



HAL
open science

Photon-photon scattering and related phenomena. Experimental and theoretical approaches: The early period

Klaus Scharnhorst

► **To cite this version:**

Klaus Scharnhorst. Photon-photon scattering and related phenomena. Experimental and theoretical approaches: The early period. 2018. hal-01638181v2

HAL Id: hal-01638181

<https://hal.science/hal-01638181v2>

Preprint submitted on 19 May 2018 (v2), last revised 24 Oct 2023 (v5)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Photon-photon scattering and related phenomena.

Experimental and theoretical approaches: The early period

K. Scharnhorst[†]

Vrije Universiteit Amsterdam, Faculty of Sciences, Department of Physics and
Astronomy, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

Abstract

We review the literature on possible violations of the superposition principle for electromagnetic fields in vacuum from the earliest studies until the emergence of renormalized QED at the end of the 1940's. The exposition covers experimental work on photon-photon scattering and the propagation of light in external electromagnetic fields and relevant theoretical work on non-linear electrodynamic theories (Born-Infeld theory and QED) until the year 1949. To enrich the picture, pieces of reminiscences from a number of (theoretical) physicists on their work in this field are collected and included or appended.

[†]E-mail: k.scharnhorst@vu.nl,
ORCID: <http://orcid.org/0000-0003-3355-9663>

Contents

Contents	2
1 Introduction	3
2 Experiment	7
2.1 Photon-photon scattering	8
2.2 Propagation of light in strong electromagnetic fields	10
2.2.1 Influence of the intensity on the propagation of light	10
2.2.2 Propagation of light in macroscopic, constant magnetic and electrical fields	11
2.2.3 Propagation/scattering of photons in the Coulomb field of nuclei	14
2.3 Considerations related to extraterrestrial and astronomical observations	17
3 Theory	20
3.1 Born-Infeld electrodynamics	21
3.2 Quantum electrodynamics	29
Acknowledgements	35
Appendices	36
Appendix A	36
Appendix B	41
Appendix C	44
Appendix D	48
Appendix E	49
Appendix F	53
Appendix G	55
Appendix H	57
Appendix I	59
Appendix J	61
Main list of references	63
List of full author names	86
Further literature	94

1 Introduction

Light has caught the imagination of men since the earliest historic times and theories on its nature probably have an even long history. The physical world around us to a large extent is being perceived by us by means of (visible) light and also the discourse of physical theory-building is strongly influenced by our anthropomorphic capability “to see”. A hypothetic intelligent species without such a visual capability probably would approach physical theory-building in a different manner than we do - trying to figure out the nature of that “dark matter” that is visible to mankind. To us, light and its interaction with (charged) matter is prototypical, and so is the most advanced to date theory describing it - quantum electrodynamics (QED). Our visual perception of the world (and also many physical experiments) is resting on two pillars: the interaction of light with charged particles whose strength is ruled by the fine structure constant, and the (for all practical purposes) non-interaction of light with itself described by the superposition principle. In the literature, various formulations of the superposition principle can be found. As we are both concerned with experimental and theoretical work to us the superposition principle is more than a mathematical rule related to the linearity of the equations describing a physical phenomenon. Quite generally, we understand the superposition principle as the principle that two physical influences can be superposed without impeding each other. In this sense, in our context it can be understood as a specific non-interaction principle – a non-self-interaction principle.

That the superposition principle for light is something special to be thought about has been recognized long ago. In 1604, Johannes Kepler wrote in his book *Ad Vitellionem Paralipomena, Quibus Astronomiae Pars Optica Traditur*^[a]: “The rays of light neither mutually color each other, nor mutually illuminate each other, nor mutually impede each other in any way. . . . This is just like one physical motion’s not impeding another.” It seems that Kepler is the first to explicitly mention this property of light: His intellectual predecessor, Erazmus Vitello (Vitellio, Vitello; 13th century) Kepler refers to in the title of his book does not mention this property in book II of his *Perspectiva* [161, 162] where one would expect such an observation to occur. Somewhat later than Kepler, in 1690, Christiaan Huygens commented in his *Traité de la Lumière*^[b]: “Another property of waves of light, and one of the most marvellous, is that when some of them come from different or even from opposing sides, they produce their effect across one another without any hindrance.” At the beginning of the 19th century the superposition principle led Thomas Young to the formulation of the principle of interference within the framework of the wave theory of light [168]. However, while within the framework of the wave theory of light the superposition principle must have been more or less natural, for the proponents of

^[a]Proposition 26 ([157], p. 23, reprint (1859) p. 142, reprint (1939) p. 32: cf. our App. A, p. 36; English transl. [160], p. 37)

^[b][163], p. 20: cf. our App. A, p. 36; English transl. [164], pp. 21/22 (We owe this quote Vavilov [8]).

the rival model – the corpuscular model of light developed by Descartes and Newton – the superposition principle required special consideration. Sergeï I. Vavilov writes on this account^[c]: “The reproach often made in the XVIII. century to the Newtonian theory consisted just in saying that the collisions of the light corpuscles, i.e., a violation of the superposition, should be observed. An answer to this difficulty was the admission of the extreme smallness of the corpuscles: «I know – Ломоносов³^[K.S.: orig. footn.] wrote addressing the defenders of the corpuscular hypothesis – that you divide the material of light into such fine particles and place them in universal space with so little density that the whole quantity can be compressed and packed in the porous crevices of one grain of sand.»”.

In the 19th century, the wave theory of light got the upper hand over the corpuscular model in explaining optical phenomena and James Clerk Maxwell with his equations finally brought mathematical clarity to electromagnetic phenomena, including the propagation of electromagnetic waves and specifically visible light. The phenomenon of the non-self-interaction of light (in vacuum) described, for example, by Kepler and Huygens received its mathematical explanation in terms of the linearity of the Maxwell equations in vacuum. However, as Faraday discovered in material (polarizable) media electromagnetic fields may exert an influence on each other: In a magnetic field, a rotation of the plane of polarization of linearly polarized light propagating along the direction of the magnetic field can be observed (Faraday effect). Consequently, the linearity of the Maxwell equations has a restricted range of validity. Besides the Faraday effect, the discovery of the (electro-optic) Pockels and Kerr effects and the (magneto-optic) Voigt, Majorana, and Cotton-Mouton effects (birefringence in a magnetic field for propagation of light perpendicular to it) showed that material media can be polarized by electromagnetic fields in such a manner that the superposition principle for electromagnetic fields does not apply for material (polarizable) media in general (For references concerning these effects see [170, 171]). However, matterless space, the vacuum, remained immune from such effects in the view of physicists.

With the advent of the quantum theory at the beginning of the 20th century the corpuscular model of light made a surprising return with the emergence of the concept of the photon. The particle-wave dualism characteristic for all quantum phenomena led to a dialectical synthesis of the wave and corpuscular models of light. However, once the particle-like aspects of light had been recognized the century-old problem of the corpuscular model of light reemerged. What about the scattering of corpuscles of light – photons – among each other? Qualitative investigations of this problem began to emerge. The first theoretical attempt to study this prob-

^[c][8], p. 555, reprint [165], p. 234: cf. our App. A, p. 36; English transl. p. T-1 (= P-3). Also see [166], part 2, § 2.

