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# Quantum foundations of special relativity: evidence of a dilatant vacuum

Marco Fedi \*

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## Abstract

By exactly solving two known anomalies, Mercury's perihelion precession and the Pioneer anomaly in a dilatant vacuum, it is demonstrated that the physical vacuum actually behaves as a dilatant fluid, as shear stress increases toward a relativistic regime. Lorentz factor is reinterpreted as vacuum's rheogram, relativistic mass increase as actually the action of vacuum's apparent viscosity on accelerated bodies and a general formula for exactly computing vacuum's friction is presented, as a modified nonlinear Stokes's formula. Applied to planetary orbits negligible values over billions of years are obtained, making a dilatant vacuum compatible with observations, while for accelerated particles results are consistent with the resistance to acceleration encountered in synchrotrons.

**Keywords**— special relativity, dilatant vacuum, perihelia precession, Pioneer anomaly, quantum cosmology.

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## 1 Introduction

Several authors considered the possibility that the physical vacuum may actually be a superfluid, i.e. a special Bose-Einstein condensate [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 21]. In the present investigation it is found that quantum vacuum rather acts as a *dilatant* fluid as shear stress increases.

As shown below, even for velocities in the classical limit, vacuum's nonlinear apparent viscosity emerges and it is demonstrated that this is the real cause of the anomalous precession of perihelia and of the Pioneer anomaly, which here finds its most direct and accurate solution, without simulations, compelling us to reopen this old question Sect.3. As regards Mercury, it is shown that the anomalous (relativistic) positive contribution to perihelia precession is due to the variable interaction with a dilatant vacuum, caused by variable orbital velocity, which lets the centrifugal force oscillate perturbing the orbit's equation. In the calculations, Stokes's law for motion through a viscous fluid and Lorentz factor reinterpreted as the rheogram of quantum vacuum are used, to obtain a modified Stokes's law for computing friction in a dilatant vacuum. Exact results are obtained. Lorentz factor (Sect.2) therefore emerges as vacuum's nonlinear viscosity factor and indicates the phenomenon of relativistic mass increase as illusory, being actually due to the action of vacuum's shear-dependent viscosity. Once it has transiently solidified under shear stress (quasi-lattice condition [24]), vacuum's interstices are likely finer than a tiny electron. This would explain the limit to acceleration proven in synchrotrons, where particles face like an impenetrable wall. On the contrary, for more massive bodies, such as planets, the formula yields negligible deceleration values and planetary orbits are therefore stable over billions of years (Sect.3.1). Indeed, unexpectedly, while a proton accelerated to almost the speed of light undergoes huge decelerations, as the mass of the accelerated bodies increases, negative acceleration due to vacuum's friction decreases, implying unexpected opportunities for aerospace engineering.

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## 2 Lorentz factor as the rheogram of a dilatant vacuum

The behavior of the physical vacuum is shown to range from superfluid up to dilatant, according to applied shear stress. Here Lorentz factor actually corresponds to the rheogram of a fluid quantum vacuum, expressing its shear-dependent nonlinear apparent viscosity (Fig.1). Here motion in the flat spacetime of special relativity (S-R) is therefore considered as occurring in a non-classical (quantum) vacuum. We know that apparent viscosity in dilatant fluids is nonlinear and once the medium has transiently solidified within Frenkel's relaxation time [26] due to applied shear stress, further positive acceleration through the medium is then possible only in two cases: a) passing through its interstices or b) cracking its lattice. Reasonably assuming that the texture of the physical vacuum is the finest in nature, case a) is excluded even for a tiny electron or neutrino. Once accelerated to relativistic velocities, particles face an impenetrable wall, as verifiable in synchrotrons. Through the locally and transiently solidified vacuum (quasi-lattice condition) and due to its finest texture, only phonon (acoustic) propagation would be possible and that lets us reflect on a probable photon-transverse phonon identity in a dilatant vacuum, as analyzed in [10, 23]. Lorentz factor in a dilatant vacuum therefore emerges as

$$\gamma \equiv \frac{d}{d\beta} \arcsin \beta = \frac{1}{\sqrt{1-\beta^2}} \quad (1)$$

