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Singer Cyclic Difference Sets for an Energy-Efficient Joining Scheme

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Abstract—This paper considers the problem of joining a network of constrained IoT devices. We propose to take advantage of Singer Cyclic Difference Sets (S-CDS) for a joining scheme that results in low duty cycles and apply the proposed scheme to the joining problem in 802.15.4e TSCH networks. S-CDS distributes the active periods of nodes over time so that a joining node does not suffer from long scanning periods in contrast to other schemes. We compare S-CDS through simulation with other state-of-the-art schemes adapted to TSCH networks. The comparisons show that S-CDS achieves a better trade-off between energy consumption and joining delay than the best joining schemes while being particularly suitable for devices with strong energy constraints such as energy harvesting nodes.

Keywords- Cyclic Difference Set, Internet of Things, Joining Protocol, Network Discovery, 802.15.4e TSCH

I. INTRODUCTION

Internet of Things (IoT) networks face new challenges such as energy efficiency, mobility management, and scalability. Although crucial, the joining phase in which a node enters an existing network and starts to operate according to the established configuration of the network, was often neglected. This phase only becomes important when people start to deploy networks in real-world use cases.

To save energy, IoT nodes usually operate at low *duty cycles*: they go to sleep most of the time and only wake up at some instants to communicate. For efficient communication, nodes are synchronized (e.g., beacon-enabled 802.15.4 or 802.15.4e TSCH [1]) and intermittently send beacons (short frames with network parameters) to enable other nodes to join the network. Such an operation makes the discovery of the network particularly difficult because the joining node does not know the channel of a beacon nor the instant at which other nodes will wake up. The situation is even more complex in the case of *energy harvesting* nodes that obtain energy from the environment and only rely on a small capacity battery. Such devices do not have a fixed initial amount of energy for operation, so they cannot stay awoken during long periods scanning for beacons—they need to distribute the scanning periods over time and interleave them with the periods of energy harvesting.

The objective of a joining scheme is to minimize the scan duration to consume the minimal amount of energy during this phase and join the network as fast as possible. An intuitive approach is to let the network nodes periodically

send beacons and a joining node continuously scan a chosen channel until it receives a beacon (e.g., in a beacon-enabled 802.15.4 network [1]). The work by Qiu et al. [2] formalized this asymmetric approach. As the joining node does not know whether it is in the communicating range of a node that sends beacons, we must take into account the scenario in which a joining node will not receive any beacons because the network nodes are too far away or they have moved. To design an energy-efficient scheme, we must focus not only on the average joining energy consumption but also on the worst case scenario in which no advertising node is in the communicating range of the joining node. For instance, a joining scheme when a node just switches its radio on to continuously scan for beacons is not feasible for an energy harvesting node: a duty cycle approach for both the joining and the advertising nodes is also required.

There is extensive literature with theoretical work on symmetric neighbor discovery in the context of ad hoc, mobile, and sensor networks assuming a *time-slotted discovery model*. In this model, time is divided into fixed-width slots and the roles of the nodes are symmetrical: they need to discover each other without prior synchronization information in bounded time. In each slot, a node can go to sleep or actively scan for a neighbor by transmitting a beacon at the beginning and the end of the slot, and listening to (scanning for) beacons sent by a neighbor [3]. In this model, the nodes successfully discover each other when two active slots in the wake up/scan schedule temporally overlap.

Meng et al. [4] proposed an approach based on Cyclic Difference Sets (CDS) [5]. They showed that the proposed scheme outperforms Searchlight, a former best performing symmetric discovery scheme [6]. However, the optimal difference codes only exist for a few values of the duty cycle in practice and are currently not realizable except for some special configurations [7].

In this work, we focus on the joining problem for nodes that cannot stay active for long periods of time, i.e., nodes that need to use their radio according to a low duty cycle pattern, not only during the operational phase, but also during the joining phase. For this purpose, we extend the approach based on CDS [4] by taking advantage of Singer CDS [8] and we apply our scheme for the joining problem in 802.15.4e TSCH networks. We evaluate the proposed scheme with a simulation based comparison with other

protocols adapted to TSCH networks: Searchlight [6], B-Nihao, and a duty cycle version of B-Nihao [2]. The comparison shows that for low duty cycles, the proposed scheme consumes less energy than Searchlight for similar joining latency. Moreover, it results in shorter joining latency than B-Nihao for similar energy consumption.