³[K.S.: orig. footn.] М. В. Ломоносов, Слово о происхождении света. Собрание разных сочинений, ч. III [M. V. Lomonosov, Slovo o proiskhozhdenii sveta. Sbranie raznykh sochinenii, ch. III], 1803, p. 155. (K.S.: [169])

lem was published in 1925 by Konstantin N. Shaposhnikov [6] by writing down the quantum mechanical equations for energy-momentum conservation for the photon-photon scattering process. Shaposhnikov's main purpose was to emphasize that these equations allow solutions which correspond to a non-violation of the superposition principle. In 1926, Louis de Broglie then pointed out ([7], Chap. XI, Sec. 2, pp. 96-98; cf. our Appendix B, p. 41) that these equations also allow solutions which correspond to nontrivial scattering events between two photons in deviation from the superposition principle. This point has further been elaborated in a short contribution by Arthur L. Hughes and George E. M. Jones [10] in 1929. Vavilov, in 1928, discussed the status of the superposition principle for light from an experimental point of view by reporting on laboratory experiments of his own and invoking extraterrestrial considerations by means of an analysis of the problem of the solar corona ([8], cf. our subsecs. 2.1 and 2.3). The article by Vavilov was followed by a public exchange of comment and response between Shaposhnikov and Vavilov [11, 12]. In his reply ([12], p. 395, p. T-3 (= P-5) of the English transl.), giving credit for this consideration to Yakov I. Frenkel Vavilov pointed out that the superposition principle for light should be violated, in principle, in any case due to the (negligibly small) gravitational interaction of two quanta of light. The same argument had been made in 1928 already by Léon Rosenfeld and Enos E. Witmer ([9], p. 521) in an article in which they considered from a qualitative point of view the role of collisions among photons for the discussion of black body radiation.

However, the physical mechanism for photon-photon interaction processes and their quantitative details remained beyond consideration for a couple of further years. In 1933, Otto Halpern [24] finally proposed that virtual electron-positron pairs could be at the origin of photon-photon collisions. While this clarified the qualitative picture to be applied for the description of photon-photon scattering, only the early development of quantum electrodynamics (QED) provided physicists with the theoretical tools for a quantitative answer: It was finally given by two students of Werner Heisenberg, Hans Euler and Bernhard Kockel, who calculated in 1935 the leading nonlinear corrections to the Maxwell equations in vacuum [45]. Within the framework of QED, it turned out that photon-photon scattering as a characteristic feature of a nonlinear electrodynamic theory has a very low probability for all practical purposes. Once the theoretical picture had been established the challenge emerged to demonstrate experimentally consequences of the violation of the linearity of Maxwell equations in vacuum, i.e., of the violation of the superposition principle for light (or, speaking more generally, for electromagnetic fields). The processes of Delbrück scattering (elastic scattering of a photon in the Coulomb field of a nucleus) and photon splitting (in an external field) have meanwhile experimentally been confirmed (cf. [172, 173]). However, up to the present day demonstration of such a fundamental physical phenomenon as photon-photon scattering remains a problem at the edge of current experimental and observational capabilities (for some recent developments see [174, 175]).

There is a continuous current, varying in intensity over time, of experimental and theoretical research in the field of nonlinear electrodynamic phenomena in vacuum. The main motivation for experimental work derives from the prominent role electromagnetic phenomena play in the physical world we live in. Theoretical studies often have a wider set of motivations which are often linked to the changing directions of theoretical thinking. New research can successfully only be based on the knowledge of the achievements and failures of past generations of scientists. While the main publications from the past in the field of nonlinear electrodynamics in general and of QED in particular are well known many details of the early thinking and experimentation concerning photon-photon scattering and related phenomena seem to be largely forgotten. It is the purpose of the present review to collect as completely as possible the early literature on this subject as a source and inspiration for further research. The time span covered reaches from the earliest publications that can be linked to the problem of nonlinear electrodynamics up to 1949, i.e., the end of the 1940's, which roughly marks the begin of a new period in the history of theoretical physics with the emergence of renormalized quantum electrodynamics. Of course, the year 1949 is somewhat arbitrarily chosen based on an analysis of the occurrence of relevant literature references. In a certain way, the short review article by Kunze in 1949 [152] marks the end of the period the present exposition is concerned with.

The present review has a mixed character. On one hand, primarily it is written as a literature review. On the other hand, it is also a piece of science history. To illuminate various social and science history aspects we quote extensively from a number of different sources. This rather unconventional set-up serves a twofold purpose. First of all, it provides the reader with a more complex and first-hand picture of the early developments. The second purpose consists in collecting the widely scattered pieces of text for the convenience of the reader in one place. Some of the early pieces of text have been translated into English to make them more easily accessible to the current and future generations of physicists.

The review is split in a section on early experimental work and a section on theoretical studies in nonlinear electrodynamics. Corresponding to the historic course of research, we cover both Born-Infeld electrodynamics as a (mainly) classical field theory and quantum electrodynamics which gives rise to an effective action by means of which contact can be made with a classical theory as Born-Infeld electrodynamics. However, the relation between both theories is more than a historic one. Also modern research benefits from viewing both theories as special instances of nonlinear electrodynamics because this allows to elucidate more easily general principles. The early literature on Born-Infeld electrodynamics has been included in the list of references somewhat more broadly than that from QED. As there does not seem to exist any comprehensive review of the early developments of Born-Infeld theory this choice seemed to be appropriate.

The list of references is split into two parts. One part arranged according to the

year of publication contains the main body of references related to photon-photon scattering and related nonlinear electrodynamic phenomena up to 1949. Most of these references are cited and commented in the main body of our text, very few references are only listed in the bibliography for completeness and are not mentioned elsewhere [58, 65, 153–156]. The references in the main bibliography represent the present literature review in the narrow sense. All further references cited are collected in a separate list of (auxiliary) references. We deliberately cite literature after 1949 only once this seems absolutely to be necessary for the presentation or discussion. Consequently, most of the more recent relevant literature from 1950 onwards is absent. The main list of references is supplemented by a list of author names where for identification purposes the first names and, if known, the dates of birth and death are given. If available, we provide for each name a link to further information. For the convenience of the reader, we rely primarily on Wikipedia articles. Preference has been given to English language articles. If an English language article was not existent we have tried to make some other reasonable language choice. If no Wikipedia article was available at all, a link to some other information page on the Internet has been selected. In a number of cases, no appropriate information on the Internet was available. We then make reference to some printed obituary. In a small number of cases, we had to restrict ourselves to links to VIAF (Virtual International Authority File, OCLC) or some related source. Finally, for very few author names, we were unable to trace any further author information.

2 Experiment

In the period under consideration, experimental work concerning the study of photon-photon scattering and related phenomena has been fueled by a number of different developments in physics. There have been two main directions of experimental work:

1. Experimental work concerning the study of photon-photon scattering in the narrow sense, i.e., direct searches for experimental signatures of the scattering of light by light.
2. The experimental study of the propagation of light (photons) in strong (electromagnetic) fields. This main direction of research can historically be divided into 3 subcategories:
 - A. The dependence of the propagation of light on the intensity of the light wave.
 - B. The propagation of light (photons) in macroscopic, constant magnetic and electrical fields.
 - C. The propagation/scattering of photons in the Coulomb field of nuclei.

Finally, it seems to be appropriate to add as a further direction of experimental work:

3. Considerations related to extraterrestrial and astronomical observations.

In the following subsections arranged according to the above classification we will give an overview over the experimental and observational work up to the end of the 1940's. Early experimental and observational work on photon-photon scattering and related phenomena has been scarce. We have tried to cover the relevant literature as completely as possible and in the subsections below we will refrain from making any further statements to the effect that no further relevant work is known to us.

2.1 Photon-photon scattering

The first attempt to directly observe the scattering of photons by photons in an experiment seems to have been undertaken in 1928 in the Soviet Union by S. I. Vavilov (Institute of Biological Physics, Moscow). In a contribution read to the 6. Congress of Russian Physicist in Moscow on August 6, 1928 Vavilov reports^[d]: “Under laboratory conditions, the highest instantaneous radiation densities can certainly be obtained by means of the light of a condensed spark. Concentrating this light with a lense, instantaneous values of the light energy density that exceed the corresponding value on the surface of the sun can be achieved easily. In this case, the average density is small due to the short duration and rareness of the sparks, but the hypothetical effect of the “collisions” of light quanta must be proportional to the square of the instantaneous density, therefore, a spark turns out to be considerably more advantageous than, for example, an arc. Preliminary experiments with a spark the light of which met in an evacuated container did not uncover any noticeable scattering. These observations have been carried out with the usual precautions, in front of distant container walls covered with black velvet; for control, the experiments have been repeated with the light of an incandescent lamp that delivered the same average radiation density; in both cases the result has been equally negative.”.