with  $\beta \equiv v/a$ , being  $a$  the speed of acoustic perturbation in vacuum's quasi-lattice [24]. In the case that photons were not particles finer than vacuum's interstices, but more likely quasi-particles possessing wave-particle behavior such as phonons [25],  $a = c$  would be valid, where  $c$  is the speed of light. When we consider SR endowed with a quantum vacuum, if the apparent viscosity of this acted onto accelerated bodies, we should consequently reinterpret the phenomenon of relativistic mass increase as actually the action of vacuum's viscous force  $F_{\text{vacuum}}$ . Indeed, in both cases, we see that a positive acceleration provided may be cancelled: respectively as  $m_{\text{rel}}$  or  $F_{\text{vacuum}}$  increase, where the first is the relativistic mass and the latter the nonlinear viscous force exerted by the physical vacuum which defeats a linearly increasing force applied

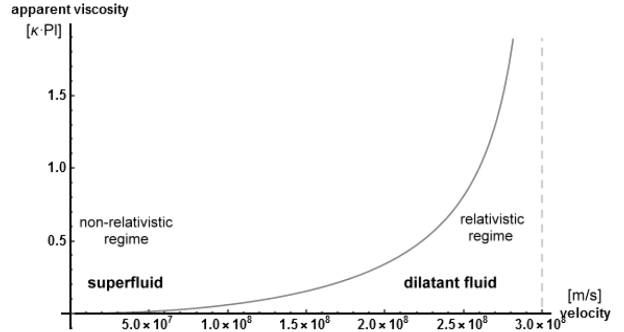


Figure 1: Lorentz factor reinterpreted as the rheogram of a fluid quantum vacuum ranging from superfluid to dilatant regime, according to shear stress. The asymptote indicates the velocity of both photons and (transverse) phonons, suggesting a possible identity in a dilatant vacuum [10, 23, 25].

in the opposite sense

$$a = \frac{F}{m_{\text{rel}}} \xrightarrow{\text{actually}} a = \frac{F - F_{\text{vacuum}}}{m}. \quad (2)$$

In fact, resorting to the demonstrations below, we prove that in (2) the correct equation is that on the right: in a flat spacetime when measuring with respect to a non-inertial reference frame the mass of a body which is accelerated in relativistic regime actually remains unaltered even from the point of view of the inertial observer, and what occurs is the action on the body of a viscous force exerted by a dilatant vacuum, a preferred frame of reference. About relativistic time dilation in a flat spacetime due to velocity, its quantum justification would be then linked to the action of apparent viscous forces, even at Planck scale, onto the dynamic physical processes from which time emerges. Finally, as far as the Lorentz-Fitzgerald contraction is concerned, it is justified in the same way, since it is due to time dilation affecting measurements. In short, relativistic mass increase is an illusory effect due to quantum vacuum's dilatant behavior, which also provokes time dilation and, as a direct consequence, length contraction (as unequal measurements due to different clocks' pace). Let us therefore prove the existence of a dilatant vacuum and its role in relativity, by solving the following anomalies.

### 3 First evidence: the Pioneer anomaly recalculated in a dilatant vacuum

Being the anomalous negative acceleration of the Pioneer spacecrafts 10 and 11 well-known (concrete investigations of the anomaly started in 1994 [12]), it is not necessary to summarize here this issue. In the light of the present results it appears evident that this problem was not correctly solved yet, despite copious previous investigations. At the moment, it is generally accepted that the solution to this anomaly is due to the anisotropic recoil of thermal photons [11, 12, 13, 14, 15, 16, 17]. However, by carefully analyzing these works, we realize that due to the difficulty of building an exact thermal model of the probes, especially as far as their internal heat dynamics is concerned, many numerical approximations and assumptions have been made and that weakens the scientific solidity of the conclusions. On the contrary, an exact and direct solution is made available from the interaction of the Pioneer with a viscous quantum vacuum. For doing this, we resort to Stokes's formula for motion in a viscous fluid

