complete picture however, we need to observe the linear system where the largest region is in peripheral position with respect to the external borders of the overall system. We expect to see again a leading role of the large region in smoothing the convergence process, but to a smaller extent given the distance. Again, this is not

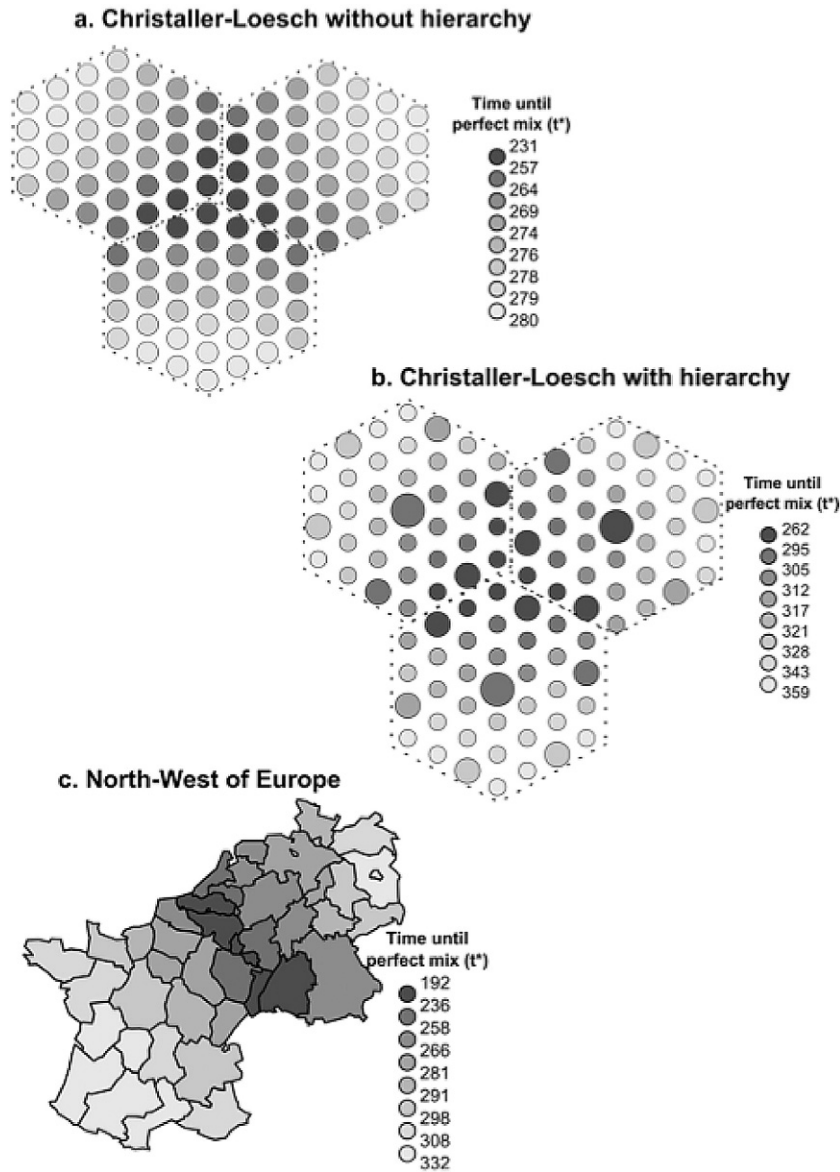


Fig. 5. Spatial distribution of regional convergence time to perfect mix (\hat{t}_i) within the different spatial systems.

what we find: it is again the central region despite its smallest size that converges more quickly. Overall, we therefore find that the effect of geographical proximity overtakes the effect of population differences.

In terms of overall convergence time, we find that the introduction of population differences increases the speed of convergence to perfect mix and steady-state because the agents in less populated regions have a greater incentive to travel to a foreign region and therefore to participate to the mixing of coins.

4.2.2. Christaller-Loesch and North-West EU cases

For the more complex structure of the Christaller-Loeschian system (Fig. 5-a and -b), the spatial distribution of countries is similar to the one in the circular model at 3 regions as every pair of countries is equidistant. The national aggregates are however disaggregated in regions, leading to the introduction of border effects within countries, where peripheral regions can either be located at the core of the international system or at its edges. Note that this distribution of regions is symmetrical between countries.

