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► **To cite this version:**

Alireza Shams Shafigh, Panayotis Mertikopoulos, Savo Glisic, Yuguang Michael Fang. Semi-cognitive radio networks: A novel dynamic spectrum sharing mechanism. *IEEE Transactions on Cognitive Communications and Networking*, 2017, 3 (1), pp.97-111. 10.1109/TCCN.2017.2681081 . hal-01643344

**HAL Id: hal-01643344**

**<https://hal.science/hal-01643344v1>**

Submitted on 6 Jan 2025

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# Semi-Cognitive Radio Networks: A Novel Dynamic Spectrum Sharing Mechanism

Alireza Shams Shafigh, *Student, IEEE*, Panayotis Mertikopoulos, *Member, IEEE*, Savo Glisic, *Senior, IEEE* and Yuguang Michael Fang, *Fellow, IEEE*

**Abstract**—In conventional cognitive radio networks (CCRN), channels that are in use by opportunistic secondary users (SUs) can be recaptured by the network’s licensed primary users (PUs) at will, thus interrupting the connectivity of the former. To compensate for this, we propose here a *semi-cognitive radio network* (SCRN) paradigm where PUs are constrained to first use all *free* channels in the network before being allowed to capture channels that are currently in use by SUs. By imposing a monetary (or other) penalty to the network’s secondary spectrum owners when opportunistic channel use becomes excessive, this additional constraint only induces a slight drop in the PUs’ performance while offering significant benefits to the network’s SUs. In this paper, we provide a game-theoretic analysis of such systems and we derive both centralized and decentralized adaptive algorithms that allow the system control process to converge to a stable equilibrium state. Our numerical results show that, with the same channel efficiency, SCRNs provide increased profits to the primary network and significantly reduced interruption rates to the secondary network.

**Keywords.** Semi-cognitive radio networks, dynamic spectrum sharing, convex optimization, game theory.

## I. INTRODUCTION

Current design specifications for 5th generation (5G) wireless systems target a massive increase in network capacity, fiber-like connection speeds (well into the Gb/s range), and an immersive overall user experience with ultra low latency and response times. So, to balance the projected spectrum crunch with the needs of next-generation wireless networks, the ICT industry has turned to more flexible wireless network paradigms such as that of cognitive radio (CR) [1]. At its most basic form, CR networks introduce a two-level hierarchy between wireless users induced by spectrum licensing: primary owners (POs) have purchased spectrum rights which are made partially available to secondary owners (SOs), but with no quality of service (QoS) guarantees to the latter. Thus, by opening up the unused part of the spectrum to opportunistic user access, overall spectrum utilization is greatly increased [2].

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There is an extensive corpus of literature on dynamic spectrum sharing focusing on improving the performance of cognitive radio networks. In [3], [4], the authors considered an auction-based spectrum sharing mechanism to maximize the profit for primary owner in a network with multiple primary (PU) and secondary (SU) users. In [5], [6], dynamic spectrum sharing was analyzed within a general game-theoretic framework for dynamic spectrum leasing (DSL) and by carefully identifying requirements for the coexistence of primary and secondary systems. Economic interactions between secondary users and primary operators in conventional cognitive radio network (CCRN) scenarios are studied in detail in [7]–[9]. Therein, users are charged a fixed price per unit of the bandwidth being used, and face spectrum access costs. In [10]–[13] channel allocation in cognitive radio networks is considered as a resource allocation problem under the assumption that the allocation of transmission rate and transmission power for secondary users is restricted. In [14], [15], the authors investigate cognitive spectrum sharing scheme for LTE-Advanced and 5G systems respectively, while [16] studies the long- vs. short-term market effects between a single PO and multiple unlicensed SUs in spectrum trading. In [17], the authors develop a price-based spectrum access control with service provisions to delay-sensitive SUs with different PO pricing strategies. In [18]–[20], the issues of joint pricing and spectrum allocation in CCRNs are addressed. A quantitative description of SU spectrum demand is formulated and a novel joint spectrum pricing and spectrum allocation scheme based on cooperative game theory is proposed to achieve the maximization of joint utility of all PUs. [21] investigates spectrum sensing imperfections, one of the most important challenges in CCRNs. Spectrum sensing errors by a secondary user cause false alarm and missed-detection events, which can potentially degrade the quality-of-service experienced by primary users. In [22], the authors design an algorithm for simultaneous transmissions of POs and SOs via efficient power control so as to improve the performance of cognitive radio networks. SOs try to minimize their effect on primary users’ channels by minimizing the sum of interference. As it was shown in [23], efficient spectrum handoff techniques can help the interrupted secondary user vacate the occupied licensed channel and find a suitable target channel to resume its unfinished data transmission. These methods are not applicable for dynamic networks because SOs need more reliable access to channels.

Thus, despite their very promising features, CCRNs suffer a major drawback in that they allow POs to capture/recapture channels blindly, regardless of whether the channels are being

temporarily used by SUs or not [1], [2]. POs may interrupt the SUs' transmissions even though there might be still free channels available, a factor which can significantly degrade the network's overall performance. In addition, SOs must consume resources for sensing the channels in order to access the network, while POs capture/recapture those channels immediately; as a result, rational SOs would not be inclined to participate in this dynamic spectrum sharing mechanism. Consequently, these mechanisms must be further improved in order to motivate SOs to participate in CR schemes and make the application of cognitive networks feasible.

Our point of departure is the observation that the network's SUs would benefit from a significantly enhanced network experience if the POs made the costless – but *beneficial* – effort of avoiding collision with ongoing SU transmissions whenever it is possible to employ other, unused channels. With this in mind, we propose here a modification of the cognitive radio paradigm where, by avoiding the use of occupied channels when possible, the network's POs significantly improve the SUs' quality of service and overall network performance. In this setting, POs and SOs define their working spaces through a novel long-term/short-term contract-based strategy scheme. Specifically, POs make long-term and short-term contracts with the PUs and the SOs respectively, and every PO periodically revises the short-term contracts with new terms in order to be able to respect the long-term contract.

More specifically, we propose a *semi-cognitive radio network* (SCRN) paradigm where a portion of the channels is reserved for the real-time traffic needs of the network's POs. The rest of the channels are divided into a pool that is shared by both SOs and POs, and a pool which is used exclusively by POs; the relative size of these pools then evolves over time based on the dynamics of the primary network. This system is analyzed by means of a game-theoretic model where the network's SOs and POs seek to maximize their individual utilities. These utilities are defined so as to capture the trade-off between the financial benefits of leasing a part of the spectrum against the costs involved in this lease. To reach an equilibrium, we then propose a distributed learning algorithm that allows the system to converge to a stable state. We also provide a centralized solution addressing the PUs' constraints (as determined by their connectivity demands), which we then implement using a global optimization approach. Finally, we combine the two algorithms into a hybrid method that concurrently addresses both issues.

The paper is organized as follows: Section II introduces our system model and preliminary definitions concerning SCRNs. Short-term contract negotiation is explained in detail in Section III including a distributed learning algorithm. Section IV describes long-term contract profit maximization problems including three centralized profit maximization schemes for the network's POs while Section V investigates the spectrum utilization in SCRNs and CCRNs. Section VI presents numerical results by comparing different approaches to the solution. Conclusions are summarized in Section VII.

TABLE I  
TABLE OF NOTATION

Symbol	Meaning
$\mathcal{P}$	set of primary operators (POs)
$\mathcal{S}$	set of secondary operators (SOs)
$\alpha$	number of POs
$\beta$	number of SOs
$B_i^{PU}$	number of channels for high priority and real time primary users (PUs)
$B_i$	number of channels in the pool of the $i$ -th PO
$C_i$	number of channels shared between POs and SOs
$b_{ij}$	number of channels demanded by the $j$ -th SO from the $i$ -th PO
$\Delta$	long-term contract (LTC) length
$\epsilon_{ij}$	channel efficiency between the $i$ -th PO and the $j$ -th SO
$\vartheta$	<i>channel substitutability</i> parameter
$p_i$	price per unit of bandwidth set by the $i$ -th PO
$\bar{\tau}$	average access delay
$d_i$	probability of empty channel detection by the $i$ th PO
$\eta$	maximum budget of SOs
$Z_k$	number of primary users in the $k$ th PO
$A$	PU/SU arrival rate

## II. SYSTEM MODEL

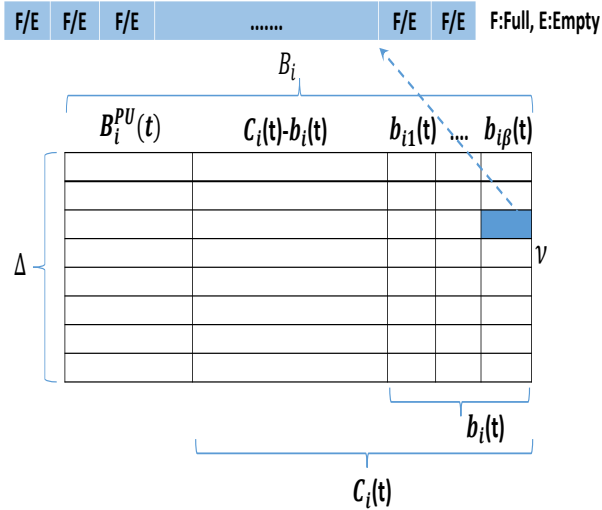
### A. Cognitive Spectrum Sharing Through Contracts

We consider networks where the spectrum is owned by a primary owner (PO) who makes long-term contracts (LTCs) with each primary user (PU) and short-term contracts (STCs) with each secondary owner (SO). In a LTC, the PO assigns its channels to PUs on demand while reserving the right to lease (opportunistically) any unused part of the spectrum to SOs under a STC. If the PUs' QoS degrades due to spectrum sharing with SOs, POs can charge SOs (based on the terms of the negotiated STC) so as to reimburse the network's PUs. The conditions with respect to PUs do not change in time so the contract is valid throughout a long horizon.

On the other hand, the PO must negotiate constantly with SOs in order to make the best possible STC, which defines the rules of spectrum sharing for a specific period. Within a STC, freedom and dominance of PUs may change according to PO traffic conditions, which have an impact on the corruption probability of the assigned channels to SOs. For this reason, the contract should be revised more often in order to provide more reliable spectrum access for SOs.

