



HAL
open science

A Computational Damage Micromodel of Laminated Composites

Pierre Ladevèze, Gilles Lubineau, David Violeau

► **To cite this version:**

Pierre Ladevèze, Gilles Lubineau, David Violeau. A Computational Damage Micromodel of Laminated Composites. *International Journal of Fracture*, 2006, 137 (1-4), pp.139-150. 10.1007/s10704-005-3077-x. hal-01648334

HAL Id: hal-01648334

<https://hal.science/hal-01648334>

Submitted on 19 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A computational damage micromodel of laminated composites

Pierre Ladevèze, Gille Lubineau and David Violeau

LMT-Cachan, ENS de Cachan, Paris 6 Univ., CNRS, 61 Avenue du Président Wilson, 94235 Cachan, France

Abstract. A new computational damage micromodel for laminates, which takes into account classical experimental micro- and macro-observations for various stacking sequences, is described. The first computational examples are shown.

Key words: Composite, computation, damage, laminate, micromechanics.

1. Introduction

The last quarter-century has witnessed considerable research efforts in the mechanics of composites aimed to understand their behavior and to model or calculate them – the ultimate goal being the design of the materials/structures/manufacturing processes. Even in the case of laminated composites, which are the most studied and, therefore, the most understood, the prediction of the evolution of damage up to and including final fracture remains a major challenge. Today, the use of laminated composites in the aerospace industry always involves characterization procedures consisting of huge numbers of tests, which shows the low level of confidence in models. Significant improvements in this situation could be achieved if one could create a real synergy among the approaches on different scales which, today, are followed quite independently of one another in the case of laminated composites. One could jokingly say that there is, on the one hand, the micromechanics of laminates where one counts cracks and, on the other, the meso- or macromechanics of laminates where one measures stiffness. Previous works have proved that such a bridge exists and can be built (Ladevèze and Lubineau, 2001, 2002). A state-of-the-art review can be found in Ladevèze (2004, 2005).

The question discussed here is the impact of such a bridge on the models themselves, especially the micromodels, the objective being to calculate the intensities of the damage mechanisms at any point of laminated structures subjected to complex loading and at any time until final fracture resulting from strain and damage localization. We propose and describe a computational damage model which is rather simple, yet semi-discrete and probabilistic. In part, it is discrete and minimum cracking surfaces are defined for each elementary cracking area thanks to well-known micromechanics considerations (Nairn, 2000; Berthelot, 2003); one can say that the proposed micromodel belongs in what is known as “finite fracture mechanics” (Hashin, 1986, 1996). For the rest, diffuse damage and (visco)plasticity are described through continuum mechanics models.

Unfortunately, the application of this micromodel to engineering structures leads to prohibitive calculation costs if one uses current industrial codes. The use of a multiscale computational strategy is absolutely essential. Here, we use the technique with homogenization in space and in time developed at Cachan. This technique is not detailed in this paper, which focuses only on the first computational examples obtained with the proposed micromodel. For different stacking sequences, we describe micro-observations such as the thickness effects, the edge effects or the difference between initiation and propagation of intralaminar cracks.

2. Microphenomenology

Let us assume that any complex state of degradation of a laminated composite is due to the accumulation and subsequent localization of a family of elementary mechanisms. Of course, depending on the material properties controlling each mechanism and on the loading, the sequence of these elementary mechanisms can differ and lead to very different states of apparent degradation.

These mechanisms (Figure 1) can be classified into two groups, according to their apparent morphology on the ply's scale. The first group consists of discrete phenomena, in which cracked surfaces can be clearly identified on the ply's scale and which correspond to the classical mechanisms introduced in micromechanics. The second group consists in "continuous" mechanisms, which on the ply's scale lead to a seemingly homogeneous, but damaged, material. Since such mechanisms cannot be observed directly through cracked surfaces, they are often ignored in micromechanics.

2.1. THE CLASSICAL DISCRETE MECHANISMS

Here we are considering two mechanisms that are involved in most experimental and theoretical works done in micromechanics.

The first is called "transverse microcracking". Transverse microcracks spread throughout the thickness of the plies and develop mainly in plies subjected to transverse loading. This mechanism has been studied by the micromechanics community mostly in the framework of finite fracture mechanics (see Nairn, 2000; Berthelot,

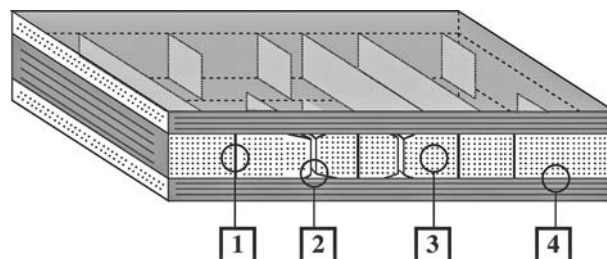


Figure 1. The mechanisms of degradation on the microscale: 1: transverse microcracking, 2: local delamination, 3: diffuse damage, 4: diffuse delamination.