From Table 2, we see that the disaggregation of the system, leading to the redistribution of the national populations within regions, increases the time necessary to reach the perfect mix t^* . This increase in

the convergence time is greater and more heterogeneous for smaller regions than for the most populated ones.

When looking at the spatial distribution of these local convergence times (Fig. 5-a and -b), we see that, despite the equality in the number of agents and coins per country in the two models, the value of t^* varies from 280 (Christaller-Loesch without hierarchy) to 359 (Christaller-Loesch with hierarchy). Likewise, the minimum value of t_i^* in the Christaller-Loesch without hierarchy ($\min(t_i^*) = 231$) is much smaller than the one obtained when population heterogeneity is introduced in the system ($\min(t_i^*) = 262$).

This is explained by the fact that the spatially-constrained motion of agents and the resulting relocation of coins are perturbed by the attraction of regions from higher hierarchies. As these regions are either located at the centre of each national system or at their extremities, the incentive for crossing the international boundaries is -in average- reduced. Indeed, since there is no particular incentive for crossing international boundaries, agents will be more likely to work inside their home country than abroad, thus reducing the amount of coins crossing the border at each time step. In the end, it is hard to visually distinguish the relative influence of the proximity of an international boundary or the attractiveness of regions on the spread of coins from borders.

Table 3

Effect (coef. estim.) of spatial interaction factors and distance to borders on convergence time (Signif.: ***<0.001).

	Christaller-Loesch with hierarchy		Eurozone (partim)	
Intercept	384.2(45.064)***	322.1(43.352)***	332.884(21.540)***	210.2(11.036)***
Opportunity to move (O)	−0.001(−7.838)***	−0.001(−5.116)***	−1.122(−3.606)***	n.s.
Attractiveness (A)	−2.735(−5.528)***	−2.224(−6.925)***	n.s.	n.s.
Border (B)		814.5(12.393)***		0.015(7.594)***
Adjusted R ²	0.386	0.746	0.233	0.673

This is particularly true with the North-West eurozone regional system (Fig. 5-c) which shows clearer center/periphery effect on convergence times and in which the effect of population is much harder to find: in many cases the most central regions are also the most populated ones. If the convergence times are in general hardly comparable with the ones obtained in the Christaller-Loesch systems - as the number of countries issuing the coins is here equivalent to 5—the fact that the minimum value of \bar{t}_i is lower ($\min(\bar{t}_i) = 192$) than the previous ones may be explained by the relatively small disaggregation of the system: the total amount of agents is dispersed within 45 regions despite of 111 in the two theoretical spatial systems.

4.3. Border vs spatial interaction effects

In order to distinguish distance to international border effects from standard spatial interaction effects beyond the cartographic analysis, we fit an OLS model to explain convergence time (\bar{t}_i) in the NW Europe—case and hierarchical Christaller-Loesch case.

Our first two regressors are derived from spatial interaction principles. Following the definition of population potentials (Fotheringham & O’Kelly, 1989; Stewart, 1947) we define for each region i the opportunity for moving out of the region (O_i) and then its relative attractiveness (A_i). Both derive from populations (P_i, P_j) and interregional Euclidean distances (d_{ij}).

$$O_i = \sum_{j=1}^n \frac{P_j}{d_{i,j}} \quad (9)$$

$$A_i = \sum_{j=1}^n \frac{P_i}{d_{i,j} O_j} \quad (10)$$

The third regressor is the distance to the closest international border, denoted by B_i . Distance units are not similar between the Christaller-Loesch and the NW eurozone case, hence the related coefficients are not to be compared directly (only the sign and the level of significance are important).

The overall fit and estimated coefficients are displayed in Table 3. It is striking to see that the two spatial interaction variables are significant in the theoretical Christaller-Loesch space but not for the eurozone configuration. The first visual intuition we got from mapping \bar{t}_i is thus reinforced here statistically. Given that we gave the same total population in the two settings and the same mobility rules, the lesson is that the topology of the eurozone and the relative location of regions not only matter in explaining the convergence, but also tend to prioritise distance to international borders over hierarchical effects and spatial interaction effects. It is not that spatial interaction does not play a role (it is explicit in the mobility of agents) but its overall effect follows the international border effects (as one can see by comparing the model with and without the border variable B) and therefore cannot be significantly distinguished in the output. As a result, the overall fit of the model is also reduced for the eurozone.

