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A Visco-hyperelastic Model With Damage for the Knee Ligaments Under Dynamic Constraints

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The aim of this study was to identify the behaviour laws governing the knee ligaments, accounting for the damage incurred by the structure under dynamic constraints. The model is developed using a thermodynamic formulation based on the coupling between a viscoelastic model and a damage model. Identification is carried out using the results of dynamic traction tests performed on a bone ligament/bone complex to which traction velocities of around 1.98 m/s were applied. The results show the ability of the model to account for the brittle and ductile failure processes occurring in the cruciate and lateral ligaments, respectively.

Keywords: Knee ligaments; Viscoelasticity; Damage model; Dynamic constraints

INTRODUCTION

In automobile accidents and sports injuries, the ligaments of the knee are often subjected to dynamic constraints involving high strain rate, which results in damage or failure of the structure. Both experimental and numerical approaches have been performed to investigate the human behaviour and tolerance in case of impacts and to develop realistic finite element model of the knee. It is therefore necessary to include a damage model in the behaviour laws governing the knee ligaments under dynamic loading. Many advances have been made in characterising the mechanical behaviour of connective tissues, using thermodynamic [1–3], structural [4–7] or phenomenological formulation [8,9]. However, most of the previous models have dealt only with the mechanical behaviour of these tissues under low strain rates or focusing on the elastic response of the structure. Few models for soft tissues taking into account damage were developed [10–12]. These models were based on a structural formulation taken into account fibres bundles with different initial lengths and different ultimate forces at failure with, for some authors a statistic distribution of these properties. The aim of the present study was to develop a thermodynamic visco-hyperelastic model with finite transformation damage, by combining a viscoelastic model [3] and a damage model [13]. In the following part, the theoretical basis of the model is presented.

Parametric identification is performed with experimental data obtained on 17 human knee ligaments subjected to dynamic traction in the fibres axes. The experimental measurements are briefly discussed in the second part. The results of the identification procedure are presented in the third part. The last section contains the discussion and the conclusion.

THEORETICAL FORMULATION

General Principles

As ligaments are assumed to be submitted to large displacements, the current strain tensor considered, is the Cauchy Green tensor ($\mathbf{C} = \mathbf{F}'\mathbf{F}$). The density is assumed to be time independent. The material satisfies the balance conditions (mass and movement conservation) and the two thermodynamics principle [14].

Viscoelastic Constitutive Law

Based on the local action principle, the principle of material frame-indifference and the principle of fading memory, the stresses occurring in the material can be divided into an elastic stress response, a viscous stress and

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a long-term memory stress [15].

$$\mathbf{S}(t) = S_e(\mathbf{C}(t)) + S_v(\dot{\mathbf{C}}(t); \mathbf{C}(t)) + \int_{\delta}^{\infty} \Sigma\{G(t - \tau), \tau; \mathbf{C}(t)\} d\tau, \quad (1)$$

where $G(t - \tau) = \mathbf{C}(t - \tau) - \mathbf{C}(t)$ describes the space of the strain history, Σ is a general tensor-valued function with the variable $G(t - \tau)$ and τ . δ is a positive real value in the neighbourhood of zero. $\dot{\mathbf{C}}$ is the strain rate tensor. Since the fading memory contribution is negligible compared to the elastic and viscous terms in Eq. (1) and to the damage effects which will be introduced further, the stresses we consider in this study will be reduced to the elastic and viscous contributions.

Free Energy and Viscous Potential

In the case of an elastic deformation, it has been shown [16] using the second thermodynamic principle, that the elastic contribution (S_e) to the response of the model can be derived from a hyperelastic potential.

$$S_e = 2\rho_0 \frac{\partial W_e}{\partial \mathbf{C}} \quad (2)$$

In the same way, the viscous response (S_v) was introduced using a convex dissipative pseudo potential [16].

$$S_v = \mathbf{S} - 2\rho_0 \frac{\partial W_e}{\partial \mathbf{C}} = 2\rho_0 \frac{\partial W_v}{\partial \dot{\mathbf{C}}} \quad (3)$$

Isotropy and Incompressibility

In this work, the ligament is assumed to be homogenous and isotropic. With the representation theorem, the free energy W_e is written as a function of the three invariant of \mathbf{C} (I_1, I_2, I_3).