2003 for reviews). These works helped us to identify several key points of this mechanism (the distinction between initiation and propagation, the thickness effects ...). These and their integration into the model will be discussed in the rest of the paper.

When the loading increases, “local delamination” appears at the interfaces between plies (Nairn and Hu., 1992; Ogihara and Takeda, 1995; Zhang et al., 1999) due to the induced overloading at the tips of the transverse microcracks. In practice, the development of this mechanism is responsible for the saturation of the microcracking state, and participates in the complete failure of the structure which results from the localization of the degradation.

Another key point is that microcracking is a stochastic phenomenon. Various models have been proposed in the literature, see Berthelot and Le Corre (2000). In Ladevèze (2005), we proposed a rather simple model which requires no additional parameter.

2.2. THE NECESSARY “CONTINUOUS” COMPLEMENTARY MECHANISMS

Now we are considering two mechanisms which are not usually introduced in micromechanics, but which were included a long time ago on coarser scales, e.g., in the damage mesomodel for laminates (Ladevèze, 1986; Ladevèze and Le Dantec, 1992; Ladevèze et al., 2005).

The first, called “diffuse damage”, is associated with quasi-homogeneous fiber-matrix debonding within the layer. This mechanism can be naturally homogenized on the ply’s scale. This is the main mechanism for layers subjected to shear loading. In this case, a sharp decrease in shear modulus is observed, although no transverse cracking occurs. A clear and definitive experimental proof of the existence of this mechanism was given recently in Lagattu and Lafarie-Frénot (2000). Therefore, models that take into account only classical discrete mechanisms are valid only for specific loading configurations and fail in case of complex transverse/shear loading.

The second mechanism, “diffuse delamination”, is associated with microvoids within the interlaminar interface and results in the seemingly reduced stiffness of the interlaminar interface, with no observable delamination.

2.3. SEQUENCE OF THE MECHANISMS LEADING TO FRACTURE

In most practical cases, the sequence of the mechanisms is the following: diffuse damage and diffuse delamination occur first, leading to progressive degradation within the plies and interfaces, which can still be considered homogeneous. Then, transverse microcracking occurs and develops through a percolation phenomenon. As the loading increases, local delamination appears at the crack’s tips. The competition between these last two mechanisms ends in the saturation of the microcracking level and is relayed by the catastrophic development of local delamination. Finally, general fracture occurs with fiber breakage and global delamination.

Let us note that fiber breakage can be described as either a discrete or a “continuous” phenomenon. Here we will take the discrete option.

3. The computational damage micromodel for laminates – basic aspects

Let us start with the initial state in which residual stresses occur. These can be calculated from the simulation of the process, but a more pragmatic and standard approach is simply to apply a uniform negative temperature variation, and calculate the corresponding residual stresses under elastic assumptions.

On the microscale, the structure is described as an assembly of layers, made of the fiber-matrix material, and interfaces, which can be cracks. The proposed computational micromodel for laminates is hybrid (Figure 2).

The fiber-matrix material follows the classical continuum mechanics framework. The surfaces of the cracks are described using a discrete model by introducing “minimum cracked surfaces”. This belongs in what is called “finite fracture mechanics” (Hashin, 1996).

The minimum cracked surfaces related to transverse microcracking are square surfaces, parallel to the fiber, whose dimension is the thickness of the ply. This choice of the lateral dimension derives from the fact that for values equal to or greater than the thickness the tunneling energy release is nearly constant (Figure 3). For minimum cracked delamination surfaces localized at the interfaces, the corresponding areas are about $0.1h \times 0.1h$. The results should vary only slightly depending on this choice.

Another key point concerning fiber breakage is the introduction of a minimum volume for fracture. This is a cube whose dimension is the thickness of the ply. The cubes follow the fiber’s direction in the ply. Today, after the pioneering work of Bažant and Pijaudier-Cabot, strain and damage localization are rather

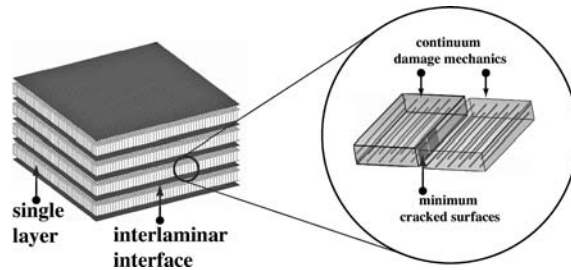


Figure 2. The computational damage micromodel: minimum cracking surfaces.