To illustrate this concept, Fig. 1 shows the logical structure of the proposed model. In the figure,  $B_i$  denotes the total number of channels in the pool of the  $i$ -th PO and  $B_i^{PU}$  the number of channels that are reserved for the network's high priority PUs (assumed constant over time for simplicity). The remaining channels of the  $i$ -th PO are assumed to be in a common pool of size  $C_i = B_i - B_i^{PU}$ , and they can be shared by PUs and SOs (whenever there is a vacancy in the spectrum). To account for this, we let  $b_{ij}$  denote the number of channels demanded from the  $i$ -th PO by the  $j$ -th SO and we write  $b_i = \sum_j b_{ij}$  for the total demand from the  $i$ -th PO by all SOs in the system.

In our semi-cognitive radio context, the freedom of the PUs is constrained by requiring them to first use the set of *unshared* sub-channels of size  $C_i - b_i$ . Only if the unshared pool is occupied, are PUs allowed to behave as in the conventional cognitive radio case and take the channel back from the SOs



(a) Channel Allocation Scheme



(b) Contract and negotiation phases

Fig. 1. Channel Allocation Mechanism of  $PO_i$ .

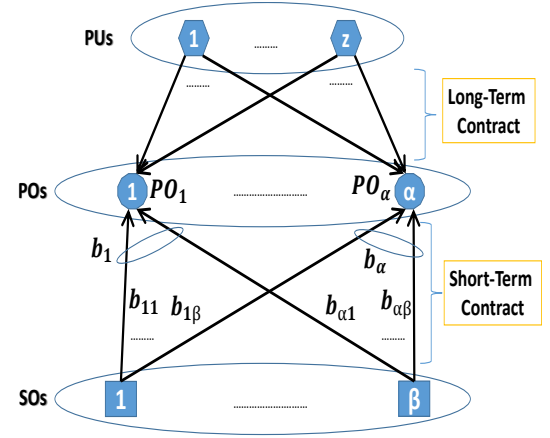
that employ them. Thus, in a typical STC, PO and SOs agree on the number of shared channels  $b_i$  (not specific frequencies) and the remaining  $C_i - b_i$  channels are considered as unshared. The spectrum sharing ratio is defined as  $b_i/C_i$ .

In view of the above, the POs of a semi-cognitive radio network (SCRN) must balance their individual benefits and responsibilities under a LTC in order to develop a STC properly. Thus, the spectrum sharing process of SCRN consists of two different phases: negotiation and contract. In the negotiation phase (Phase 1), POs and SOs make a STC according to their traffic conditions and QoS requirements. In the contract phase (Phase 2), the PO controls the access of PUs based on the terms of the negotiated STC. Based on the negotiated LTC, POs have to reimburse the PUs due to the longer network access delay [24] and possible channel quality degradation. During one LTC, POs can periodically make several STCs which depend on the network dynamics e.g. PU arrival rates  $\mathbf{A} = (A_1, A_2, \dots, A_\alpha)$ , where  $\alpha$  is the number of POs.

Formally, we consider a network consisting of a set  $\mathcal{P}$  of POs that can lease out bandwidth (in the form of wireless channels) to a set  $\mathcal{S}$  of SOs that demand channels from the POs. Fig. 2 presents an extension of Fig. 1 to multiple-primary and multiple-secondary operators (MP-MSO) spectrum sharing systems.<sup>1</sup>

From their end, SOs make individual bandwidth demands from the POs. Of course, if the demand  $b_i$  is high, the bandwidth available to PUs – and, hence, their QoS – will be decreased accordingly. In particular, the degradation of QoS is due to

<sup>1</sup>For simplicity, we assume here that bandwidth can be parceled out in a continuous fashion; the discrete case is analyzed in Section IV.

Fig. 2. multiple-primary and multiple-secondary operators (MP-MSO) sharing system ( $Z$  is the number of PUs).

two reasons: 1) since only the number of rented channels  $b_i$  is specified, not the specific frequencies (slots), for a new arriving PU the PO has to search for the channel which is not temporally used by SOs leading to increased PU network access delay. 2) the good channels might be occupied by secondary users (SUs) so that the incoming PU has to use a channel of lower quality.

### III. SHORT-TERM CONTRACT NEGOTIATION

#### A. Game-theoretic formulation

In a semi-cognitive framework, we assume that SOs and POs negotiate the price of channels in order to maximize their throughput and revenue respectively. To model this, we will consider a utility-based formulation which, in the case of SOs, takes the form [25]:

$$U_j(p; b) = \sum_{i=1}^{\alpha} \epsilon_{ij} b_{ij} - \sum_{i=1}^{\alpha} p_i b_{ij} - \frac{1}{2} \sum_{i=1}^{\alpha} b_{ij}^2 + \vartheta \sum_{i \neq j} b_{ij} b_{lj}, \quad (1a)$$

where  $p = (p_i)_{i \in \mathcal{P}}$  is the POs' price vector (more on this below), the matrix  $b = (b_{ij})_{i \in \mathcal{P}, j \in \mathcal{S}}$  represents the demand of channels by the  $j$ -th SO from the  $i$ -th PO,  $\vartheta \in [0, 1]$  is the *channel substitutability* parameter introduced in [18], [19], and  $\epsilon_{ij}$  denotes the channel efficiency of SUs depending on channel corruption rates.<sup>2</sup> This utility model has been widely used in the literature (see e.g. [18], [19] and references therein), and it can be interpreted as follows:

- First, each summand  $\epsilon_{ij} b_{ij}$  in (1a) represents the utility derived by obtaining the demanded channels from PO  $i$ . Since spectrum efficiency is the SOs' primary concern, the bandwidth obtained is weighed by the corresponding channel efficiency coefficient  $\epsilon_{ij}$  which determines the channels' quality.
- Second, the term  $\sum_{i=1}^{\alpha} p_i b_{ij}$  is the total price paid for obtaining said channels; as such, it is subtracted from the derived utility.
- Finally, the quadratic term  $\frac{1}{2} \sum_{i=1}^{\alpha} b_{ij}^2 - \vartheta \sum_{i \neq j} b_{ij} b_{lj}$  is a widely used surrogate term which reflects indirect, non-linear (second-order) saturation effects that arise as an

<sup>2</sup>In Appendix A, we calculate channel corruption probabilities for SCRN and CCRN systems

SO acquires more bandwidth and increases transmission rates. In particular, the channel substitutability parameter  $\vartheta$  represents the degree of flexibility in switching between different parts of the spectrum (leased by different POs); for a detailed discussion, see [18], [19].

On the PO side, a penalty function determines how much a PU must be compensated for its performance degradation. Thus, the POs' utility function will be given by:

$$U_i(p; b) = R_i^{PU} + R_i^{SU}(b_i, p) - \xi_i^{PU}(b_i), \quad (1b)$$

where  $R_i^{PU}$  represents the revenue from PUs of  $PO_i$ , the term  $R_i^{SU} \equiv p_i \min\{b_i, C_i\}$  is the revenue obtained by assigning bandwidth to SOs at the price of  $p_i$  per unit of bandwidth and the penalty  $\xi_i^{PU}$  is a nonnegative cost function that represents the average reimbursement to the PUs due to performance degradation when  $b_i$  units of bandwidth have been allocated to SOs. The first term in (1b) is not related to dynamic spectrum sharing, but the second and the third terms reflect precisely that.

At this point, we are making no specific assumptions for the dependence of the cost function  $\xi_i$  on the amount of bandwidth  $b_i = \sum_{j \in \mathcal{S}} b_{ij}$  leased to SOs. In realistic scenarios, this cost function may have a complicated dependence on the number of channels leased to SOs so, for a detailed analysis, we refer the reader to Section IV. Instead, what is important here is to note that the POs' utility function depends *linearly* on the price that the POs charge to SOs per unit of bandwidth, and in a – potentially – more complicated way on the SOs' individual bandwidth demands  $b_{ij}$ .

With all this in mind, the above system can be modeled as a non-cooperative game where POs aim at maximizing their net profit by adjusting the price that they charge to SOs in their short-term contracts, while SOs seek to maximize their individual utility by properly adapting their bandwidth demands. Formally, this can be formulated as a normal form game, defined as follows:

- 1) The set of players  $\mathcal{P} \cup \mathcal{S}$  is comprised by the system's POs and SOs.
- 2) The players' action sets are defined as follows:
  - 2a) The action set of the  $i$ -th PO is  $\mathcal{X}_i = [0, p_{\max}]$ , representing the channels' price range.
  - 2b) The action set of the  $j$ -th SO is  $\mathcal{X}_j = [0, b_{\max}]^\alpha$ , representing the possible channel demand vectors of the  $j$ -th SO from the network's POs.
- 3) The utility functions of the POs and the SOs are given respectively by (1a) and (1b).

In this multi-agent context, the most widely used solution concept is that of Nash equilibrium (NE), a notion which describes action profiles where no player has an incentive to deviate from equilibrium point [26], [27]. Formally, we say that a profile  $(p^*, b^*)$  is a Nash equilibrium if

$$\begin{aligned} U_i(p^*; b^*) &= \max_{p_i \in \mathcal{X}_i} U_i(p_i, p_{-i}^*; b^*), \\ U_j(p^*; b^*) &= \max_{b_j \in \mathcal{X}_j} U_j(p^*; b_j, b_{-j}^*), \end{aligned} \quad (\text{NE})$$

for all  $i \in \mathcal{P}$  and all  $j \in \mathcal{S}$  respectively.

Our first result is that Nash equilibria always exist in the above framework:

**Proposition 1.** *The defined non-cooperative game always admits a Nash equilibrium.*

*Proof:* See Appendix B. ■

Proposition 1 guarantees the existence of Nash equilibrium solutions but, of course, there could be multiple such solutions. Since equilibrium multiplicity reduces the system's predictability (due to the fact that there would be several stable system states with a priori different performance characteristics), it would be desirable to be able to tell under what conditions the system admits a unique equilibrium solution. To that end, following [28], [29], let

$$v_i(p; b) = \partial_{p_i} U_i(p; b) \quad i \in \mathcal{P}, \quad (2a)$$

$$v_j(p; b) = \nabla_{b_j} U_j(p; b) \quad j \in \mathcal{S}, \quad (2b)$$

denote the players' individual payoff gradients with respect to their individual action variables.<sup>3</sup> Then, Rosen [28] showed that the game admits a unique Nash equilibrium provided the following diagonal strict concavity (DSC) condition holds:

$$\begin{aligned} & \sum_{i \in \mathcal{P}} \lambda_i \left\langle v_i(p'; b') - v_i(p; b) \mid p'_i - p_i \right\rangle \\ & + \sum_{j \in \mathcal{S}} \lambda_j \left\langle v_j(p'; b') - v_j(p; b) \mid b'_j - b_j \right\rangle \leq 0 \end{aligned} \quad (\text{DSC})$$

for some  $\lambda > 0$ , for all  $p_i \in \mathcal{X}_i$ ,  $b_j \in \mathcal{X}_j$ , and equality holding if and only if  $(p; b) = (p'; b')$ .