From Eq. (2), the isotropic stress S_e is obtained by deriving W_e with respect to \mathbf{C} :

$$S_e = 2\rho_0 \left[\frac{\partial W_e}{\partial I_1} \mathbf{Id} + \frac{\partial W_e}{\partial I_2} (\text{tr} \mathbf{C} \mathbf{Id} - \mathbf{C}) + \frac{\partial W_e}{\partial I_3} I_3 \mathbf{C}^{-1} \right]. \quad (4)$$

For an incompressible material, the viscoelastic behaviour law of the ligament becomes:

$$\mathbf{S} = -p \mathbf{C}^{-1} + 2\rho_0 \left[-\frac{\partial W_e}{\partial I_2} \mathbf{C} + \frac{\partial W_v}{\partial \dot{\mathbf{C}}} + \left(\frac{\partial W_e}{\partial I_1} + \text{tr} \mathbf{C} \frac{\partial W_e}{\partial I_2} \right) \right]. \quad (5)$$

Viscoelastic Model With Damage

Damage evolution law is introduced with a scalar ranging from 0 to 1 and based on a Lemaitre and Chaboche

formulation [17,18]. The thermodynamic pairs of variables become (\mathbf{C}, \mathbf{S}) and (D, Y) , where $Y \in R$ is the thermodynamic force associated with the damage D . In this study, damage is assumed to be time independent, not to vary after the ultimate deformation and to affect both the viscous and the elastic properties of the material. The damage energy in the material is written as follows:

$$W(\mathbf{C}, \dot{\mathbf{C}}, D) = (1 - D)(W_e + W_v) \quad (6)$$

The associated thermodynamic force Y ($Y \in R$) is expressed as:

$$Y = -\frac{\partial W}{\partial D} = W_e + W_v \geq 0.$$

The evolution damage law is written using a pseudo potential $\Omega_D(Y)$ which is a convex, positive closed function of Y . As damage is assumed to be time independent, $\Omega_D(Y)$ is replaced by an indicatrice function of a non damage convex, $f_D(Y, D) \leq 0$ [13]. With the normality rule, this becomes:

$$\dot{f}_D = \frac{\partial f_D}{\partial D} \dot{D} + \frac{\partial f_D}{\partial Y} \dot{Y} = 0. \quad (8)$$

As Y depends on the left Cauchy Green tensor \mathbf{C} , \dot{D} is expressed by:

$$\dot{D} = \begin{cases} 0 & \text{if } f_D < 0 \\ \delta \frac{\partial f_D}{\partial Y} & \text{if } f_D = 0 \end{cases} \quad (9)$$

with

$$\delta = \left\langle \frac{\partial f_D}{\partial Y} \frac{\partial Y}{\partial \mathbf{C}} : \dot{\mathbf{C}} - \frac{\partial f_D}{\partial Y} \frac{\partial f_D}{\partial D} \right\rangle, \quad \text{and } \langle \dots \rangle \text{ the positive part.}$$

Application to the Ligaments

The parameters of an isotropic incompressible viscoelastic model accounting for damage model of the knee ligaments are calculated on the basis of tensile traction tests with a constant velocity of 1.98 m/s in the fibre directions. Considering l_0 the initial length, l the current length and v the velocity (which is assumed to be constant), the elongation of the ligament is defined by $\lambda = l/l_0$ and $l = l_0 + vt$. The tangent linear application is also given by $F_{ij} = 0$ if $i \neq j$, $F_{11} = \lambda$ and $F_{22} = 1/\sqrt{\lambda}$.