Shortly thereafter, in 1930, A. L. Hughes and G. E. M. Jauncey (Washington U., St. Louis, USA) published a somewhat more elaborate description of a similar experiment to detect collisions of photons [14, 15]. In difference to Vavilov, they used (intense) light from the sun to perform the experiment. Two light beams were allowed to meet at an angle of 120° and the observation for scattered light was performed with the dark-adapted eye in the plane of the scattering beams at the (forward) direction of the bisector of the angle between them. In their experiment, Hughes and Jauncey took into account that with the chosen geometric set-up light

^[d][8], § 2, pp. 556/557, reprint [165], p. 236: cf. our App. A, p. 36; English transl. pp. T-2/T-3 (= P-4/P-5). Also cf. his note [16].

resulting from photon-photon collisions would emerge with a higher frequency according to the quantum-mechanical laws of energy-momentum conservation in the collisions. In the experiment, no experimental sign of photon-photon collisions was found and Hughes and Jauncey give as bound for the cross section σ of photon-photon scattering $\sigma < 3 \times 10^{-20} \text{ cm}^2$.

Interestingly, the article by Hughes and Jauncey [15] is not only followed by a short note by Vavilov [16] pointing out the results of his earlier experiments [8] but also by a short note by A. K. Das (Alipore Observatory, Calcutta, India) [18] giving details of an earlier (unpublished) idea for performing an experiment to study photon-photon scattering. Noting that due to the (likely) miniscule size of the effect photon-photon scattering cannot be observed by optical methods (cf. the analogous comment by Vavilov: [8], § 2, p. 558, p. 237 of the reprint, p. T-4 (= P-6) of the English translation), Das proposed to observe single (scattered) photons by means of “Elektronenzählrohren” (electron counter tubes) he had constructed for the investigation of gamma and cosmic rays. From a modern point of view, it is also interesting to note that Das envisioned the experiment to be performed with X-rays or gamma-rays (and not with visible light for which the photon-photon cross section is smaller).

The experiment in 1930 by Hughes and Jauncey was followed in 1933 by an analogous experiment performed by F. L. Mohler (Bureau of Standards, Washington, D. C., USA) [25] which differed by that of Hughes and Jauncey geometrically by the angle between the scattering light beams (180° in the experiment by Mohler versus 120° in the experiment by the former; also see the corresponding comment by Hughes and Jauncey [37]). Mohler relied in his experiment on the light of projection lamps and the observation for scattered photons was performed with the human eye in the direction perpendicular to the joint axis of the two scattering light beams. The result of the experiment was negative and Mohler expressed the result in his article in terms of a limit on the cross section σ for photon-photon scattering ($\sigma < 6 \times 10^{-17} \text{ cm}^2$).

The following logical step in the experimental search for the existence of photon-photon collisions has been made in two Master theses: the human eye as detector of light (radiation) is being replaced by a photographic film. In 1937, B. Castaldi (Clark University, Worcester, MA, USA) investigates the problem of photon-photon scattering for visible light [103] (thesis advisor: P. M. Roope). In the experiment of Castaldi, a sodium Lab-arc is used as source of light (with a wavelength of approximately 589 nm). Two beams of light met at an angle of 60° and the photographic film used as detector of scattered light was placed in the plane of the scattering beams at the (forward) direction of the bisector of the angle between them (as in the experiment by Hughes and Jauncey). In the (final) trial (after the design phase), the photographic plate was exposed for 28 days (672 hours) and then developed. No trace of scattered light was found. Castaldi estimates that his method was 661.6 times as sensitive as that of Hughes and Jauncey relying on the human eye as de-

tector. In 1940, to study the problem of photon-photon collisions R. V. Wiegand (Montana State College, Bozeman, MT, USA) uses X-rays for which, according to Euler and Kockel [45], the cross section ($\sim \omega^6$, where ω is the frequency of the scattering light quanta) is larger than for visible light [135] (thesis advisor: A. J. M. Johnson). The experiment has been performed with X-rays of 1.473 \AA ($= 0.1473 \text{ nm}$) and for an collision angle of $28^\circ 8, 6'$. “X-ray film was placed on the six sides of the collision point and long exposures taken to determine if radiation was given off in any direction.” The maximal exposure time was 25 hours but “no “collision” radiation of sufficient amount to register on the photographic film was found” (both quotes are from the abstract of [135]).

2.2 Propagation of light in strong electromagnetic fields

2.2.1 Influence of the intensity on the propagation of light

In the late 19th/early 20th century the question has been studied if the speed of light might depend on the intensity of the propagating light wave. Experiments by J. J. Müller (University of Leipzig, Germany) performed with visible light and published in 1871 [1] indicated that the speed of light (in air) increases with increasing frequency (cf. the tables on pp. 107, 120 of [1]). The intensity variations studied by Müller were not larger than 1:10. Müller used interference experiments to investigate the dependence of the wave length λ of an incident (monochromatic, with frequency f) light beam on its intensity and inferred from it a corresponding variation in the speed of light $c = f\lambda$. Such a phenomenon of the intensity dependence of the speed of light (in vacuum) would clearly represent a violation of the superposition principle and would – on the theoretical side – correspond to a Lagrangian for the (gauge) field describing the light wave which is not quadratic in this field.

Contemporary physicists apparently had been aware of the potential significance of the results of Müller. Not long after the publication of his work [1] they have checked his assertion in other experiments with higher precision and found no indication for any intensity dependence of the speed of light hereby ruling out any noticeable self-interaction of a light beam with itself (F. Lippich, University of Prague, Austria: [2]; H. Ebert (University of Erlangen, Germany): [3]; Th. E. Doubt, University of Chicago, USA: [4]). Ebert [3] besides giving the results of his experiments also uses astronomical arguments (pp. 381-383) to infer that the speed of light cannot depend on the intensity of the light wave in any significant way (cf. our subsection 2.3). Doubt achieving in his experiments light intensity variations between 1:43000 and 1:290000 concluded that any corresponding variation in the speed of light, if any at all, must be smaller than roughly one part in one billion.

2.2.2 Propagation of light in macroscopic, constant magnetic and electrical fields^[e]

We start this subsection with a reminiscence by P. L. Kapitsa of a conversation he has had with Albert Einstein on the occasion of a visit^[f] of the latter to the Cavendish Laboratory of the University of Cambridge. His report provides us with an interesting glimpse into the thinking of Albert Einstein. Kapitsa recalls^[g]: “In the 1930s, in Cavendish’s laboratory, I developed a method of obtaining magnetic fields one order stronger than had previously been attained. In a conversation Einstein tried to persuade me to study experimentally the influence of a magnetic field upon the velocity of light. Such experiments had been conducted, and no effect was discovered. In my magnetic fields it was possible to raise the limit of accuracy of measurement by two orders of magnitude, because the effect was dependent on the square of the intensity of the magnetic field. I protested to Einstein that according to the existing picture of electromagnetic phenomena, I could not see from whence such a measurable phenomenon would come. Having found it impossible to prove the need for such experiments, Einstein finally said, “I think that *der liebe Gott* could not have created the world in such a fashion that a magnetic field would be unable to influence the velocity of light.” Of course, it is hard to counter that kind of argument.”

A first experiment investigating the propagation of light in a (constant homogeneous) transverse magnetic field had been carried out in 1929 by W. H. Watson [13] at the Cavendish Laboratory in Cambridge, UK. This experiment predates the development of the theory of quantum electrodynamics. The main motivation for the experiment has been to look for the existence of a magnetic moment of the photon. The experiment has been carried out with a magnetic field strength (magnetic induction) of 10 000 gauss (= 1 tesla = 1 T) (for reference, the QED critical field strength is $B_{\text{cr}} = \frac{c^2 m_e^2}{e\hbar} \sim 10^9 \text{ T}$, m_e is the electron mass). The interaction region between the strong magnetic field and the light beam used for observation was located in a Fabry-Pérot cavity from which air had been pumped out (The quality of the obtained vacuum is not described in [13]). The interference pattern whose potential change has been observed was produced by polarized, monochromatic (visible) light of a wavelength of 585.2 nm. The null result that was obtained in the experiment yields a bound on the magnetic moment of the photon μ_γ of $\mu_\gamma < 1.4 \cdot 10^{-22} \text{ emu}$

^[e]For a different discussion of the first three experiments dealt with in this subsection see p. 11 of [176].