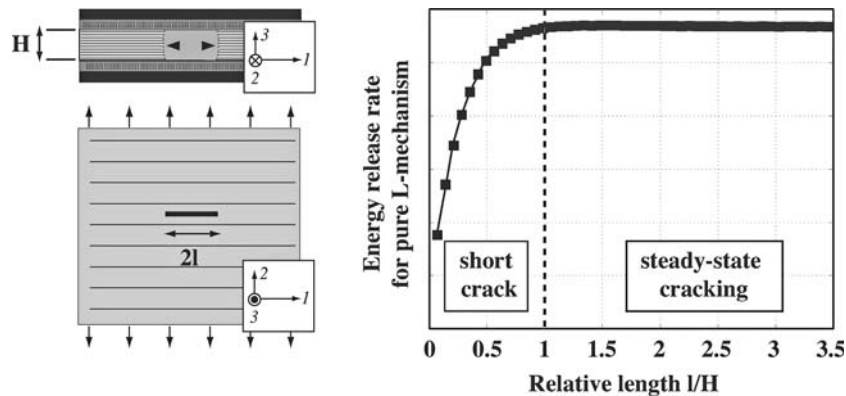


Figure 3. Tunneling energy release rate as a function of the lateral dimension.

well-understood phenomena (Bažant and Pijaudier-Cabot 1988; Pijaudier-Cabot and Bažant, 1987). Here, we are following the discrete approach, which is robust and mechanically sound.

For the rest, diffuse damage and plasticity are described through continuum mechanics models. Consequently, the fiber-matrix material which constitutes elementary homogeneous volumes between cracking surfaces follows the classical damage mesomodel for laminates: diffuse damage (Ladevèze and Le Dantec, 1992) and, if necessary, (visco)plasticity.

Thus, the continuous part of the model is completely classical and will not be detailed further in this paper. From now on, we will focus on the description of the discrete part for transverse microcracking and local delamination.

4. The damage micromodel for microcracking

4.1. SOME KEY POINTS OF MICROMECHANICS

Key point 1: modeling of initiation/propagation – the thickness effect.

Most basic papers on the subject are not recent (Garrett and Bailey, 1977; Parvisi and Bailey, 1978; Wang and Crossman, 1980; Boniface et al., 1997; Yang et al., 2003). For tension tests of stacking sequences built with 0° and 90° plies, two main observations have been made: first, the behavior of thick 90° plies is different from that of thin 90° plies: in thick plies, the transverse microcracks always run throughout the width of the specimen, whereas in thin plies they may stop near the edges. The second observation is related to the thickness effects (see Figure 4): the transition thickness \bar{h} is about twice that of the elementary ply. A general discussion of these effects can be found in Bažant (2002).

The theoretical explanation of the existence of transverse microcracks was given a long time ago and is well known. Let us consider a flaw in the form of a penny-shaped crack which can propagate either in the longitudinal or in the transverse direction. It has been proven that the transverse energy release rate is much higher than the longitudinal energy release rate and, therefore, that the flaw propagates in the transverse direction, which is the thickness of the layer (Dvorak and Laws, 1987).

Another point is that there is a difference between the initiation of a crack or a minimum cracking surface and the propagation of an existing crack. The criteria should be different: the latter is an energy release rate criterion and depends on

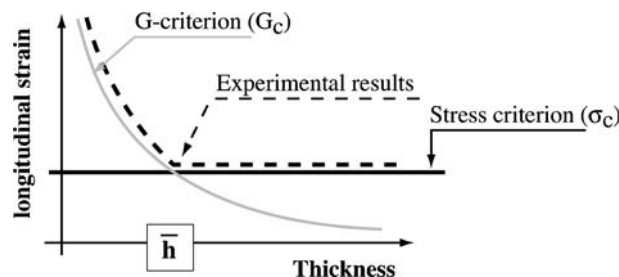


Figure 4. Failure strain as a function of the number of 90° plies.

the thickness; the former is a stress criterion which does not depend on the thickness. Consequently, one can take a new look at the curve of Figure 4: the right part for thick ply is, in fact, related to the initiation criterion; initiated cracks propagate immediately. The left part is associated with the propagation criterion, in this case for thin plies.

Key point 2: microcracking as a stochastic phenomenon.

Several probabilistic models have already been proposed (Wang et al., 1984; Fukunaga et al., 1984; Laws and Dvorak, 1988; Manders et al., 1983; Berthelot and Le Corre, 2000). For high-cracking densities, heuristic coefficients were introduced in order to characterize the imperfect periodicity (Dvorak and Laws, 1987; Nairn et al., 1993; Ladevèze and Lubineau, 2002). This was necessary in order to obtain reasonable agreement with experiments (Yahvac et al., 1991).