Here the free energy function W_e [19], the viscous pseudo-potential W_v [20] and the damage f_D function are expressed, respectively as:

$$W_e = \alpha \exp[\beta(I_1 - 3)] + \gamma(I_2 - 3), \quad (10)$$

$$W_v = \frac{\eta}{4} \text{tr}(\dot{\mathbf{C}})^2 (I_1 - 3), \quad (11)$$

$$f_D = Y - Q^{D+D_0} \sqrt{n} \leq 0. \quad (12)$$

Initially the ligament is assumed to be stress free $\mathbf{S}(\mathbf{Id}) = 0$. The experimental traction tests were performed without applying any lateral forces to the specimen so that $\mathbf{S}_{22} = 0$. Under these conditions, the constitutive equations become after some calculations:

$$\mathbf{S}_{11} = -p \frac{1}{\lambda} + (1 - D)\eta K \lambda^2 + 2\alpha\beta(1 - D) \times \left[\exp(\beta K) - \frac{1}{\lambda} \right], \quad (13)$$

with

$$p = +2\alpha\beta C_{22}(1 - D)\exp(\beta K) - \alpha\beta C_{22}(1 - D) \times \left(\lambda^2 - \frac{1}{\lambda} \right) - (1 - D)\eta K \frac{\lambda^2}{2\lambda^4} \quad (14)$$

and

$$\gamma = -\frac{\alpha\beta}{2} \quad \text{and} \quad K = \left(\lambda^2 + \frac{2}{\lambda} - 3 \right) \quad (15)$$

If $f_D = 0$, then:

$$\dot{D} = \frac{n(D + D_0)\dot{\lambda}}{Q^{D+D_0}\sqrt{n}} \left[-\frac{\alpha\beta}{\lambda} + \alpha\beta \exp\left(\beta\left(\lambda^2 + \frac{2}{\lambda} - 3\right)\right) \right] \quad (16)$$

If $f_D < 0$, then:

$$\dot{D} = 0. \quad (17)$$

The complete viscoelastic model with damage introduced above, is characterised by six parameters, two accounting for the elastic behaviour (α , β), one for the viscosity (η) and three for the damage (n , Q , D_0). The stress and the hydrostatic pressure are expressed as a function of the time variable and parameters (α , β , η , n , Q , D_0) after analytically integrating the differential Eqs. (16) and (17). Identification of these parameters is performed by minimizing the distance between the simulated and experimental force/displacement curve. Minimization is performed using a conjugate gradient algorithm.

Usually, identification of the viscous component cannot be performed with only one type of experimental test. It is necessary to perform tensile testing with different strain rates or to perform relaxation tests. As we focused on the damage behaviour in the present study, cyclic loading before traction up to failure can damage the material and might therefore affect the mechanical behaviour. One solution to this problem might consist of performing further tensile tests specifically for the purpose of studying the viscous behaviour. Since experimentally the prescribed initial forces have attenuated the viscous effects and since from a numerical point of view the value of η did not significantly affect the results, η was taken to be small. The behaviour law is written as a relation between the

stress and the strain, whereas the identification process is performed on the force displacement relations in order to be in agreement with the experimental measurements. Ligaments are a highly non-homogeneous material in which it is very difficult to determine the deformation and stress because of the ligaments length and section.

Experimental Device

Identification was performed with the knee ligaments of four human donors obtained from the Faculty of Medicine of the Mediterranean University (Marseille, France). All the knee cadavers were treated using Winckler's conservation solution and refrigerated at $+3^\circ\text{C}$ [21]. The dissection consisted in removing a bone/ligament/bone complex with each ligament. The biometry consisted in evaluating the length, section (in the middle and at the two insertion points) and relative orientation of the ligaments insertions surfaces before and after removal. According to the study [22], the ligaments were assumed to have elliptic sections. The bone insertions were embedded with resin in the spherical part. The sample was fixed in a specially designed apparatus (Fig. 1) to perform tensile traction tests in the direction of the fibre axes with velocities of 1.98 m/s up to the failure of the structure. The special feature of this apparatus [23] is that it can be used to perform dynamic tensile traction tests (using a hydraulic Schenck jack) in all the possible relative orientations of the ligaments (corresponding to the anatomical axes or other). The tests were carried out at room temperature (around 23°C), and a pre-load comprised between 5 and 25 N was applied to each ligament prior to each test. In