$$F_v = 6\pi r v \eta \quad (3)$$

where  $v$  is the translational velocity,  $r$  the radius of the object (the law considers a spherical object) and  $\eta$  a coefficient of dynamic viscosity expressed in  $\text{Pa} \cdot \text{s}$ . Differently from Stokes's law, in our case the fluid is however dilatant (non-Newtonian) and its apparent viscosity nonlinear. We therefore resort to Lorentz factor ( $\gamma$ ) as the expression of vacuum's apparent viscosity (Sect. 2) and we insert it into the formula, in place of the viscosity coefficient  $\eta$ , using the form  $\gamma - 1$ . The velocity which appears in the linear form of Stokes's law (3) is now included in the nonlinear viscosity factor (Lorentz factor)

$$F_{v\text{vacuum}} = 6\pi r (\gamma - 1) \kappa. \quad (4)$$

Here  $\kappa$  is a unitary constant expressed in  $\text{Kg} \cdot \text{s}^{-2}$ . When this modified Stokes's formula for friction in a dilatant vacuum is put into Newton's second law, we can exactly calculate the Pioneer anomalous deceleration as  $a_P = -F_{v\text{vacuum}}/m$ , where  $m$  is the mass of the spacecraft and the

sign minus is due to the frictional (braking) force

$$\begin{aligned} a_P &= -\frac{F_{v\text{vacuum}}}{m} = -\frac{6\pi r \left( \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1 \right) \kappa}{m} = \\ &= -\frac{6\pi \cdot 1.37 \text{ m} \cdot \left( \frac{1}{\sqrt{1-\left(\frac{36737 \text{ m}\cdot\text{s}^{-1}}{299792458 \text{ m}\cdot\text{s}^{-1}}\right)^2}} - 1 \right) \cdot 1 \text{ Kg}\cdot\text{s}^{-2}}{222 \text{ Kg}} = \\ &= -8.73 \times 10^{-10} \text{ m} \cdot \text{s}^{-2} \end{aligned} \quad (5)$$

As we see, the result is in perfect agreement with the measured value expressing the anomalous negative acceleration of the Pioneer

$$a_P = -(8.74 \pm 1.33) \times 10^{-10} \text{ m} \cdot \text{s}^{-2} \quad (6)$$

The data used for the calculation are: the mass of the spacecrafts (258 kg) minus the mass of the burned fuel (36 kg), the radius of the antenna (1.37 m) and the maximum speed of the probe ( $36737 \text{ m/s} = 132252 \text{ Km/h}$  [18]) after the swing-by caused by Jupiter. This exact and direct result cannot be ignored and the Pioneer issue should be therefore reopened. In addition, several critical remarks to the currently accepted thermal explanation can be expressed. Even at the end of mission, the heat radiated by the probe was in excess of 2 kW. Since the power needed to produce the reported acceleration is only  $\sim 65 \text{ W}$ , the anisotropy is only  $\sim 3\%$  and a small error in computing the total radiated heat produces an unacceptable discrepancy in the recoil force [16]. Moreover, every revolution of the spinning spacecraft adds further discrepancies to measurements. Third, the largest amount of heat was radiated from the radioisotope thermoelectric generators, not from the louver, being useless as regards the computation of the anomalous deceleration. Finally, without accurately knowing the dynamics of heat diffusion inside the probe [15], we cannot be sure of the role of the anisotropic thermal photons' recoil. Scheffer [14] analyzes in detail four sources of anisotropic heat radiation and their possible contribution to the anomaly. The author assumes (a) a uniform internal temperature with closed louvers but, as said, the modelling of the internal heat diffusion is considered difficult due to a lack of precise information, (b) the contribution from the not reflected photons coming from the radio antenna (at about 45 degree angle to the spin axis), (c) the radiation from the radioisotope heater

units and from the radio-thermal generators are considered by the author difficult to calculate and are hypothesized, (d) the antenna's solar reflectivity, for which however the same uncertainties, assumptions and hypothetical modelling have to be taken into account, especially since dealing with extremely low discrepancies. In a last analysis, we can therefore conclude that the thermal cause of the anomaly is possible by opting for suitable values in the calculations among different scenarios of thermal diffusion, while the numerical computation from a viscous vacuum is direct and exact.

