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# Evidence of a dilatant vacuum

Marco Fedi\*

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## Abstract

A formula to exactly calculate vacuums viscous force is presented as a modified, nonlinear Stokes formula for a shear-thickening vacuum. It is shown that the formula is successful in exactly calculating the anomalous deceleration of the Pioneer probes, therefore challenging the previously accepted explanation of the anomaly from approximated thermal simulations. Moreover, resorting to the formula, the equation of general relativity for perihelia precession (classical test of general relativity) is derived, indicating that perihelia precession is actually due to the presence of a shear-thickening quantum vacuum. When the formula is applied to planetary orbits, it indicates stability over billions of years, making it overall a useful tool for astronomy, astrophysics and aerospace engineering, while it sheds new light on the nature of the physical vacuum and on the phenomenon of relativistic mass increase.

## 1 Introduction

Some authors considered the possibility that the physical vacuum may actually be a superfluid, a special Bose-Einstein condensate [1, 2, 3, 4, 5, 6, 7, 8]. Here it is shown that it rather behaves as a dilatant fluid as shear stress increases. Sect. 2 introduces a modified nonlinear Stokes's formula for motion in a dilatant vacuum. In Sect. 3, divided in three subsections, by applying this formula, even for non-relativistic speed, vacuum's nonlinear apparent viscosity emerges as the real cause of

the anomalous precession of perihelia and of the Pioneer anomaly. The latter is currently considered solved after the thermal simulations presented in 2012 but assumptions and approximations were adopted [9, 10], which are now challenged by the direct result presented in Sect. 3.1, which exactly matches the NASA deceleration value of  $-8.74 \times 10^{-10} \text{m} \cdot \text{s}^{-2}$ , while thermal simulations gave  $-7.4(\pm 2.5) \times 10^{-10} \text{m} \cdot \text{s}^{-2}$ , considering the uncertainties they entail. In Sect. 3.2 the output of the modified Stoke's formula says that due to the large masses of planets (large momentum), planetary orbits are stable over billions of years. A third correct result concerns Mercury's perihelion precession. Indeed, Sect. 3.3 shows that the anomalous positive contribution to perihelia precession (a classical test of general relativity) is actually due to the interaction of the planet with a dilatant vacuum, since Einstein's formula for perihelia precession is rederived in a dilatant vacuum. Sect. 3.4 shows that the relativistic phenomenon of mass increase apparently detected in synchrotrons may actually correspond to the braking action of a dilatant vacuum on accelerated particles, which lets detect an increased inertia and implies a maximum speed always lower than that of light and depending on a synchrotron's maximum energy. Sect. 4 discusses about the consequences of the results presented in this study as regards the field of aerospace engineering, emphasizing the importance of objects of great mass to approach as much as possible and with less energy expenditure the speed of light, reversing the current beliefs coming from special relativity, which actually resorted to the insuperability of the speed of light in Lorentz factor as a matter of fact, without adducing quantum explanations. Finally Sect. 5 presents the conclusions of this work and reflects on the possible causes of vacuum's shear-thickening behavior.

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## 2 Methods: modified Stokes's law for a dilatant vacuum

Stokes' formula, derived in 1851, to calculate the viscous force acting on a body traveling through a viscous, Newtonian fluid [11] reads

$$F_v = -6\pi r v \eta \quad (1)$$

where  $v$  is the translational velocity,  $r$  the radius of the object (the law refers to spherical shape) and  $\eta$  is a coefficient of dynamic viscosity expressed in Pa·s. Since here we want to hypothesize and verify that the physical vacuum behaves as a shear-thickening (dilatant) fluid, its apparent viscosity has to nonlinearly increase according to the applied shear stress, thus the simple coefficient of linear apparent viscosity,  $\eta$ , in Eq. (1), which is valid for Newtonian fluids, is not appropriate. To identify the right mathematical behavior of vacuum's apparent viscosity, there is first an element for us to consider. When a dilatant fluid transiently becomes solid-like within a relaxation time [12] due to shear stress, its quasi-lattice [13, 14] can be traveled either by shattering it (but let us assume this does not occur for the vacuum) or by passing through its interstices. Since the physical vacuum likely possesses the finest structure in nature, nothing is smaller than its interstices, so motion through them is excluded. But sound can still travel in this finest quasi-lattice (phonon modes can emerge [13, 14]). This means that we need to put a vertical asymptote at the speed of sound in the physical vacuum as a scalar field ( $v_{svac}$ ). The mathematical behavior of vacuum's apparent viscosity can be therefore expressed as

