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Analyzing health quality survey using constrained co-clustering model for ordinal data and some dynamic implication

Margot Selosse

Université de Lyon, Lyon 2, ERIC EA 3083, Lyon, France.

Julien Jacques

Université de Lyon, Lyon 2, ERIC EA 3083, Lyon, France.

Christophe Biernacki

Inria, Université de Lille, CNRS, Lille, France.

Florence Cousson-Gélie

Université Paul Valéry Montpellier 3, Université Montpellier, EPSLYON EA 4556, F34000, Montpellier, France.

Summary. The dataset which motivated this work is a psychological survey on women affected by a breast tumor. Patients replied at different moments of their treatment to questionnaires with answers on ordinal scale. The questions relate to aspects of their life called dimensions. To assist the psychologists in analyzing the results, it is useful to emphasize a structure in the dataset. The clustering method achieves that by creating groups of individuals that are depicted by a representative of the group. From a psychological position, it is also useful to observe how questions may be grouped. This is why a clustering should be performed also on the features, which is called a co-clustering problem. However, gathering questions that are not related to the same dimension does not make sense from a psychologist stance. Therefore, a constrained co-clustering has been performed to prevent questions from different dimensions from getting assembled in a same column-cluster. Then, evolution of co-clusters along time has been investigated. The method relies on a constrained Latent Block Model embedding a probability distribution for ordinal data. Parameter estimation relies on a Stochastic EM-algorithm associated to a Gibbs sampler, and the ICL-BIC criterion is used for selecting the numbers of co-clusters.

1. Introduction

Persons with cancer usually go through traumatizing hardships as chemotherapy and intense stress. The disease and its treatment has an impact on different domains of their environment like social life, or emotional state. In psychology, these domains are divided in dimensions. For example, in Table 1 the domain *quality of life* is divided into six dimensions (physical functioning, role functioning, social functioning, emotional functioning, cognitive functioning, global health evaluation). Differently, the domain *emotional state* is defined with the two dimensions anxiety and depression Zigmond and Snaithe (1983). Others psychological dimensions have been identified as a quality of life predictor like perceived control of the illness, which corresponds to the general belief whereby evolution of the disease depend either on internal factors (action, effort, personal abilities) or on external factors (hazard, destiny) (Cousson-Gélie, 2014), or

social support which assesses perceived availability (number of people on whom the individual thinks they can count if necessary) and the degree of satisfaction relating to this support (Sarason et al., 1983).

The dataset which has initiated this work is a survey realized on women affected by breast cancer (Cousson-Gélie, 2014). The patients were asked to reply to various questionnaires related to distinct dimensions, the answers being of the ordinal kind with different numbers of levels (Agresti, 2012). They repeated this exercise at six different moments of their treatment. Therefore the resulting dataset is a set of six tables, the lines representing the patients, and the columns representing the questions. First of all, the psychologists are interested in identifying psychological profiles. Particularly, they are willing to analyze the mutual influence of the different dimensions for each profile. To help them on this task, a constrained co-clustering was performed. Co-clustering is a technique which operates simultaneous clustering of the rows and columns of a matrix (Govaert and Nadif, 2014). As a result, a co-clustering emphasizes an internal structure in the dataset, which in this case allows to detect typical psychological profiles and the groups of questions that differentiates them. The term “constrained” is used because the co-clustering was forced to keep separated the questions (columns) that did not relate to a common dimension. In a second time, the experts want to investigate the evolution of their patients. Indeed, the notion of trajectory is also important in psychology (Annema et al., 2017): not only does it look at the psychological state of a patient, but it also focuses on the changes of this state. Realizing a co-clustering at each time the patients had to answer the questionnaires gives a better idea of the evolution of the patients on different perspectives. On a global scale, it shows how groups of persons evolve, and how replies changed along the study period. On a more precise scale, the co-clustering makes possible to analyze the behavior of a single patient, by noticing her row-clusters changes over time.

The dataset exclusively contains values of the ordinal type, which is a particular type of categorical data, occurring when the levels are ordered (Agresti, 2012). Unlike categorical data, ordinal data have received less attention from a clustering aspect. Therefore, confronted to such data, the practitioners often transform them into continuous data, by associating an arbitrary number to each level (Kaufman and Rousseeuw (2008), Lewis et al. (2005)) or into nominal data (Vermunt and Magidson, 2005). These choices allow to use well-known distributions but lose the information given by the existing order among levels. More recent contributions have defined clustering algorithms specific to ordinal data (Jollois and Nadif (2009), Giordan and Diana (2011)). In a co-clustering context, Jacques and Biernacki (2017) defines a model-based algorithm relying on the Latent block Model (Govaert and Nadif (2014)). It embeds a recent distribution for ordinal data (BOS for Binary Ordinal Search model, Biernacki and Jacques (2016)) on an SEM-Gibbs algorithm. This model presents strong advantages through its parsimony and the significance of its parameter. Nevertheless, the weakness of this model is its inability to treat variables with different numbers of levels, which is an actual issue for this dataset.

In this work, the Latent Block Model is adapted as a constrained version, so that certain questions cannot be part of the same column-cluster. This extension solves two issues. First, it allows to force the column-clusters to be formed with questions of a

same psychological dimension. Furthermore, it entitles to separate the questions that do not have the same number of levels, so that the BOS distribution can be used. Indeed, this distribution is defined with two parameters (μ, π) . It makes sense to compare the parameters of two samples generated from different (μ, π) only when the number of levels of these samples is equal. The paper is organized as follows: Section 2 presents the dataset and the notations, while Section 3 explains the statistical models that were used. At last, Section 4 describes the obtained results on the psychological dataset.

2. Material

2.1. Dataset

2.1.1. Inquiry population description

Several questionnaires were given to $N = 161$ women who had their first surgery for suspicious breast tumor. These patients were from 31 to 77 years old with an average age of 56,25 years (standard deviation=9.99). Most were married or lived maritally (77,0%). Near half of the patients were active professionally (49,7%) and 38.5% were retired the moment they got into the study. These 161 patients were asked to answer several questionnaires, at different moments of their treatment: one at their first surgery, and 1, 4, 7, 10, 13 months after this assessment. As a result, the patients replied 6 times to 134 questions and each answer was given on an ordinal scale (with a number of levels varying from 4 to 7). Therefore, the dataset is a set of 6 matrices of ordinal data such that the observations (rows) correspond to the patients, and the variables (columns) correspond to the questions. Furthermore, sometimes some patients did not reply to a couple of questions: therefore, there is a little proportion of missing data (18 values) in the dataset.

2.1.2. Psychological dimensions

The questionnaires that were given to the patients were not made with casual questions. As a matter of fact, the conception of questionnaires is a highly-specialized work in psychology. Each questionnaire relates to precise domains of life that are themselves divided into dimensions (e.g: MaloneBeach and Zarit (1995)), Table 1 lists the ones that were broached in the study. Most of the questions are actually associated to a dimension. The few questions that are not related to one of these psychological dimensions actually concern the treatment and disease symptoms (nausea, tiredness...).

2.2. Data representation and conventions

First of all, it has been agreed that for all the questions, the most positive answer is given the level "1". For example, for the question: "*Have you had trouble sleeping?*" with possible responses: "*Not at all.*" "*A little.*" "*Quite a bit.*" "*Very much.*", the following levels number are assigned to the replies: 1 "*Not at all.*", 2 "*A little.*", 3 "*Quite a bit.*", 4 "*Very much.*", because it is perceived as more positive not to have had trouble sleeping.

Secondly, a graphical way of representing the data has been accorded. Figure 1 exposes it: the women are projected on lines and the questions are projected on columns. Therefore, the cell (i, j) is the reply of patient i to question j . The shades of gray

Table 1. Table of domains and dimensions approached by the questionnaires.