II. BACKGROUND ON NEIGHBOR DISCOVERY

Pozza et al. presented a global survey of proposed neighbor discovery protocols [9]. In their taxonomy, mobility agnostic, asynchronous, temporal overlapping neighbor discovery protocols fall into two categories: probabilistic and deterministic protocols. In probabilistic methods, nodes listen, transmit, or sleep with different probabilities, which leads to unpredictable rendezvous delays and long tails of the joining delay distribution [10]. Deterministic protocols provide bounds on delay based on the Chinese Remainder Theorem (DISCO, U-Connect) [3], [11], quorum-based overlaps (Quorum, Searchlight, Hello) [12], [13], [14], [6], [15], or combinatorial techniques with CDS [16], [4]. In this paper, we focus on the lowest consuming protocols of the literature: Searchlight [6] and B-Nihao [2].

To characterize the duty cycle of the considered joining schemes, we define the Slot Duty Cycle (SDC) that corresponds to the proportion of active slots (in reception Rx or in transmission Tx) during a frame and the Duty Cycle (DC), the proportion of the time the node radio is active during a frame. SDC may be different from DC, if the radio is not active during a whole slot. We use the Rx and Tx time as the metrics of energy consumption—we assume that the radio energy consumption varies linearly with the time the radio stays in these modes, the slope only being different for different hardware platforms.

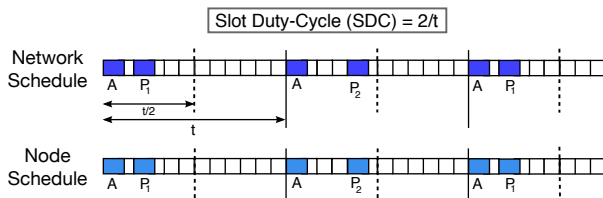


Figure 1: Searchlight-S principle. Anchor slots (A), Probe slots (P).

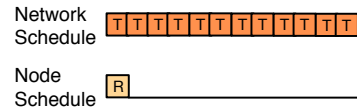
A. Searchlight-S

The striped version of Searchlight (Searchlight-S) [6] is an energy-efficient, well-proven, and fully deterministic quorum-based protocol. In this variant, a node only wakes up twice in a frame of t consecutive slots, so SDC experienced by the network and the joining node is $\frac{2}{t}$. We can choose frame size t to obtain a given target duty cycle both for the network and the joining node. Two active slots are distributed in the first half of a frame as follows:

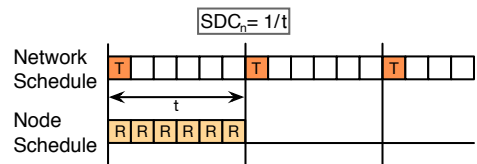
- Anchor slots (A): the first slot of the frame is always an active slot.
- Probe slots (P): the second active slot changes place each frame iteration to “search for” the anchor slot of the other node.

In systematic probing, Probe slot P is initialized as the slot just after Anchor slot A, i.e., the second slot of the frame. It then selects the next slot for each frame repetition. Hence, the probing slots will be in the third slot of the frame at the second repetition, the fourth one at the third repetition, and so on.

To handle non-alignment of slots, Searchlight uses the Disco solution consisting of sending beacons not once per slot, but at the beginning and the end of a slot. Bakht et al. [6] showed that, as nodes transmit two beacons per slot and select their probing slots in a consecutive way, Searchlight can skip one probing slot out of two without impacting performance. Hence, in the striped version of Searchlight (Searchlight-S), the probing slots are only selected as one out of two slots in the frame. To cover the case of perfect slot alignment, Bakht et al. proposed to slightly increase the slot size while in activity and compensate with a reduction of the next inactive slot. Fig. 1 illustrates the principle of Searchlight-S. In this example, the Searchlight pattern is repeated after two frames and active slots are slightly larger than other slots. The following slots are reduced to compensate this increase.



(a) Simplified version of Nihao (S-Nihao).



(b) Principle of B-Nihao for the target network slot duty cycle SDC of $1/t$.

Figure 2: Nihao principles. T stands for Transmitting, R for Receiving.

B. B-Nihao

Nihao [2] adopts a different model with respect to time-slotted protocols by defining dedicated receive and transmit slots. In a transmit slot, a node sends a short beacon at the beginning of the slot so that the node listening in a receive slot can get the beacon. In the S-Nihao variant, a node transmits beacons in each slot with the joining delay and Rx time of only one slot in the worst case (see Fig. 2a).