5. Empirical validation

In the previous section, we have shown the effect of spatial structure and relative population endowment on the diffusion process of coins, we now complement our analysis by contrasting simulation output to observed data. We particularly aim to determine whether the evolution of the mix in the part of the eurozone considered above can be predicted from our model, and whether and when the perfect mix can be reached.

5.1. Data and calibration

The model is calibrated against sixteen surveys of individual wallets conducted by the ESDO in France, Belgium and Germany, by the LISER PSELL survey in Luxembourg, and by the Eurodiffusie project in the Netherlands (see Table 4 for source and dates of survey).³ For statistical significance, we only kept regions where more than 20 wallets were surveyed. Data for the Netherlands were only available at the national level. The model was therefore run at regional level to avoid bias in the destination choice process but results were aggregated afterwards using population-weighted averages. For the other cases the data correspond to the finest scale of the model. There are limitations in our dataset related to the fact that the number of surveys differs among countries and scale problems. Yet this is a regional dataset that offers quite a unique calibration potential through time.

The time-frame of the ESDO surveys being organised per months, we choose to calibrate the model according to a monthly definition of the time step t . In addition, as explained above, regional populations have been reduced by a ratio of 1000 for computational reasons. For computational convenience, we also reduce the number of runs of the model with a given set of parameters from 100 in Section 4 to 20 in this section. Le Texier and Caruso (in press) have described the relationship between the mobility parameter (μ), population and time units. Reducing the number of agents (thousands) compared to the observed population (millions) leads to lowering mobility rates or enlarging time increments. Once the time step is chosen, we need to vary mobility rates μ to find a good empirical fit. In addition, we need to calibrate both the distance decay parameter α and the population size effect β , that were both set to unity in the theoretical experiments above.

Calibration consists in minimizing an error term Err defined as the sum of the squared differences between the simulated and observed shares of foreign coins at every surveyed point in time and across the regions. The three parameters α , β and μ have been varied systematically. We started from the following coarse ranges and increments: $\alpha \in [0.010, 2.000[$ by 0.398, $\beta \in [0.010, 2.000[$ by 0.398, and $\mu \in [0.010, 1.000[$ by 0.198; which were then refined following a Newton-Raphson method, given the monotonic behaviour of the exploration. The quality of the adjustment to all euro coins surveyed since 2002 in NW Europe can be assessed from Supplementary Fig. S1 where we report the average errors obtained for every pair of parameters during the coarser and finer calibration stages.

Historians have shown that the higher the value of a coin, the wider is its spatial diffusion (see for instance Bursche, 2002). A similar effect is

³ The program code for the model can be obtained from <https://github.com/MarionLeTexier/Coins> and data is available upon request to the authors.

Table 4
Empirical data used for calibration. ESDO (www.esdo.prd.fr); LISER (www.liser.lu); Eurodiffusie (www.eurodiffusie.nl).

	France	Belgium	Germany	Luxembourg	The Netherlands
Source	ESDO	ESDO	ESDO	LISER	Eurodiffusie
Surveys selected	15 (Jun 2002–Dec 2011)	1 (Dec 2003)	1 (Dec 2005)	1 (Mar–Jul 2006)	10 (Dec 2002–2011)
Survey type	At-home <i>Omnibus</i>	At-home <i>Omnibus</i>	At-home <i>Omnibus</i>	Annual survey (PSELL-3)	Online
Scale of analysis	NUTS2	NUTS1	NUTS1	NUTS0	NUTS0
Number of territorial units selected	15	3	16	1	1
Number of coins observed	217,994	9083	27,417	48,104	92,678

found for euro coins (Grasland et al., 2012). Therefore we expect important differences in the estimates of the three parameters according to the value of the coins. In Table 5, we provide the calibrated values of α , β and μ respectively for all the coins, for large denominations only (50 cents, 1 and 2 euros), and for small denominations only (1, 2, 5 cents). The range of parameters for the calibration is refined accordingly.