Domains				
Quality of life	Social Support	Specific Social Support (for a close person)	Emotional State	Control perception regarding the disease
Dimensions				
Physical functioning, Role functioning, Emotional functioning, Cognitive functioning, Social functioning, Global health evaluation.	Satisfaction, Quantity.	Intensity, Perception of availability Conflicts.	Anxiety, Depression.	Causal attribution, Control perception, Religion control.

indicates how positively the person replied. For example, for the question *"Have you had trouble sleeping?"*, if the patient answers *"Not at all."*, the corresponding cell will be white, whereas a response like *"Very much."* would imply a black cell.

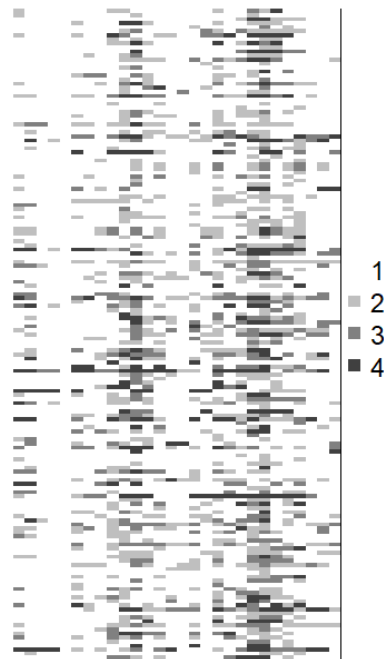


Fig. 1. Graphical representations of the patients replies. The women are in lines and the questions are in columns. A cell is the answer of a person to a question. The darker the cell is, the more pessimistic the patient responded.

2.3. Notations

First of all, an ordinal variable x with m levels $\{l_1, \dots, l_m\}$ is a categorical variable whose levels order is significant. The total order relation between the levels is quoted in the sequel " $<$ ": $l_1 < \dots < l_m$. Furthermore, for simplicity the levels are numbered $\{1, \dots, m\}$ according to their order. Following this notation, an ordinal variable x is therefore an element of $\{1, \dots, m\}$.

The representation of the questionnaires responses at a given time is now detailed. The questions are separated according to two criteria: the number of levels m and the dimension it is related to. Indeed, the variables that do not have the same number of levels and the variables that are not related to the same dimension are pulled apart. This results in a matrix split up in D tables, such that the d^{th} table is a $N \times J_d$ matrix written \mathbf{x}^d , made of ordinal data with number of levels m_d , as illustrated in Figure 2.

$$\mathbf{x} = \left[\begin{array}{c} \left[\begin{array}{c} \mathbf{x}^1 \end{array} \right] \quad \dots \quad \left[\begin{array}{c} \mathbf{x}^D \end{array} \right] \end{array} \right], \text{ with } \mathbf{x}^d = (x_{ij}^d)_{i=1, \dots, N; j=1, \dots, J_d}$$

Fig. 2. Representation of the patients and questions at a given time. Questions related to different dimensions or with different number of levels m are separated

The dataset contains missing data. The whole dataset will be written $\mathbf{x} = (\check{\mathbf{x}}, \hat{\mathbf{x}})$, $\check{\mathbf{x}}$ being the observed data, and $\hat{\mathbf{x}}$ being the missing data. Consequently a cell of \mathbf{x} will be annotated as follows: \check{x}_{ij} , whether x_{ij} is observed, \hat{x}_{ij} otherwise.

Finally, the limits of variation $1 \leq i \leq N$, $1 \leq j \leq J$, $1 \leq g \leq G$, $1 \leq h \leq H$ (or $1 \leq h \leq H_d$ from Section 3.2.2) will be implicit. For example, the matrix $\mathbf{x} = (x_{ij})_{1 \leq i \leq N, 1 \leq j \leq J}$ will be written $(x_{ij})_{i,j}$. Furthermore, the sums and the products relating to rows, columns, row-clusters and column-clusters will be subscripted respectively by the letters i , j , g , and h . So the sums and products will be written \sum_i , \sum_j , \sum_g and \sum_h and \prod_i , \prod_j , \prod_g and \prod_h .

3. Methods

3.1. The BOS distribution for ordinal data

The BOS model (Biernacki and Jacques, 2016) is a probability distribution for ordinal data parametrized by a position parameter $\mu \in \{1, \dots, m\}$ and a precision parameter $\pi \in [0, 1]$. It was built by assuming that an ordinal variable is the result of a stochastic binary search algorithm within the ordered table $(1, \dots, m)$. This distribution rises from the uniform distribution when $\pi = 0$ to a more peaked distribution around the mode μ when π grows, and reaches a Dirac distribution at the mode μ when $\pi = 1$. Figure 3 illustrates the shape of the BOS distribution with different values of μ and π . It is shown in Biernacki and Jacques (2016) that the BOS distribution is a polynomial function of π with degree $m - 1$, whose coefficients depend on the position parameter μ . For a

univariate ordinal variable, the path in the stochastic binary search can be seen as a latent variable. Therefore, maximum likelihood estimation of model parameters can be simply performed using an EM algorithm (Dempster et al., 1977).

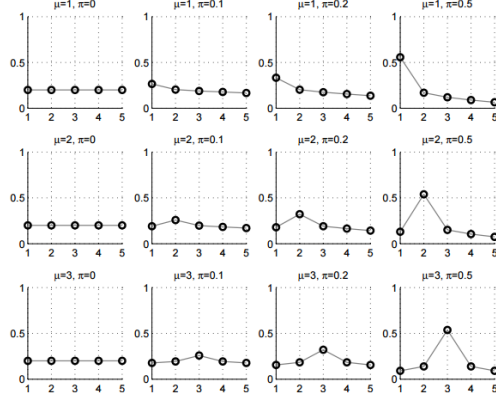


Fig. 3. BOS distribution $p(x; \mu, \pi)$: shape for $m = 5$ and for different values of μ and π

3.2. Latent Block Model extension

In this section, the Constrained Latent Block Model is described but first the Latent Block Model concepts are recalled (Govaert and Nadif, 2014).

3.2.1. Latent Block Model

Let $\mathbf{x} = (x_{ij})_{i,j}$ be a data matrix. It is assumed that there exists a partition $\mathbf{v} = (v_{ig})_{i,g}$ and a partition $\mathbf{w} = (w_{jh})_{j,h}$ such that each element x_{ij} is generated under a parameterized probability density function $f(x_{ij}; \alpha_{gh})$ where g denotes the cluster of row i while h denotes the cluster of column j . The univariate random variables x_{ij} are assumed to be conditionally independent given the row and column partitions \mathbf{v} and \mathbf{w} . Therefore, the conditional probability density function of \mathbf{x} given \mathbf{v} and \mathbf{w} can be expressed in the following form:

$$p(\mathbf{x}|\mathbf{v}, \mathbf{w}; \boldsymbol{\alpha}) = \prod_{i,j,g,h} f(x_{ij}; \alpha_{gh})^{v_{ig}w_{jh}},$$

considering that $v_{ig} = 1$ if i belongs to cluster g , whereas $v_{ig} = 0$ otherwise, and that $w_{jh} = 1$ when j belongs to cluster h , but $w_{jh} = 0$ otherwise.