In its extended version called B-Nihao, nodes send a beacon after t slots and the joining node listen during t slots. Fig. 2b presents the main concept of B-Nihao.

III. S-CDS JOINING SCHEME

Our proposal is inspired by the work of Meng et al. [4]. They showed that a discovery scheme based on perfect Cyclic Difference Sets (CDS) consumes a small amount of energy with bounded delay. However, the generation of perfect CDS is difficult and only a small number of sequences are known: the lowest duty cycle we could find was for a sequence generated from an optimal Golomb Ruler with a 5% duty cycle. We propose to extend the CDS approach to handle low duty cycles (up to 0.3%) with the use of Singer Cyclic Difference Sets, CDS based on Singer cycles [8].

A. Background on Cyclic Difference Sets

Cyclic Difference Sets (CDS) [5] are defined as follows:

Definition 1. A v, k, λ -difference set $D = \{d_1, d_2, \dots, d_k\}$ is a collection of k residues modulo v such that for any residue $\beta_{ij} \not\equiv 0 \pmod{v}$, the congruence

$$d_i - d_j \equiv \beta_{ij} \pmod{v}$$

has exactly λ pairs (d_i, d_j) with d_i and d_j in D .

In other words, $D = \{d_1, d_2, \dots, d_k\}$ is a set of k positive numbers (less than v), with the property that all the difference $\beta_{ij} \pmod{v}$ (for $i \neq j$) are different.

Definition 2. Perfect CDS is a Cyclic Difference Set for which $\lambda = 1$.

To apply CDS to the neighbor discovery problem, let us assume that the network sends beacons at the slots defined by a CDS sequence $D = \{d_1, d_2, \dots, d_k\}$ and the joining node only listen at the slots that follow the same CDS sequence D . In our joining scheme, we exploit the shift property of perfect CDS:

- Every number from 0 up to $v - 1$ occurs in exactly k shifts
- Two different shifts of CDS have $\lambda = 1$ element in common.

Hence, CDS guarantee that for any offset at which the joining node starts the scanning (α), there will be an overlap of an active slot of the network and a listening slot of the joining node.

B. Neighbor Discovery Example

Fig. 3 shows an example of neighbor discovery with a perfect 7,3,1-CDS sequence: $\{0, 1, 3\} \pmod{7}$ [5]. We assume that time is slotted and consider the network following a periodic schedule of a frame repeating in time. The size of the frame is set to CDS parameter v (7 in this case) and the active slots are selected from the sequence: slots 0, 1, and

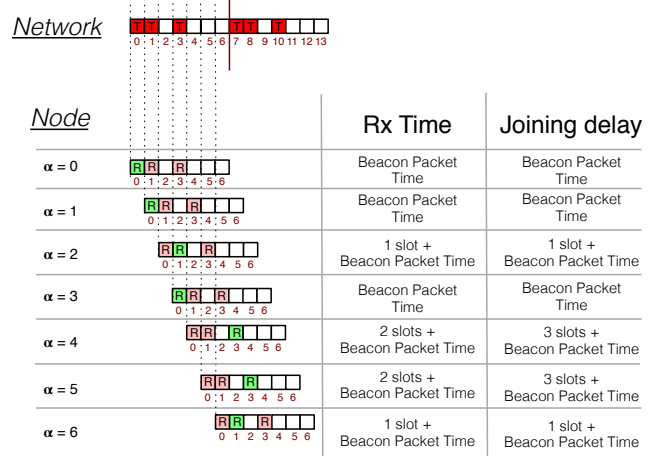


Figure 3: Example for the perfect 7,3,1-CDS: $\{0,1,3\}$.

3 are active while the other slots are in sleep mode. The listening schedule of the joining node is the same. Figure 3 shows different offsets α of the joining node corresponding to possible instants at which the node wakes up and starts scanning. We also present the joining delay and the total time the node consumes energy when receiving (Rx time) until the node succeeds. Note that for legibility of the figure, offsets are full slot values (we deal with non-aligned slots later on).

The example shows the main advantage of a CDS sequence: it gives the worst case delay bound (CDS size v) and allows the network and the joining node to distribute active (advertising or scanning) and inactive (radio switched off) slots to save energy. As all joining schemes, S-CDS presents a trade-off between energy consumption and low joining delay: more sleeping slots means lower energy consumption and longer delay.