For the all coins case, the total error is minimized ($Err = 1.950$) at $\alpha = 1.000$, $\beta = 0.260$ and $\mu = 0.011$. Surprisingly α is estimated to 1.000. This linear behaviour was chosen in the theoretical part. Our earlier results are therefore quite general. More interestingly, compared to this distance effect, we see that the population effect in our theoretical section was too generous: the attractive effect from population is positive but far from being linear, thus dampening the effect of hierarchical diffusion. We find also that the estimated value of μ is very much in line with the share of cross-border commuters in France, which amounts to 1.1% of the active population (Floch, 2011). Given that cross-border commuting does not account for business and leisure trips, the parameter value obtained from our calibration suggests that these additional trips do not contribute much to the diffusion of coins.

As expected, if one considers only the coins ≥ 50 cents, the distance decay parameter obtained is half of the all coins effect. Conversely if only the coins ≤ 5 cents are considered, the distance decay effect doubles. Regarding the effect of population size, it is higher in the case of small coins than in the case of large coins. Mobility effects are much more constant across cases.

These results show that the circulation of small denominations is more constrained by spatial interaction processes than the high denominations of coins. This can be explained by a differentiated use of coins according to mobility types: commuters may go shopping close to their workplace and use small denominations for their daily and frequent consumption. Conversely, a person on a rarer business trip may be less inclined to pay with cash or to give the exact value for a less usual place of purchase.

5.2. Convergence

After calibration with all coin values, the model predicts a perfect mix of coins 2,448 months after the introduction of the euro, i.e. in the year 2190 (Table 6). This seems surprisingly long but we should not forget that this is a perfect mix in every single region and for every coin type (country face). At national level, the perfect mix is obtained earlier: in year 2153. With a system of two spatial units only (France vs the rest

Table 5
Refined calibration ranges and estimations per values of coins.

	α	β	μ	Err
All coins estimates	[0.800,1.200] by 0.050	[0.010,0.410] by 0.050	[0.009,0.011] by 0.001	1.950
Small coins estimates	1.000	0.260	0.011	
Large coins estimates	[0.400,0.600] by 0.020	[0.010,0.210] by 0.020	[0.009,0.011] by 0.001	0.567
	0.480	0.110	0.011	

of the eurozone) and no regional breakdown, Le Texier and Caruso (in press) obtained a convergence in year 2064. With more countries considered, the mix is more complicated to get since, within the average wallet of every region, the coins minted by each of these countries should be proportionate to their population. Besides, and as expected, the smaller the values of the coins used for the calibration, the slower the convergence process in the mix of coins across regions and countries. Using only large denominations of coins (50 cents, 1 and 2 euros), the perfect mix is expected in year 2089 at the country scale and in year 2116 at the regional scale.

Fig. 6 shows the regional convergence time to perfect mix after model calibration by values of coins. We see that border regions converge more quickly toward the perfect mix of coins than peripheral regions (up to 104 years later). One can note that, in principle, even if all regions tend to be closer to the perfect mix as time increases, nothing prevent that they move away from the perfect mix in later times. This case is not observed in our simulations.

From the comparison of the three maps in Fig. 6, we see as expected that border effects are reinforced in the case of small denominations. Because of a high distance decay parameter, the coins diffuse through proximity. For larger denominations, coins diffuse more quickly across all regions and therefore border effects are decreased. Interestingly, the negative effect of population size on the convergence speed echoes the results found in the case of the theoretical 3 regions (Section 4.2). When interregional mobilities are made easier by a low distance decay effect, the convergence speed is slower in regions with many inhabitants compared to those with fewer inhabitants at similar location. There are more national coins in those highly populated areas, and these coins flow away very slowly.

5.3. Residuals

Beyond the convergence time of the different regions, we can also analyse the share of foreign coins at a certain point of time as an indication of regional integration. Are there any regions in which the mixing of coins is quicker (resp. slower) than expected by our model? In other words, are there any regions that attract and/or diffuse more (resp. less) residents (from) abroad than their population and relative location would have suggested? On Fig. 7 we report on whether there is a mismatch between empirical observation and simulated results. We compare simulated shares of large foreign coins (≥ 50 cents) to the last survey available in each region (December 2003 for Belgium, December 2005 for Germany, June 2006 for Luxembourg and December 2011 for France).