^[f]This meeting took place at an unknown date, somewhen during the period between 1922 and 1934 when Kapitsa worked at the University of Cambridge - probably, after 1929 when the experiment by W. H. Watson [13] had been performed (see further below). The question when this conversation took place precisely must rest with historians of science.

^[g][177], p. 31, [178], p. 40, reprints: 1. [178], p. 40. 2. [179], 3. ed. p. 375, 4. ed. p. 374, English transl. [180], p. 9, reprint [181], p. 317; we quote here the English translation, for the original Russian text see our App. A, p. 36. I owe this quote Yu. M. Poluektov [182], p. 6.

($= 1.4 \cdot 10^{-25} \text{ Am}^2$). Equivalently, the null result entails that the change in the refractive index of the vacuum under the influence of the magnetic field is less than $4 \cdot 10^{-7}/\text{T}$.

Three years later, in 1932, C. C. Farr and C. J. Banwell of the Canterbury University College, Christchurch, New Zealand, published the description of an experiment designed to study the “Velocity of propagation of light in vacuo in a transverse magnetic field” [19]. This experiment relied on a different experimental set-up than that performed by Watson. A Jamin refractometer was used where one unpolarized, monochromatic light beam (of a wavelength of 546.1 nm) passed through the strong magnetic field while the other (parallel, of the same wavelength) beam only experienced a weaker leakage field. Using this set-up Farr and Banwell achieved a roughly one order of magnitude higher sensitivity than Watson. The strength of the magnetic field amounted to 17 992 gauss ($= 1.7992 \text{ tesla}$). The quality of the obtained vacuum in the interaction region is given as 0.005 mm of mercury ($= 0.005 \text{ torr} = 0.67 \text{ Pa}$) and the authors conclude from the null result they obtained that the change in the refractive index of the vacuum under the influence of the magnetic field is less than $2.7 \cdot 10^{-8}/\text{T}$.

Further eight years later, in 1940, Banwell and Farr published an account of an improved experiment of the same type performed using a yet different set-up [132], namely, a Michelson interferometer. Here one of the two perpendicular optical arms was placed in the strong magnetic field while the other did not experience the influence of any significant magnetic field at all. In the Michelson interferometer unpolarized, monochromatic light of a wavelength of $\lambda = 546.1 \text{ nm}$ was propagating. The intensity of the applied magnetic field is given as 19 917 oersted (corresponding to a magnetic induction of 1.9917 T in vacuo). The quality of the vacuum in the Michelson interferometer is stated as 0.05 mm of mercury ($= 0.05 \text{ torr} = 6.7 \text{ Pa}$). With the probable error taken into account, Banwell and Farr obtain as result of their Michelson interferometer experiment a non-null result. In the applied magnetic field of roughly 2 T they find an increase in the speed of light by $(0.3431 \pm 0.1856) \text{ m/s}$. The estimated error corresponds to a change in the refractive index of $3.1 \cdot 10^{-10}/\text{T}$. Banwell and Farr conclude their article [132], p. 25, with the following sentences: “The authors consider that they have taken all precautions to eliminate causes tending to give a spurious effect greater than the probable error of observation quoted above. They would however be very hesitant in accepting the above final result as real. All that can be said definitely is that in such a field (20,000 oersted) the effect is less than 1 part in about 5×10^8 .” (For a discussion of this non-null result obtained by Banwell and Farr see our Appendix C.).

The influence of strong (inhomogeneous) electric fields on the propagation of light has been studied by J. Stark (Traunstein, later Eppenstatt, Germany) in the years after World War II (For the role of Stark in fascist Germany see, e.g., [183], chap. 6, pp. 103-122, [184]). The motivation for such a study Stark derived from

a light vortex model for photons (which, of course, implicitly entails the existence of nonlinearities in the corresponding electromagnetic theory); cf. [185], sec. 13, pp. 40-44. In his private laboratory he has conducted – among others – the following experiment ([142]; also see [185], sec. 14, pp. 44-50 (reprinted from [144]), [186]): A razor blade is positioned in a parallel trough cut out of a metal plate (perpendicular to the razor blade; cf. fig. 2 on p. 507 of [186]) at close distance. A narrow beam of polarized light passes along the razor blade in the space between blade and trough and consequently falls on a screen (photographic plate). The razor blade/metal plate system is placed in a vacuum tube. The image produced by the light beam (mostly yellow light) is compared for the situation without any electric field applied and the situation where a strong electric field is applied between razor blade and metal plate. Stark estimates the strength of the electric field close to the razor blade to be 1 to 1.5×10^6 V/cm at maximum. He reports on the existence of a slight difference in the images for the cases with and without electric field applied once the electric field of the polarized light is perpendicular to the razor blade (cf. fig. 1 on p. 47 and fig. 2 on p. 48 of [185] and fig. 3 on p. 508 of [186]). For the case of the electric field of the polarized light parallel to the razor blade Stark reports a null result. The experiment of Stark has been repeated independently by J. Sperling (University of Kiel, Germany) [150] who finds no effect whatsoever. For a discussion of other, related experiments of Stark see [145, 147, 148]. From out the modern point of view of quantum electrodynamics a null result does not come unexpected. Comparing the maximum value of the electric field given by Stark as 1 to 1.5×10^6 V/cm with the QED critical field strength $E_{\text{cr}} = \frac{c^3 m_e^2}{eh} \sim 10^{16}$ V/cm (m_e is the electron mass) one immediately recognizes that the strong electric field used by Stark is too weak to give rise to any nonlinear quantum electrodynamic effects.

We conclude this subsection with the description of a somewhat speculative experimental proposal by H. Bauer (University of Kiel, Germany) made in 1931 [17]. The proposal starts with the theoretical observation of the existence of a source-free (Poynting) energy current $\mathbf{S} \sim \mathbf{E} \times \mathbf{B}$ in the simultaneous presence of (say, constant) magnetic (\mathbf{B}) and electric (\mathbf{E}) fields. Concerning the existence of such an energy current Bauer refers (see his footnote 3 on p. 38), among others, to Planck [187], second part, chap. 2, § 13, p. 26 (p. 34 of the English translation). It should be mentioned that the literature on this subject in later decades is controversial, however, Feynman, for example, expresses the same view as Planck [188], chap. 27, sec. 27-5, p. 27-8. Bauer notes that the existence of such an energy current (for constant cross-fields) had not (yet) been demonstrated experimentally. He then points out that a momentum current \mathbf{S}/c^2 is associated with the energy current \mathbf{S} . He then goes on to reason that if a non-vanishing cross section for photon-photon scattering would exist in nature (the constant magnetic and electric fields correspond to a coherent state of photons) one could light of frequency ν let propagate in (opposite) line with the momentum current \mathbf{S}/c^2 . The phenomenon of photon-photon scattering would then result in a (tiny) frequency change of the light photons which could be measured. He estimates that a frequency change related to a cross section of $2 \cdot 10^{-27}$ cm² should be

observable. Bauer points out that a non-negative experimental result would entail a violation of the superposition principle (characteristic for the Maxwell theory), i.e., it would be related (effectively) to a nonlinear electromagnetic theory. Bauer further points out that a non-negative experimental result would show the physical reality of the Poynting energy current (in the constant cross-field situation) and at the same time prove the existence of the phenomenon of photon-photon scattering.