C. Singer Cyclic Difference Sets

Singer Cyclic Difference Sets (S-CDS) are CDS based on Singer cycles [8]. They are defined as follows.

Definition 3. For all prime power q , a Singer difference set exists.

Let $G = \text{GF}(q^{n+2})^*/\text{GF}(q)^*$, where $\text{GF}(q)$ is the Galois Field of order q , and $\text{GF}(q)^*$ is the multiplicative group of non-zero element.

The set $D = \{x \in G \mid \text{Tr}_{q^{n+2}/q}(x) = 0\}$ is a Singer v, k, λ -difference set with the following parameters:

$$v = \frac{q^{N+1} - 1}{q - 1}, k = \frac{q^N - 1}{q - 1}, \lambda = \frac{q^{N-1} - 1}{q - 1},$$

where $\text{Tr}_{q^{n+2}/q} : \text{GF}(q^{n+2}) \rightarrow \text{GF}(q)$ is the trace function: $\text{Tr}_{q^{n+2}/q}(x) = x + x^q + \dots + x^{q^{n+1}}$, and $N = 2n$.

We propose to use S-CDS with parameter $N = 2$ and q relatively large to achieve low duty cycles, hence low energy

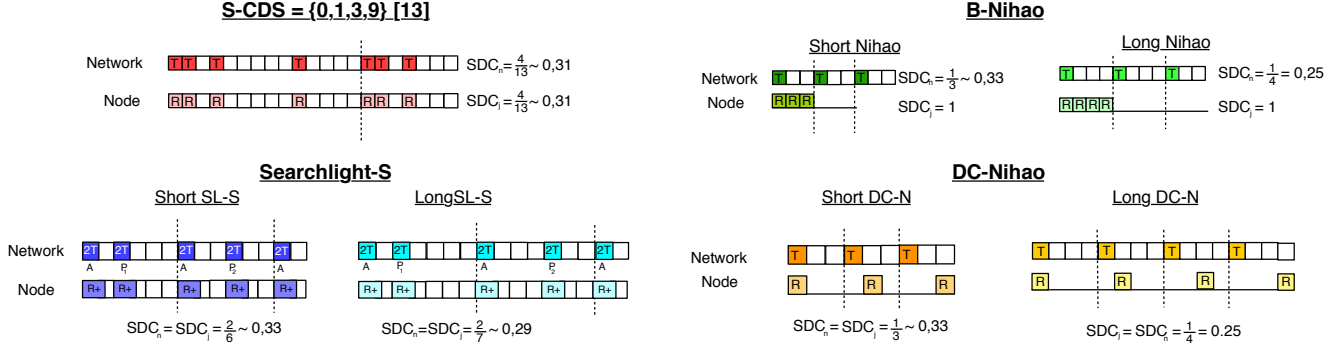


Figure 4: Compared schemes and their duty cycles.

consumption. For instance, the S-CDS of size $v = 3783$ corresponds to the following sequence of active slots: $\{0, 1, 73, 159, 205, 343, 427, 507, 549, 568, 734, 791, 845, 876, 879, 884, 981, 1010, 1058, 1108, 1164, 1170, 1177, 1179, 1197, 1207, 1260, 1307, 1469, 1572, 1589, 1647, 1663, 1707, 1742, 1820, 1824, 1996, 2064, 2257, 2401, 2493, 2515, 2602, 2616, 2640, 2661, 2710, 2861, 2873, 3081, 3107, 3148, 3214, 3362, 3385, 3417, 3592, 3603, 3628, 3668, 3732\} \bmod 3783$ [17], leading to a slot duty cycle SDC of $\frac{62}{3783} \sim 1.64\%$.

D. Application to 802.15.4e TSCH

As mentioned in the introduction, a joining scheme based on CDS intrinsically fits networks with a time-slotted duty cycle schedule like TSCH [1]. TSCH organizes the network operation as a time-frequency map in which the time is structured in *timeslots*. TSCH nodes enable discovery of the network by other nodes with advertisement of Enhanced Beacons (EB). EBs provide the necessary information to actually join the network (such as Absolute Slot Number, timeslot template, hopping sequence, slot allocation in the slotframe, etc.). The joining scheme currently defined for TSCH networks [18] proposes a non duty cycle operation for the joining node—when a node scans for a beacon, it will remain active until it receives EB, which may drain a large amount of energy.