Different univariate distributions can be used regarding the type of data (e.g: Gaussian, Bernoulli, Poisson...). In the present case, the BOS distribution is chosen. The label of row i is called v_i and belongs to $\{1, \dots, G\}$. Similarly, the label for column j is called w_j and belongs to $\{1, \dots, H\}$. They are latent variables assumed to be independent so $p(\mathbf{v}, \mathbf{w}; \boldsymbol{\gamma}, \boldsymbol{\rho}) = p(\mathbf{v}; \boldsymbol{\gamma})p(\mathbf{w}; \boldsymbol{\rho})$ with:

$$p(\mathbf{v}; \boldsymbol{\gamma}) = \prod_i p(v_i; \boldsymbol{\gamma}) = \prod_{i,g} \gamma_g^{v_{ig}} \text{ and } p(\mathbf{w}; \boldsymbol{\rho}) = \prod_j p(w_j; \boldsymbol{\rho}) = \prod_{j,h} \rho_h^{w_{jh}},$$

knowing that $\gamma_g = p(v_{ig} = 1)$ with $g \in \{1, \dots, G\}$ and $\rho_h = p(w_{jh} = 1)$ with $h \in \{1, \dots, H\}$. This implies that, for all i , the distribution of v_i is the multinomial distribution $\mathcal{M}(\gamma_1, \dots, \gamma_G)$ and does not depend on i . In a similar way, for all j , the distribution of w_j is the multinomial distribution $\mathcal{M}(\rho_1, \dots, \rho_H)$ and does not depend on j . From these considerations, the parameter of the latent block model is defined as $\boldsymbol{\theta} = (\boldsymbol{\gamma}, \boldsymbol{\rho}, \boldsymbol{\alpha})$, where $\boldsymbol{\alpha} = (\alpha_{gh})_{g,h}$, with $\alpha_{gh} = (\mu_{gh}, \pi_{gh})$ being the position and precision BOS parameters of the distribution of block (g, h) . Additionally, $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_G)$ and $\boldsymbol{\rho} = (\rho_1, \dots, \rho_H)$ are the mixing proportions. Therefore, if V and W are the sets of all possible labels \mathbf{v} and \mathbf{w} , the probability density function $p(\mathbf{x}; \boldsymbol{\theta})$ of \mathbf{x} can be written:

$$p(\mathbf{x}; \boldsymbol{\theta}) = \sum_{(\mathbf{v}, \mathbf{w}) \in V \times W} \prod_{ig} \gamma_g^{v_{ig}} \prod_{jh} \rho_h^{w_{jh}} \prod_{i,j,g,h} f(x_{ij}; \alpha_{gh})^{v_{ig} w_{jh}}.$$

3.2.2. Constrained Latent Block Model

In this section, the Latent Block Model is extended as in Robert (2017) so that the questions from different dimensions are kept separated, as well as the questions with different number of levels. In this co-clustering framework, \mathbf{x} is a $N \times (J_1 + \dots + J_D)$ matrix, and it is seen as matrices $\mathbf{x}^1, \dots, \mathbf{x}^D$ stored side by side as explained in Section 2.3. It is supposed that there is a row-partition \mathbf{v} , and that for all $d \in \{1, \dots, D\}$, there exists a column-partition \mathbf{w}^d such that each element x_{ij}^d is generated under a parameterized probability density function $f(x_{ij}^d; \alpha_{gh})$. Here, h denotes the cluster of column j , with $j \in \{1, \dots, J_d\}$ and $h \in \{1, \dots, H_d\}$. The univariate random variables x_{ij}^d are assumed to be conditionally independent given the row and column partitions \mathbf{v} and \mathbf{w}^d . Therefore, the conditional probability density function of \mathbf{x} given \mathbf{v} and $\mathbf{w}^d_{d \in \{1, \dots, D\}}$ can be written in the following form:

$$p(\mathbf{x} | \mathbf{v}, \mathbf{w}^1, \dots, \mathbf{w}^D; \boldsymbol{\alpha}) = \prod_{i,j,d} f(x_{ij}^d; \alpha_{v_i w_j^d}) = \prod_{d,i,j,g,h} f(x_{ij}^d; \alpha_{gh})^{v_{ig} w_{jh}^d},$$

knowing that: $\forall d \in \{1, \dots, D\}$, $w_{jh}^d = 1$ when j belongs to cluster h , but $w_{jh}^d = 0$ otherwise.

The labels $v_1, \dots, v_N, (w_1^d, \dots, w_{J_d}^d)_{d \in \{1, \dots, D\}}$ are latent variables assumed to be independent: $p(\mathbf{v}, \mathbf{w}^1, \dots, \mathbf{w}^D; \boldsymbol{\gamma}, \boldsymbol{\rho}) = p(\mathbf{v}; \boldsymbol{\gamma}) \prod_d p(\mathbf{w}^d; \boldsymbol{\rho}^d)$ with:

$$p(\mathbf{v}; \boldsymbol{\gamma}) = \prod_i p(v_i; \boldsymbol{\gamma}) = \prod_{i,g} \gamma_g^{v_{ig}} \text{ and } p(\mathbf{w}^d; \boldsymbol{\rho}^d) = \prod_j p(w_j^d; \boldsymbol{\rho}^d) = \prod_{j,h} \rho_h^d {w_{jh}^d}^{w_{jh}^d},$$

knowing that $\rho_h^d = p(w_{jh} = 1)$ with $h \in \{1, \dots, H\}$. Again, for all i , the distribution of v_i is the multinomial distribution $\mathcal{M}(\gamma_1, \dots, \gamma_G)$ and does not depend on i . Evenly, for all j and for all d , the distribution of w_j^d is the multinomial distribution $\mathcal{M}(\rho_1^d, \dots, \rho_{H_d}^d)$ and does not depend on j . By analogy with the classic Latent Block Model, the probability density function $p(\mathbf{x}; \boldsymbol{\theta})$ is written:

$$p(\mathbf{x}; \boldsymbol{\theta}) = \sum_{(\mathbf{v}, \mathbf{w}^1, \dots, \mathbf{w}^D) \in V \times W_1 \times \dots \times W_D} \prod_{i,g} \gamma_g^{v_{ig}} \prod_{d,j,h} \rho_h^d {w_{jh}^d}^{w_{jh}^d} \prod_{i,j,g,d,h} f(x_{ij}^d; \alpha_{gh})^{v_{ig} w_{jh}^d}.$$

3.3. Model inference with an SEM-Gibbs algorithm

This section details the model inference in the case of the constrained latent block model. The aim is to estimate $\boldsymbol{\theta}$ by maximizing the observed log-likelihood $l(\boldsymbol{\theta}; \tilde{\mathbf{x}}) = \sum_{\tilde{\mathbf{x}}} p(\mathbf{x}; \boldsymbol{\theta})$. In a co-clustering context, the EM algorithm is not computationally feasible (see Govaert and Nadif (2014)). Indeed, the E step requires the calculation of the joint conditional probability of the missing labels $p(v_{ig} = 1, w_{jh} = 1 | \mathbf{x}; \boldsymbol{\theta}^{(q)})$ for $1 \leq i \leq N$, $1 \leq g \leq G$, $1 \leq d \leq D$, $1 \leq j \leq J_d$, $1 \leq h \leq H_d$, with $\boldsymbol{\theta}^{(q)}$ the current value of the parameter. Therefore, this step implies to compute $N \times G \times (J_1 \times H_1 + \dots + J_D \times H_D)$ terms that cannot be factorized as for a standard mixture, due to the dependence of the row and column labels conditionally to the observations. There exists several alternatives to the EM algorithm like variational EM algorithm the SEM-Gibbs algorithm or Bayesian inference (Govaert and Nadif, 2014). The SEM-Gibbs is known to avoid spurious solutions (Keribin et al., 2010), this is why it is used in this paper. Starting from an initial value for the parameter $\boldsymbol{\theta}^{(0)}$, the q^{th} iteration of the algorithm is composed of two steps.

3.3.1. SE-step

The SE-step consists in estimating the joint conditional probability of the missing labels by a Gibbs sampling. Therefore, it repeats several times these steps:

First step. Generate the row partitions with $v_{ig}^{(q+1)} | (\mathbf{x}^d)_{d \in \{1, \dots, D\}}, (\mathbf{w}^d)_{d \in \{1, \dots, D\}}^{(q)}$:

$$p(v_{ig} = 1 | \mathbf{x}^d, \mathbf{w}^{d(q)}; \boldsymbol{\theta}^{(q)})_{d \in \{1, \dots, D\}} \propto \gamma_g^{(q)} \times \prod_d t_g^d(x_{i.}^d | \mathbf{w}^{d(q)}; \boldsymbol{\alpha}^{(q)}), \quad (1)$$

where: $t_g^d(x_{i.}^d | \mathbf{w}^{d(q)}; \boldsymbol{\alpha}^{(q)}) = \prod_{j,h} f(x_{ij}^d; \mu_{gh}^d, \pi_{gh}^d) w_{jh}^{d(q)}$.