### 3.1 Negligible orbital decay for planets

As far as orbital stability in time despite the existence of a dilatant vacuum is concerned, by using the modified Stokes's formula (4) in Newton's second law, exactly as for the case of the Pioneer probes analyzed above, and putting in it the data of Mercury (mean radius, mean orbital velocity and mass), we see that Mercury decelerates because of vacuum's viscosity by the following negligible value

$$-\frac{6\pi \cdot 2.44 \times 10^6 \text{ m} \cdot \left( \frac{1}{\sqrt{1 - \left( \frac{47362 \text{ m} \cdot \text{s}^{-1}}{299792458 \text{ m} \cdot \text{s}^{-1}} \right)^2}} - 1 \right) \kappa}{3.3 \times 10^{23} \text{ Kg}} = \quad (7)$$

$$= -1.74 \times 10^{-24} \text{ m} \cdot \text{s}^{-2}$$

which corresponds to a plausible orbital velocity decrease of  $5.48 \times 10^{-8} \text{ m} \cdot \text{s}^{-1}$  per  $10^9$  years. On the contrary, the much smaller momentum of the Pioneer probes let them more evidently decelerate under the action of vacuum's viscosity (Sect.3). Indeed, the orbital deceleration for Jupiter is for instance lower than that of Mercury and exactly only  $6.59 \times 10^{-28} \text{ m/s}^2$ . Planetary orbits can be therefore considered stable over billions of years.

The micro-asteroid 2004 FH, having  $m = 2.8 \times 10^7 \text{ Kg}$ ,  $r = 12 \times 10^3 \text{ m}$  and  $\bar{v} = 32237 \text{ m/s}$ , should undergo a negative acceleration  $a_{2004fh} = -4.67 \times 10^{-11} \text{ m/s}^2$ , losing  $1.09 \times 10^{-3} \text{ m/s}$  per orbital revolution (270 days), each prolonged by 0.79 s. The modified Stokes's formula (4) also tells us that the more massive and thin a body is, the less deceleration it experiences under the action of vacuum's apparent viscosity. A spherical body with radius 1 m and mass 10 Kg traveling at 83% the speed of light would experience a deceleration of  $-1.5 \text{ m/s}^2$ , while with

a mass of  $10^3 \text{ Kg}$  only  $-0.015 \text{ m/s}^2$ . On the contrary a proton (using the charge radius in the calculations<sup>1</sup>) accelerated to 99.999999%  $c$  would undergo a huge negative acceleration of  $-6.97 \times 10^{16} \text{ m/s}^2$ . While a hypothetical big spacecraft with radius of 300 m and a mass of  $10^5$  tons traveling at the same quasi-luminal speed would undergo a negative acceleration of just  $-0.021 \text{ m/s}^2$ , implying a significant restructuring of our thinking and opening new scenarios for aerospace engineering.

## 4 Second evidence: Mercury perihelion precession

Net of classical gravitational contributions, perihelia precessions show an anomalous positive contribution, which is particularly evident for the planet Mercury. The correct calculation of this anomaly is one of the most solid proofs for general relativity (GR). Here, we aim to demonstrate that though relativity is safe – indeed Einstein's formula for perihelia precession is *quantitatively* correct – the role of curved spacetime is actually played by a flat dilatant quantum vacuum, yielding a different qualitative explanation. In GR [19], the anomalous perihelia precession is represented by a formula which can be observed in three equivalent forms

$$\Delta\phi = \frac{24\pi^3 a^2}{T^2(1-e^2)c^2} = 6\pi \left( \frac{\bar{v}_{orb}}{c} \right)^2 \frac{1}{1-e^2} = 6\pi \frac{GM}{a(1-e^2)c^2} \quad (8)$$

where  $\Delta\phi$  expresses the relativistic contribution to perihelia precessions in radians per revolution corresponding, using the data of Mercury, to the known value of  $\sim 43''$  per century (or  $5.022 \times 10^{-7} \text{ rad/rev.}$ ),  $a$  is the semi-major axis,  $T$  the orbital period and  $e = 0.205$  the orbital eccentricity. In the expression in the center of Eq.(8) the mean orbital velocity  $\bar{v}_{orb} = 2\pi a/T$  has been used, then  $T^2 = 4\pi^2 a^2 / \bar{v}_{orb}^2$  and in that on the right we have used the stable second cosmic velocity squared,  $v^2 = GM/a$ , where  $GM/a$  is the classical gravitational potential, putting the radius  $r = a$ . Below the quantum foundations of this phenomenon are demonstrated via the interaction of the planet with a dilatant vacuum.