Along with (Ladevèze, 2005), we support the idea that the process is stochastic, but quasi-independent of the probabilistic law. We thus introduce simply uncertainty on the critical energy release rate $\{G_c\}$. It is assumed to vary near a mean value: $\{G_c\} \in \{G_c^{\text{mean}}\}_{-\Delta}^{+\Delta}$, where Δ is a small parameter. One then prescribes a uniform probability density over the domain:

$$\{\underline{M} | \max_{\underline{M}'} \{G(\underline{M}')\} - \{G(\underline{M})\} \leq \Delta\}. \quad (1)$$

Figure 5 shows several samples for different values of Δ ; the “mean” curve appears to be insensitive to the probabilistic parameter Δ and to the samples and is a quasi-deterministic curve, which is quite different from the curve related to a periodic pattern.

4.2. THE MODEL

Let us consider a minimum surface which can crack and let $\{Y\} = \{Y^I, Y^{II}, Y^{III}\}$ be the corresponding discrete damage force. The failure criterion is expressed as

$$\Phi(\{Y\}, \{Y_c\}) = \left[\left(\frac{Y^I}{Y_c^I} \right)^\alpha + \left(\frac{Y^{II}}{Y_c^{II}} \right)^\alpha + \left(\frac{Y^{III}}{Y_c^{III}} \right)^\alpha \right]^{1/\alpha} \geq 1. \quad (2)$$

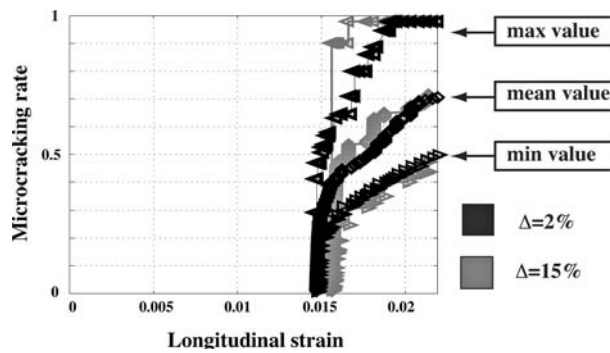


Figure 5. Max, mean and min values of the microcracking rate vs. the longitudinal strain.

For initiation, i.e., in the case of a minimum surface which is not connected to other, already cracked, surfaces, one introduces:

- the damage force $\{Y\}_{\text{init}} = \min\{\frac{\{G\}}{h}, r \frac{\{G\}}{\bar{h}}\}$, where $\{G\}$ is the discrete energy release rate for the Modes I, II, III, h the thickness of the ply, \bar{h} the transition thickness and r a material coefficient;
- the critical damage force $\{Y_c\}_{\text{init}} = \frac{\{G_c\}}{\bar{h}}$, where \bar{h} is transition thickness and $\{G_c\}$ the discrete critical energy release rate.

In summary, for initiation, one has

$$\Phi(\min\{\frac{\{G\}}{h}, \frac{\{G\}}{\bar{h}}\}, \frac{\{G_c\}}{\bar{h}}) \geq 1. \quad (3)$$

In the case of propagation, i.e., when the minimum surface being studied is connected to existing cracking surfaces, one introduces:

- the damage force $\{Y\} = \{G\}$, where $\{G\}$ is the discrete energy release rate;
- the critical damage force $\{Y_c\} = \{G_c\}$.

Thus, the propagation criterion is:

$$\Phi(\{G\}, \{G_c\}) \geq 1 \quad (4)$$

Let us note that these criteria are equivalent to those described in Ladevèze (2005). Presently, we use the same critical value for Modes II and III. Moreover, for Mode I, an additional criterion can be introduced by separating traction from compression.

When an elementary surface is cracked, unilateral contact conditions with friction apply. The critical values $\{G_c\}$ are stochastic fields for which we assume that the correlation length is not greater than the thickness h . Then, after discretization, they can be replaced by independent stochastic variables for which a modified normal law is introduced.

Remark: The energy release rate related to microcracking could be calculated simply by using the tunneling value.

5. The damage micromodel for delamination

Here again we introduce minimum surfaces for cracking. We also introduce a single criterion which connects:

- the damage force $\{G_{\text{del}}\}$ (the finite energy release rate for Modes I, II, III);
- the critical force $\{G'_c\}$.

This criterion is:

$$\Phi'(\{G_{\text{del}}\}, \{G'_c\}) \geq 1. \quad (5)$$

6. The damage micromodel for fiber breakage

Fiber breakage is assumed to be brittle. We introduce a minimum volume for fracture in the form of a cube of height h made of fiber-matrix material, for which we define:

- the mean traction energy Y_f^+ ;
- the mean compression energy Y_f^- ;
- the mean shear energy Y_d .