We propose to schedule the transmission of EBs on one well-known frequency channel dedicated to network discovery with active slots defined by S-CDS: the joining node uses the active slots in the S-CDS sequence to scan for beacons during a full TSCH slot.

E. Slot Non-Alignment

A joining node may wake up at any instant, so its slot may be not aligned in time with the slots used by the network. Prior work on neighbor discovery protocols handles slot non-alignment by sending a beacon at the beginning and the end of each active slots [3], [6]. However, Qiu et al. showed that there is no need for two beacons: eliminating the beacon at the end of an active slot will not affect the discovery [2].

Another indirect cause of slot non-alignment may be the clock drift experienced by nodes due to hardware imperfection. In our context, the imprecision of hardware clocks is not relevant since the joining node waking up is fully asynchronous, hence the initial offset may take any values with the same probability. We assume that the clock drift experienced during scanning is negligible because the joining delays we consider are of the order of tens of seconds. Besides, if needed, the node can wake up a little bit earlier than the next scheduled slot: the duration of the required guard interval directly depends on the time spent in sleep state and the hardware clock precision [19].

IV. EVALUATION

We evaluate the performance of the proposed joining scheme with a Python simulator that mimics the joining procedure of a node trying to join an operational TSCH network at random offset α of the network schedule. The simulator computes the join delay and the time spent in reception (Rx time) of the joining node for all possible initial offsets between the network and node schedule.

For a given initial offset, the parameters influencing energy consumption and the joining delay are the following:

- joining strategy,
- strategy for sending beacons,
- DC_j , the radio Duty Cycle of the joining node,
- DC_n , the radio Duty Cycle of the node sending beacons (network Duty Cycle).
- SDC_j (respectively SDC_n), the Slot Duty Cycle of the joining node (the network Slot Duty Cycle, respectively).

This section gives the details of the strategies for sending beacons and joining with which we compare the proposed scheme. They are all adapted to operate in TSCH networks. The adaptation consists first of differentiating the role of a transmitter and a receiver for each strategy. During their active slots, nodes of the network transmit EB according to a TSCH transmission schedule. The joining node turns its

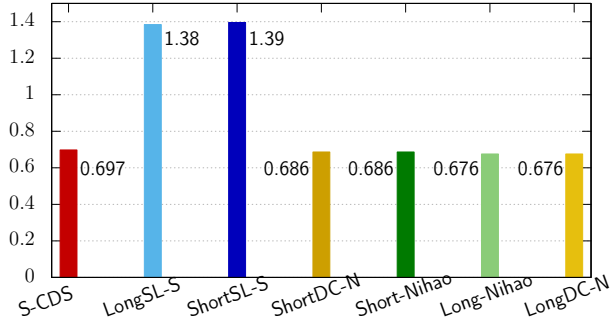


Figure 5: Network radio duty cycle DC_n (%): avg/min/max.

radio on at the beginning of a slot and scans for the entire slot duration fixed to 10 ms.

Slots in Searchlight-S are slightly longer than in other protocols due to handling slot non-alignment. Hence, we slightly increase the Searchlight receiving slot size. Moreover, while all other schemes only transmit one EB per slot, which strictly follows the TSCH timeslot template, we have modified the transmitting slot template for Searchlight-S so that a node will send two consecutive beacons.

Fig. 4 presents the schemes we use for comparisons for a target network slot duty cycle SDC_n of 31% (a value just for understanding the principle), which corresponds to the $\{0, 1, 3, 9\} \bmod 13$ S-CDS sequence [17]. To set all schemes on an equal basis for comparisons, we fix network slot duty cycle SDC_n to the same value for all considered schemes.

Note that the advertiser radio is not in Tx mode during the whole TSCH slot [1], leading to a difference between SDC_n and DC_n . From the joining node point of view, considering the radio duty cycle DC_j or the slot duty cycle SDC_j will not change the performance of most of the studied protocols since the joining node stays in Rx mode during the whole slot. Only Searchlight increases the slot size in Rx mode, so it will exhibit some difference between SDC_j and DC_j .

Moreover, for some schemes, it is not possible to achieve the same value of SDC_n : for instance, the Searchlight schedules have the form of $SDC_n = \frac{2}{t}$ whereas those of Nihao are $SDC_n = \frac{1}{t}$, where t is the schedule period. So, $SDC_n|_{S-CDS}$, the duty cycle for a given S-CDS sequence is not exactly the slot duty cycle of Searchlight $SDC_n|_{SL}$ nor Nihao $SDC_n|_{Nihao}$. To address this issue, we define two other schemes—we choose two SDC_n values closest to given SDC_n : a smaller one (denoted by *short*) and a larger one (denoted by *long*) (see Fig. 4).