$$\gamma \equiv \frac{d}{d\beta} \arcsin \beta = \frac{1}{\sqrt{1-\beta^2}} \quad (2)$$

with  $\beta \equiv v/v_{svac}$ . Returning to Stokes's law and considering Eq. (2) in the form  $\gamma-1$ , vacuum's apparent viscosity can be now nonlinearly expressed for the case of a shear-thickening fluid with asymptote at the speed of sound and Eq. (1) becomes

$$F_{vac} = -6\pi r (\eta \gamma - 1) \kappa = -6\pi r \left( \frac{\eta}{\sqrt{1 - \left(\frac{v}{v_{svac}}\right)^2}} - 1 \right) \kappa \quad (3)$$

where  $\eta = 1 \text{ Kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$  is a unitary coefficient of dynamic viscosity and  $\kappa$  is a unitary constant expressed in m/s, working as a dimensional correction. It is very interesting, and that should let us reflect, to notice that if we use the equivalence  $v_{svac} = c$ , where  $c$  is the speed of light, the issues illustrated in the following sections are correctly solved thanks to the modified Stokes's formula for a dilatant vacuum (4) and this fact cannot be ignored. But can we use this equivalence? Yes. Indeed, by stating that in a dilatant vacuum there is an asymptote at the speed of sound and since we neither detected asymptotes at a speed lower than that of light, nor of course over the speed of light, we must logically conclude that Eq. (2) is Lorentz factor. Our modified Stokes formula (3) therefore changes into

$$F_{vac} = -6\pi r \left( \frac{\eta}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) \kappa \quad (4)$$

and is ready to be used to correctly solve the following anomalies.

## 3 Results: Pioneer, Mercury, orbital stability and particle acceleration

### 3.1 Exact value for the Pioneer acceleration

Being the anomalous negative acceleration of the Pioneer spacecrafts 10 and 11 well-known (concrete investigations of the anomaly started in 1994 [15]), it is not necessary to summarize here this issue. In the light of the result presented below, it appears evident that this problem has not been correctly solved yet, despite copious previous investigations. At the moment it is indeed generally accepted that the solution to this anomaly is due to anisotropic recoil of thermal photons [9, 10, 15, 16, 17, 18, 19]. However, by carefully analyzing these works, we see that due to the absence of an exact thermal model of the probes, especially as far as their internal heat dynamics is concerned, several numerical approximations and assumptions have been made, as for instance in [10], and that weakens the scientific solidity of the thermal explanation, whose result [9] may be approximated to  $-7.4(\pm 2.5) \times 10^{-10} \text{ m} \cdot \text{s}^{-2}$ . On the contrary, the exact and direct solution is made

available from the interaction of the Pioneer spacecrafts with a dilatant quantum vacuum. This is evident by resorting to Newton's second law to express acceleration, where the force is Eq. 4, and entering accurate data of the Pioneer

$$\begin{aligned}
 a_P &= -\frac{F_{vac}}{m} = -\frac{6\pi r \left( \frac{\eta}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1 \right) \kappa}{m} \\
 &= -\frac{6\pi \cdot 1.371 \text{ m} \cdot \left( \frac{1 \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}}{\sqrt{1-\left(\frac{36737 \text{ m} \cdot \text{s}^{-1}}{299792458 \text{ m} \cdot \text{s}^{-1}}\right)^2}} - 1 \right) \cdot 1 \text{ m} \cdot \text{s}^{-1}}{222 \text{ kg}} \quad (5) \\
 &= -8.74 \times 10^{-10} \text{ m} \cdot \text{s}^{-2}.
 \end{aligned}$$

As we see, the result exactly corresponds to the by NASA detected anomalous negative acceleration of the Pioneer

$$a_P = -(8.74 \pm 1.33) \times 10^{-10} \text{ m} \cdot \text{s}^{-2} \quad (6)$$

The data used for the calculation are: the mass of the spacecrafts (258 kg) minus the mass of the burned fuel (36 kg), the radius of the antenna, 1.371 m (antenna diameter is 9 ft) and the maximum speed of the probe (36737 m/s = 132252 Km/h, as indicated by NASA Scientific and Technical Information Office [20]) after the swing-by caused by Jupiter. This exact and direct result cannot be ignored and the Pioneer issue should be therefore reopened. Moreover, NASA should of course consider to Doppler track other probes which are expected to reach a speed not lower than that of the Pioneer after the Jupiter flyby. Unfortunately this was not the case of the New Horizons spacecraft, for which data are missing.