Thus, we have the two criteria:

$$l_t(Y_f^+, Y_d) \geq 1, \tag{6}$$

$$l_c(Y_f^-, Y_d) \geq 1. \tag{7}$$

7. First implementations and examples

The possible cracks are defined *a priori* in the computations in progress and are connected to the mesh.

7.1. A WORD ON THE NUMERICAL DIFFICULTIES AND COMPUTATION REQUIREMENTS

Let us consider the example of the low-velocity impact problem defined in Figure 6.

In the classical finite element method, a reasonable mesh would lead to $2 \cdot 10^{10}$ degrees of freedom. It is, therefore, indeed a research challenge to derive alternative computational strategies capable of solving such engineering problems. This is a hot topic referred to as “multiscale computational strategies with uncertainties”. Several approaches have already been proposed (Fish et al., 1997; Zohdi et al., 1999; Feyel, 2003; Ladevèze et al., 2001; Devries et al., 1989).

The examples presented below are based on the approach by (Ladevèze et al., 2001; Ladevèze and Nouy, 2003). The description of the solution using multiple scales reduces the number of degrees of freedom drastically. Two scales lead to $4 \times$

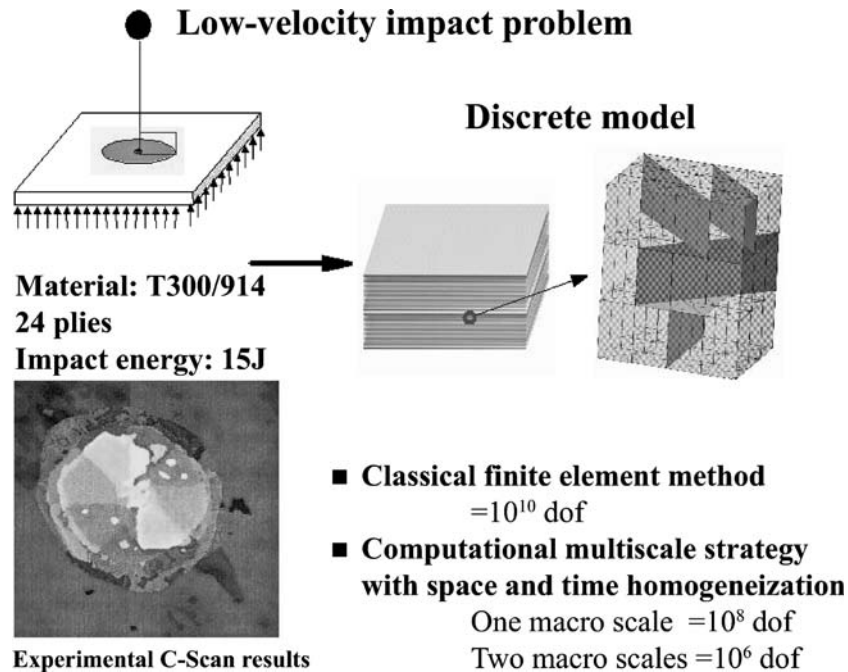


Figure 6. A low-velocity impact problem.

10^8 degrees of freedom. In order to get reasonable calculations, it is necessary to introduce an additional scale (a macro scale), which for the problem of Figure 6 would lead to 10^6 degrees of freedom.

7.2. ILLUSTRATION: STATIC AXIAL LOADING ON CROSS- AND ANGLE-PLY LAMINATES

We carried out basic simulations on cross- and angle-ply laminates with material properties representative of carbon–epoxy composites. These samples were assumed to be subjected initially to thermal loading due to the process ($\Delta T = -95^\circ\text{C}$), then to pure mechanical loading in tension.

Figure 7 shows the mesh used in this calculation. For the sake of simplicity, “minimum fracture areas” were introduced *a priori* into the mesh. Thus, the sample is composed of elementary cells which are assumed to be homogeneous, inside which diffuse damage can develop. The interfaces between elementary cells may fail to reproduce transverse microcracking.

Figures 8 and 9 show that the model seems capable of reproducing the main basic features of classical experimental results.

- It is well known that cross-ply laminates have a behavior depending of the thickness of the ninety-degree ply. In thick ninety-degree plies, cracks initiated near the edge propagate immediately towards the core of the laminate, whereas in thin ninety-degree plies one can observe partial cracks. Figures 9a and 9b, show comparative simulations of $[0_2/90_8/0_2]$ and $[0/90/0]$. Partial cracks can indeed be

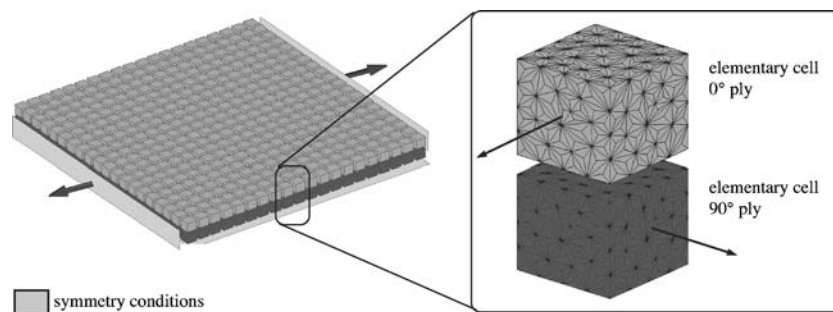


Figure 7. The mesh used in the calculation, based on the subdivision of the sample into elementary cells.