2.2.3 Propagation/scattering of photons in the Coulomb field of nuclei

While the experiments discussed further above have intentionally been devised to study the properties of light, the experiments referred to in this subsection have been performed historically with another objective, namely to advance the understanding of cosmic rays, to gain further insight into the nuclear structure of atoms, and to explore experimentally the predictions of (relativistic) quantum mechanics. In the early decades of the 20th century numerous experiments have been made to study the absorption and scattering of γ -rays emitted by radioactive materials by various targets. We will not review these experiments here, the interested reader is referred to [189, 190] (For a related historic account see [191]). Here, we are concerned with such γ -ray experiments performed by L. Meitner and collaborators (Hupfeld, Kösters; Kaiser Wilhelm Institute of Chemistry, Berlin, Germany) in the early 1930s [23, 192–197] (For a historic discussion of these experiments see [198]). In these γ -ray absorption and scattering experiments performed to explore the validity of the Klein-Nishina formula (one of the consequences of relativistic quantum mechanics) Meitner and collaborators found that for targets with large nuclear number (nuclear charge) the Klein-Nishina formula is not sufficient to correctly describe the experimental results. Apparently, besides the scattering of γ -rays off electrons (described by the Klein-Nishina formula) another physical process related to the atomic nuclei of the target material was also to be considered. Referring to the just discovered creation of positive electrons (positrons) by γ -rays in various materials, M. Delbrück (Kaiser Wilhelm Institute of Chemistry, Berlin, Germany) hypothesized in 1933 (in an addendum to an article by Lise Meitner – whose assistant he was by then – and H. Kösters)^[h], “. . . that it involves a *photoeffect* on one of the infinitely many electrons in states of negative energy, which according to Dirac’s theory fill the entire space with infinite density and would well be capable of such an absorptive process, by virtue of their interaction with the nucleus.” He then concludes: “Such a proposal has also the consequence that that these electrons of negative energy are capable of *scattering* γ -rays, and in fact *coherently*, analogous to the phenomenon of the *unshifted* Compton line.”

In appreciating the contribution by Delbrück one has to remind oneself that at

^[h][23]: cf. our App. A, p. 37, p. 37; English transl. [198], Appendix, p. 135. The English translation given here slightly deviates from the English translation printed there in order to make the translation more adequate.

the time of writing of this addendum the theory of quantum electrodynamics was still in its birth phase but Delbrück relied on the model picture of the Dirac sea to qualitatively describe an advanced quantum field theoretic process whose detailed nature Delbrück did not sketch. Today, to lowest order of QED perturbation theory contributions to Delbrück scattering – as it is now being called – can be depicted by Feynman diagrams as shown in Fig. 1.

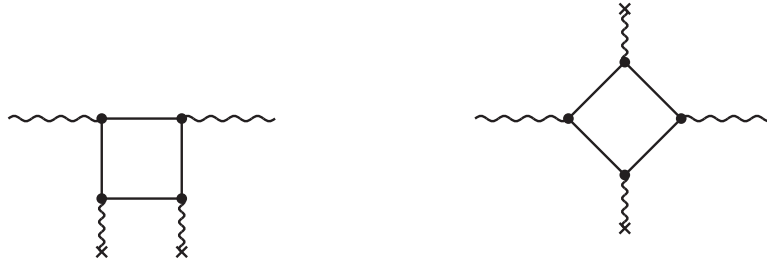


Figure 1: Typical (lowest order) Feynman diagrams contributing to Delbrück scattering (The wavy line ending in a cross depicts the Coulomb field of a nucleus.).

Interestingly, in 1978 Delbrück has described his proposal and its further fate in an oral history interview of the California Institute of Technology Archives ([199], pp. 52-54 = Delbrück-45 - Delbrück-47) from which we quote in the following (with kind permission by the Caltech Archives):

“... One of the graduate students of Lise Meitner had studied the scattering by lead of gamma rays of ThC^{11} ; ThC^{11} is a gamma ray source with relatively hard gamma rays, as I recall, 2.6 million electron volts. If you scatter these gamma rays on lead, then, according to then current theory, you should find very little coherent scattering. Most of the scattered light should be Compton-scattered -- that means scattering where the electron acts as if it were a free electron. And after scattering you find, at right angles then, a Compton-scattered light quantum which is very much less energetic than the incoming one. This student, H. Kösters had found a scattered component which was much harder than the expected one. I put out the conjecture that this had something to do with the new theory of the electron that Dirac had proposed, according to which the negative energy states of an electron (with energies below minus mc^2) were all filled, and the electron never jumped from plus energy to minus energy because these were filled (because of Pauli’s exclusion principle). I made the conjecture that these negative energy electrons in the vicinity of the nucleus are not free electrons, but that their wavefunction was distorted by the nucleus of the atom and therefore that they could scatter. If they are free electrons then they wouldn’t scat-

ter, but if they are disturbed by the field of the nucleus then there could be virtual transitions from minus to plus energy, and there would be corresponding scattering.

This problem is related to the problem of scattering of light by light. In the classical theory, two light beams just go right through each other and don't interact, but in quantum electrodynamics if you take into account these negative energy electrons, then the first light beam polarizes the vacuum, and the second light beam then is scattered on the first one. So I made a conjecture that these hard scattered rays should be due to this scattering of underground electrons. The fate of this conjecture was that it turned out that the scattered light, the scattered quanta observed by Kösters, were not due to that effect. Instead, they were due to the effect that the negative energy electrons actually absorbed a quantum, and thereby created a hole there, a positive electron. This positive electron then could recombine with some other electron and make annihilation radiation, and that is very much harder than the Compton-radiation. Actually that was an obvious implication that I had overlooked. And that came out very quickly. Nevertheless the effect that I predicted ought to be there also, and the question was how to calculate it, and I slaved on that and it turned out to be a nightmare to calculate that.^{[1][footn. K.S.]} With the help of some advice by Hans Bethe I got so far as to predict that this effect should be proportional to the fourth power of the nuclear charge, Z^4 , and that's about all that I predicted; it was published in a short appendix, I think, to the paper by Kösters.^{12[K.S.: orig. footn.]}

That's where my contribution ended to this problem, and I never heard of it again until about 20 years later, in the fifties, when I was long since in biology. Somebody told me that there had been published two papers in Physical Review on "Delbrück scattering," by Bethe and some graduate students of his who had made some progress in calculating them.^{13[K.S.: orig. footn.]} So since then this name, "Delbrück scattering" exists, and if you ask theoretical physicists then I am known scurrilously for that little incident. I understand that the actual calculation of

^[1]K.S.: In this context, W. Heisenberg reported in a letter of June 16, 1935 to W. Pauli: "Delbrück recently was here and reported about his fruitless attempts to calculate the coherent scattering. He certainly would be glad if he could do this work together with somebody else, e.g., Weisskopf (Sauter is now in Göttingen)." ([200], letter [374] of June 16, 1934, pp. 331-333, specifically p. 332: cf. our App. A, p. 37; English transl.: K.S.).

¹²[K.S.: orig. footn.] M. Delbrück, "Zusatz bei der Korrektur," in L. Meitner and H. Kösters, "Über die Streuung Kurzweiliger γ -Strahlen," Zeitschrift für Physik 84:137-144 (1933), 144. (K.S.: cf. [23])

¹³[K.S.: orig. footn.] F. Rohrlich and R. L. Gluckstern, "Forward Scattering of Light by a Coulomb Field," Physical Review 86:1-9 (1952); H. A. Bethe and F. Rohrlich, "Small Angle Scattering of Light by a Coulomb Field," Physical Review 86:10-16 (1952.) (K.S.: cf. [201, 202])

this effect, and experimental verification of it, still has been lingering on for the next 20 years after that, because it turned out to be just very, very difficult to calculate; also, in order to observe it you need to go to much higher energies -- I think the optimal energy is about 10 million electron volts rather than 2.7 -- and I think now it has been confirmed to exist. ...”.