Specifically, under (DSC), we have:

**Proposition 2.** *Assume that (DSC) holds for some  $\lambda > 0$ . Then, the game admits a unique Nash equilibrium.*

*Proof:* By the proof of Proposition 1, it follows that each player's payoff function is individually concave. Our claim then follows from Theorem 2 in [28]. ■

To verify whether (DSC) holds, we present below a simple second-order condition based on the *Hessian matrix* of the game, defined here as the block matrix  $H = (H_{k\ell})_{k, \ell \in \mathcal{P} \cup \mathcal{S}}$  with blocks:

$$H_{k\ell}(p; b) = \begin{cases} \partial_{p_i} v_k(p; b) & \text{if } \ell \in \mathcal{P}, \\ \nabla_{b_\ell} v_\ell(p; b) & \text{if } \ell \in \mathcal{S}. \end{cases} \quad (3)$$

More succinctly, if we let  $x_k = p_k$  when  $k \in \mathcal{P}$  and  $x_k = b_k$  when  $k \in \mathcal{S}$  (a notational shorthand which we also follow in Appendix B), each block of  $H$  can be written as

$$H_{k\ell}(x) = \frac{\partial^2 U_k}{\partial x_\ell \partial x_k} \quad \text{for all } k, \ell \in \mathcal{P} \cup \mathcal{S}. \quad (4)$$

In other words, each constituent block  $H_{k\ell}$  of  $H$  is simply the Hessian matrix of the utility function  $U_k$  of player  $k \in \mathcal{P} \cup \mathcal{S}$ , thus justifying the name ‘‘Hessian matrix’’ for  $H$ .

As we show in Appendix B, the SCR game under study is *concave* in the sense that each player's set of actions is convex and compact, and their utility functions are concave in each player's individual action variables (for a precise statement, see the proof of Proposition 1). Thanks to this observation, Theorem 6 in [?] provides the following straightforward sufficient condition for (DSC):

<sup>3</sup>Recall here that  $p_i$  is a scalar while  $b_j$  is a vector variable.

**Proposition 3.** *With notation as above, suppose that the symmetrized Hessian matrix  $H(x) + H(x)^\top$  of the game is negative-definite for all  $x$ . Then, (DSC) holds with  $\lambda = 1$  and the game admits a unique NE.*

*Proof.* Simply note that the symmetrized Hessian matrix of the game as defined above coincides with the  $G$ -matrix in the statement of Theorem 6 in [?] with  $r = 1$ .  $\square$

Given the parameters of a real-life SCRN model, the symmetrized matrix  $[H(x) + H(x)^\top]$  is easy to calculate. As such, Proposition 3 provides a straightforward verification criterion which is similar to the standard second-order derivative test used to establish convexity in ordinary calculus. This result can be further extended to arbitrary  $\lambda$  by asking that the so-called  $\lambda$ -weighted Hessian matrix  $H(x; \lambda)$  with blocks  $H_{k\ell}(x; \ell) = \lambda_k H_{k\ell}(x)$  be negative-definite for some  $\lambda > 0$ . In practice however, the simple criterion outlined in Proposition 3 is easier to verify, so we do not treat the more general case here.

### B. Distributed Learning and Convergence to Equilibrium

Of course, even if the game admits a unique Nash equilibrium it is not clear how players can converge to it. To that end, we propose below a distributed price-and-demand adjustment scheme based on the following exponentiated gradient ascent (EGA) scheme:

$$\begin{aligned} y_i(n+1) &= y_i(n) + \gamma_n v_i(p(n); b(n)), \\ y_j(n+1) &= y_j(n) + \gamma_n v_j(p(n); b(n)), \\ p_i(n+1) &= p_{\max} \frac{e^{y_i(n+1)}}{1 + e^{y_i(n+1)}}, \\ b_j(n+1) &= b_{\max} \frac{e^{y_j(n+1)}}{1 + e^{y_j(n+1)}} \end{aligned} \quad (\text{EGA})$$

where  $\gamma_n$  is a variable (nonincreasing) step size parameter, and we are using vectorization to extend the definition of a scalar function of a scalar argument to a vector function of a vector argument.

Intuitively, the variables  $y$  in (EGA) represents a step along each player's direction of individually steepest payoff ascent, captured here by the individual payoff gradients  $v$  of (2). However, because such a step could end up violating the feasibility constraints of the players' action spaces as identified in the definition of the game, this variable is subsequently exponentiated and normalized so that the resulting action profile remain feasible [30]–[32].

With all this in mind, we can now state our main result:

**Theorem 1.** *The stationary points of the price-and-demand adjustment scheme (EGA) coincide with the game's interior Nash equilibria. Moreover, if (DSC) also holds, the iterates of (EGA) converge to the game's (necessarily) unique Nash equilibrium, provided that the step-size sequence  $\gamma_n$  is chosen such that  $\sum_{n=1}^{\infty} \gamma_n^2 < \sum_{n=1}^{\infty} \gamma_n = \infty$ .*

*Proof:* See Appendix B.  $\blacksquare$

This theorem shows that the stability condition (DSC) can be guaranteed by the Hessian criterion of Proposition 3. Furthermore, the STC negotiation converges to equilibrium which can then be affected by the long-term profit maximization of

the POs. In this way, Theorem 1 provides the operational basis for short-term contract optimization between SOs and POs. The region of validity of the stability condition (DSC) and the interplay of this phase with the POs' long-term profit maximization is examined in more detail in the following sections.

## IV. LONG-TERM CONTRACTS AND PROFIT MAXIMIZATION

In the previous section, we focused on a game-theoretic formulation of the problem where the network's primary owners react to the secondary owners' demands by adjusting the price in the negotiated short-term contract so as to maximize their revenue. In Section III we are making no specific assumptions for the form of the penalty function  $\xi_i^{PU}(b_i)$  that reflects the amount of reimbursement to PUs due to degradation of their performance. In this section several forms of this function are elaborated representing different applications. We focus on the optimization of the POs long-term contracts, based on the PUs demands and characteristic parameters included in the penalty function  $\xi_i^{PU}(b_i)$ .

### A. SO Best Response Function

We develop first a SCRN case study with  $\alpha$  primary spectrum owners and  $\beta$  secondary owners, where  $i \in \mathcal{P}$  opportunistically shares its own channel pool of size  $B_i$  at price  $p_i$  per channel. In (1a), the  $j$ -th SO tries to maximize his individual utility by choosing the optimum demand from different primary operators. By differentiating the utility function (1a) and solving for its critical point, optimum demand ("best response") of the  $j$ -th SO from the  $i$ -th PO is given by

$$\bar{b}_{ij}^*(p) = \frac{(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{lj} - p_l)}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)}. \quad (5)$$

In the above,  $(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1)$  depends on  $p_i$  and the channel efficiency parameter  $\epsilon_{ij}$  of the current primary network while the term  $\vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{lj} - p_l)$  accounts for all other POs in the network. The channel efficiency parameter depends on the channel corruption rate which is a function of user arrival rates in both PO and SO networks. SOs have to adjust their demands according to the current corruption rates of POs' network. To this end, every SO can incorporate corruption rates in its optimum demand function as

$$\bar{b}_{ij}(p) = \frac{(\bar{\epsilon}_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - \vartheta \sum_{l \neq i}^{\alpha} (\bar{\epsilon}_{lj} - p_l)}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)}. \quad (6)$$

where  $\bar{\epsilon}_{ij} = (1 - \psi_i^{(-)})\epsilon_{ij}$  and  $\psi_i^{(-)}$  is the channel corruption probability of the  $i$ -th PO either in CCRN or SCRN models given by (A1) and (A2), respectively (See Appendix A for details). For simplicity, we assume that service rate is the same for all SOs, so we can replace  $\psi_{ij}^{(-)}$  with  $\psi_i^{(-)}$ . In the dynamic demand function, the average achievable data rate of every channel is  $\bar{\epsilon}_{ij} = (1 - \psi_i^{(-)})\epsilon_{ij}$ . In a congested network POs would lose their SUs due to higher corruption rates.

Note that only the number of shared channels  $b_i$  is specified by STC not specific frequencies so that each PO must determine which one out of  $C_i - b_i$  channels are still free. The  $i$ -th PO can share some portion of the available channels  $C_i$  (see Fig. 1) with SOs who are willing to rent the channels. In this case, a PO

broadcasts to SOs the number of available channels, channel corruption probabilities, and their prices. If a SO wants to rent some channels, it finds its optimum demand value considering to the price and channel corruption probabilities. Then, it sends its demand to the PO. After receiving the message, the PO recalculates price and sends it to the SO again.

In what follows, we address the PU performance degradation problem by studying different applications in the SCRNM model. In general, we can first solve the profit maximization problems for only one time slot, then we can extend it over the period of  $\Delta$  time slots due to the independence of PUs and SUs arrival rates in different time slots ( $\nu$ ). In the sequel, we use  $\nu$  to represent the length of one time slot, and the specific number of consecutive time slots  $\Delta$  (in Fig. 1,  $\Delta = 8\nu$ ) represents the long-term cycle defining the length of LTCs.

### B. Profit Maximization with Delay-Sensitive PUs in IoT (Internet of Things) Networks

In this section, we further elaborate the models discussed so far to account for delay-sensitive applications that cannot tolerate an extra delay caused by the spectrum sharing mechanism in SCRNs. This case study is interesting for IoT applications requiring low latency solutions : specifically, if a PO has a LTC with delay-sensitive primary users, it cannot share a lot of channels because it has to pay back a significant penalty to its delay-sensitive primary users.