Second step. For all $d \in \{1, \dots, D\}$ generate the column partitions for the d^{th} table \mathbf{x}^d with $w_{jh}^d | (\mathbf{x}^d)_{d \in \{1, \dots, D\}}, \mathbf{v}^{(q+1)}$:

$$p(w_{jh}^d = 1 | \mathbf{x}^d, \mathbf{v}^{(q+1)}; \boldsymbol{\theta}^{(q)}) \propto \rho_h^{d(q)} \times s_h^d(x_{d.j} | \mathbf{v}^{(q+1)}; \boldsymbol{\alpha}^{(q)}), \quad (2)$$

where: $s_h^d(x_{d.j} | \mathbf{v}^{(q+1)}; \boldsymbol{\alpha}^{(q)}) = \prod_{i,g} f(x_{ij}^d; \mu_{gh}^d, \pi_{gh}^d) v_{ig}^{(q+1)}$.

3.3.2. M-step

The M-step consists in maximizing the completed log-likelihood by updating the co-clusters parameters according to the results of the last SE step. It relies on the EM algorithm used in Biernacki and Jacques (2016) for the estimation of the BOS distribution on each block.

3.3.3. Imputation of missing values

The SEM-algorithm is able to take into account the missing data and to estimate them. It is assumed that the whole missing process is Missing At Random (see Little and Rubin (1986)). First, the notation of \mathbf{x} becomes $\mathbf{x}^{(q)}$ since the missing variables are going to be imputed. Then, a third step is added to the SE-step. For all $d \in \{1, \dots, D\}$, it generates the missing data $\hat{x}_{ij}^{d^{(q+1)}} | \check{\mathbf{x}}^d, \mathbf{v}^{(q+1)}, \mathbf{w}^{d^{(q+1)}}$ as follows:

$$p(\hat{x}_{ij}^{d^{(q+1)}} | \check{\mathbf{x}}^d, \mathbf{v}^{(q+1)}, \mathbf{w}^{d^{(q+1)}}; \boldsymbol{\theta}^{(q)}) = \prod_{g,h} p(\hat{x}_{ij}^{d^{(q)}}; \mu_{gh}^{d^{(q)}}, \pi_{gh}^{d^{(q)}}) v_{ig}^{(q+1)} w_{gh}^{d^{(q+1)}}.$$

3.3.4. Estimation of partitions and model parameter

The SEM-algorithm repeats several times the aforementioned steps. The first iterations are called the burn-in period, which means the parameters are not stable yet. Consequently, the iterations that occurred after this burn-in period are taken into account, they are called the sample distribution. The final estimation of the position parameter μ_{gh} is the mode of the sampling distribution. The final estimation of the continuous parameters $(\pi_{gh}^d, \gamma_g, \rho_h^d)_d$ is the median of the sample distribution. It corresponds to a final estimation of $\boldsymbol{\theta}$ that is called $\hat{\boldsymbol{\theta}}$. Then, a sample of $(\hat{\mathbf{x}}, \mathbf{v}, (\mathbf{w}^d)_d)$ is generated by a SE-step with $\boldsymbol{\theta}$ fixed to $\hat{\boldsymbol{\theta}}$. The final partitions $(\hat{\mathbf{v}}, \hat{\mathbf{w}})$ and the missing observation $\hat{\mathbf{x}}$ are estimated by the mode of their sample distribution.

3.4. Model Selection

3.4.1. ICL Criterion

To select the number of clusters, G in rows and H_1, \dots, H_D in columns, a model selection criterion must be used. The most classical ones, like BIC (Schwarz, 1978) rely on penalizing the maximum log-likelihood value $l(\hat{\boldsymbol{\theta}}; \mathbf{x})$. However, due to the dependency of the observed data, this value is not available in a co-clustering context. Instead, an approximation of the ICL information criterion (Biernacki et al., 2000) called ICL-BIC is employed. Keribin et al. (2015) details the ICL-BIC expression for the case of categorical data. In Jacques and Biernacki (2017), the BOS distribution is considered as a specific model for categorical data and the ICL-BIC expression is easily deduced from Keribin et al. (2015). In the present work, the ICL-BIC is therefore adapted for the constrained latent block model:

$$\text{ICL-BIC}(G, H_1, \dots, H_D) = \log p(\check{\mathbf{x}}, \hat{\mathbf{v}}, \hat{\mathbf{w}}^1, \dots, \hat{\mathbf{w}}^D; \hat{\boldsymbol{\theta}}) - \frac{G-1}{2} \log N - \sum_d \frac{H_d-1}{2} \log J_d - \sum_d \frac{G \times H_d}{2} \log(N \times J_d),$$

where $\hat{\mathbf{v}}, \hat{\mathbf{w}}^1, \dots, \hat{\mathbf{w}}^D$ are the row and column partitions discovered by the SEM-algorithm, and $\hat{\boldsymbol{\theta}}$ is the corresponding estimated model parameter.

On a dataset, the co-clustering is performed several times, with different (G, H_1, \dots, H_D) . The configuration with the highest ICL-BIC is retained.

3.4.2. *Highest ICL-BIC search*

In order to find the best (G, H_1, \dots, H_D) , the co-clustering has to be performed for each possible values of G and H_d , $d \in \{1, \dots, D\}$, then the result with the highest ICL-BIC is retained. Let n_G be the number of candidate values for G , while n_{H_d} is the number of candidate values for H_d , $d \in \{1, \dots, D\}$. Thus, the number of co-clustering to execute is $n_G \times n_{H_1} \times \dots \times n_{H_D}$. As an example, if $D = 3$ and the user wants to try 3 values for G and for each H_d , then it would require to execute $3^4 = 81$ co-clusterings. Depending on the dataset, it might take too much time to find the best solution. With the Bi-Km1 algorithm, Robert (2017) offers an efficient way to find a good (G, H_1, H_2) in the case of binary data, and with $D = 2$. In this work, D can get much larger, which significantly increases the execution time. Therefore, a strategy that simplifies the Bi-Km1 is proposed here. Algorithm 1 sums up this strategy.

Let call (G_{min}, G_{max}) the boundaries of the candidate values for G . Similarly, for all $d \in \{1, \dots, D\}$, $(H_{d_{min}}, H_{d_{max}})$ are the boundaries of the candidate values for H_d . The algorithm starts with the set $(G_{min}, H_{1_{min}}, \dots, H_{D_{min}})$. At iteration r , the current best set (G, H_1, \dots, H_D) is called $(G, H_1, \dots, H_D)^{(r)}$ and is made of values: $(G^{(r)}, H_1^{(r)}, \dots, H_D^{(r)})$. At r^{th} iteration, $D+1$ different co-clusterings are performed with sets $(G^{(r)}+1, H_1^{(r)}, \dots, H_D^{(r)})$, $(G^{(r)}, H_1^{(r)}+1, \dots, H_D^{(r)})$, ..., $(G^{(r)}, H_1^{(r)}, \dots, H_D^{(r)}+1)$. Then, the ICL-BIC is computed for each result. If none of the ICL-BIC is better than for the set $(G, H_1, \dots, H_D)^{(r)}$, the algorithm finishes and $(G, H_1, \dots, H_D)^{(r)}$ is the set to use. In the other case, the set with the highest ICL-BIC is retained, and becomes $(G, H_1, \dots, H_D)^{(r+1)}$. The algorithm afterwards reiterates the same steps.

```

 $(G, H_1, \dots, H_D)^0 = (G_{min}, H_{1_{min}}, \dots, H_{D_{min}})$  ;
ICL-BIC=-inf ;
better-ICL = TRUE ;
r=0 ;
while better-ICL do
    r = r+1 ;
    Perform  $D + 1$  different co-clusterings with sets  $(G^{(r+1)}, H_1^{(r)}, \dots, H_D^{(r)})$ ,
     $(G^{(r)}, H_1^{(r)} + 1, \dots, H_D^{(r)})$ , ...,  $(G^{(r)}, H_1^{(r)}, \dots, H_D^{(r)} + 1)$  ;
    Compute the ICL-BIC for each result ;
     $(G, H_1, \dots, H_D)^{(r+1)} =$  set with highest ICL-BIC ;
    if  $ICL-BIC((G, H_1, \dots, H_D)^{(r+1)}) \geq ICL-BIC((G, H_1, \dots, H_D)^{(r)})$  then
        | better-ICL = TRUE ;
    else
        | better-ICL = FALSE ;
    end
end
return  $(G, H_1, \dots, H_D)^{(r)}$ 
Algorithm 1: Algorithm for finding a good set  $(G, H_1, \dots, H_D)$ .