It remains to add that L. Meitner in 1934 [203] refers to a qualitative calculation by Delbrück himself of the particular scattering process described by him in [23] ^[j]. She quotes Delbrück as having calculated the angular distribution of the Delbrück scattering process and mentions that this calculation will be published in the near future. However, apparently this calculation never got published and also a preserved manuscript of it is not known. Related results have been published in 1937 by N. Kemmer and G. Ludwig (University of Zurich and ETH Zurich, Switzerland) [112]. Kemmer and Ludwig ([112], p. 184) give for the cross section q for the scattering of light of (long) wave length λ ($\gg \hbar/mc$, $m = m_e$) off a nucleus with charge $Z|e|$ the formula (c_1, c_2 are certain unknown constants and θ is the scattering angle)

$$q = \left(\frac{e^2}{mc^2}\right)^6 \cdot \left(\frac{Z}{\lambda}\right)^4 \cdot [(c_1^2 + c_2^2)(1 + \cos^2 \theta) + 2c_1c_2 \cos \theta] . \quad (1)$$

Related results have been published in 1937 by Akhiezer and Pomeranchuk [100] (cf. subsec. 3.2). As M. Delbrück in his Caltech Archives oral history interview recalls (see above quote) the further study of the Delbrück scattering process continued in the 1950's only.

2.3 Considerations related to extraterrestrial and astronomical observations

Since the earliest times of mankind, light has naturally played a prominent role in developing an understanding of the physical world we live in. It comes as no surprise that also the problem of photon-photon interactions (or, nonlinear electrodynamics, or possible violations of the superposition principle) has been approached by means of results of extraterrestrial and astronomical observations. In the period under consideration, three comments that belong to this line of research are known to us. One comment has been made by H. Ebert in 1887 [3] concerning the possible dependence of the speed of light on the intensity of the propagating beam, a second consideration on the subject of using the solar corona for obtaining limits on the violation of the (optical) superposition principle has been published in 1928 by S. I. Vavilov, and a third comment concerning a possible frequency dependence of the speed of light is due to F. Zwicky (California Institute of Technology, Pasadena, USA) in 1937 [120]. In the following, we will let Ebert and Zwicky speak for themselves by

^[j]The memory of Max Delbrück in his above quoted interview apparently slightly mixes the course of events. His note [23] does not contain any calculation.

simply quoting from their articles while the somewhat longer discussion by Vavilov is being summarized only.

Let us first quote the relevant part of the paper by Ebert^[k] in which he makes contact between his laboratory experiments (cf. subsection 2.2.1) and astronomical observations (an early example of astroparticle physics ...):

“At the end, I am allowing myself to apply the above result to some astrophysical questions.

Is the velocity of the propagation of light dependent on its intensity to some noticeable degree, beyond the circumstance that the phenomena in the sky occurring to us at the same moment belong to very different times in reality, the further complication would step in that these times even for equal spatial distance would be very different for the different objects. In view of the large differences in intensity that are being met here and in view of the large distances which the light rays have to pass before they can reach us, even small differences in the velocity of the propagation of light can make themselves noticeable to a large degree. For example, this should occur for physical double stars; here, we have got two sources of light that, in some rough approximation, are equally far away from us but whose intensity of light, in general however, considerably differs from each other. Despite this we find that - if we succeeded in calculating the trajectories of double stars - the two components indeed show up at the same time at corresponding points of their trajectories and together with their joint center of gravity always lie on a straight line. Right from this fact, we can also draw new support for the experimental result obtained by us.^{[l][footn. K.S.]} One example might incidentally show yet that the bound proved by me for the independence of the two quantities in question^{[m][footn. K.S.]} is completely sufficient for the astronomical practice.

According to calculations by Auwers and Peters, the companion of Sirius, calculated by Bessel and discovered by Clark, has an extraordinary large mass ($1/2$ to $2/3$ of the mass of Sirius itself). At the same time, it occurs to us as a star of about class 9 only, also compared with its main star, its brightness is a very small one, according to Schönfeld^{1)[K.S.: orig. footn.]} appr. $1/1000$ of that of Sirius only. Further above, for rays of medium wave length (for sodium light) it was found that for an attenuation of light from intensity 1 to intensity $1/33$ the velocity of the propagation of light does not change even by $1/500\,000$. Relying on this limit, i.e. assuming that for differences in brightness of 33:1 the velocity of propagation could differ just yet by this amount and that the latter would grow proportionally with the difference in intensity, in the present example, in each second the light of the

^[k][3], pp. 381-383: cf. our App. A, p. 37; English transl.: K.S.. To some extent the original German text exhibits a historic style used at the end of the 19th century, the present English translation does not deliberately attempt to imitate this historic style.

^[l]K.S.: Cf. our subsec. 2.2.1.

^[m]K.S.: i.e., the velocity of light and the intensity of light.

¹⁾^[K.S.: orig. footn.] Schönfeld, Die dunklen Fixsternbegleiter. Mannheimer Verein für Naturkunde. 30. Jahresbericht. (K.S.: cf. [204])

companion would stay behind the light of the main star by $30 \times 0,6$ or 18 km. In light-time, the distance of the Sirius system amounts to about 30 years. Assuming it were precisely 30 years, it would take the light of the companion:

$$\frac{300\,000}{299\,982} \times 30 = 30,0018$$

years to reach us, i.e., of two simultaneously emanated light rays the one emitted by the companion would always arrive about 0.7 days later only than the one emitted by Sirius itself. As, however, the period of revolution of both bodies around their joint center of gravity is 50 years, this difference in time and the difference in location corresponding to it can be neglected. The assumptions made, however, are very unfavourable ones; for all orbit determinations, we can disregard any dependence, existent yet perhaps, of the velocity of propagation on any intensity whose value lies below the limits established by the experiment.

Further, the obtained result is of relevance for applications of the Doppler principle where the relative velocities of the stars in the direction of the line of sight are determined from changes in the average refractivity of isolated spectral lines. Here, in the different cases very large differences in brightness exist and, consequently, for the applicability of this principle it is important to have shown directly that these differences in brightness do not also have an influence.”

As described in subsection 2.1, Vavilov [8, 16] had performed in 1928 a laboratory experiment to attempt the direct (optical) observation of photon-photon scattering in vacuum. This experiment ended with a negative result: no photon-photon scattering has been observed. It should be noted that in 1928 the experiment had been based on a qualitative understanding of the quantum nature of light only, the later qualitative predictions of QED for the scattering of photons by photons were not yet made. To Vavilov it was clear that the superposition principle for electromagnetic radiation is fulfilled to a high degree, consequently, the experimental search for any signatures of photon-photon scattering is confronted with a tiny effect (if any at all). To Vavilov, a high radiation density was therefore the key for a possible observation of photon-photon scattering (Only the later QED calculations, see subsection 3.2, further clarified the situation: the size of the photon-photon scattering cross section is critically dependent on the photon energy.). In his article of 1928 Vavilov, therefore, also turned his attention to the sun with its large radiation density. From data concerning the solar corona he concluded that in any photon-photon scattering at most $1.8 \cdot 10^{-17}$ of the beam energy is being scattered ([8], § 2, p. 558, p. 237 of the reprint, pp. T-4 (= P-6) of the English translation; also see [16]). From this estimate he concludes that even if photon-photon scattering existed it could not be observed in any terrestrial experiment because the human eye would not be sensible enough to detect the low intensity of scattered photons (At that time, the human eye was the most sensitive photon detector in such a type of optical experiments, cf. e.g. [15], p. 777, last sentence.). Finally, noting that there was no satisfactory theory of the solar corona (in 1928) he then tried to explain (in a somewhat more

hypothetical way) certain known features of the solar corona by means of photon-photon collisions (For a current treatment of the solar corona see, e.g., [205]).