We reformulate  $\xi_i^{PU}(b_i)$  in (1b) in order to define the profit function of the  $i$ -th PO with delay-sensitive PUs as

$$U_i(p; b) = \iota_1 Z_i + p_i b_i - \iota_2 Z_i \bar{\tau}_i^{\zeta_3} \quad \forall i \in \mathcal{P}, \quad (7)$$

where  $\iota_1$  is a constant benefit from serving a single primary user and  $\iota_2$  is a penalty value to be paid to each PU for not having immediate access to the channel but having to search for it ( $\iota_3$  is a positive constant). In the above,  $Z_i$  is the number of primary users and  $\bar{\tau} = \{\bar{\tau}_1, \dots, \bar{\tau}_\alpha\}$  is the vector of average channel access delays of primary users for the whole system. Overall, the first term in (7) represents the revenue from primary users through LTCs ( $R_j^{PU}$ ), the second term is the revenue from secondary users ( $R_j^{SU}$ ) and the last term is the penalty depending on the average access delay. Note that the second and third terms reflect the dynamic spectrum sharing mechanism between POs and SOs defined by STCs. Therefore, the critical issue in the profit optimization problem is to find the optimum price vector compensating the penalty values.

By approximating  $\bar{\tau}_k^{\zeta_3}$  with  $C_i/(-b_i(2d_i - 1) + C_i d_i)$  (For details refer to [33]), we reformulate the objective function (7) to

$$U_i(p; b) = \iota_1 Z_i + p_i b_i - \frac{C_i \iota_2 Z_i}{-b_i(2d_i - 1) + C_i d_i} \quad \forall i \in \mathcal{P}, \quad (8)$$

where parameter  $d_i$  is probability of correct detection of empty channel [34]. The profit function (8) is still a non-convex function due to the second term  $p_i b_i$ .

With this in mind, the profit optimization of each PO  $i$  with delay-sensitive PUs takes the general form:

### Problem 1-1.

$$\begin{aligned} \max_{p_i} \quad & U_i(p; b) \quad \text{s. t.} \\ & a) b_i \in \{0, 1, \dots, C_i\} \\ & b) p_i \geq 0 \\ & c) \sum_{i \in \mathcal{P}} b_{ij} p_i \leq \eta_j, \forall j \in \mathcal{S} \end{aligned} \quad (9)$$

where  $b_i = \sum_{j \in \mathcal{S}} b_{ij}$  represents the number of shared channels of the  $i$ -th PO with all SOs. In the above,  $b_{ij}$ ,  $b_i$  and  $b$  are functions of  $p$  and  $\eta_j$  represents the maximum budget of the  $j$ -th SO given as  $\sum_{i \in \mathcal{P}} b_{ij} p_i \leq \eta_j$ . This combinatorial problem is complicated to solve even for a small network. We incorporate (9) into sum-utility optimization and simplify the problem by relaxing the combinatorial variable  $b_i$  to a continuous real value to obtain

### Problem 1-2.

$$\begin{aligned} \max_p \quad & \sum_{i \in \mathcal{P}} U_i(p; b) \quad \text{s. t.} \\ & a) 0 \leq \frac{(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{lj} - p_l^{(l)})}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)} \leq C_i, \\ & b) p_i \geq 0, \\ & c) \sum_{i \in \mathcal{P}} \frac{p_i(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - p_i \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{lj} - p_l^{(l)})}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)} \leq \eta_j, \forall j \in \mathcal{S}, \\ & d) p_i^{(l)} = p_i, \forall i \in \mathcal{P}, \end{aligned} \quad (10)$$

where  $p_i^{(l)}$  is a local copy of price value of PO  $i$  by PO  $l$ . In (10), the coupled constraint d),  $p_i^{(l)} = p_i, \forall i \in \mathcal{P}$  implies that all local copies must be equal with the real ones.

In general, the price optimization problem is non-convex and we have to use some simplification techniques (e.g. first order approximation approach) to convert the non-convex budget constraint ( $-p_i^2$  is concave) to convex form. We linearize the non-convex term  $-p_i^2$  using the first order approximation method [35].

An approach to solve Problem 1-2 is to use a dual decomposition method. This approach splits the problem with coupling constraints  $p_i^{(l)} = p_i, \forall i \in \mathcal{P}$  into several independent subproblems. Then, we form the Lagrangian as

$$L = \sum_{i \in \mathcal{P}} U_i - \sum_{i \in \mathcal{P}} \sum_{l \in \mathcal{P}, l \neq i} \zeta_{il} (p_i - p_i^{(l)}) \quad (11)$$

where  $\zeta_{\alpha \times \alpha}$  is the Lagrange multiplier associated with the coupled constraint ( $\zeta_{il}$  is the Lagrange multiplier for the price consistency between POs  $i$  and  $l$ ). We denote with  $\zeta_i$  the list of Lagrange multipliers of PO  $i$ . Thus, the independent subproblems that can be solved in distributed way by every PO will be given by

### Problem 1-3.

$$\begin{aligned} \max_{p_i} \quad & U_i - \sum_{l \in \mathcal{P}, l \neq i} \zeta_{il} (p_i - p_i^{(l)}) \\ \text{s. t.} \quad & a) 0 \leq \frac{(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{lj} - p_l^{(l)})}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)} \leq C_i, \\ & b) p_i \geq 0, \\ & c) \sum_{i \in \mathcal{P}} \frac{(p_i \epsilon_{ij} - f(p_i))(\vartheta(\alpha - 2) + 1) - p_i \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{lj} - p_l^{(l)})}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)} \leq \eta_j, \forall j \in \mathcal{S}, \end{aligned} \quad (12)$$

where  $f(p_i) = -p_i^2(n) - 2p_i(n)(p_i - p_i(n))$  is the first order approximation of  $-p_i^2$  and  $n$  is the iteration index.

The dual function of the main problem can be formulated as

$$g(\zeta) = \sum_{i \in \mathcal{P}} g_i(\zeta_i) \quad (13)$$

where  $g_i(\zeta_i)$  is the optimum value of (11) for a given  $\zeta_i$ . The master problem for the dual decomposition approach will thus be to minimize  $g(\zeta)$  with respect to  $\zeta$ . This master problem can be iteratively and independently solved by subgradient method with the following updates

$$\zeta(n+1) = \zeta(n) + s_n \left( \sum_{i \in \mathcal{P}} \sum_{l \in \mathcal{P}, l \neq i} \zeta_{il}(p_i - p_i^{(l)}) \right) \quad (14)$$

where  $s_n$  is a positive step-size. In the above,  $p_i(n)$  is solution of the  $i$ -th subproblem in the  $n$ -th iteration (see Algorithm 1 for a pseudocode implementation).

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**Algorithm 1** Distributed Long-Term Contract Profit Maximization with Delay-Sensitive PUs

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- 1: Initialize  $\zeta(n)$  with some values and set  $n=0$ .
- 2: At each iteration  $n$
- 3: 1) Every PO solves (14) to update  $\zeta(n)$ .
- 4: 2) Each PO solves (10) independently to find  $p_i(n)$  and broadcast it to other POs.

As an alternative approach to solve the Problem 1-I in (9), we find Nash equilibrium point by best response strategy of POs. Every PO  $i$  unilaterally maximizes its own profit by finding optimum price  $p_i(n)$  for a given price list  $p_{-i}(n)$ , where  $n$  is the iteration index. Every PO sequentially solves the optimization problem defined as **Problem 1-4**.

**Problem 1-4**  $\equiv$  **Problem 1-2**

$$\text{with } \sum_{i \in \mathcal{P}} U_i(p; b) \rightarrow U_i(p; b). \quad (15)$$

to update its optimum price  $p_i(n)$  for the current setting in the iteration  $n$ . These optimizations and price updating processes continue until all POs converge to the NE point such that no player can improve its profit unilaterally.

**C. Load Balancing via Dynamic Demand Function in SCR N**

The dynamic spectrum sharing model helps SUs select the most reliable channel with minimum probability of collision with PUs. SOs with dynamic demand function - as in (6) - must decide about how to distribute the secondary arrival calls between different sub-bands of POs. In this context, the channel corruption predominantly affects the performance of both POs and SOs.

The objective of load balancing aims here to force SOs to request more channels from the lower-overloaded PO networks. In general, by including the channel corruption probability in the demand function, the new form of dynamic load balancing optimization problem (DLBOP) is proposed as

**Problem 2.**

$$\text{Problem 2} \equiv \text{Problem 1-1} \quad (16)$$

$$\text{with } b_{i,j} \rightarrow \bar{b}_{i,j}.$$

In this setting, SOs avoid requesting spectrum from the highly congested primary networks due to higher channel corruption rate. The only difference between this problem and *Problem 1* is that here we replace  $b_{ij}$  with  $\bar{b}_{ij}$  given by (6).

**D. Profit Maximization under Channel Quality Degradation**

In the SCR N model, better channels could be allocated to SUs instead to PUs. To model this phenomenon, it is assumed that high quality channels are uniformly distributed in  $C_i$ . Therefore, if  $b_i$  is the number of assigned channels to SOs by the  $i$ -th PO, the probability of assigning better channels to SOs is  $b_i/C_i$ . POs should find out the optimum value of  $b_i$  leading to the maximum profit including the penalty due to lower quality channel assignment to PUs. Therefore, the new format of the objective function is

$$\bar{U}_i = \iota_i Z_i + \theta_i p_i b_i - \varsigma_i \frac{Z_i b_i}{C_i}, \quad i \in \mathcal{P} \quad (17)$$

where  $\theta_i$  and  $\varsigma_i$  are constants. Although POs benefit from assigning channels to SOs they must also pay higher reimbursement prices to the network's PUs due to possible performance degradation of PUs. In view of all this, we replace  $b_i$  with the optimum demand function (5) to obtain

**Problem 3.**

$$\begin{aligned} & \max_{p_i} \bar{U}_i \\ & \text{s. t.} \\ & \text{a) } 0 \leq \frac{(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{l,j} - p_l^{(i)})}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)} \leq C_i, \\ & \text{b) } p_i \geq 0, \\ & \text{c) } \sum_{i \in \mathcal{P}} \frac{p_i(\epsilon_{ij} - p_i)(\vartheta(\alpha - 2) + 1) - p_i \vartheta \sum_{l \neq i}^{\alpha} (\epsilon_{l,j} - p_l^{(i)})}{(1 - \vartheta)(\vartheta(\alpha - 1) + 1)} \leq \eta_j, \forall j \in \mathcal{S}, \\ & \text{d) } p_i^{(i)} = p_i, \forall i \in \mathcal{P}. \end{aligned} \quad (18)$$

The same approach (e.g. first order approximation and dual decomposition) as in the previous sections can be used to solve the problem defined by (18).