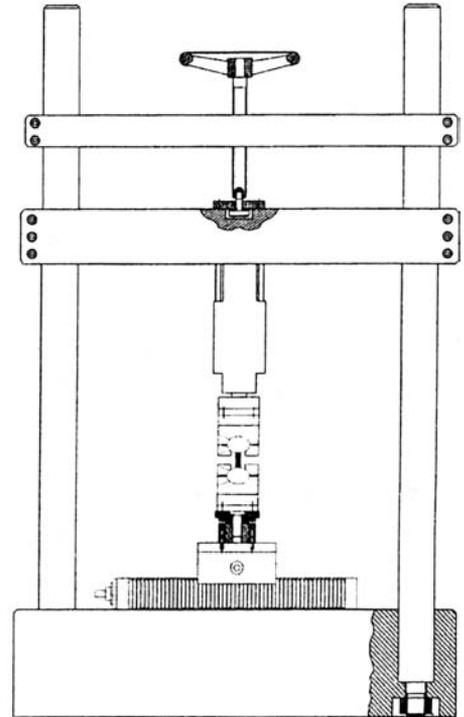


FIGURE 1 The experimental device.

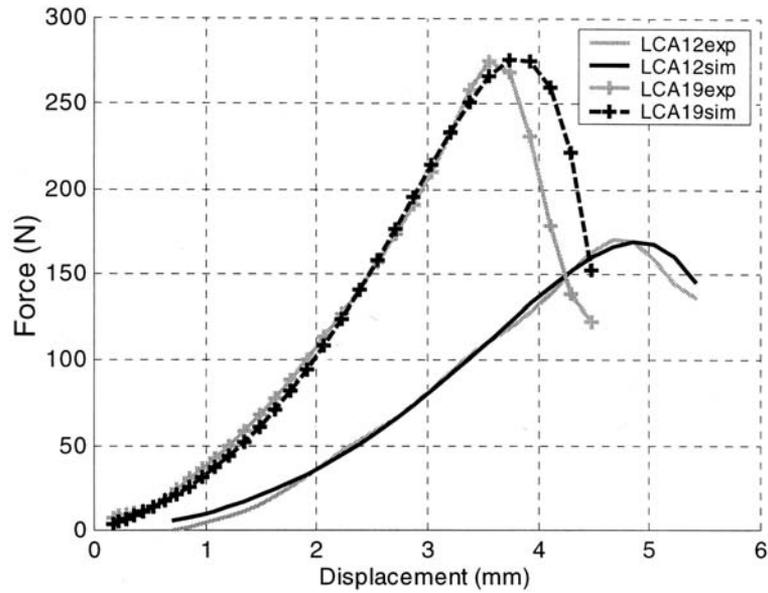


FIGURE 2 Identification of the behaviour in the case of two anterior cruciate ligaments 12 (LCA12) and 19 (LCA19).

order to keep the ligament sufficiently moist, Winckler solution was regularly applied. Measurements were performed with a 3D Kistler force transducer, an accelerometer and a displacement transducer. Data acquisition were performed at a sampling frequency of 10 kHz and corrected using a 1 kHz filter.

RESULTS

The force versus displacement obtained curves plotted (Figs. 2–5) can be divided into three parts.

First they contain a non linear elastic region with a low modulus, which was negligible because of the force initially applied. Secondly an elastic linear region for which Young’s modulus can be calculated, and lastly damage region up to failure. The damage phase of cruciate ligaments is very brittle, occurs at deformation levels of

around 18% (in the case of equally pre-stressed ligaments), and consists of the two-stage failure of the two main fibres. With lateral ligaments, this phenomenon is more ductile, occurs at deformation levels of around 24%, and involves the failure of only one bundle of fibres.