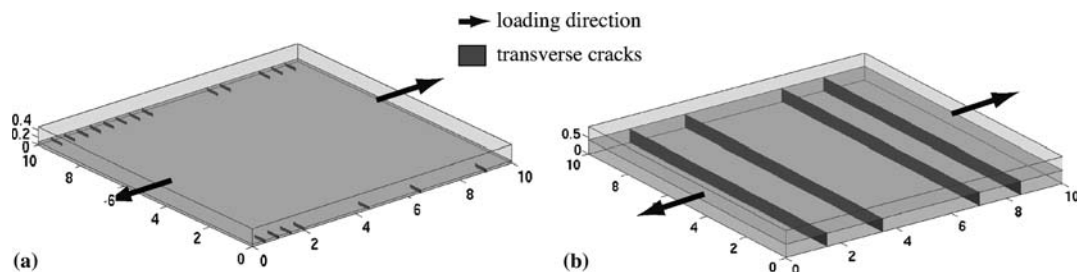


Figure 8. Comparison of the cracking patterns (a) in thin laminates ($[0_6/90]_5$) and (b) in thick laminates ($[0_3/90_3]_5$).

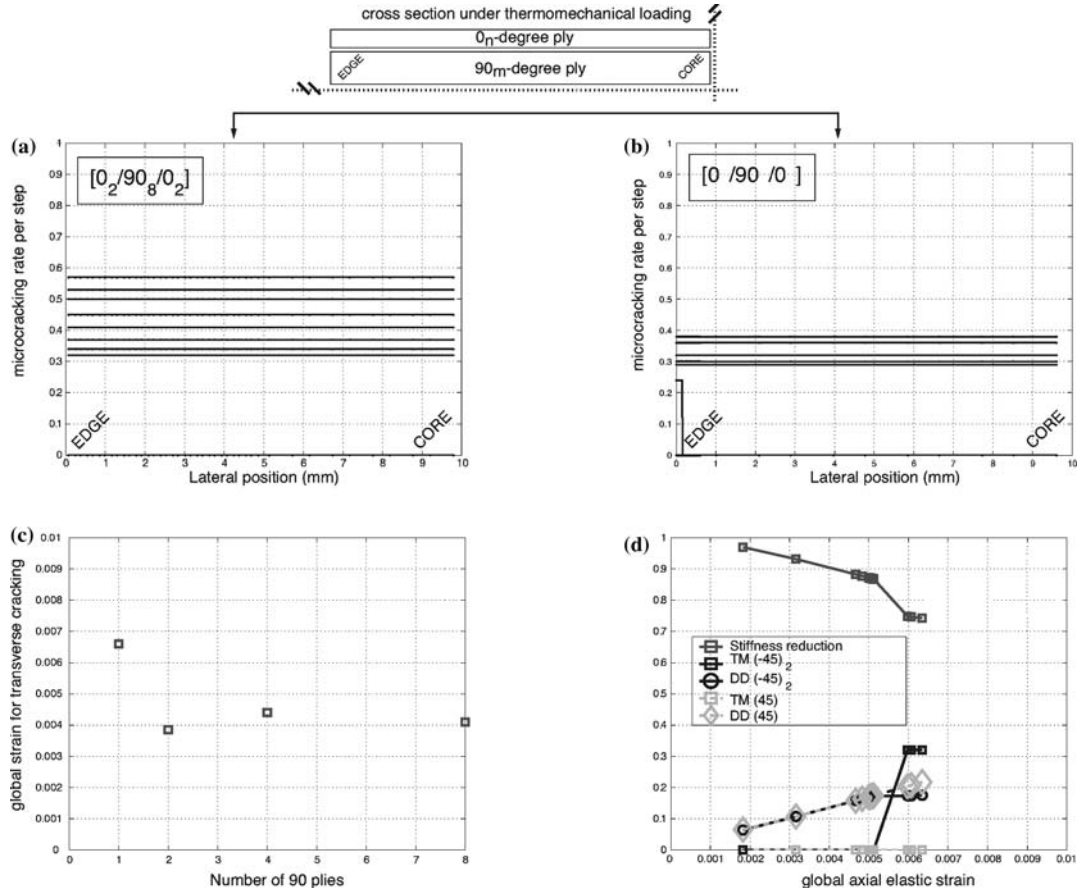


Figure 9. (a and b) microcracking rates from edge to core in the central ply of cross-ply composites at each step of the calculation. (c) strain at the beginning of microcracking vs. the ply's thickness. (d) simulated damage mechanisms (diffuse damage and microcracking rate) and global modulus drop for a $[45/-45]_{2s}$ sample.

observed in the second case. An illustration is given in Figure 8, which shows the cracking patterns for a very thin ($[0_3/90]_5$) laminate and a very thick ($[0_3/90_3]_5$) laminate at two loading levels.