**V. SPECTRUM UTILIZATION**

In general, the concept of cognitive networks is introduced in advanced communications in order to improve spectrum utilization. In this section, we study simplified SCR N and CCR N models with a single PO and multiple SOs in order to quantify gains in spectrum utilization obtained by deploying SCR Ns rather than CCR Ns. We model this scenario as a discrete-time Markov chain with states  $(e, f)$  referring to the state with  $e$  active PUs and  $f$  active SUs. In the first step, we find different state transition probabilities for CCR N and SCR N models as given in Table II.

TABLE II  
STATE TRANSITION PROBABILITIES.

Transition	CCR N/SCR N
$(e, f) \rightarrow (e + 1, f)$	$\left(\frac{C_i - (e+f)}{C_i - e}\right) A_i$
$(e, f) \rightarrow (e, f + 1)$	$\frac{\sum_j^{\beta} A_j b_{ij}}{\sum_l^{\alpha} b_{l,j}}$
$(e, f) \rightarrow (e - 1, f)$	$e \mu_i$
$(e, f) \rightarrow (e, f - 1)$	$f \sum_{j=1}^{\beta} \mu_j^{SU}$
$(e, f) \rightarrow (e + 1, f - 1)$ See discussion below	$f \sum_{j=1}^{\beta} \mu_j$

The state transition probabilities for SCR N are the same as for CCR N except for the transition  $(e, f) \rightarrow (e + 1, f - 1)$ , in which a SU is replaced by the arriving PU. In the CCR N model, PUs of the  $i$ -th PO force out SUs with probability



$(f/(C_i - e))A_i$ . Let parameters  $A_i$  and  $A_j$  represent the arrival rate of PUs and SUs of  $PO_i$  and  $SO_j$ , respectively. Furthermore,  $\mu_i$  and  $\mu_j$  represent PU and SU service rates in system respectively. In SCRN, these PUs do not interrupt SUs unless there is no free channel while some SUs have occupied channels. In general, the decision is made when one primary user call arrives and PO wants to select a channel for transmission. At this point, POs in the SCRN model check the mentioned conditions before assigning the channels. In Table II,  $\sum_j^\beta A_j b_{ij} / \sum_i^\alpha b_{i,j}$  represents a simple load balancing scheme run by the  $j$ -th SO trying to distribute the arriving secondary calls according to the amount of its optimum demands (5) and (6) from different POs.

In general, SCRN provides benefits for primary owners by improving their spectrum utilization if they accept to be slightly tolerant to the presence of SUs. This is based on the fact that when POs let SOs transmit data in a more reliable way they are more willing to use the network. The spectrum utilization for POs is given now by

$$H = \frac{\sum_{i=1}^\alpha (h_i^p + h_i^{PO} + \sum_{j=1}^\beta h_{i,j}^{SO})}{\sum_{i=1}^\alpha B_i}, \quad (19)$$

where  $\alpha$  and  $\beta$  are the number of POs and SOs, respectively and  $h_i^p$  and  $h_i^{PO}$  represent the number of channels used by  $PO_i$  in the reserved and unshared channels, respectively. The parameter  $h_{i,j}^{SO}$  represents the number of the shared channels of  $PO_i$  used by  $SO_j$ . In (19), the term  $\sum_{i=1}^\alpha \sum_{j=1}^\beta h_{i,j}^{SO} / \sum_{i=1}^\alpha B_i$  represents the extra spectrum utilization of the SCRN system. When there is no more available channels for the new arrival calls, the call is blocked. Thus, the blocking probability can be calculated by

$$\begin{aligned} Pr_{block}^{SU} &= \sum_{e+f=C} \phi(e, f) \sum_{j=1}^\beta A_j \frac{b_{ij}}{\sum_{i=1}^\alpha b_{ij}}, \\ Pr_{block}^{PU} &= \sum_{e=C} \phi(e, f) A_i. \end{aligned} \quad (20)$$

where  $\phi(e, f)$  is the probability that there is  $e$  PUs and  $f$  SUs in the system and  $0 \leq b_{ij} / \sum_{i=1}^\alpha b_{ij} \leq 1$  is a coefficient used for distributing arriving SUs to different POs. In both SCRN and CCRN systems, PU is not dropped by SUs due to the perfect channel sensing assumption. The interrupt probability of SU is

$$\begin{aligned} Pr_{interrupt}^{SU,CCRN} &= \sum_{e+f \leq C} \phi(e, f) \frac{f A_i}{(C_i - e)}, \\ Pr_{interrupt}^{SU,SCRN} &= \sum_{(e+f=C) \vee (f > b_i)} \phi(e, f) \frac{f A_i}{(C_i - e)}. \end{aligned} \quad (21)$$

Note that in SCRN POs only interrupt SU's ongoing transmissions in a few states which provides more reliable transmission for SUs. This brings more reliability and encouragement for SOs to increase their demands.

## VI. NUMERICAL RESULTS

In this section, we present numerical results in order to validate the theoretical analysis of the previous sections and provide some further insights into the operation of SCRNs. In addition, we study the efficiency of a hybrid optimization algorithm (HOA) combining optimization methods presented

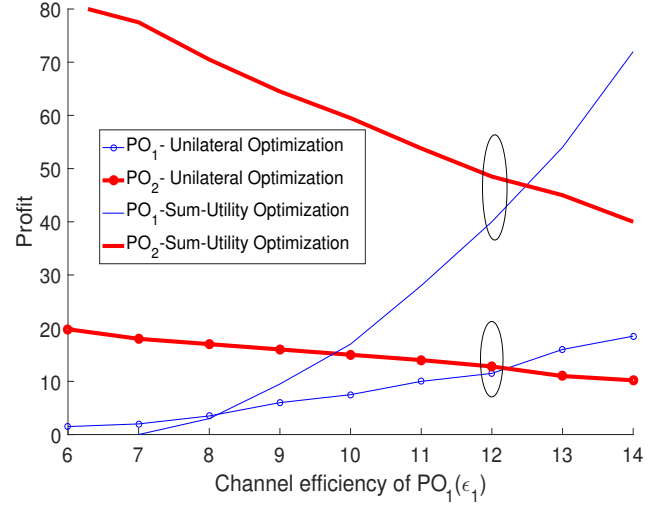


Fig. 3. Sum-Utility (10) vs Unilateral (15) optimization under Long-Term Contracts ( $d_1 = 0.8$ ,  $d_2 = 1$ ,  $\nu=0.5$ ,  $\epsilon_2 = 10$ ) in SCRN.

in Section IV, referred to as convex optimization algorithms (COA), and methods presented in Section III, referred to as distributed learning algorithms (DLA). Here, each PO occasionally runs CO algorithm while DLA is used to track continuously the changes in the network over time. The former provides a good initialization state so that the latter may converge faster and more reliably.

### A. Optimization Problems of Long-Term Contracts

In the sequel we quantify the benefits for SCRNs. To show the results of LTC optimization, we assume that there are two different POs and one SO. In the considered scenarios, the channel efficiency of the first player ( $PO_1$ ) is between 6 and 14 Mbps while the second player ( $PO_2$ ) has a fixed channel efficiency equal to 10 Mbps. Players deploy a SCRN model unless mentioned otherwise.

1) *Profit Optimization*: We first analyze the impact of channel efficiency and channel state detection probability on the performance of SCRN operators. The results are obtained by solving (10). Fig. 3 presents the profit of the SCRN system versus the channel efficiency  $\epsilon_1$  of  $PO_1$ . As expected, the performance of  $PO_1$  is better as  $\epsilon_1$  increases. Since  $\epsilon_2$  is kept fixed ( $\epsilon_2 = 10$ ) the performance of  $PO_2$  decreases as  $\epsilon_1$  becomes more dominant with respect to  $\epsilon_2$ . The figure also demonstrates the importance of correct channel detection  $d_1$  and  $d_2$  on the system performance, where  $PO_2$  has more accurate channel state detection. For this reason,  $PO_1$  should have higher channel quality to have the same profit as  $PO_2$ . As expected, sum-utility optimization defined by (10) always outperform the Nash Equilibrium option obtained by unilateral optimization of the POs' individual utilities as defined by (15). For the unilateral maximization scheme, both players achieve the same profit 10 with  $\epsilon_1 = 12$  and  $\epsilon_2 = 10$ . This means Player 1 ( $PO_1$ ) with lower channel detection probability  $d_1 = 0.8$  needs 20% higher channel efficiency to get the same profit as Player 2 ( $PO_2$ ). It can be even worse for the sum-utility optimization scheme (10), where Player 1 needs 25% higher channel efficiency to

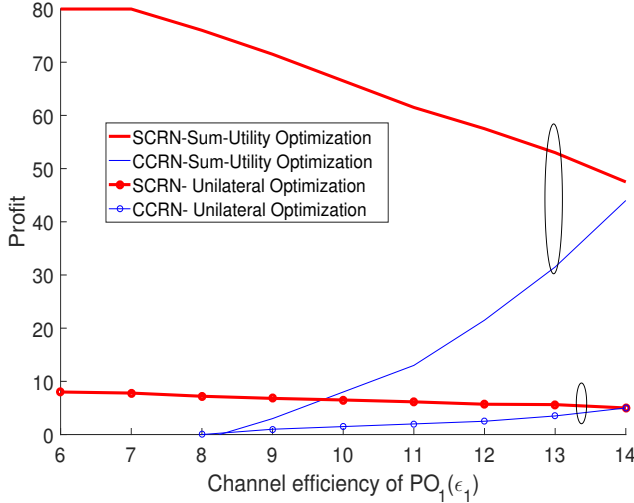


Fig. 4. Profit of SCR/CCR operators under Long-Term Contracts ( $d_1 = d_2=1, \nu=0.5, \epsilon_2 = 10$ ).

get the same profit as Player 2. Taken together, these results suggest that there is a strong relation between channel detection probability and operator profit.