Parameters of the behaviour law were identified on each ligaments. These parameters were determinate considering the whole behaviour for lateral ligaments and considering only the first damage step for cruciates ligaments. For these results, comparisons between the average error observed between the simulated and experimental behaviour showed that the error with parametric identification was less than 8% for all the ligaments. In regard of the pre-stress applied, the viscous part of the model has not a sensible influence on its response. Nevertheless, it could be considered as a mean of regularising the behaviour at the end of the elastic phase.

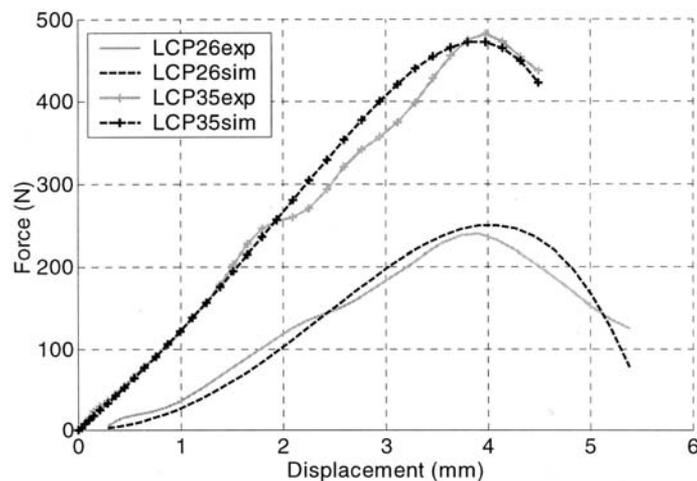


FIGURE 3 Identification of the behaviour of two posterior cruciate ligaments 26 (LCP26) and 35 (LCP35).

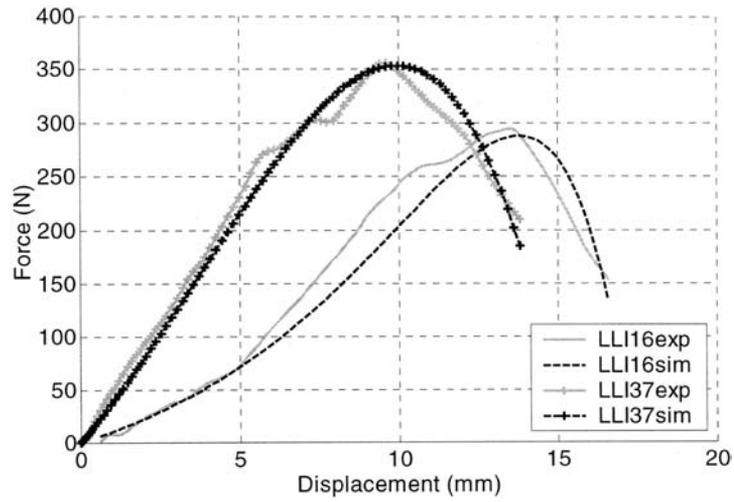


FIGURE 4 Identification of the behaviour of two medial collateral ligaments 16 (LLI16) and 37 (LLI37).