In B-Nihao, the joining node continuously scans for beacons, which is not suitable for most devices. Hence, we define another variant of B-Nihao denoted by *DC-Nihao* in

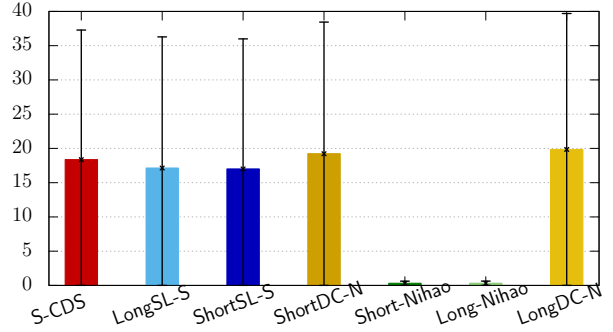


Figure 6: Joining delay (s): avg/min/max.

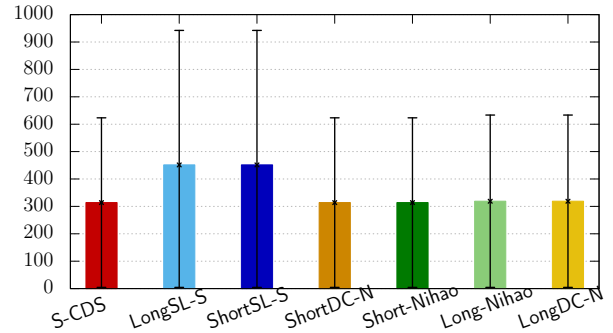


Figure 7: Rx time (ms): avg/min/max.

which the joining node limits its scan to t consecutive slots and it distributes them over t repetitions of the schedule of size t , which results in the same duty cycle for the network and the joining node. Fig 4 also presents *DC-Nihao*: the joining node experiences the same slot duty cycle as the network, i.e., $SDC_j = SDC_n$, similarly to Searchlight and S-CDS.

V. EVALUATION RESULTS

We start with the results for a network slot duty cycle set to $SDC_n = 1.6\%$, an arbitrary low value, and then present the comparisons for varying SDC_n .

A. Low Duty Cycle ($SDC_n = 1.6\%$)

Fig. 5 presents DC_n , the network duty cycle corresponding to the proportion of the Tx time of the advertising network node.

All values are similar except for Searchlight since the DC_n and SDC_n parameters are tightly linked for most schemes: the actual radio duty cycle DC_n is equivalent to

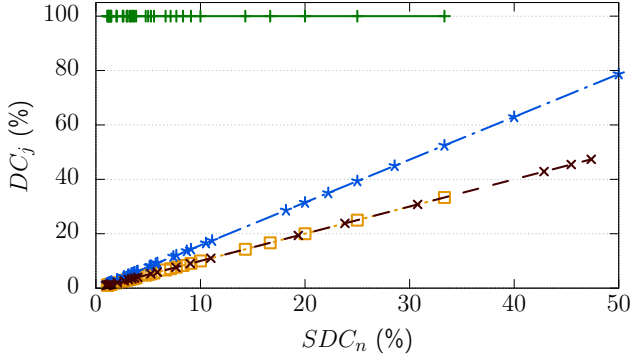


Figure 8: Joining node duty cycle DC_j vs. network slot duty cycle SDC_n .

DC-N —□— B-Nihao —+— Searchlight —*— S-CDS —x—
Figure 9: Legend for Figs. 8, 10, and 11

the SDC_n value of 1.6 divided by the radio Tx activity during a slot. Hence, we can find the expected value with the expression:

$$DC_n = SDC_n \times \frac{4.256}{10} \sim 0.68\%.$$

Note that this expression cannot be applied to Searchlight, since a node transmits two beacons in a slot, which gives the value of $DC_n \sim 1.36$.

Figs. 6 and 7 present the performance of the considered schemes for given $SDC_n = 1.6\%$: the joining delay and the Rx time for a joining node.

Compared to the B-Nihao scheme (Short-Nihao and Long-Nihao), S-CDS achieves similar energy consumption. However, with respect to the joining delay, B-Nihao is significantly faster because the joining node in this scheme operates as an always-on device, which guarantees fast discovery, but may be impossible for battery-powered or energy harvesting nodes.