2) *Dynamic Load Balancing*: Here we show that using SCR as the spectrum sharing model rather than CCR, operators can benefit in terms of revenue and system performance. In this analysis, we consider the impact of the channel corruption probability on the performance of POs and SOs, assuming that the SOs distribute the incoming SUs adaptively based on the current condition of the POs' networks. We assume that  $PO_1$  (Player 1) and  $PO_2$  (Player 2) deploy CCR and SCR spectrum sharing models respectively. Player 2 is a SCR operator that provides more reliable access for SUs using a dynamic demand function (6). Fig. 4 demonstrates that the SCR-player outperforms the CCR-player. CCR-Player would achieve similar profits as SCR operator if it could have much higher channel quality about 14Mbps.

3) *Profit Optimization under Low Quality Channel Allocation*: In this section, we evaluate the performance of SCR scheme where the operators have a contract with their PUs to assign better channels to them. So, capturing a channel by SO under SCR agreement can violate the items of the contract with PUs. To fix this issue, SCR operators promise to the PUs to compensate their performance degradations due to assigning low-quality channel to them. In the context of optimizing the allocation of low-quality channels in Problem 3 formulated by (18), POs optimize their prices to compensate the penalty that they have to pay back due to allocating the stronger channel to SUs instead to PUs. Fig. 5 shows the results for the low quality channel allocation optimization. Again, as expected, sum-utility optimization outperforms the Nash equilibrium solution of unilateral optimization.

### B. Optimization Problems of Short-Term Contracts via Distributed Learning

In this section, we apply the game-theoretic analysis of Section III to show the efficiency of the proposed SCR model for

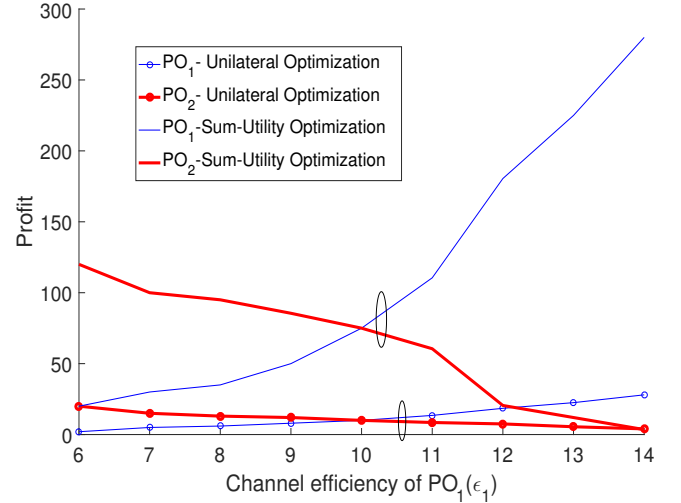


Fig. 5. Sum-Utility (10)/Unilateral (15) optimum profit of the low quality channel allocation problem ( $\epsilon_2 = 10, \theta = 0.6$ ).

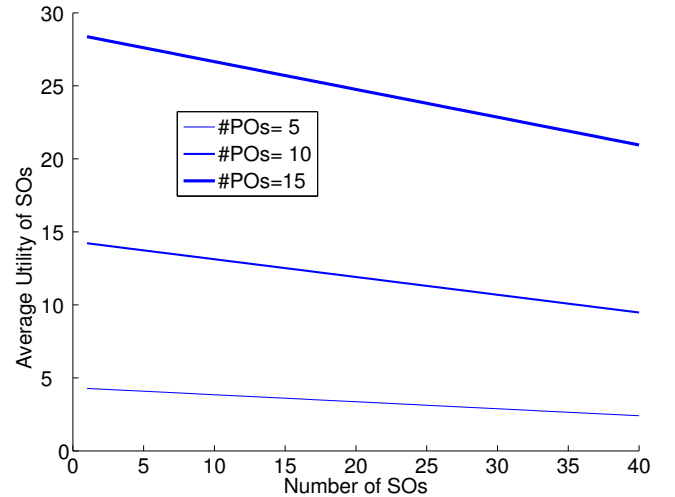


Fig. 6. Normalized per SOs' average Utility versus the number of SOs ( $\gamma = 0.01$ ) in SCR after the convergence of distributed algorithm.

profit and utility maximization. Figs. 6 shows the performance of the EGA model for semi-cognitive network for different numbers of SOs and POs. In the figure, we can see that the utility per SO at the equilibrium points of the network decreases as the number of SO (competitors) increases. As can be seen from the figure, the average utility of SOs can be improved at least by a factor of 7 if the number of POs  $\alpha$  is increased from 5 to 15. In particular for  $\alpha = 15$ , the average utility of SOs is decreased by 18% if the number of SOs  $\beta$  is increased from 5 to 40.

In Fig. 7, we analyze a typical SCR network model (eq. EGA) with 5 POs and 10 SOs as a function of different number of iterations in order to study the impact of parameter  $\gamma$  on profit functions. As can be seen from the figure, the DLA algorithm with learning rate  $\gamma > 0.4$  does not converge even during 20 iterations while with learning rate  $\gamma \leq 0.4$  needs at most 10 iterations to converge. Furthermore, for the given set of the

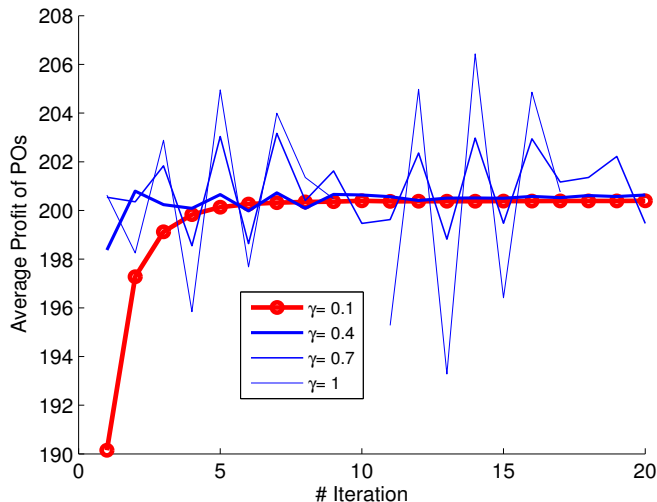


Fig. 7. Profit versus #Iteration for different Step Size  $\gamma$  for distributed learning algorithm (DLA).

network parameters, the minimum number of iterations for the DLA algorithm is 5 in order to generate precise results. These results indicate that the proposed DLA can quickly be adapted to network dynamics.

### C. Hybrid Optimization Algorithm (HOA)

Here, we use the distributed learning algorithm (DLA) for tracking the changes of the optimum solution of Problem 1 in (9) (for other problems procedure is the same) when making multiple STCs (see Fig.8). HOA combines both distributed learning algorithm (DLA) and convex optimization (CO) in such a way that CO is launched occasionally for the initialization of DLA. In DLA, it is simpler (faster) to constantly calculate the optimum points in order to track the changes in the solutions. DLA is able to efficiently find a suboptimal solution in a reasonable time (See Fig. 7) while CO can find better solutions in a longer time. Therefore, combining DLA and CO results into the HOA, which benefits from the advantages of both algorithms. Fig. 9 shows the efficiency of the distributed learning game in tracking the optimum points. Note that the optimum points are provided by solving the convex optimization Problem 1. Once an optimum is calculated for a LTC, it is used in the distributed learning algorithm (EGA) as initial state to track the evolution of this solution over time for STCs. Comparing the results for pure CO and HOA in Fig. 9, it can be seen that HOA is effectively able to track the optimal points of CO with high accuracy. In Fig. 10, new STCs are negotiated at each epoch. As we see, HOA is considerably faster while providing the same performance as Convexified scheme solving repeatedly Problem 1. For HOA scheme in Fig. 10, the average running time is reduced by factor 1-4, where HOA alternatively uses CO scheme to calibrate the DLA algorithm.

### D. Utilization Rate Analysis

In this section, we study the spectrum utilization rate of a scenario with a PO and multiple SOs. The SCRNN spectrum sharing scheme aims to improve the system utilization while

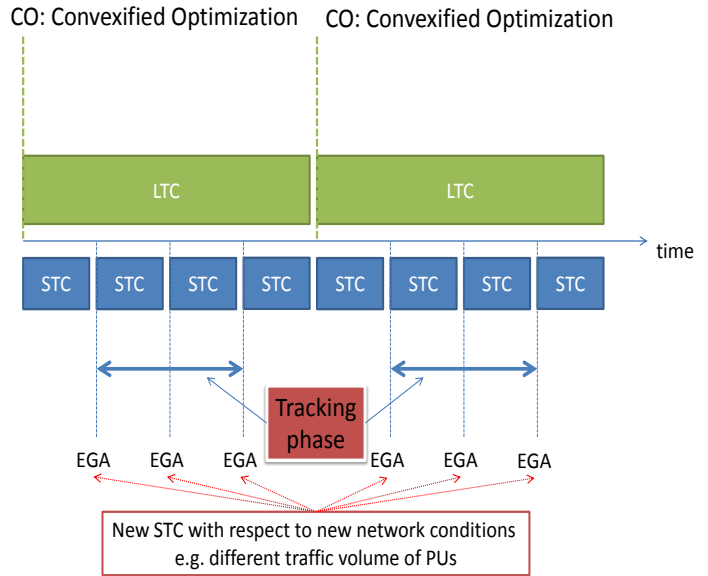


Fig. 8. Hybrid Optimization Algorithm (HOA).