### 3.2 Stability of planetary orbits

As far as orbital stability over time despite the existence of a dilatant vacuum is concerned, by using the modified Stokes's formula (4) in Newton's second law, exactly as for the case of the Pioneer probes analyzed above, and putting in it the data of a planet (mean radius, mean orbital velocity and mass), as for instance Mercury, we see it decelerates because of vacuum's viscosity by the fol-

lowing negligible value

$$\begin{aligned}
 a &= -\frac{6\pi(2.44 \times 10^6 \text{ m}) \left( \frac{1 \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}}{\sqrt{1-\left(\frac{47362 \text{ m} \cdot \text{s}^{-1}}{299792458 \text{ m} \cdot \text{s}^{-1}}\right)^2}} - 1 \right) \cdot 1 \text{ m} \cdot \text{s}^{-1}}{3.3 \times 10^{23} \text{ Kg}} \quad (7) \\
 &= -1.74 \times 10^{-24} \text{ m} \cdot \text{s}^{-2}
 \end{aligned}$$

which corresponds to a decrease in mean orbital speed of  $5.48 \times 10^{-8} \text{ m} \cdot \text{s}^{-1}$  over  $10^9$  years. That means planetary orbits are stable over billions of years despite the existence of a dilatant vacuum. On the contrary, the much smaller momentum of the Pioneer probes let them more evidently decelerate under the action of vacuum's viscosity. Indeed, orbital deceleration of more massive planets is even lower than for Mercury: for the case of Jupiter, from Eq. (4), it is only  $6.59 \times 10^{-28} \text{ m/s}^2$ .

### 3.3 Deriving Einstein's formula for the precession of perihelia in a dilatant vacuum

Net of classical gravitational contributions, perihelia precessions show an anomalous positive contribution, which is particularly evident for the planet Mercury. The correct calculation of this anomaly is one of the classical proofs for general relativity (GR). However, here it is demonstrated that the role of curved spacetime is actually played by a dilatant vacuum, yielding a different qualitative explanation able to build a bridge from quantum physics. In GR [21], the anomalous perihelia precession is represented by a formula which can be observed in three equivalent forms

$$\Delta\phi = \frac{24\pi^3 a^2}{T^2(1-e^2)c^2} = 6\pi \left( \frac{v}{c} \right)^2 \frac{1}{1-e^2} = 6\pi \frac{GM}{a(1-e^2)c^2} \quad (8)$$

where  $\Delta\phi$  expresses the relativistic contribution to perihelia precessions in radians per revolution corresponding, using the data of Mercury, to the known value of  $\sim 43''$  per century (or  $5.022 \times 10^{-7} \text{ rad/rev.}$ ),  $a$  is the semi-major axis,  $T$  the orbital period and  $e = 0.205$  orbital eccentricity. The expression in the center of Eq. (8) is obtained via the equivalence  $T^2 = 4\pi^2 a^2 / v^2$ , as mean orbital velocity is used, and in the expression on the right we see the stable second cosmic velocity squared,  $v^2 = GM/a$ , where  $GM/a$  is the classical gravitational potential, putting the radius  $r = a$ . As for the case of the Pioneer, we resort to

the modified Stokes's law for a dilatant quantum vacuum (4). In this case, the treatment of the planet as a point mass is respected, so the direct proportionality to planetary radius is not taken into account. Resorting to the nondimensionalized norm of Eq. (4) after dividing by  $r$  yields

$$6\pi \left( \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) = 6\pi(\gamma - 1) \quad (9)$$

which can be expressed in radians. Resorting to Taylor we proceed using the approximation

$$2(\gamma - 1) \approx \left(\frac{v}{c}\right)^2 \quad (10)$$

and Eq. (9) now reads

$$3\pi \left(\frac{v}{c}\right)^2 = 3\pi \frac{GM}{ac^2} \quad (11)$$

where, on the right, the planet's velocity is written as stable second cosmic velocity squared, as in the rightmost expression in Eq. (8). As we refer to motion in elliptic orbit, we have to use the elliptic parameter, correcting  $a$  into  $a(1 - e^2)$  and we obtain a formula which exactly gives 1/2 the result of GR (8)

$$\Delta\phi = 3\pi \frac{GM}{a(1 - e^2)c^2} = 3\pi \left(\frac{v}{c}\right)^2 \frac{1}{1 - e^2} \quad (12)$$