Zwicky in 1937 [120] is concerned with a frequency dependence of the speed of light possibly arising within theories of nonlinear electrodynamics. He writes: “Certain effects have recently come to our knowledge which suggest that the differential equations governing the propagation of light through empty space are not strictly linear. *Nonlinearity* of these equations results in a *dependence of the velocity of light on frequency*. One reason for the existence of slight deviations from the superposition principle of light lies in the potential possibility of the formation of pairs of positive and negative electrons by interacting photons. This interaction necessitates a generalization of Maxwell’s field equations through the introduction of non-linear terms in the field strengths.⁵[*K.S.: orig. footn.*] As a result, light traveling through space which is free of matter but filled with radiation will have a velocity depending on its frequency. In addition similar effects arise from the gravitational interaction of light with light and matter. Although the effects to be expected are in all probability small the possibility of an experimental test may be kept in mind. An obvious way of investigating effects of the kind mentioned lies in the observation of light signals of different frequency which have traveled through space for a long time. If we knew of any signals which have started *simultaneously* from a very distant source the dependence of the velocity on frequency could perhaps be demonstrated by checking up on the times of arrival of these signals. A unique case to perform this test is afforded by the observation of distant nova outbursts. For such an outburst we may safely assume that photons in the various emission lines of, say, hydrogen have started out simultaneously on their long journey. We must therefore attempt to determine whether or not the intervals in which the different hydrogen lines flare up in the spectrum of a distant super-nova are zero or not. With the present telescopic equipment it is probably possible to find super-novae to distances as great as 10^8 light-years. With reasonable luck it should be possible to detect differences in the time of arrival of various emission lines amounting to as little as a few days. Observations of this kind will therefore enable us in the most favorable cases to determine the ratio of the velocity of violet light to that of red light traveling through internebular space with a relative accuracy of 10^{-10} .”. He then goes on to describe certain astronomical observations concerning novae in the Andromeda galaxy M31 which - at the time of writing - might be considered being related to a possible frequency dependence of the speed of light.

3 Theory

In classical electrodynamics the superposition principle holds true, consequently, direct interaction processes (without mediation by charged particles) of electromag-

⁵[*K.S.: orig. footn.*] Euler and Kockel, *Naturwiss.*, **23**, 246 (1935). (K.S.: cf. [45])

netic fields such as photon-photon scattering do not occur within this framework. Its theoretical description, Maxwell electrodynamics, is based on the following Lagrange density for the electromagnetic field

$$\mathcal{L}_0 = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2} \epsilon_0 (\mathbf{E}^2 - c^2 \mathbf{B}^2) = \frac{1}{2} \left(\epsilon_0 \mathbf{E}^2 - \frac{1}{\mu_0} \mathbf{B}^2 \right) \quad (2)$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor and the velocity of light (squared) is given by $c^2 = 1/(\epsilon_0\mu_0)$ with ϵ_0 , μ_0 being the dielectric permeability and the magnetic permittivity of the vacuum, respectively. A characteristic feature of the Maxwell theory is that the Lagrange density of the electromagnetic field is quadratic in the electric (\mathbf{E}) and magnetic (\mathbf{B}) fields. Electromagnetic theories which are to model direct interaction processes of electromagnetic fields have to be described by Lagrange densities which contain higher powers of the electromagnetic fields. The first theoretical study which considered Lagrange densities of the electromagnetic field of a more general form has been published by G. Mie (University of Greifswald, Germany) in 1912 [5]. However, Mie considered not only a dependence of the Lagrange density on the electric and magnetic fields themselves but also explicitly and separately on the electromagnetic potentials. In his famous review of the theory of general relativity [206] in 1920, W. Pauli (University of Munich, Germany) pointed out (§ 64, pp. 754-769, pp. 188-192 of the English translation) that the non-gauge invariant ansatz of Mie is physically not viable. The work of Mie does not contain any explicit reference to such processes as photon-photon scattering we are interested in in the present review but it has to be considered as the earliest theoretical approach in principle entailing such processes.

3.1 Born-Infeld electrodynamics

The following step in the further development of nonlinear Lagrangians for electromagnetism is closely related to a political-historic situation – the rise to power of the Nazi party in Germany in 1933. M. Born (University of Göttingen, Germany) describes in his autobiography [207] the situation he found himself in. After he had been dismissed from his chair at the University of Göttingen as a result of the fascist Law for the Restoration of the Career Civil Service [Gesetz über die Wiederherstellung des Berufsbeamtentums] of April 7, 1933 he immediately left Germany for Italy in May 1933. He writes about this time in Italy ([207], part 2, chapter IV, pp. 254-255): “I soon began to miss my accustomed work. But I had neither books nor periodicals. So I looked for a subject for which no literature was needed, something in quite a new line. I started from my old favourite problem, the electromagnetic mass of the electron, which I had treated in my thesis^[n]^[footn. K.S.] for admission as a lecturer. According to Coulomb’s law, the energy of a charged particle becomes infinite when the radius shrinks to nothing. One has therefore to assume that the

^[n]K.S.: i.e., habilitation thesis [208].

electron has a finite size. But if it is a rigid sphere one gets into trouble with the theory of relativity because there, in general, rigidity does not exist. In my thesis (see Part 1, Chapter XI)^{[o][footn. K.S.]}, I had discovered that there are special movements for which rigidity can be defined, and I had obtained for these a definite expression for the self-energy (electromagnetic mass). But for arbitrary movements this was not possible. The only way out of this difficulty seemed to be the assumption that the ordinary Maxwellian theory of the electromagnetic field holds only approximately for dimensions large compared with the radius of the electron, while for smaller distances it should be replaced by another, more subtle theory. A quite general frame for modifying Maxwell's equations had been given, many years before, by Gustav Mie^{[p][footn. K.S.]}. I remembered these and tried to fill out this frame by special, plausible assumptions. But I was not successful though I spent many hours on my verandah pondering about it. At last I succeeded by abandoning Mie's formalism and inventing another one. The usual linear theory of the electromagnetic field was replaced by a non-linear one, in which point charges existed which carried a finite electrostatic energy.

I had no means (tables, etc.) to work it out numerically, and I remember the tension with which I waited for the letter from a colleague whom I had asked to evaluate the decisive elliptic integral. When it came I was very happy and proud^{[q][footn. K.S.]}; for I was filled with the intense desire to discover something fundamental to regain my self-confidence after the loss of my job in Göttingen, and to 'show the Germans what they had lost'. Alas, these ambitions were not fully satisfied. My non-linear field theory made some stir, but did not lead to the solution of the problem of the structure of elementary particles, although I spent a great deal of time and effort on it during the following years, together with a number of collaborators, in the first place the Pole Infeld. I shall return to this later. The so-called Born-Infeld theory was one of the first attempts at a non-linear field theory and worked perfectly in the 'classical' (non-quantum) domain, but failed in the quantum domain. Today one has a wide empirical knowledge about elementary particles, and it is clear that the electron cannot be treated separately. The most promising theory, that of Heisenberg, also uses non-linear field equations and follows thus the direction indicated by my work, but rests on quantum theory right from the beginning. In any case, this work helped me to overcome the bitter feelings produced by the loss of my position and my expulsion."

In his Letter to *Nature* [20] (followed by a full length paper [26] submitted for publication at about the same time, August 1933) M. Born proposed as a gauge-invariant nonlinear Lagrangian of electromagnetism the expression (b is some di-

^[o]K.S.: Part 1, Chapter XI of [207].

^[p]K.S.: [5].

^[q]K.S.: The first Letter of Max Born on this subject appeared in the journal *Nature* in August 1933 [20].

mensionful constant; [26], p. 426, eq. (6.1))

$$\mathcal{L} = -\frac{b^2}{\mu_0} \sqrt{1 + \frac{\mathcal{F}}{b^2}}, \quad (3)$$

$$\mathcal{F} = \frac{1}{2} F^{\mu\nu} F_{\mu\nu},$$

which leads to a finite value of the self-energy of a point charge (in contrast to Maxwell electrodynamics) as shown by M. Born and L. Infeld (both at the University of Cambridge, UK) shortly thereafter [21]. Ya. I. Frenkel' (Physico-Technical Institute, Leningrad, USSR) immediately devoted two sections in volume 1 of his new teaching book on classical electrodynamics to this nonlinear electrodynamic theory by Born ([33], chap. X, § 11-12, pp. 407-422).