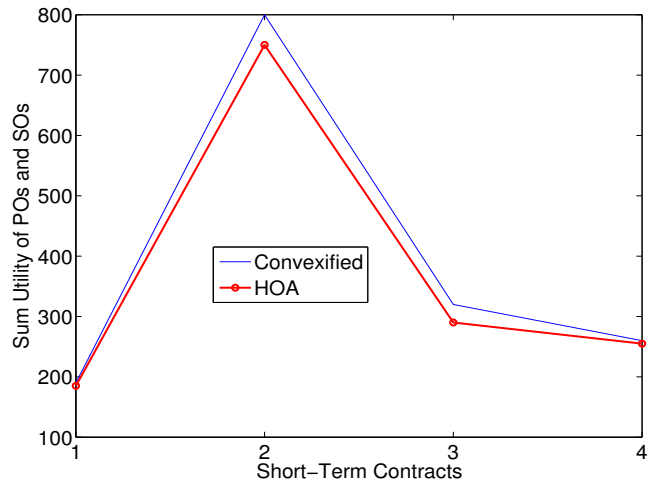


Fig. 9. Tracking by HOA ( $\alpha = 2$  and  $\beta = 1$ ).

requirements of both PUs and SOs are met properly. The system performance is evaluated under different SOs and POs arrival rates. Fig. 11 presents the spectrum utilization rate for different  $b$  that is the number of shared channels by the PO (we drop the index  $i$  because we consider a single PO scenario). As shown in the figure, the higher  $b$  results in a notable improvement in the utilization rate of the order of 20% and higher. In SCRNN, the increase of arrival rate of primary user leads to the increase in the number of collisions with secondary user (interruption rate). Therefore, the spectrum utilization rate due to the higher block and interruption rates would be decreased. In Fig. 12, the impact of SO arrival rate is investigated on interruption rate of SUs. Significant improvements in SCRNN are also noticeable. As can be seen from the figure, SCRNN model reduces interruption rate by factor 4-16 if the SO arrival rate is increased from 1 to 10. In particular, for  $b = 5$  the interruption rate is less than 3%. Notice that SCRNN with  $b = 0$  is exactly the same model as CCRNN.

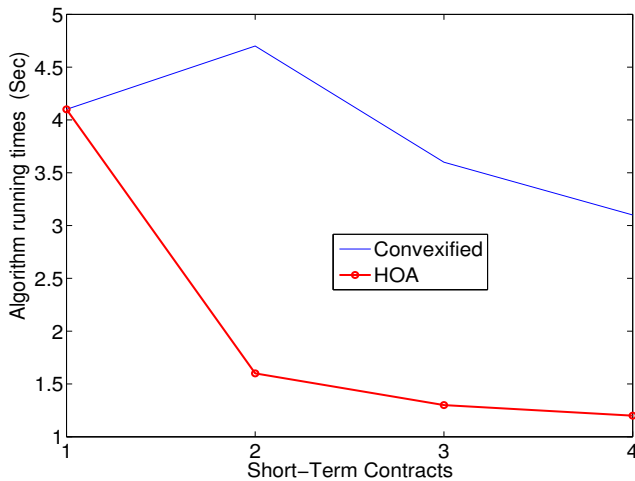


Fig. 10. Algorithm running times ( $\alpha = 2$  and  $\beta = 1$ ).

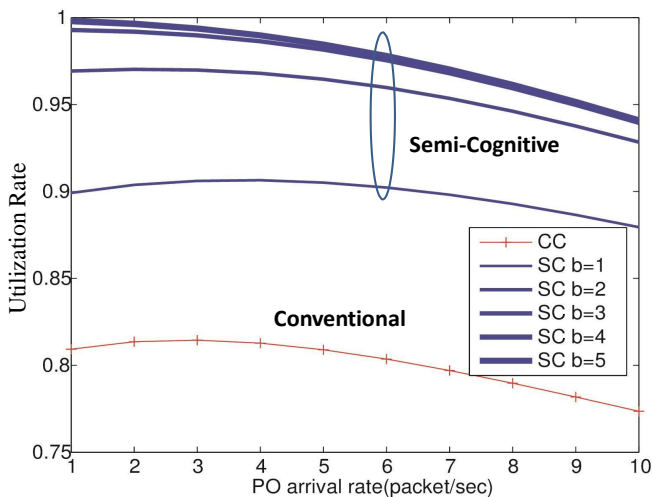


Fig. 11. Spectrum utilization rate versus PO packet arrival rate(Packet/sec).

## VII. CONCLUSION

In this paper, we propose a novel cognitive radio paradigm referred to as semi-cognitive radio networks (SCRNs), where POs are not absolutely dominant users of the spectrum. Here, SOs can actively negotiate the channel prices and quality, thus having a sufficient incentive to participate in the dynamic spectrum sharing mechanism. To support this concept, a novel channel management policy is proposed to support different network applications with different requirements.

The dynamic spectrum sharing between SOs and POs is controlled according to a short-term contract in order to capture the dynamics of the networks. This is achieved by considering the channel corruption rate as a parameter that drives spectrum supply-and-demand process. To analyse the impact of the POs' strategies on the SCRn model, we use both individual and sum utility optimizations of POs.

To achieve equilibrium in this general setting, we also provide a distributed learning algorithm based on exponentiated gradient ascent (EGA). Three different SCRn case studies

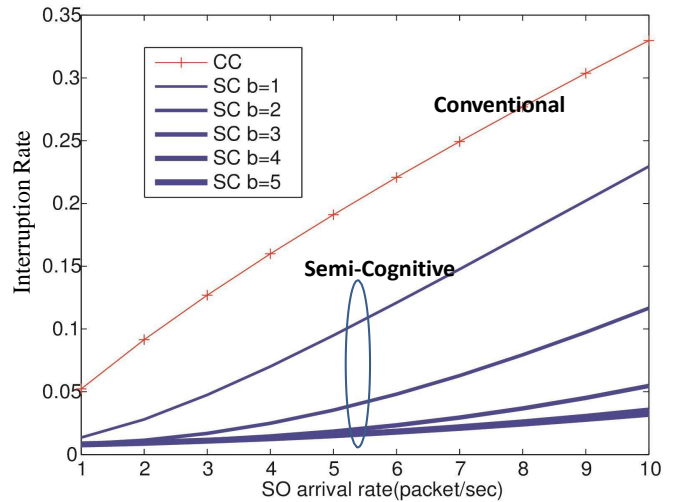


Fig. 12. Interruption rate (given by equation (21)) of SO vs SO packet arrival rate(Packet/sec).

(pricing, load balancing and quality degradation problems) are investigated to analyze the efficiency of SCRn in different network applications. We also analyze the system utilization by a Markov chain model to study the behavior of SCRn in more detail.

Our results demonstrate a significant performance improvement for SCRn operators (both primary and secondary) in the dynamic spectrum sharing process. In general our numerical results show that with the same channel efficiency SCRn provides higher profit for POs, improved system utilization and significantly reduced interruption probability in SO network. As an illustration, for a specific set of the network parameters and the same channel efficiency, SCRn provides at least five times higher profit for PO, the network utilization improves from 80% to close to 100% and depending on SO packet arrival rates the interruption rate is reduced by factor 2-10 in SO's network. In addition, we also propose for SCRn-based POs a hybrid spectrum management mechanism which combines both convexified optimization and learning algorithms to manage simultaneously LTCs and STCs.

Finally as an observation with respect to the applicability of the results to the standardized TV white space spectrum utilization in cognitive networks one should be aware of the following. In the case of the TV white space spectrum the dynamics of the PU arrivals is much lower and predictable so that the conflict situations might be avoided by simple scheduling. That means that the spectrum segments where TV program is scheduled at a given time will be omitted in short term contracts from leasing to SO in the period of the program duration. Our models are still valid although in such applications the probability of corruption will be zero. The delays in accessing the network by PU will be eliminated by scheduling since the dedicated spectrum will be waiting for the start of the program. In future work, we intend to focus on extending SCRn to heterogeneous wireless networks, where SUs and PUs have different requirements. We will also consider SCRn model in higher frequency bands e.g. mmWave communications.

APPENDIX A  
PU RETURN PROBABILITY IN SCRNs

Once a PU recaptures a channel employed by a SU, the ongoing transmission of the SU might be interrupted even if there are still empty channels available. In this context, from the point of view of the network's SOs, the main parameter in the spectrum trading process is the PUs' *return probability*, i.e. the corruption probability of an ongoing SU transmission by a new PU in a given channel of the  $i$ -th PO. In CCRNs, this return probability can be calculated as

$$\psi_i^{(CC)} = \sum_{z=1}^{C_i} \frac{z \rho_i^z \exp(-\rho_i)}{C_i z!}, \quad (\text{A1})$$

where  $\frac{\rho_i^z \exp(-\rho_i)}{z!}$  is the probability of  $z$  PU arrivals which follows a Poisson distribution with mean  $\rho_i = A_i^{PO}/\mu^{SO}$ . Parameter  $A_i^{PO}$  represents the arrival rate of primary users and  $\mu^{SO}$  is the service rate of secondary users.

On the other hand, in the semi-cognitive regime (where PUs only recapture a channel if there are no unused channels to take), the corresponding channel corruption probability will be given by

$$\psi_i^{(SC)} = \sum_{z=1}^{b_i} \frac{z \rho_i^{C_i - b_i + z} \exp(-\rho_i)}{b_i (C_i - b_i + z)!}, \quad (\text{A2})$$

where  $b_i$  represents the number of the demanded channels by SOs. If  $z$  additional channels are required and all unshared sub-channels are occupied by PUs, then the SO transmissions would be corrupted with probability  $z/b_i$ . In the SCRn model,  $b_i/C_i$  is an important parameter representing the *tolerance level* of the  $i$ -th PO.

APPENDIX B  
GAME-THEORETIC ANALYSIS

To simplify notation, in what follows, we write  $\mathcal{K} = \mathcal{P} \cup \mathcal{S}$  to denote the set of all players of the game (i.e. both POs and SOs), indexed by  $k \in \mathcal{K}$ . In the same spirit, we write  $x_k = p_k$  for  $k \in \mathcal{P}$  and  $x_k = b_k$  for  $k \in \mathcal{S}$ ; we maintain the indices  $i$  and  $j$  (and  $p_i$  and  $b_j$  respectively) only where there is danger of confusion.

*Proof of Proposition 1.* Note first that the players' action sets are convex and compact; as a result, by the general theory of [27], [28], it suffices to show that each player's payoff function  $U_k$  is individually concave in  $x_k$ .