This result would be wrong (1/2 compared to that of general relativity, as also in [22]) if we considered  $v$  as the mean orbital velocity. Actually, the real situation is represented in Fig.1 on the left, with variable orbital velocity. Since mean orbital velocity is  $v = (v_{max}/2) + (v_{min}/2)$ , we are allowed to use the reduced model on the right side of Fig.1. For describing the precession occurring in a semi-orbit, Eq.(12) is now correct. Then we use it twice (respectively with  $v_{max}$  and  $v_{min}$ ) to express the precession during a full orbit and that yields the full result of GR (8)

$$\begin{aligned} \Delta\phi &= 3\pi \left(\frac{v_{min}}{c}\right)^2 \frac{1}{1 - e^2} + 3\pi \left(\frac{v_{max}}{c}\right)^2 \frac{1}{1 - e^2} \quad (13) \\ &= 6\pi \left(\frac{v}{c}\right)^2 \frac{1}{1 - e^2} = \frac{24\pi^3 a^2}{T^2(1 - e^2)c^2} \end{aligned}$$

where  $v$  is mean orbital velocity. Merging in a single formula the steps presented in this subsection, we can now

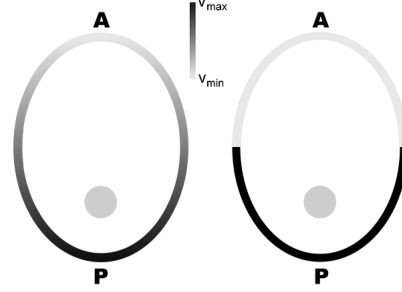


Figure 1: Left: variable orbital velocity in the elliptic orbit (P and A are respectively perihelion and aphelion). Right: a reduced model used in the present study which considers half an orbit at max orbital velocity and the other one at minimum velocity.

look at the relationship between Eq. (4), which expresses the viscous force in a dilatant vacuum, and the anomalous contribution to the precession of perihelia now reads

$$\Delta\phi_{anomalous} \equiv \left\| \frac{2F_{vac}}{\kappa\eta r(1 - e^2)} \right\| \quad (14)$$

In this way it is demonstrated that perihelia precession actually occurs via the interaction with a dilatant vacuum, of which Einstein's curved spacetime is a mathematical representation. After all, this is in agreement with the stress-energy tensor of the field equation, in which  $T^{00}$  is vacuum's density ( $\rho_{vac}$ ), also present in the cosmological constant  $\Lambda = \kappa\rho_{vac}$ , and the remaining components can be as well hydrodynamically interpreted, being pressure, shear stress, momentum flux and momentum density. Interestingly, also a recent study by Conti and Marcucci [23] shows that the interaction at Planck-scale with a quantum fluid may be cause of precession and this may be considered as a first investigation of the micro-scale reasons behind the macroscopic effects exerted by the dilatant vacuum which are here analyzed.

### 3.4 Particle acceleration in a shear thickening vacuum: revisiting relativistic mass

Taking into account a dilatant vacuum whose apparent viscosity obeys Lorentz factor (Sect. 2), it is obvious that particles which are accelerated toward the speed of light are subject to a nonlinearly increasing viscous force

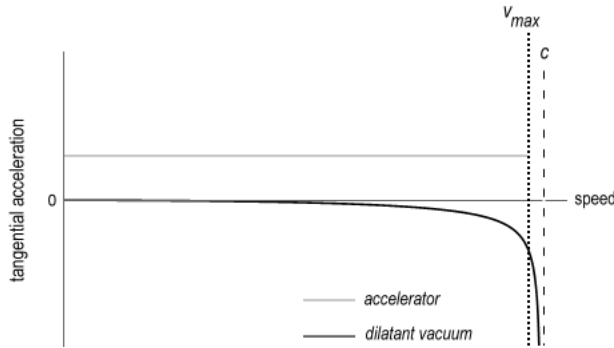


Figure 2: Total tangential acceleration is zero when the negative acceleration of the particles due to the presence of a dilatant vacuum equals the acceleration applied by the synchrotron,  $|a_{T_{vacuum}}| = |a_T|$ . Particles reach in this way their maximum speed ( $v_{max}$ ).