```

Table 2. Co-clustering result on dimensions anxiety, depression, symptoms: estimated BOS parameters (μ_{gh}, π_{gh}) for each cluster (g, h) .

	Anxiety		Depression		Symptoms	
	col. cluster 1	col. cluster 2	col. cluster 1	col. cluster 2	col. cluster 1	col. cluster 2
row cluster 1	(2,0.77)	(2,0.77)	(1,0.70)	(2,0.83)	(2,0.46)	(1,0.74)
row cluster 2	(2,0.68)	(3,0.72)	(2,0.47)	(2,0.79)	(3,0.39)	(1,0.42)
row cluster 3	(1,0.64)	(2,0.44)	(1,0.77)	(2,0.70)	(2,0.58)	(1,0.71)
row cluster 4	(1,0.67)	(2,0.47)	(1,0.79)	(2,0.71)	(1,0.80)	(1,0.93)
row cluster 5	(2,0.72)	(3,0.55)	(2,0.64)	(2,0.75)	(2,0.66)	(1,0.77)

4. Application on the survey dataset

4.1. Constrained co-clustering on different dimensions

Several constrained co-clusterings were performed on the dataset, with different dimensions and at different times. This section presents some significant results that were obtained. In the following experiments, Algorithm 1 was executed with $G_{min} = 3$ and $H_{d_{min}} = 1$ to choose the numbers of row-clusters and column-clusters (G and $(H_d), d \in \{1, \dots, D\}$). All the ICL-BIC values are available in the appendix. The choice for a good number of iterations for the SEM-algorithm and for the burn-in period was made empirically. It was noticed that the parameters would stabilize after 150 iterations (or less). Therefore, the burn-in period was set to 400 iterations and the total number of iterations was fixed to 500. In the appendix, Figure A1 shows how some example parameters converges through iterations.

4.1.1. Anxiety, depression and symptoms.

As a first experiment, it was decided to investigate the responses that were given at time $T5$, at the end of the treatment. For the symptoms dimension, it is a key moment because the patient has been going through chemotherapy for one year. A constrained co-clustering was realized by fetching the questions related to the dimensions anxiety, depression and symptoms. In this case, all the questions have a number of levels m equals to 4. Therefore, the only constraint is the separation of the questions that are from different dimensions. The execution time of this set up is about 12 seconds with an Intel Xeon E5-2620 CPU 2.00 GHz and 8Go RAM. The result of the constrained co-clustering is illustrated by Figure 4. For all the figures, clusters are read from left to right and from top to bottom. Furthermore, Table 2 details the estimated BOS parameters $(\mu_{gh}$ and $\pi_{gh})$ for $g \in \{1, \dots, G\}$ and $h \in \{1, \dots, H_d\}, \forall d \in \{1, \dots, D\}$.

Five row-clusters are highlighted by the co-clustering results. Table 2 shows that the positions parameters of the second row-cluster $(\mu_{2h})_{d,h}$ are globally greater than (or equal to) those of the other row-clusters. It means that the second group presents more signs of anxiety and depression, and senses more intensively the disease symptoms than the other ones. It is also noticed that the fourth row-cluster is less inclined to anxiety and depression and suffers less from the symptoms than the others groups: indeed, parameters $(\mu_{4h})_{d,h}$ are globally the lowest. Furthermore, the precision parameters $(\pi_{4h})_{d,h}$ are quite high for this row-cluster, which means that the answers do not disperse much around the position $(\mu_{4h})_{d,h}$. By observing these two groups results, one could tell that the sense of symptoms is closely associated to signs of anxiety and depression, which

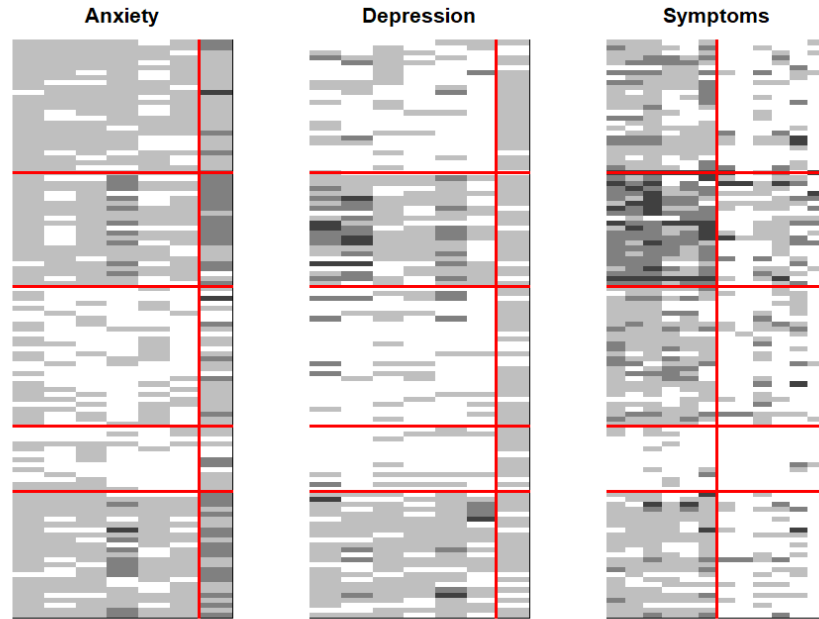


Fig. 4. Results from constrained co-clustering on dimensions anxiety, depression and symptoms.

is a pretty logical and intuitive result. Yet, the first, the third and the fifth groups bring more information. They are effectively very similar about how much they suffer the disease symptoms dimensions. However, they differ a lot in the first column cluster of anxiety, and in the first column cluster of depression. It means than even if a link between symptoms, anxiety and depression can be deduced from the first observations, it is not totally confirmed when people do not sense the symptoms at the extremes (very much or not much). Moreover the column-clusters offer interesting result as well: there is a clear separation among the symptoms. By examining the questions in each cluster, it turns out that questions in the first cluster exclusively deal with pain and fatigue, while the second cluster deals with other symptoms such as nausea or loss of appetite. The co-clustering therefore detected two sub-dimensions for the symptoms dimension. What's more there is a big difference on how the patients sense these two clusters: it is easily noticed that the position parameters $(\mu_{g1})_{(symptoms)}$ are globally higher than $(\mu_{g2})_{(symptoms)}$. Therefore all the patients in general suffers more from pain and fatigue than the others symptoms.

4.1.2. *Social support: satisfaction, availability, intensity and conflicts.*

As a second experiment, questions related to the social support were used. The responses come from the fourth moment of the experiment: it is in the middle of the treatment for the patients, so they have gone through a lot, but know they have to keep on going

Table 3. Co-clustering result on social support dimensions: estimated BOS parameters (μ_{gh}, π_{gh}) for each cluster (g, h) .