- In Figure 9c, the strain at which microcracking starts is plotted for several ply's thicknesses. As observed experimentally, the strain necessary to initiate transverse cracks is greater in thin plies.
- The introduction of a diffuse damage mechanism explains the reduction of modulus under shear loading, even though only a few microcracks are observed. A partial simulation of a tension test on a $[45/-45]_{2s}$ (Figure 9d) shows that the modulus drops by 20% without any transverse crack. Even then, very few microcracks propagate only very late just prior the final fracture in the thick central layer, which is consistent with experimental observations. Here so, the modulus drop of 30–40% is due to diffuse damage mechanisms.

8. Discussion and conclusion

The computational micromodel developed in this paper seems to be a very promising approach because it enables one to take into account all classical micro- and macro-information. Further research is necessary, especially in computational mechanics, to develop this model into a true engineering tool.

References

- Bažant, Z.P. (2002). *Scaling of Structural Strength*. Hermes Penton Science, London.
- Bažant, Z.P. and Pijaudier-Cabot, G. (1988). Nonlocal continuum damage, localization instability and convergence. *ASME Journal of Applied Mechanics* **55**, 287–293.
- Berthelot, J.M. and Le Corre, J.F. (2000). Statistical analysis of the progression of transverse cracking and delamination in cross-ply laminates. *Composites Science and Technology* **60**, 2659–2669.
- Berthelot, J.M. (2003). Transverse cracking and delamination in cross-ply glass-fiber and carbon-fiber reinforced plastic laminates: static and fatigue loading. *Applied Mechanics Reviews* **56**(1), 1–37.
- Boniface, L., Smith, P.A., Bader, M.G. and Rezaifard, A.H. (1997). Transverse ply cracking in cross-ply CFRP laminates. Initiation or propagation controlled. *Journal of Composite Materials* **31**(11), 1080–1112.
- Devries, F., Dumontet, H., Duvaut, G. and Ln, F. (1997). Homogenization and damage for composite structures. *International Journal for Numerical Methods in Engineering* **27**, 285–298.
- Dvorak, G.J. and Laws, N. (1987). Analysis of progressive matrix cracking in composite laminates. Part 2: First ply failure. *Journal of Composite Materials* **21**, 309–329.
- Feyel, F. (2003). A multilevel finite element method FE^2 to describe the response of highly non-linear structures using generalized continua. *Computer Methods in Applied Mechanics and Engineering* **192**, 3233–3244.
- Fish, J., Sheck, K., Pandheeradi, M. and Shepard, M.S. (1997). Computational plasticity for composite structures based on mathematical homogenization: theory and practice. *Computer Methods in Applied Mechanics and Engineering* **148**, 53–73.
- Fukunaga, H., Chou, T.W., Peters, P.W.M. and Schulte, K. (1984). Probabilistic failure strength analyses of graphite/epoxy cross-ply laminates. *Journal of Composite Materials* **18**, 339–356.
- Garrett, K.W. and Bailey, J.E. (1977). Multiple transverse fracture in 90° cross-ply laminates of a glass fibre-reinforced polyester. *Journal of Materials Science* **12**, 157–168.
- Hashin, Z. (1996). Analysis of stiffness reduction of cracked cross-ply laminates. *Engineering Fracture Mechanisms* **25**, 771–778.
- Hashin, Z. (1996). Finite thermoelastic fracture criterion with application to laminate cracking analysis. *Journal of the Mechanics and Physics of Solids* **44**(7), 1129–1145.
- Ho, S. and Suo, Z. (1993). Tunneling cracks in constrained layers. *Journal of Applied Mechanics* **60**, 890–894.
- Hoover, J.W., Kujawski, D. and Ellyin, F. (1997). Transverse cracking of symmetric and unsymmetric glass-fibre/epoxy-resin laminates. *Composites Science and Technology* **57**, 1513–1526.
- Ladevèze, P. (1986). Sur la mécanique de l'endommagement des composites. In: *Comptes-rendus des JNC5* (edited by C. Bathias and D. Menkès), Pluralis Publication (in French), 667–683.
- Ladevèze, P. and Le Dantec, E. (1992). Damage modeling of the elementary ply for laminated composites. *Composite Science and Technology* **43**(3), 257–267.
- Ladevèze, P., Loiseau, O. and Dureisseix, D. (2001). A micro-macro and parallel computational strategy for highly heterogeneous structures. *International Journal for Numerical Methods in Engineering* **52**, 121–138.
- Ladevèze, P. and Lubineau, G. (2001). On a damage mesomodel for laminates: micro-meso relationships, possibilities and limits. *Composite Science and Technology* **61**(15), 2149–2158.
- Ladevèze, P. and Lubineau, G. (2002). An enhanced mesomodel for laminates based on micromechanics. *Composite Science and Technology* **62**, 533–541.
- Ladevèze, P. and Nouy, A. (2003). On a multiscale computational strategy with time and space homogenization for structural mechanics. *Computer Methods in Applied Mechanics and Engineering* **192**, 3061–3087.

