

# Wythoff Wisdom Report

I will first focus primarily on an overview, and then focus more on specific details.

Sections 1 and 2 provide some direct enlightenment for non-specialists. However, a significant part of the rest amounts to a sophisticated guide to the literature that is of specialized rather than general appeal. If this work could assume just a bit less of the reader it would be more valuable. For example, section 3 might at least include some explicit closed form formulas for the P-positions (e.g. those already known to Connell [20]) of the oldest and simplest variations of Wythoff's Nim. Some hints as to why they are valid would even be better.

Section 4 rapidly covers an intimidatingly large number of topics. If it is to be more than an expertly annotated guide to the literature, it should be a bit more selective. In other words, fewer topics with more illumination.

Section 5 is clearly written and provides enough detail to give some feeling for the subject prior to tracking down the references for a more comprehensive view. Section 6.1 seems aimed at experts, but even so I think FSM should be defined immediately rather than about 5 sentences later. Some references to Sprague-Grundy functions (and perhaps an indication as to which reference would be most useful for a non-expert reader of this section) would be appropriate. This would also help with section 6.2.

Section 7 is much more relaxed and charming than much of what I find here. One can get a fair amount from it even before tracking down references. I was surprised and enlightened by the Mancala connection! Section 7 does cover many topics that are more easily explained than what is covered in many of the other sections.

The diagrams, figures, and illustrations of section 8 seemed initially appealing to me, but I found the text rather elliptical and hard to follow. It would set a better initial tone if the writer followed the sentence "It turns out that the P-positions correspond to those of Wythoff Nim" with an explanation (or if it is obvious, say so). Also, what is "Property W"? What is the "\*-operator"? I think this section would greatly benefit from fewer topics, less generality, and more detail, including explanatory detail, not just data-incorporating displays. For example, going into more detail on MAHARAJA NIM at the expense of other topics would be nice.

Now I will give a number of more specific comments. Even before the introduction the structure of the paper should be clearly indicated. That is, it is a collection of 6 articles by 6 different authors, preceded by an introductory guide.

In the first paragraph the sentence starting with "In this survey..." seems peculiar

and rather awkward. How about "...rabbit population. The subject has been receiving more attention in recent decades thanks in part to new studies of Wythoff Nim and its variants by Fraenkel, and investigations into related sequences and arrays by Kimberling. In this paper we provide surveys of six different aspects of this subject by six of its current masters."

When "MEX" is mentioned on p.1 it should be "MEX (but without giving it a name)." In the last paragraph of section 2, it should read "...Beatty sequences, for example one with Porta and Fraenkel [56], where many curious (note the spelling) identities..."

In section 7, p.28, the paragraph "A variant of the Schroeder-Mancala game..." is very charming, as is the next paragraph, "Another method..." However, I do request that a paragraph be put between them as follows.

"Another method for generating both L and U is by sending light rays through a certain 2-dimensional array of half-silvered mirrors (the type used in the Michelson-Morley experiment) and seeing which integer points on the co-ordinate axes become illuminated. See Porta and Stolarsky [P-S]."

The next paragraph would then begin "Yet another method..." I have yet another thing to add to p.28 after "See also A187950." It is the following.

"It would be of interest to see more of algebraic irrationalities of degree greater than 2 playing a role in this subject. See also section 3.2, [29], and [Ra]. But it is the case that the original Wythoff pairs have in fact a (somewhat hidden) quartic algebraic structure. See [S] and [R]."

Here are a few other concerns.

p. 32 line 20. "... the number of P-positions ..."

p. 32 line 12 (How is "palace compensation term" defined?)

p. 32 lines 8, 9 ?The general games seems harder?

p. 33 line 8 "...it was conjectured..."

p. 34 section 8.4 "In this paper, co-authored..." What paper? Is there a reference?

Finally, here are the additional references.

[P-S] Porta, H. and Stolarsky, K. B., Half-silvered mirrors and Wythoff's game, *Canad.Math.Bull.* 33 (1990), 119-125.

[R] Ridley, J. N., Fourth order linear recurrences satisfied by Wythoff pairs, *Ramanujan J.*5 (2001), 159-165.

[Ra] Rauzy, G., Nombres algébriques et substitutions, *Bull. Soc. Math. France* 110 (1982), 147-178.

[S] Stolarsky, K. B., Fourth order linearly recurrent Wythoff pairs, *Ramanujan J.* 2(1998), 441-448.