	Satisfaction	Availability		Intensity		Conflicts		
	col. cluster 1	col. cluster 1	col. cluster 2	col. cluster 1	col. cluster 2	col. cluster 1	col. cluster 2	col. cluster 3
row cluster 1	(2,0.90)	(1,0.72)	(1,0.96)	(3,0.48)	(1,0.59)	(4,0.80)	(3,0.24)	(1,0.62)
row cluster 2	(3,0.87)	(1,0.64)	(1,0.50)	(3,0.46)	(2,0.48)	(4,0.47)	(3,0.42)	(1,0.49)
row cluster 3	(1,0.73)	(2,0.72)	(1,0.86)	(3,0.52)	(2,0.63)	(4,0.59)	(3,0.51)	(1,0.44)
row cluster 4	(2,0.79)	(1,0.61)	(2,0.64)	(3,0.31)	(2,0.50)	(3,0.27)	(3,0.32)	(1,0.44)
row cluster 5	(1,0.93)	(1,0.78)	(1,0.91)	(3,0.27)	(1,0.68)	(4,0.71)	(3,0.18)	(1,0.63)

for a few months. Their perception of their social support is therefore interesting at this moment. This aspect includes questions of four dimensions: the satisfaction (with number of levels $m = 6$), the perception of availability, the intensity and the conflicts (with a number of levels $m = 4$). The questions which relate to the same dimension have the same number of levels. Again, the only constraint is the separation of the questions that are not from the same dimensions. The result of the constrained co-clustering is illustrated by Figure 5. Furthermore, Table 3 details the estimated BOS parameters (μ and π) for each co-cluster.

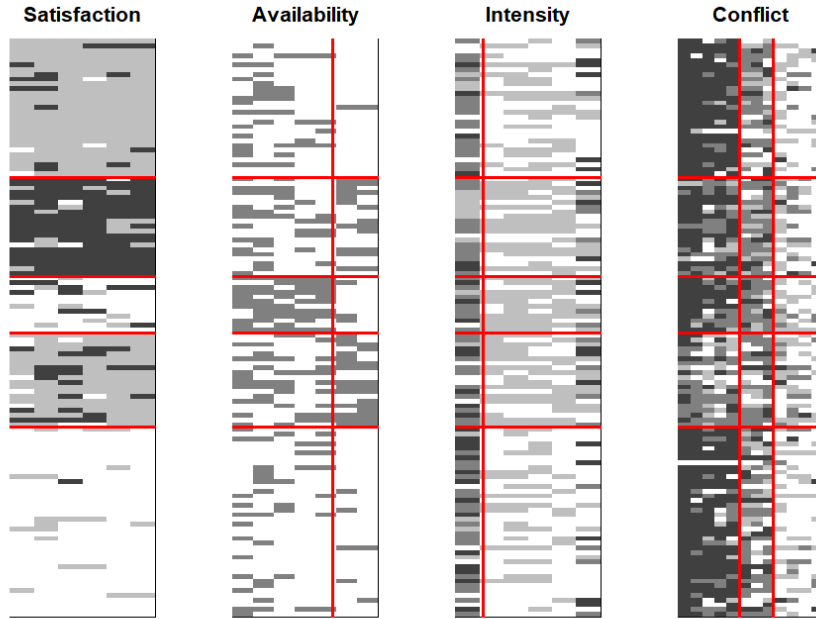


Fig. 5. Result from constrained co-clustering on dimensions related to social support.

The co-clustering detected five row-clusters. The third and the fifth ones are clearly satisfied with the social support they have. Indeed, their position parameters $\mu_{31}(\text{satisfaction})$ and $\mu_{51}(\text{satisfaction})$ are equal to 1. Furthermore, the precisions $\pi_{31}(\text{satisfaction})$ and $\pi_{51}(\text{satisfaction})$ are really high, which means that most of the patients effectively replied in the most positive way to the questions regarding their satisfaction. The women in

Table 4. Co-clustering result on symptoms dimension, at three different times: estimated BOS parameters (μ_{gh}, π_{gh}) for each co-cluster (g, h) .

	T0 Symptoms		T2 Symptoms		T25 Symptoms	
	col. cluster 1	col. cluster 2	col. cluster 1	col. cluster 2	col. cluster 1	col. cluster 2
row cluster 1	(2,0.20)	(1,0.67)	(2,0.62)	(1,0.72)	(2,0.64)	(1,0.74)
row cluster 2	(2,0.09)	(1,0.62)	(3,0.43)	(1,0.40)	(3,0.42)	(1,0.46)
row cluster 3	(1,0.66)	(1,0.84)	(1,0.58)	(1,0.85)	(1,0.54)	(1,0.84)

the first group are quite dissatisfied by their social support compared to the other ones. An interesting result is that the notion of satisfaction does not seem tightly linked to the perception of availability. Actually, the third group, which is one of the most satisfied, has one of the worst perception of availability from their close family and friends ($\mu_{31(availability)} \geq \mu_{g1(availability)}$). Similarly, the presence of conflicts does not seem to have an impact on the satisfaction neither: the BOS parameters of the different row-clusters for the conflict dimension do not significantly change from a row-cluster to another. At last, a inherent structure has been detected among the questions related to conflicts. Indeed, the dimension is divided into three groups. By reading the questions of each group, it can be interpreted as follow: the first group of questions is about the efforts the patient has to make not to enter in conflict with their close ones. The second groups gathered questions about changes in the relationship, whereas the last cluster concerns the sentiments of anger towards their close family and friends.

4.1.3. Symptoms at different times.

In this experiment, the questions related to symptoms were selected for different moments (at time $T0$, $T2$ and $T5$). The constraint is therefore not to separate the questions from different dimensions, but to separate the questions that are not from the same time. The point of performing a co-clustering on such a dataset is that the row-clusters gather people that had a similar evolution regarding this dimension. Furthermore, the column-clusters give information about how the patients symptoms globally worsens (or get better) throughout the treatment. BOS parameters for this experiment are available in Table 4, and Figure 6 illustrates the results.

The co-clustering emphasizes three row-clusters. The third one gathers people that felt less the disease symptoms than the others: the position parameters $(\mu_{3h})_{d,h}$ are all equals to 1. What's more, the precision parameters $(\pi_{3h})_{d,h}$ are pretty high, which implies that the responses do not spread a lot around the value 1. It is also interesting to investigate how the column-clusters evolve. To begin, for each time, the symptoms are separated into two column-clusters: systematically, the first one is globally worse than the second one, because $(\mu_{g1})_{(T0,T2,T5)} \geq (\mu_{g2})_{(T0,T2,T5)}, \forall g \in \{1, \dots, G\}$. It is observed that at time $T0$ there is less symptoms in the column-cluster 1 than in column-cluster 2, whereas they are equally shared at times $T2$ and $T5$.

4.2. Handling of the dynamical aspect of the data

The patients answered to the same questionnaires at 6 different moments of their treatment. Consequently, there is an interest in the evolution of the responses. To handle this

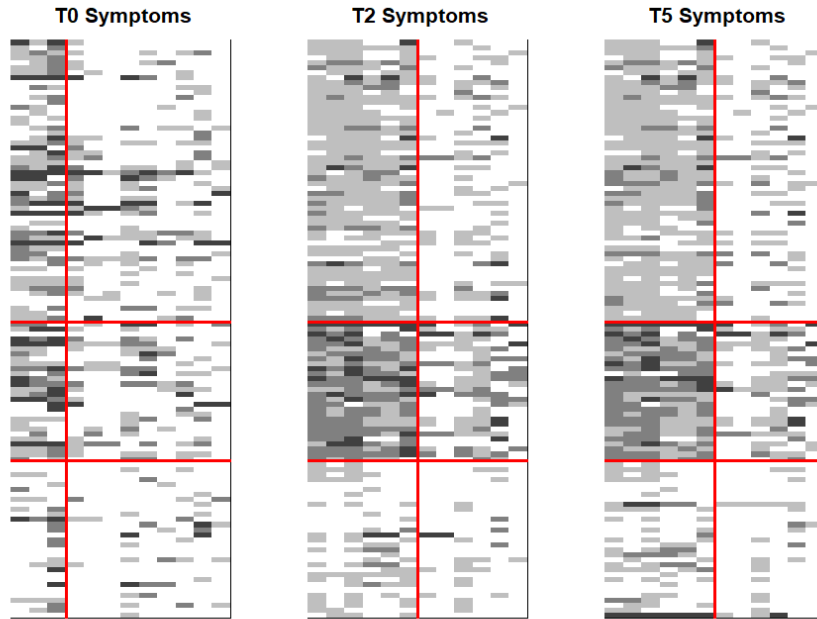


Fig. 6. Co-clustering results with questions related to symptoms, at three different times.

perspective, visualization were created so that the psychologists could have a first impression of the evolution, with the `Javascript` library `D3js`. First of all, the interesting dimensions were selected. Then, a co-clustering was performed, similarly to Section 4.1, for each time T_0 , T_1 , T_2 , T_3 , T_4 , T_5 . The visualization represents the row-clusters on the ordinate axis, and the time line on the abscissa axis: Figure 7 illustrates the home page of a visualization that was created with dimensions dealing with quality of life and emotional state. If the expert wishes to observe the evolution of a single patient, they can click on the list of patient on the right to see the row-clusters it belongs to through time, like in Figure 8. Moreover, if the expert wants to know the co-cluster BOS parameters, they click on the row-clusters, and is able to read the (μ, π) of the corresponding co-clusters, as in Figure 9.