<sup>1</sup>Comparing the data from the modified Stokes's formula with data from particle accelerators can allow us to determine the real radius of a proton

As for the case of the Pioneer, we resort to the modified Stokes's law for a dilatant quantum vacuum (4). In this case we respect the treatment of the planet as a point mass, so the direct proportionality to the radius is not taken into account. Eq. (4) becomes the following adimensional expression, which is expressed in radians

$$6\pi \left( \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) = 6\pi(\gamma - 1). \quad (9)$$

Resorting to Taylor we use the approximation

$$2(\gamma - 1) \approx \left(\frac{v}{c}\right)^2 \quad (10)$$

and (9) now reads

$$3\pi \left(\frac{v}{c}\right)^2 = 3\pi \frac{GM}{ac^2} \quad (11)$$

where, on the right, the planet's velocity is written as stable second cosmic velocity squared as in the rightmost expression in (8). As we refer to motion in an elliptic orbit, we have to use the elliptic parameter, correcting  $a$  into  $a(1 - e^2)$  and we obtain a formula which exactly gives 1/2 the result of GR (8)

$$3\pi \frac{GM}{a(1 - e^2)c^2} = 3\pi \left(\frac{v}{c}\right)^2 \frac{1}{1 - e^2} \quad (12)$$

This result would be wrong ( $-50\%$ ) if we consider  $v$  as the mean orbital velocity. Actually, the real situation is represented in Fig.2 on the left, with a variable orbital velocity. Since  $\bar{v}_{orb} = (v_{max}/2) + (v_{min}/2)$  we are allowed to use the reduced model on the right side in Fig.2, i.e. one semi-orbit with maximum orbital velocity ( $v_{max}$ ) and the other with minimum velocity ( $v_{min}$ ). For describing the precession occurring in a semi-orbit Eq.(12) is correct. Then we use it twice (respectively with  $v_{max}$  and  $v_{min}$ ) to express the precession during a full orbit and that yields the full result of GR (8)

$$\begin{aligned} \Delta\phi &= 3\pi \left(\frac{v_{max}}{c}\right)^2 \frac{1}{1 - e^2} + 3\pi \left(\frac{v_{min}}{c}\right)^2 \frac{1}{1 - e^2} = \\ &= 6\pi \left(\frac{\bar{v}_{orb}}{c}\right)^2 \frac{1}{1 - e^2} \end{aligned} \quad (13)$$

where  $v_{max}$  and  $v_{min}$  combine into the mean orbital velocity. In this way it is demonstrated that perihelia precessions actually occur via a viscous interaction with a dilatant

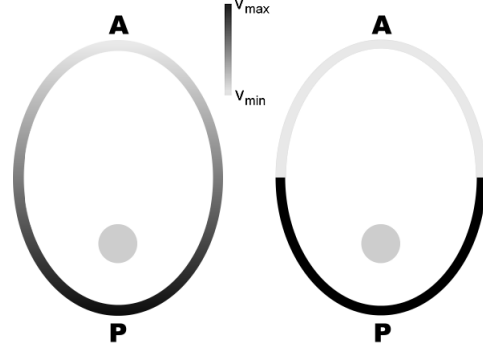


Figure 2: Left: variable orbital velocity in the elliptic orbit between perihelion (P) and aphelion (A). Right: a reduced model used in the present study which simplifies the calculations considering half an orbit at max orbital speed and the other one at minimum speed.

quantum vacuum. To understand in detail this anomalous contribution to the precession, let us consider a simple equation for the orbit

$$\frac{m\bar{v}_{orb}^2}{r} = \frac{GMm}{r^2} \quad (14)$$

where the centrifugal force (on the left) is equated to the gravitational (centripetal) force. When instead of the mean orbital velocity the maximum velocity of the planet is taken into account, vacuum's friction will be greater and this lets the centripetal force slightly decrease in favor of the gravitational force triggering 1/2 the precession. On the contrary, when the minimum orbital velocity is considered, vacuum's friction is lower than that occurring with  $\bar{v}_{orb}$ , so this time the centrifugal force slightly prevails, triggering the remaining 1/2 precession in the other semi-orbit, once the planet has reached the lowest orbital speed at the aphelion. The anomalous perihelia precession therefore occurs as a perturbation of the orbital equation due to the action of the apparent viscosity of a dilatant quantum vacuum.