For  $k \in \mathcal{P}$ ,  $U_k$  is linear – and hence concave – in  $x_k$  because the profit function  $U_k$  in (1b) of the  $k$ -th PO is itself linear in the PO's individual price variable  $x_k$ . On the other hand, for  $k \in \mathcal{S}$ , the situation is more complicated: the first and second terms of (1a) are both concave in  $b_j$  by our assumption for  $\varepsilon_{ij}$  and linearity, respectively. For the third term of (1a), let  $f(x) = \frac{1}{2} \sum_{i=1}^n x_i^2 + \vartheta \sum_{\ell \neq i} x_i x_\ell$  so it suffices to show that  $f$  is convex in  $x$ . To that end, a straightforward differentiation gives

$$\frac{\partial^2 f}{\partial x_i \partial x_\ell} = \delta_{i\ell} + \vartheta(1 - \delta_{i\ell}), \quad (\text{B1})$$

so  $\text{Hess}(f) = I + (1 - \vartheta)E$  where  $E$  is a constant matrix of ones. This implies that  $\text{Hess}(f)$  is a circulant matrix with eigenvalues of the form

$$\lambda_a = 1 + \vartheta(\omega_a + \dots + \omega_a^{n-1}), \quad (\text{B2})$$

where  $n$  is the dimension of  $x$  and  $\omega_a = \exp(2\pi a/n)$ ,  $a = 0, \dots, n-1$ , are the  $n$ -th roots of unity [36]. A simple manipulation then gives  $\lambda_0 = 1 + (n-1)\vartheta$  for  $a = 0$  and

$$\lambda_a = 1 - \vartheta + \vartheta(1 + \omega_a + \dots + \omega_a^{n-1}) = 1 - \vartheta + \vartheta \frac{\omega_a^n - 1}{\omega_a - 1} = 1 - \vartheta, \quad (\text{B3})$$

for all  $a = 1, \dots, n-1$ . Since  $0 \leq \vartheta \leq 1$ , we conclude that  $\text{Hess}(f) \geq 0$ , i.e.  $f$  is convex in  $x$ . In turn, this implies that  $U_k$  is concave in (1a) (as the sum of concave functions) so the existence of Nash equilibria follows from the general theory of [27], [28].  $\square$

We now proceed with the proof of our main convergence result:

*Proof.* Proof of Theorem 1 For the first assertion of the theorem, simply note that  $x^* \in \mathcal{X}^\circ$  is a Nash equilibrium if and only if  $v_k(x^*) = 0$  for all  $k \in \mathcal{K}$  (a consequence of the fact that the players' payoff functions are individually concave). Since this is true if and only if  $y_k(n+1) = y_k(n)$  for all  $k \in \mathcal{K}$ , it follows that  $x^* \in \mathcal{X}^\circ$  is a stationary point of (EGA) if and only if it is a Nash equilibrium of  $\mathcal{G}$ .

For our second assertion, we will use a proof technique developed in [37], [38] based on the so-called ‘‘Fenchel coupling’’ between primal and dual variables ( $x$  and  $y$  respectively). To introduce this notion, first let

$$h_k(x_k) = \begin{cases} x_k \log x_k + (C_k - x_k) \log(C_k - x_k) & \text{if } k \in \mathcal{P}, \\ \sum_{i \in \mathcal{P}} (x_{ik} \log x_{ik} + (b_{\max} - x_{ik}) \log(b_{\max} - x_{ik})) & \text{if } k \in \mathcal{S}, \end{cases} \quad (\text{B4})$$

and

$$h_k^*(x_k) = \max \{ \langle y_k | x_k \rangle - h_k(x_k) : x_k \in \mathcal{X}_k \}, \quad (\text{B5})$$

denote the convex conjugate of  $h_k$ .<sup>4</sup> The ( $\lambda$ -weighted) Fenchel coupling between  $x \in \mathcal{X}$  and  $y \in \mathbf{R}^{\mathcal{P}} \times \mathbf{R}^{\mathcal{P} \times \mathcal{S}}$  is then defined as

$$F(x, y) = \sum_{k \in \mathcal{K}} \lambda_k \left[ h_k(x_k) + h_k^*(y_k) - \langle y_k | x_k \rangle \right], \quad (\text{B6})$$

with  $\lambda$  chosen as in (DSC). By Fenchel's inequality, we then get  $F(x, y) \geq 0$  with equality if and only if  $x_k = Q_k(y_k)$  where

$$Q_k(y_k) = \arg \max \{ \langle y_k | x_k \rangle - h_k(x_k) : x_k \in \mathcal{X}_k \}. \quad (\text{B7})$$

By standard convex analysis arguments, it is easy to show that the solution of the maximization problem (B5) is

$$h_k^*(y_k) = \begin{cases} C_k \log(1 + e^{y_k}) & \text{if } k \in \mathcal{P}, \\ b_{\max} \sum_{i \in \mathcal{P}} \log(1 + e^{y_{ik}}) & \text{if } k \in \mathcal{S}. \end{cases} \quad (\text{B8})$$

Moreover, by the Legendre inversion formula [39], we will have

$$Q_k(y_k) = \nabla_{y_k} h_k^*(y_k) \quad (\text{B9})$$

and hence:

$$Q_k(y_k) = A_k \frac{e^{y_k}}{1 + e^{y_k}}, \quad (\text{B10})$$

with vectorized notation as in the update step of the recursion (EGA).

With these preliminaries at hand, assume that (DSC) holds, let  $x^*$  denote the game's unique Nash equilibrium, and set  $D_n = F(x^*, y(n))$ .

$$\begin{aligned}
D_{n+1} &= F(x^*, y(n) + \gamma_n v(x(n))) \\
&= \sum_{k \in \mathcal{K}} \lambda_k \left[ h_k(x_k^*) + h_k^*(y_k(n) + \gamma_n v_k(x(n))) - \langle y_k(n) + \gamma_n v_k(x(n)) | x_k^* \rangle \right] \\
&\leq \sum_{k \in \mathcal{K}} \lambda_k \left[ h_k(x_k^*) + h_k^*(y_k(n)) - \langle y_k(n) | x_k^* \rangle \right] + \gamma_n \sum_{k \in \mathcal{K}} \lambda_k \langle v_k(x(n)) | x_k(n) - x_k^* \rangle + \frac{M}{2} \gamma_n \|x(n) - x^*\|^2 \\
&= D_n + \gamma_n \sum_{k \in \mathcal{K}} \lambda_k \langle v_k(x(n)) | x_k(n) - x_k^* \rangle + \frac{M}{2} \gamma_n \|x(n) - x^*\|^2,
\end{aligned} \tag{B11}$$

By a Taylor expansion, we then get (B11) where, in the third line, we used the fact that  $\nabla h_k^*(y_k(n)) = Q_k(y_k(n)) = x_k(n)$  and the fact that the norm of the Hessian of  $\sum_k \lambda_k h_k^*$  is bounded from above by some positive constant  $M > 0$ . We now claim that there exists a subsequence  $x(n_j)$  such that  $x(n_j) \rightarrow x^*$ . Indeed, if this were not the case, we would have  $\sum_{k \in \mathcal{K}} \lambda_k \langle v_k(x(n)) | x_k^* \rangle \leq -a$  for some  $a > 0$  and for all  $n$  by (DSC). Hence, telescoping (B11) would yield

$$D_{n+1} \leq D_0 - a \sum_{j=1}^n \gamma_j + \frac{K}{2} \sum_{j=1}^n \gamma_j^2 \|x(j) - x^*\|^2, \tag{B12}$$

and, ultimately,  $\lim_{n \rightarrow \infty} D_{n+1} = -\infty$  because  $\sum_{n=1}^{\infty} \gamma_n = \infty$  while  $\sum_{n=1}^{\infty} \gamma_n^2 < \infty$  and  $\sup_n \|x(n) - x^*\|^2 < \infty$  (recall that  $\mathcal{X}$  is compact). This contradicts the fact that  $D_n = F(x^*, y(n)) \geq 0$  so we conclude that  $x(n)$  visits any neighborhood of  $x^*$  infinitely many times. Now, fix some  $\varepsilon > 0$  and let  $V_\varepsilon = \{x \in \mathcal{X} : \|x - x^*\| < \varepsilon\}$  be an  $\varepsilon$ -neighborhood of  $x^*$  in  $\mathcal{X}$ . Then, reasoning as above, if  $R_\varepsilon$  denotes the shell-like region  $V_\varepsilon \setminus V_{\varepsilon/2}$ , there exists some  $a \equiv a(\varepsilon) > 0$  such that  $\sum_{k \in \mathcal{K}} \lambda_k \langle v_k(x) | x - x^* \rangle \leq -a$  for all  $x \in R_\varepsilon$ . Also, let  $L = \max_{x \in \mathcal{X}} \|v(x)\|$  and assume that  $n$  is sufficiently large so that  $\gamma_n < \min\{a/(ML^2), \sqrt{\varepsilon}/(ML^2)\}$ .

Since  $x(n)$  visits  $V_\varepsilon$  infinitely many times, we may assume without loss of generality that  $x(n) \in V_\varepsilon$  for some sufficiently large  $n$  that satisfies the step-size condition above. Then, by (B11), we will have:

$$D_{n+1} \leq D_n + \gamma_n \sum_{k \in \mathcal{K}} \lambda_k \langle v_k(x(n)) | \gamma_k(n) - x_k^* \rangle + \frac{1}{2} \gamma_n^2 ML^2. \tag{B13}$$

Thus, if  $x(n) \in R_\varepsilon$ , we will have  $D_{n+1} \leq D_n - a\gamma_n + \frac{1}{2} \gamma_n^2 ML^2 < D_{n+1} - \frac{1}{2} a\gamma_n < D_n$ , so  $x(n+1) \in V_\varepsilon$ . On the other hand, if  $x(n) \in V_{\varepsilon/2}$ , we will have  $D_{n+1} \leq D_n + \frac{1}{2} \gamma_n^2 ML^2 < D_n + \varepsilon/2$  so, again,  $x(n+1) \in V_\varepsilon$ .

By induction, the above shows that for any  $\varepsilon > 0$ , there exists some  $n_0 \equiv n_0(\varepsilon)$  such that  $x(n) \in V_\varepsilon$  for all  $n \geq n_0$ , i.e.  $\lim_{n \rightarrow \infty} x(n) = x^*$ , as claimed.  $\square$

#### ACKNOWLEDGEMENT

The authors would like to thank to the editor and anonymous reviewers for very constructive suggestions in the process of paper revision.

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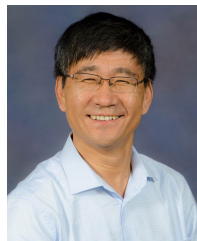
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