(4) which hinders tangential acceleration. In the light of the present study, this fact is currently interpreted as relativistic mass increase, which would therefore be an illusory effect, actually being an increasing opposite force caused by the presence of a dilatant quantum vacuum, as shown in Fig. 2. Here we see that as a particle enters a relativistic regime and its speed increases toward that of light, vacuum's apparent viscosity nonlinearly reacts by decelerating it, i.e. a negative tangential acceleration occurs,  $a_{T_{vacuum}} = F_{vacuum}/m$ , thus, as soon as total tangential acceleration is zero (dotted line in Fig. 2), the particle reaches its maximum speed ( $v_{max}$ ) allowed by the energy of a given accelerator

$$a_{T_{tot}} = a_T + a_{T_{vacuum}} = a_T + \frac{F_{vacuum}}{m_{particle}} \quad (15)$$

$$if |a_{T_{vacuum}}| = |a_T| \Rightarrow a_{T_{tot}} = 0 \Rightarrow v_{max}$$

where  $F_{vacuum}$  is Eq. (4) and is negative, with opposite sense to  $a_T$ . The more powerful the accelerator, the closer to the speed of light the dotted line representing maximum speed ( $v_{max}$ ). As we see by putting Eq. (4) in the second law of motion, the tiny mass of a proton exposes the particle to a huge deceleration, in contrast to the case of planetary orbits (3.2). In conclusion, the diagram in Fig. 2 demonstrates that if vacuum's apparent viscosity obeys Lorentz factor, as assumed in Sect. 2 then, despite the energy of the accelerator, any accelerated particle will always be subject to maximum speed  $v_{max} < c$ , as observed,

without need for relativistic mass. This leads to the considerations below.

## 4 New prospects in aerospace engineering

Since in  $a = -F_{vac}/m$  the braking action due to dilatant vacuum nonlinearly decreases as the value of mass increases in the equation, the case of heavier accelerated objects is different from what we observe in synchrotrons, a fact which aerospace engineering and space research are currently unaware of. For instance, a hypothetical spherical spacecraft with the same mass of the International Space Station (ISS) and radius 6 m (i.e. of a sphere whose volume is equal to pressurised volume in the ISS) traveling at  $0.9999991 c$ , that is at the maximum speed a proton achieves in the LHC, would undergo a negative acceleration due to dilatant vacuum of just  $-0.2 \text{ m/s}^2$ , which even reduces to only  $-0.002 \text{ m/s}^2$  by increasing 100 times the mass of the spacecraft. In short what we currently and erroneously call relativistic mass increase would not be a problem for large masses accelerated up to almost the speed of light. Space exploration at quasi-luminal speed using large spacecrafts would therefore not be a problem, if not for different reasons such as those concerning propulsion etc. Another aspect of what we might call vacuum-applied aerodynamics emerging from Eq. (4) is that the smaller the section of the accelerated body perpendicular to the direction of motion, the weaker vacuum's viscous force. Cylindrical shape would be advantageous and transportation of large amounts of fuel would not be a problem. On the contrary, that would actually help to increase mass, reducing the braking action of quantum vacuum. Progressive in-orbit assembly of large-mass spacecrafts with very capacious tanks or taking control of small asteroids on which to place scientific equipment, exploiting their mass to reduce the deceleration exerted by the vacuum by approaching the speed of light may therefore be future correct strategies for space exploration directed toward other stars.

## 5 Conclusion

Hypothesizing the existence of a shear-thickening quantum vacuum with asymptote at the speed of sound leads to Lorentz factor, shedding light on the possible crucial role of quantum vacuum in Einstein's relativity. Indeed in Sect. 3.3 we rederived the formula of general relativity for the anomalous precession of perihelia from the modified Stokes's formula for a dilatant vacuum (4). Far from pure speculations then, the validity of this approach is also demonstrated by exactly solving another anomaly, the unexpected deceleration of the Pioneer probes, for which the exact negative acceleration detected by NASA is found. Finally, relativistic mass increase observed in particle acceleration is here revealed as the braking action of vacuum's apparent viscosity onto the accelerated particles, so evident due to their tiny mass. This means that relativistic mass is an illusion and that the correct interpretation is in the role of a dilatant vacuum. Einstein used Lorentz factor which simply assumes as a matter of fact the impossibility of exceeding the speed of light. Here we say that this impossibility is due to vacuum's dilatancy and to its finest structure (Sect. 2). No contradiction then, just an explanation which missed and that seems to be the right one, considering the anomalies the modified Stokes's formula can solve, along with the correct output for orbital stability and the correct mathematical description of what we observe in particle acceleration. Taking into account the dilatant behavior of the vacuum, we may argue for a physical vacuum probably composed of two different scalar fields, a dispersed granular component, e.g. particle dark matter [24], in a preponderant scalar field which can be even superfluid [1, 25], such as dark energy. That would be another indirect confirmation of the existence of such dark fields. Alternatively, the shear-thickening behavior of the physical vacuum could be caused by the large population of particle-antiparticle pairs appearing and annihilating in the quantum vacuum. Vacuum's dilatancy should continue to be treated in subsequent works, perhaps applied to more cases.

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