We can also see that for a similar joining delay, Searchlight consumes more energy than the proposed scheme, both for the average and worst cases. Furthermore, compared to the duty cycled version of Nihao, our scheme consumes a similar amount of energy, however, it achieves a slightly shorter delay, once again both for the average and worst cases. To summarize, our proposal achieves either lower energy consumption or shorter joining delay compared to other schemes for a low value of SDC_n .

B. Varying SDC_n

To compare the schemes for varying values of the duty cycle, we have run the simulation for S-CDS of different sizes (see Fig. 8, 10, and 11). Note that the curves for the *short* and *long* versions of Searchlight, B-Nihao, and DC-

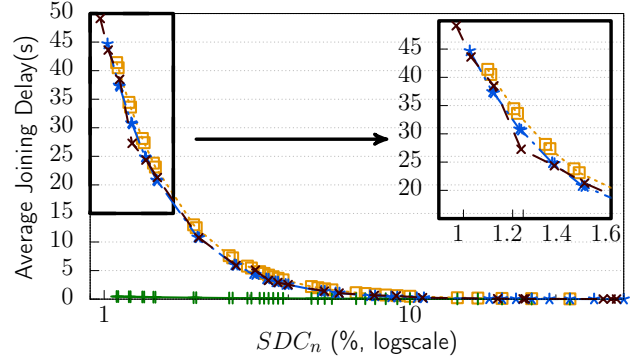


Figure 10: Joining delay of the considered schemes for varying network slot duty cycle SDC_n .

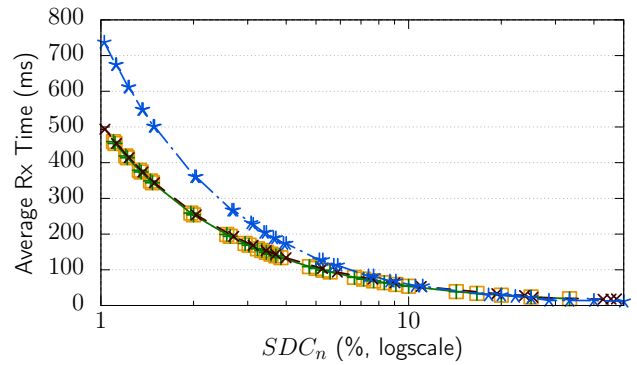


Figure 11: Rx time of the considered schemes for varying network slot duty cycle SDC_n .

Nihao correspond in fact to the same protocol, so they appear as one curve.

We can verify in Fig. 8 that the duty cycle of the joining node, imposed by a given advertising strategy, linearly varies with the network slot duty cycle for all schemes except for the scheme based on always-on scanning (B-Nihao) that results in the 100% duty cycle of the joining node. Searchlight has a slightly higher value of joining node radio duty cycle DC_j due to an increased size of the listening slots.

Figs. 10 and 11 present the performance of the considered schemes for varying SDC_n down to 1%. We can see that our proposal results in low energy consumption and short joining delay for really low duty cycles targeted in the Internet of Things (recall that 1% corresponds to an actual radio duty cycle of $DC_j \sim 0.5\%$). We can even achieve a value of SDC_n close to 0.3% for S-CDS with parameters $v = 86143$ and $k = 294$, something really good for energy harvesting platforms.

Figs. 10 and 11 also confirm the previous results: the lower the duty cycle, the greater the gain of our proposal.

VI. CONCLUSION

In this paper, we have proposed to take advantage of Singer CDS for a joining scheme that results in low duty cycles. It distributes the active periods of nodes over time so that a joining node does not suffer from long scanning periods, in contrast to other schemes.

We have compared S-CDS through simulation with other state-of-the-art schemes adapted to TSCH networks: Searchlight [6], B-Nihao, and a duty cycle version of B-Nihao [2]. The comparison shows that S-CDS performs similarly to Searchlight in terms of the joining delay, while it consumes less energy with a reduced channel occupancy. Moreover, S-CDS achieves similar low energy consumption to the duty cycle version of B-Nihao and reduces the average joining delay. Thus, S-CDS stands as a suitable alternative to B-Nihao for devices with strong energy constraints such as energy harvesting nodes.

In the future work, we plan to implement and evaluate the S-CDS scheme on an experimental testbed and extend it to a multi-channel context to exploit TSCH frequency hopping capability.