Negative residuals (simulated \geq observed) are found at the periphery of the system (e.g. South-West France and North-East Germany). Despite their high level of population those regions are more excluded from exchange than expected. Surprisingly we also find Luxembourg

Table 6
Expected year of perfect mix of coins among regions per value and scale of analysis (and t^* value).

	All	Small	Large
Regions	2182 ($t^* = 2164$)	2427 ($t^* = 5103$)	2116 ($t^* = 1375$)
Countries	2153 ($t^* = 1816$)	2372 ($t^* = 4447$)	2089 ($t^* = 1048$)

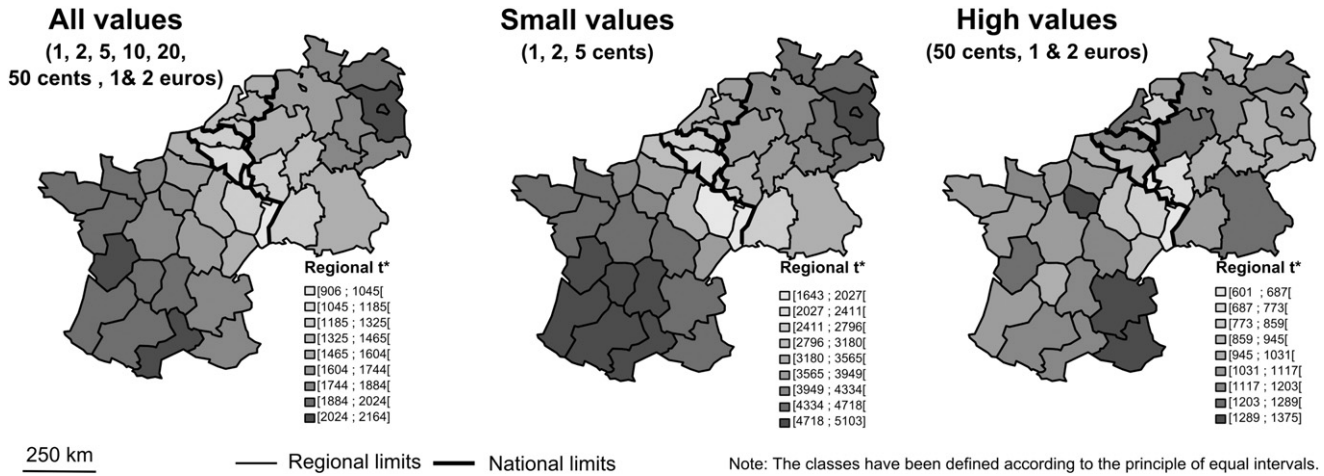


Fig. 6. Regional convergence time to perfect mix (\hat{t}_i) after model calibration by values of coins.

with a negative residual while it drives intense cross-border commuting and hosts several EU institutions. This could be due to an overestimation by the eurozone estimate of the international mobility of people who reside in Luxembourg (those who have been surveyed) while cross-border commuting is very largely due to residents in neighbouring countries who are mostly active (exchanging) in Luxembourg.

Positive residuals (observed \geq simulated) are found in regions where distance and population effects are not sufficient to support the relatively high presence of foreign coins at the time of the survey. There are three kinds of such regions: those located at the border between Belgium, France, Germany and the Netherlands; those hosting big cities (e.g. Paris and Lyons); and the very attractive tourist regions, e.g. the Mediterranean coast, the Pays-de-la-Loire and Burgundy.

Obviously the quality of the resulting convergence pattern and residuals is affected by our simplified assumptions. Especially we considered homogeneous mobility coefficients for all countries and regions. Although this seems reasonable given available stylised data, there is a probable interaction between mobility rates and opportunity of moving

(Eq. (9)), which is not treated here. Nor is the variation in time of this interaction considered.

6. Conclusion

In this paper, we have used euro coins as a marker of the international mobility of individuals within the eurozone. We have shown that the spatial distribution of coins according to their country of minting at different points of time can be reproduced by simulating the mobility of people across regions after gravity law and simple monetary exchange. Although our analysis was conducted on a limited number of countries and for a restricted time-frame, the database embodies a wide range of cross-regional mobility types. The eurozone regional system, even reduced at its core area (Belgium, France, Germany, Luxembourg, and the Netherlands) shows strong spatial asymmetries, which exacerbates center/peripheral effects and dampens agglomeration and commuting effects.