These visualizations showed that the patients globally got stabler with time. Indeed, it is noticed that whereas a lot of patients changed of row-clusters at the three first moments T_0, T_1, T_2 . these transitions get rarer after time T_2 .

5. Conclusion

In this paper, a constrained co-clustering algorithm is proposed to analyze psychological questionnaires given to women affected by breast cancer. This dataset has a lot of specificities, which makes the use of classical techniques difficult without changing the information. First, it is made of questionnaires with answers on the ordinal scale. Furthermore it included a temporal aspect because the patients answered 6 times to these

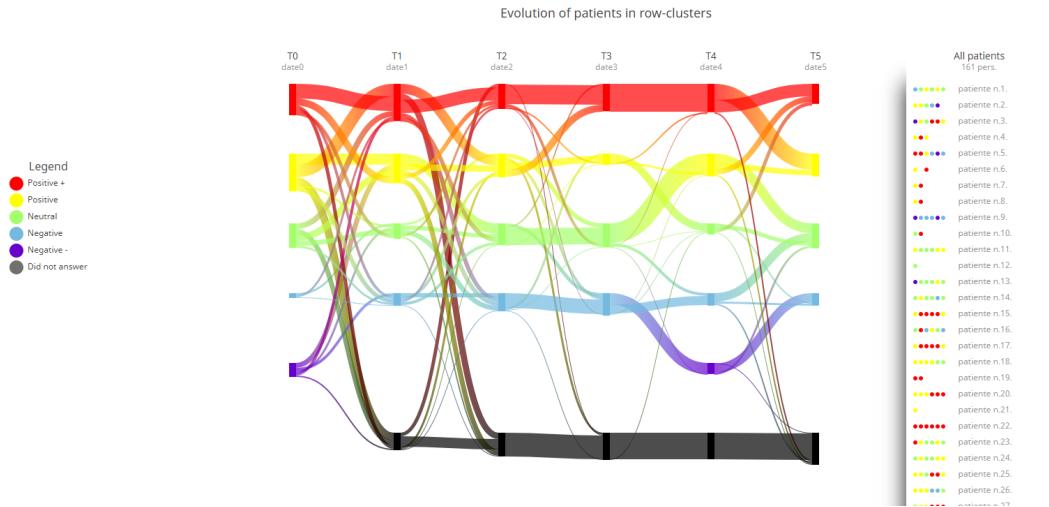


Fig. 7. Home-page: the row-clusters are represented on the ordinate axis, and the time line is on the abscissa axis.

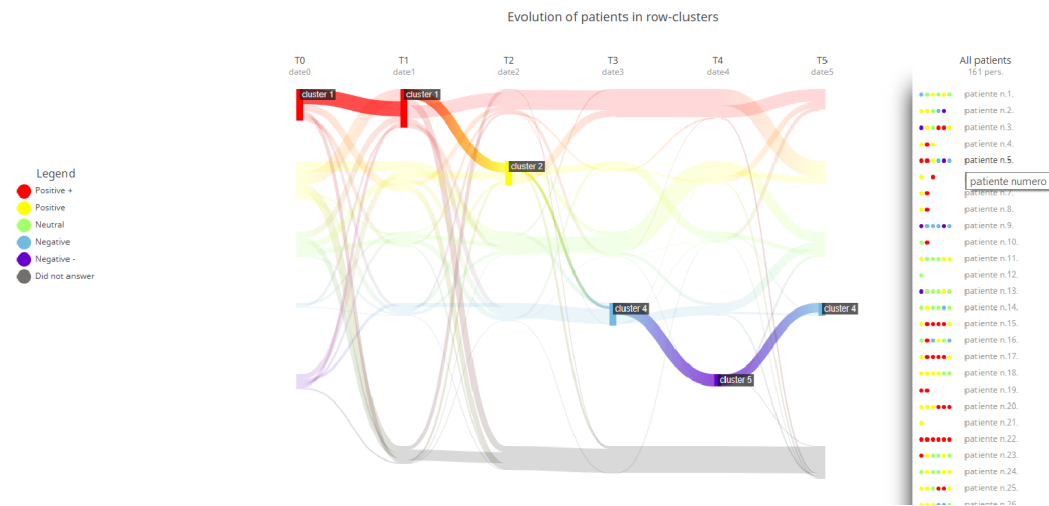


Fig. 8. When user clicks on a patient on the right list, they can observe the psychological trajectory of this patient.

questionnaires. Then, the questions are assimilated to psychological dimensions, which can not be ignored. Finally, just like a lot of real dataset, this one contains some missing values.

To adapt to the particularities of the survey, an extension of the latent block model is defined, and the parsimonious BOS distribution for ordinal data is employed. What's more, model inference is performed with an SEM-Gibbs algorithm, which allows to take missing values into account. An R package with a full implementation of this work is available upon request to the authors, and will be soon available on the CRAN. Finally,

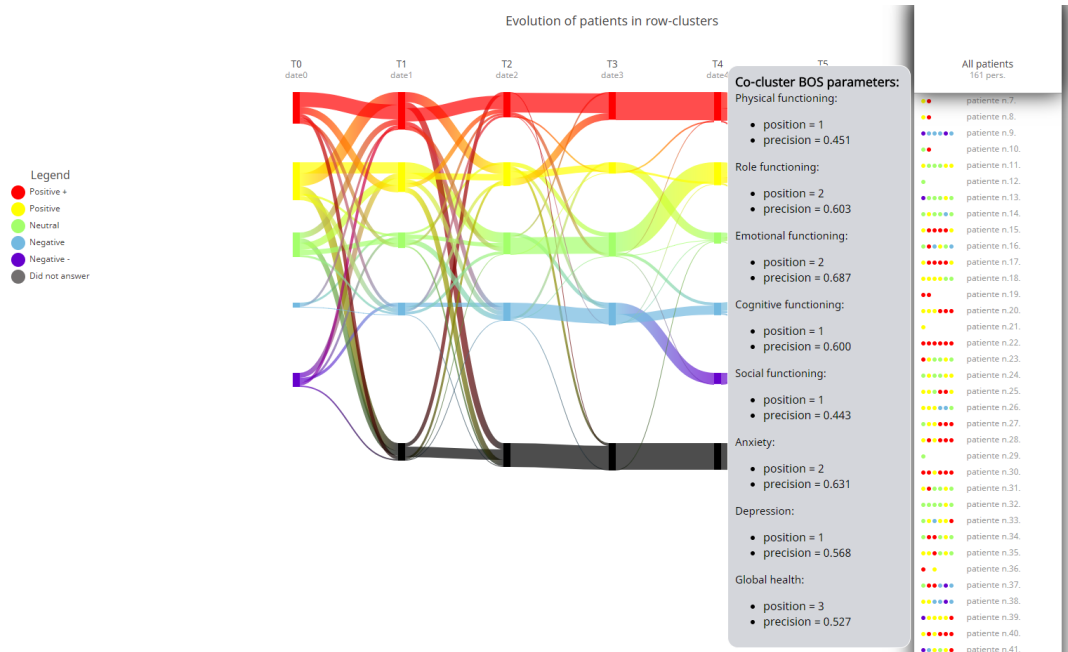


Fig. 9. When user clicks on a row-cluster they are able to see the BOS parameters of all corresponding co-clusters.

visualizations have been created to help the psychologists to observe the evolution of their patients.