- Ladevèze, P. (2005). Multiscale computational damage modelling of laminate composites. *Course CISM*, to appear.
- Ladevèze, P. (2004). A bridge between the micro- and mesomechanics of laminates: fantasy or reality? *ICTAM 2004*, to appear.
- Ladevèze, P., Lubineau, G. and Marsal, D. (1978). Towards a bridge between the micro- and the mesomechanics of delamination for laminated composites. *Composite Science and Technology* **2**(2), 155–192.
- Lagattu, F. and Lafarie-Frénot, M.C. (2000). Variation of PEEK matrix crystallinity in APC-2 composite subjected to large shearing deformation. *Composite Science and Technology* **60**, 605–612.
- Laws, N. and Dvorak, G.J. (1988). Progressive transverse cracking in composite laminates. *Journal of Composite Materials* **22**, 900–916.
- Manders, P.W., Chou, T.W., Jones, R.R. and Rock, J.W. (1983). Statistical analysis of multiple fracture in $0^\circ/90^\circ/0^\circ$ glass-fiber/epoxy-resin laminates. *Journal of Materials Science* **8**, 352–362.
- Nairn, J.A. and Hu, S. (1992). The initiation and growth of delaminations induced by matrix microcracks in laminated composites. *International Journal of Fracture* **57**, 1–24.
- Nairn, J.A., Hu, S. and Bark, J. (1993). A critical evaluation of theories for predicting microcracking in composite laminates. *Journal of Materials Science* **28**, 5099–5111.
- Nairn, J. (2000). Matrix microcracking in composites. In: R. Talreja and J.-A.E.E. Manson, editors, *Polymer Matrix Composites, Volume 2 of Comprehensive Composite Materials*, Elsevier Science, Amsterdam, 403–432.
- Ogihara, S. and Takeda, N. (1995). Interaction between transverse cracks and delamination during damage process in CFRP cross-ply laminates. *Composite Science and Technology* **54**(4), 395–404.
- Pagano, N.J., Schoeppner, G.A., Kim, R. and Abrams, F.L. (1998). Steady-state cracking and edge effects in thermo-mechanical transverse cracking of cross-ply laminates. *Composite Science and Technology* **58**(11), 1811–1825.
- Parvisi, A. and Bailey, J.E. (1978). On multiple transverse cracking in glass-fiber epoxy cross-ply laminates. *Journal of Material Sciences* **13**, 2131–2136.
- Pijaudier-Cabot, G. and Bažant, Z.P. (1987). Nonlocal damage theory. *ASME Journal of Engineering Mechanics ASCE* **113**(10), 1512–1533.
- Wang, A.S.D., Crossman, F.W. (1980). Initiation and growth of transverse cracks and edge delamination in composite laminates. Part I: an energy method. *Journal of Composite Materials* **14**, 71–87.
- Wang, A.S.D., Chou, P.C. and Lei, S. (1984). A stochastic model for the growth of matrix cracks in composite laminates. *Journal of Composite Materials* **18**, 239–254.
- Yahvac, S., Yats, L. and Wetters, D. (1991). Transverse ply cracking in toughened and untoughened graphite/epoxy and graphite/polycyanate cross-ply laminates. *Journal of Composite Materials* **25**, 1653–1667.
- Yang, T., Liu, Y. and Wang, J. (2003). A study of the propagation of an embedded crack in a composite laminate of finite thickness. *Composite Structures* **59**, 473–479.
- Zhang, J., Fan, J. and Herrmann, K.P. (1999). Delaminations induced by constrained transverse cracking in symmetric composite laminates. *International Journal of Solids and Structures* **36**, 813–846.
- Zohdi, T.I., Oden, J.T. and Rodin, G.J. (1996). Hierarchical modeling of heterogeneous bodies. *Computer Methods in Applied Mechanics and Engineering* **155**, 181–192.