Vayenas and colleagues [20], without considering the viscosity of vacuum but taking into account the flat space-time of special relativity, came to the same 1/2 partial result of GR (12). Vayenas's solution, which resorts to the illusory (Sect.2) relativistic mass increase by applying  $\gamma$  to mass [22] has to be anyway rejected in favor of the present full solution from a viscous vacuum, also because

in 14 the planetary mass cancels, so its relativistic variations cannot even be taken into account. Since Einstein calculated the anomalous contribution to Mercury's perihelion in GR, that lets us understand that the concept of curved spacetime is rather the expression of a fluid quantum vacuum, after all in agreement with the stress-energy tensor of the field equation, in which  $T^{00}$  is vacuum's density ( $\rho_{vac}$ ), also present in the cosmological constant  $\Lambda = \kappa\rho_{vac}$ , and the remaining components are indeed pressure, shear stress, momentum flux and momentum density. A fluid quantum vacuum approach to relativity may be successful for merging it with quantum physics. We also know that the equation of state of cosmology has a hydrodynamic meaning and it may represent the ratio of a fluid vacuum's pressure to its density

$$w = P_{vac}/\rho_{vac} \quad (15)$$

also useful to express vacuum's quantum potential, where spacetime curvature is replaced by vacuum's pressure. The  $\sim 70\%$  mass-energy of the universe, called dark energy, can be let correspond to the huge mass-energy of the fluid dilatant quantum vacuum itself, whose energy density generates pressure opposing gravity (indeed  $\text{J}/\text{m}^3 = \text{Pa}$ ). In this case we can no more state that dark energy does not interact with baryon matter, since it actually does it via its (=vacuum's) dilatancy, as perihelia precessions and the Pioneer anomaly demonstrate.

## Conclusion

The use of Lorentz factor as the rheogram of a dilatant vacuum combined with Stokes's law for motion in a viscous fluid yielded a modified Stokes's law for correctly computing friction in the physical vacuum (4) and the successful calculations of Mercury perihelion precession and of the anomalous deceleration of the Pioneer probes seem to confirm the existence of a dilatant vacuum. Relativistic mass-energy increase has been excluded and declared illusory, in lieu of the nonlinear action of vacuum's apparent viscosity on accelerated bodies. Also time dilation, increasing with velocity, and consequently Lorentz-Fitzgerald contraction, which directly depends on time dilation, from the point of view of quantum physics may be explainable within the action of quantum vacuum's apparent viscosity, since they obey vacuum's rheogram

(Lorentz factor). Time dilation would therefore arise from the action of viscous forces exerted by the physical vacuum both on Planck and on macroscopic scale. As far as Mercury perihelion is concerned, it has been shown (Fig. 2) that it is provoked by the dynamics of forces in the elliptic orbit, including vacuum's viscous force. This anomalous contribution to the precession of perihelia was previously calculated by Einstein resorting to the mathematical artifice of a curved spacetime<sup>2</sup>. Orbital decay due to dilatant vacuum is absolutely negligible: for Mercury about  $5.48 \times 10^{-8}$  m/s per  $10^9$  years. Vacuum's viscosity does not therefore affect orbital stability in time over billions of years, due to the high momenta of planets. While the Pioneer spacecraft has been more evidently affected. Since a greater mass of an accelerated body lets vacuum's friction decrease, calculations have been done which suggest new interesting scenarios for aerospace engineering, being a single proton much greaterly decelerated by vacuum's friction than a large, heavy spacecraft. An approach based on a fluid quantum vacuum hence shows its usefulness in several fields of modern physics and deserves further efforts and investigations.

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<sup>2</sup>Even the recently detected gravitational waves can be actually described as negative pressure waves in a fluid vacuum[21]

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