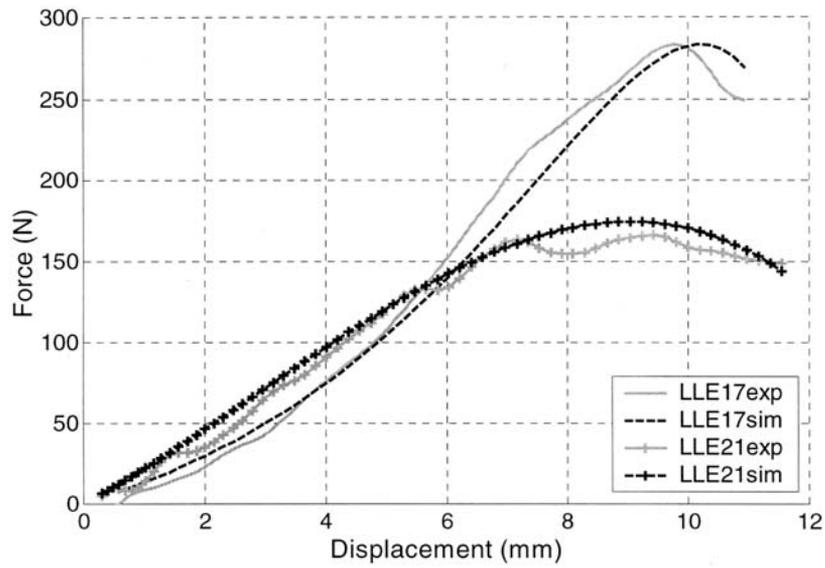


FIGURE 5 Identification of the behaviour of two fibular collateral ligaments 17 (LLE17) and 21 (LLE21).

TABLE I Characteristics of ligaments LCA12 and LCA19

Ligament	S (mm ²)	I_0 (mm)	α	β	η	D_0	Q	n	Error (N)
LCA12	50	19	0.223	7.341	1.5×10^{-5}	0.755	199.99	1	3.79
LCA19	43	20	0.480	10.863	1.35×10^{-5}	0.429	164.97	1	11.06

TABLE II Characteristics of ligaments LCP26 and LCP35

Ligament	s (mm ²)	I_0 (mm)	α	β	η	D_0	Q	n	Error (N)
LCA26	55	21	1.671	3.738	0.2	0.99	4.54	1	14.27
LCA35	55.1	26	5.09	3.84	3.8×10^{-6}	0.1	77.3	1	12

TABLE III Characteristics of ligaments LLI16 and LLI37

Ligament	S (mm ²)	I_0 (mm)	α	β	η	D_0	Q	n	Error (N)
LLI16	85	35	0.838	2.061	2.8×10^{-7}	0.20	93.85	1	16.18
LLI37	81.4	32	31.13	0.165	7.6×10^{-7}	0.07	33.69	1	11.74

TABLE IV Characteristics of ligaments LLE17 and LLE21

Ligament	s (mm ²)	I_0 (mm)	α	β	η	D_0	Q	n	Error (N)
LLE17	82	35	0.959	1.301	5.2×10^{-8}	0.0001	45.29	1	9.18
LLE21	62.8	19	2847	0.002	2.5×10^{-7}	0.703	153.65	1	6.65

Comparison between numerical and experimental response of the ligaments to dynamic tensile loading is presented in Figs. 3–5. A summary of the ligaments properties, biometry and parameters of the behaviour law obtained on each ligament was presented in Tables I–IV.

DISCUSSION

In this study, a thermodynamical formulation is used to describe the behaviour law governing knee ligaments. The model obtained accounts differently for the mechanical behaviour, depending on whether it is hyperelastic, viscoelastic or viscoelastic with damage. The structure is assumed to be isotropic and the results obtained with the model, which is written using finite transformation are in good agreement with the already published results [23–25]. The damage components is able to take into account for the variability of the behaviour observed at the occurrence of brittle and ductile failure in the cruciate and lateral ligaments, respectively.

On the Model Hypothesis

The ligaments are assumed to be isotropic because of the directions in which the loading occurs. Transverse isotropy would be an interesting possibility, but it would be very difficult to determine the response of the structure under transverse loading conditions. The material is assumed to be initially stress-free, which is equivalent to making a change of variable in the stress. \mathbf{S}_{22} is assumed to be zero, which was checked experimentally. Materials of this kind have generally been assumed to be incompressible [22], in view of their high water content (water accounts for more than 80% of the total constituents).