REFERENCES

- [1] IEEE, "IEEE Std 802.15.4-2015: IEEE Standard for Low-Rate Wireless Networks," 2015.
- [2] Y. Qiu, S. Li, X. Xu, and Z. Li, "Talk More Listen Less: Energy-Efficient Neighbor Discovery in Wireless Sensor Networks," in *Proc. IEEE INFOCOM*, 2016, pp. 1–9.
- [3] P. Dutta and D. Culler, "Practical Asynchronous Neighbor Discovery and Rendezvous for Mobile Sensing Applications," in *Proceedings of the 6th ACM SenSys*, 2008, pp. 71–84.
- [4] T. Meng, F. Wu, and G. Chen, "On Designing Neighbor Discovery Protocols: a Code-Based Approach," in *Proc. IEEE INFOCOM*, 2014, pp. 1689–1697.
- [5] L. Baumert, "Cyclic Difference Sets," *Lecture Notes in Mathematics*, vol. 182, 1971.
- [6] M. Bakht, M. Trower, and R. H. Kravets, "Searchlight: Won't You Be My Neighbor?" in *Proc. 18th ACM MOBICOM*, 2012, pp. 185–196.
- [7] P. H. Kindt, D. Yunge, G. Reinert, and S. Chakraborty, "Griassdi: Mutually Assisted Slotless Neighbor Discovery," in *Proc. 16th ACM/IEEE IPSN*, 2017, pp. 93–104.
- [8] J. Singer, "A Theorem in Finite Projective Geometry and Some Applications to Number Theory," *Transactions of the American Mathematical Society*, vol. 43, no. 3, pp. 377–385, 1938.
- [9] R. Pozza, M. Nati, S. Georgoulas, K. Moessner, and A. Gluhak, "Neighbor Discovery for Opportunistic Networking in Internet of Things Scenarios: a Survey," *IEEE Access*, vol. 3, pp. 1101–1131, 2015.
- [10] M. J. McGlynn and S. A. Borbash, "Birthday Protocols for Low Energy Deployment and Flexible Neighbor Discovery in Ad Hoc Wireless Networks," in *Proc. 2nd ACM Mobihoc*, 2001, pp. 137–145.
- [11] A. Kandhalu, K. Lakshmanan, and R. R. Rajkumar, "U-Connect: a Low-Latency Energy-Efficient Asynchronous Neighbor Discovery Protocol," in *Proc. 9th ACM/IEEE IPSN*, 2010, pp. 350–361.
- [12] W.-S. Luk and T.-T. Wong, "Two New Quorum Based Algorithms for Distributed Mutual Exclusion," in *Proc. 17th ICDCS*. IEEE, 1997, pp. 100–106.
- [13] Y. Tseng, C. Hsu, and T. Hsieh, "Power-Saving Protocols for IEEE 802.11-Based Multi-Hop Ad Hoc Networks," in *Proc. IEEE INFOCOM*, 2002.
- [14] J. Jiang, Y. Tseng, C. Hsu, and T. Lai, "Quorum-Based Asynchronous Power-Saving Protocols for IEEE 802.11 Ad Hoc Networks," *MONET*, vol. 10, no. 1-2, pp. 169–181, 2005.
- [15] W. Sun, Z. Yang, K. Wang, and Y. Liu, "Hello: a Generic Flexible Protocol for Neighbor Discovery," in *Proc. IEEE INFOCOM*, 2014, pp. 540–548.
- [16] R. Zheng, J. C. Hou, and L. Sha, "Optimal Block Design for Asynchronous Wake-Up Schedules and Its Applications in Multihop Wireless Networks," *IEEE Trans. on Mobile Computing*, vol. 5, no. 9, pp. 1228–1241, 2006.
- [17] D. Gordon. (2017) La Jolla Difference Set Repository. [Online]. Available: <https://ccrwest.org/diffsets.html>
- [18] X. Vilajosana, K. Pister, and T. Watteyne, "Minimal IPv6 over the TSCH Mode of IEEE 802.15.4e (6TiSCH) Configuration," RFC 8180, May 2017. [Online]. Available: <https://rfc-editor.org/rfc/rfc8180.txt>
- [19] E. Morin, M. Maman, R. Guizzetti, and A. Duda, "Comparison of the Device Lifetime in Wireless Networks for the Internet of Things," *IEEE Access*, vol. 5, pp. 7097–7114, 2017.