We add on previous modelling work by assessing how and why spatial interaction processes are altered by the geographical setting. Based on theoretical and empirical simulations, we have shown that the effect of population hierarchy and the relative location of regions on integration, as measured through the mixing of coins, strongly depend on the shape of the geographical system. For a given set of agents and behaviour, the complexity of the geographical system does not affect the convergence of regions toward the perfect mix but slows down the process. More importantly the geographical structure affects the diffusion channels by stressing either proximity or hierarchy effects depending on relative locations. Compared to Brownian (Seitz et al., 2009, 2012; Stoyan, 2002; Stoyan et al., 2004) or Levy flight approaches (Brockmann, 2010; Brockmann & Theis, 2008) where space is isotropic, i.e. where hierarchy and relative locations are absent, there is more interactions between similar people and the diffusion is also slowed down, somehow artificially. Compared to Le Texier and Caruso (in press) who assumed simple hierarchies in a 2 regions system, this research shows that the relative location of differentiated regions perturbs hierarchy effects. This result is not new if positioned within a dynamic spatial interaction perspective—for example after Wilson (1974) or Weidlich and Haag (1988), but it emphasizes the need for reinvestigating carefully these effects in individual-based modelling approaches.

We believe that our systematic approach to testing geographical effects for euro coin diffusion potentially bears implications about the convergence of regions and European integration. In a recent issue of *Journal of Economic Geography* dedicated to the Euro crisis, Fingleton, Garretsen, and Martin (2015) showed that the impact of the 2008

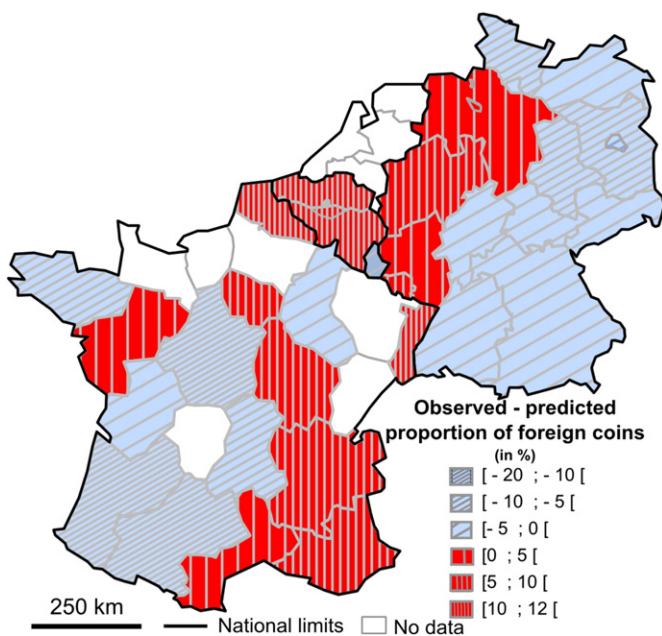


Fig. 7. Over and under prediction of the share of large foreign coins (≥ 50 cents) in NW euro regions.

financial crisis was worst in peripheral and/or geographically isolated regions of the eurozone because these regions were less embedded and had lower productivity rate. They concluded to the existence of: "a stronger and more stable core zone, and a weaker, less stable Southern and peripheral one". With our mobility point of view, we show that the stable core zone is also of two types: a strongly integrated central zone with many cross-border, metropolitan or tourist interactions and a peripheral zone where convergence is slower. Our results suggest that geographical structure may be key to understand the capacity of regions to recover after economic shocks. In particular, cross-regional mobility across national borders may serve as an adjustment (Andersson, Andersson, Harsman, & Daghbashyan, 2015), but is unlikely to be homogeneous across the monetary union, simply because of the geographical setting. Future work should compare these findings with other measures of monetary and investments flows in order to offer a larger perspective on regional integration in Europe.

More generally, our work supports the idea that a diffusion process, hence convergence and integration, cannot be reduced to a single spatial or temporal dimension (Gould, 1969; Hagerstrand, 1952). Both dimensions are interrelated and it is crucial for modellers to assess spatial biases in dynamic geographical models with cumulative effects.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <http://dx.doi.org/10.1016/j.compenvurbsys.2016.08.003>.

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