On the Thermodynamic Potential

The elastic potential used in this study is well adapted to the context of finite transformations. Although Fung (1993) or Pioletti (1997) have established the existence of a direct correlation between α , β and the material characteristics, no correlations were tested in this study. In the identification of the parameters, no particular attention

was paid to which of the solutions of the minimization problem should be chosen. It would be worth carrying out further studies on the choice of the most physically reliable solution of the minimization problem.

Short memory effects are described via the viscous pseudo potential, which is advantageous in that it entails viewing the strain rate as an explicit variable, and the short term memory effects are characterized with an independent parameter η . With the high strain rate applied to the ligaments in the present study, the viscous effects are small, which is numerically checked with η values. However, the short-term memory has regularising effects at the end of the elastic phases. In our experimental campaign the number of specimens available to be tested, was not sufficient to perform a complete identification of η . For such an identification cyclic or relaxation loadings prior to the traction up to failure are necessary, these are liable to damage the structure as well as affecting its properties. As we were limited here by the number of samples available, we decided not to risk altering the structure.

Damage laws give a good description of damage processes, except in the case of the cruciate ligaments, in which only the first bundle of fibres is subject to failure. In this study, all the ligaments were tested with the same velocity, which made it difficult to determine its effects.

Further studies require to be performed at various velocities in order to determine the time dependence. The indicatrice function giving the nondamage convex depends only on the elastic component. This shows that the damage is not time dependent and cannot evolve any further after the ultimate strain. The damage is controlled here using the damage yield D_0 , the intensity of the damage Q and the linearity parameter n (which is fixed and taken to equal one). These parameters are not directly associated with the physics of the structure. In further studies, the physics of the evolution law damage variables might be investigated by developing a structural model for the ligaments.

In order to complete the identification of the viscous parameters and the description of the damage in the structure, further experiments now require to be carried out with various strain rates.

On the Identification Algorithm

Fitting the parameters requires minimizing the distance between the simulated and experimental responses. This optimisation is performed with a conjugate gradient algorithm, using quadratic estimation on the central derivative. The relative accuracy and convergence criteria amount were 10^{-5} and 10^{-3} , respectively, with a maximum number of iterations of less than 150. The average error obtained with all the ligaments did not exceed 8% of the maximum force. Although this algorithm gives good results, each set of parameters is the first local minimum in the identification process.

CONCLUSION

The present thermodynamic approach, which was based on an extension of the Pioletti model to large displacements with high strain rates, combined with a damage model [13], was found to satisfactorily describe the failure process at work in the cruciate and lateral knee ligaments. In this model, there is no correlation between the parameters (α , β , η , n , Q , D_0) and the physics or the measurements performed during the experimental testing. It would be interesting to attempt in the future to find a relation between the parameters and physical principles governing the behaviour of the material. In this case, the model would have to take the anisotropy of the material into account. Other experiments will also be required to determine the time dependence of the damage evolution law.

NOMENCLATURE

\mathbf{S}	second stress Piola Kirschhoff tensor
\mathbf{C}	Cauchy Green strain tensor
\mathbf{C} .	Strain rate tensor
t	time
ρ_0	density
W_e	specific free energy
W_v	viscous pseudo potential
\mathbf{Id}	identity tensor
I_1, I_2, I_3	the three invariants of the Cauchy Green tensor
α, β, γ	free energy parameters
η	viscous parameter
p	hydrostatic pressure
D	damage
\dot{D}	damage rate
Y	thermodynamic force associated with the damage
D_0	non damage yield
Q	parameters contributing to the intensity of the damage

n	linearity of the damage evolution
λ	elongation
l_0	initial length
l	actual length
s	section
$V = 1.98 \text{ m/s}$	velocity
tr	trace operator
S_e	Elastic stress contribution
S_v	Viscous stress contribution (short memory effects)
f_D	indicatrice function of a non damage convex
\dot{f}_D	time derivative of the indicatrice function of a non damage convex
R	Set of real numbers

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