The results were particularly satisfying to the psychologist. The proposed technique provides a parsimonious way to cluster the patients by gathering the questions in a small number of groups, and the BOS parameters meaningfulness allows to easily interpret the resulting co-cluster. Furthermore, the constrained co-clustering overtakes two matters: the different numbers of levels for the questions, and the fact that the questions refer to different psychological dimensions.

With the proposed approach, features with different number of levels can be treated in the co-clustering execution, but they are not allowed to be part of the same column-cluster. As a future work, it would be interesting to make that possible, and ideally, to perform co-clusterings with data of different kinds (continuous, functional...). At last, although the dynamical aspect of the data has been approached with visualizations it would be advantageous to define a mathematical model on this aspect.

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Table 1. ICL-BIC values for experiment with dimensions: anxiety, depression and symptoms.

iteration number	tested set	ICL-BIC value	iteration number	tested set	ICL-BIC value
0	3111	-3182.636	4	4222	-2897.574
1	3211	-3202.003		3322	-2899.502
	4111	-3178.63		3232	-2915.243
	3121	-3135.95		3223	-2914.43
	3112	-2981.384	5	5222	-2887.428
2	4112	-2955.255		4322	-2901.103
	3212	-3012.662		4232	-2902.765
	3122	-2931.426		4223	-2911.143
	3113	-3003.892	6	6222	-2890.224
3	4122	-2909.457		5322	-2890.043
	3222	-2907.208		5232	-2898.423
	3132	-2937.025		5223	-2900.492
	3123	-2941.77			

Table 2. ICL-BIC values for experiment with dimensions of social support.

iteration number	tested set	ICL-BIC value	iteration number	tested set	ICL-BIC value	iteration number	tested set	ICL-BIC value
0	31111	-4159.044	4	51113	-3745.543	7	61223	-3654.905
1	41111	-4148.478		42113	-3792.785		52223	-3664.249
	32111	-4169.625		41213	-3807.943		51323	-3660.886
	31211	-4167.269		41123	-3708.452		51233	-3670.111
	31121	-4109.996		41114	-3782.372		51224	-3665.377
	31112	-3890.939		5	51123		-3684.643	
2	41112	-3861.417	42123		-3723.946			
	32112	-3901.316	41223		-3710.339			
	31212	-3966.906	41133		-3814.453			
	31122	-3847.206	41124		-3807.466			
	31113	-3792.995	6	61123	-3672.869			
3	41113	-3759.687		52123	-3689.939			
	32113	-3800.504		51223	-3646.392			
	31213	-3793.978		51133	-3670.815			
	31123	-3760.164		51124	-3674.937			
	51113	-3803.808						

Appendix

The following tables present the ICL-BIC obtained by executing Algorithm 1 on the applications described in Section 4. At each iteration, the values in bold represents the highest ICL-BIC values of the iteration. The underlined values are the final chosen values for (G, H_1, \dots, H_D) .

Figure A1 presents the evolution of some parameters through the SEM-algorithm iterations.

Table 3. ICL-BIC values for experiment with symptoms dimensions at times (T_0, T_1, T_2).

iteration number	tested set	ICL-BIC value
0	3111	-4479.951
1	4111	-4473.039
	3211	-4419.312
	3121	-4278.773
	3112	-4259.51
2	4112	-4206.192
	3212	-4200.733
	3122	-4021.611
	3113	-4262.249
3	4122	-4033.372
	3222	-3967.913
	3132	-4012.178
	3123	-4103.417
4	4222	-3981.241
	3322	-4082.588
	3232	-4080.828
	3223	-4046.097

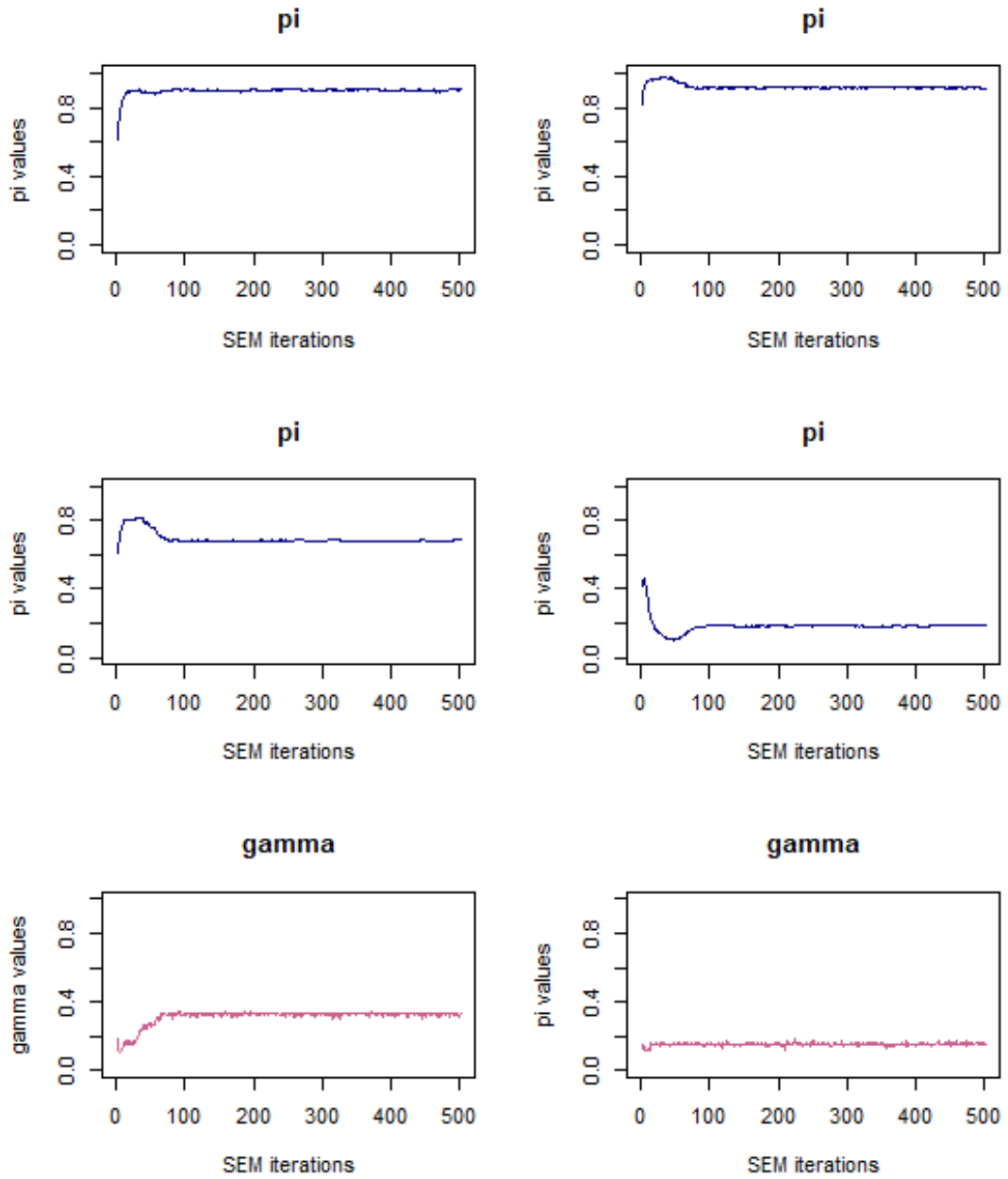


Fig. A1. Parameters evolution along time in the SEM-algorithm.