Once (arbitrary) nonlinear Lagrangians of the electromagnetic field are allowed the natural question arises by which principles the functional shape of the Lagrangian should be chosen. Born and Infeld formulated two general principles which should rule the choice of the Lagrangian: the *principle of the finite field* [22] (the corresponding full length paper is [28])/ *principle of finiteness* (“... a satisfactory theory should avoid letting physical quantities become infinite.” [28], § p. 427) and the postulate (principle) of the *invariance of action* (under space-time transformations, [28], § 2, p. 429)). Following earlier considerations by Eddington ([210], § 88, p. 206, eq. (88.4), § 97, p. 223, § 101, p. 232), Born and Infeld write down the expression (an additive constant is being omitted)

$$\mathcal{L} = -\frac{b^2}{\mu_0} \sqrt{1 + \frac{\mathcal{F}}{b^2} - \frac{\mathcal{G}^2}{b^4}}, \quad (4)$$

$$\mathcal{F} = \frac{1}{2} F^{\mu\nu} F_{\mu\nu}, \quad \mathcal{G} = \frac{1}{4} F^{\mu\nu} F^*_{\mu\nu} = \frac{1}{c} \mathbf{E} \mathbf{B},$$

as “... simplest Lagrangian satisfying the principle of general invariance” ([28], p. 432, below of eq. (2.26)). Contrasting eq. (4) with eq. (3) Born and Infeld state: “Which of these action principles is the right one can only be decided by their consequences.” ([28], p. 432, below of eq. (2.28)). A discussion of a large class of invariant Lagrangians in generalization of the eqs. (3), (4) has been given in 1935 by W. H. Erskine (Johns Hopkins University, Baltimore, USA) in his Ph.D. thesis [43, 44]. In the following years, the choice of the functional form of the Lagrangian has further been discussed in [64, 76–79, 84, 107–109, 115]. Infeld in his autobiography reports on his own view of the problem in late 1935 by saying: “... I recently discovered that Maxwell’s theory can be generalized in many different ways. This is not astonishing. But I no longer believe that the generalization presented by Born is the simplest. I don’t like the arbitrariness of the whole problem.” ([136], book two, sec. 13, p. 223 (p. 177 of the reprint # 1)). For the reader interested in science history we would like to mention that both the autobiographies of Born [207] and Infeld [136] contain

much interesting material which provides the reader with a unique insight into the development of the Born-Infeld theory.

The introduction of nonlinear Lagrangians of the electromagnetic field has resulted in a number of directions of thought whose significance is not limited to the particular Lagrangians considered by Born and Infeld. Some of these are:

- The nonlinear electromagnetic Lagrangians (3), (4) contain a dimensionful constant b which sets the scale for the nonlinearity and, consequently, for the deviation of phenomena from the predictions of (linear) Maxwell electrodynamics. Considerations concerning the value of b have been given by Born and Infeld [28], § 8, p. 446, eq. (8.8), and Born and E. Schrödinger (University of Oxford, UK)[41].
- One of the physical consequences of choosing a nonlinear Lagrangian of the electromagnetic field consists in modifications of the Coulomb law (potential) in the vicinity of a charged particle. Resulting effects in spectroscopic problems have been discussed for the first time by G. Heller and L. Motz (Columbia University, New York, USA) [35] right in 1934 concluding, however, that these are “barely observable” ([35], p. 505) in the problem under consideration (fine structure of the Balmer series for hydrogen). Later, J. Meixner (University of Gießen, Germany) [52], Z. Chrapływyj (Jan Kazimierz University, Lvov, Poland) [71, 105], and B. S. R. Madhava Rao (Indian Institute of Science, Bangalore, India) [113] have further extended these investigations drawing the same final conclusion, however. A. Bramley (Bartol Research Foundation, Swarthmore, USA) [67] explored possible consequences of Born-Infeld electrodynamics on the scattering of charged particles off each other.
- Nonlinear electromagnetic theories exhibit a rich field-theoretic structure differing from that of standard Maxwell electrodynamics. Therefore, formal field-theoretic studies to explore the mathematics related to nonlinear electromagnetic Lagrangians are necessary. Besides Born and Infeld themselves [26, 28] a group of other authors has studied formal aspects of nonlinear electrodynamics. A number of investigations are dealing with the description of the dynamics and the status of charged particles in nonlinear electrodynamics [29, 32, 42, 46, 69, 70, 90, 91, 96, 104, 126, 128, 131, 139]. Others studied the introduction of complex field variables into the formalism [59, 63, 81, 86, 118, 119], investigated solutions of the field equations [55, 56, 87, 114, 126, 130, 141], and explored further questions [60, 66, 73, 83, 85, 88, 97, 114, 121, 123, 127].
- The quantization of the nonlinear electromagnetic theory has received some attention (in the time period under consideration quantum electrodynamics was still in its infancy). Here, besides the studies by Born and Infeld [27, 30, 31, 40, 110] few further investigations exist [51, 57, 89, 92, 116, 140].

- Born and Infeld have considered the electromagnetic theory in Minkowski space. However, in the time period under consideration in the present review the problem of the unification of gravitation and electromagnetism has been explored widely. In the context of the Born-Infeld theory, this problem has been considered by B. Hoffmann (University of Rochester, USA) [36], [49] (this is the full length paper related to the Letter [36]), [47, 48] and other authors [38, 124, 127, 143, 146].

In the following we would like to discuss in somewhat greater length three such further subject that are more closely related to the phenomenon of photon-photon scattering.

1. The scattering of light by light

The first comment on the scattering of light by light within the framework of Born-Infeld theory can be found in the classical article by Euler and Kockel [45] on photon-photon scattering in QED where they point out that the lowest order terms in the QED (effective) Lagrangian of the electromagnetic field responsible for this process are fairly similar to those in the Born-Infeld Lagrangian (The most obvious difference lies in the numerical value of some coefficients involved. For the view of W. Pauli in this respect cf. our App. F.).

In 1936, a first thorough analysis within (classical) nonlinear electrodynamics of the interaction of two light waves is given in Chap. II, § 5 (pp. 59-66, pp. T-59 - T-66 (= P-63 - P-70) of the English translation) of the candidates (Ph.D.) thesis of A. A. Smirnov (Ural Physico-Technical Institute, Sverdlovsk, USSR) [94]. Smirnov relies in his analysis on the action (3) initially chosen by Born [26] which only contains the invariant \mathcal{F} of the electromagnetic field. The analysis is performed to leading nontrivial order in the constant b (the critical field strength). In principle, this kind of analysis is also applicable to the Euler-Kockel-Heisenberg Lagrangian (which however depends on both of the field invariants \mathcal{F} and \mathcal{G}). For counterpropagating electromagnetic waves (for the choice of polarization see the original thesis [94]) he finds as main results a change in the phase velocities and the emergence of small wave components with significantly different frequencies compared with the initial waves (e.g., for initial waves of the same frequency ω wave components with triple frequency 3ω emerge, i.e., higher-harmonics generation is found). The above analysis of Smirnov is published in 1940/41 [133, 137] only in a slightly generalized form (cf. the footnotes on p. 449 of [133] and on p. 48 of [137]).

In 1942, as part of a larger study, Schrödinger (Dublin Institute for Advanced Studies, Ireland) also published an approximate analysis of the interaction of two light waves [138], Part I, Sec. 4, pp. 85-88 (pp. 317-320 in [256]) within the framework of the Born-Infeld theory (Schrödinger relies on the form of the action (4)). He also finds as main result, as Smirnov did, a change in the phase velocities of the

two waves. Schrödinger also studied the interaction of three light waves [138], Part I, Sec. 5, pp. 88-92 (reprint [256], pp. 320-324). Somewhat later, in 1943, Schrödinger also found an exact two-wave solution of the equations of motion of Born-Infeld theory [141].

It should also be mentioned that in late 1936 an erroneous analysis of the interaction of two light waves in Born-Infeld theory has been published by C. D. Thomas (University of Chicago, USA) [95]. He assumed unchanged phase velocities in his Ansatz for solving the wave equation (e.g., cf. [95], p. 1048, eq. (16)). This incorrect assumption, however, leads to (physically not acceptable) wave solutions whose amplitude is linearly growing in time (cf. [95], p. 1048, eqs. (14), (14')) invalidating hereby the further analysis. The papers of Smirnov and Schrödinger both contain comments on the danger of such unphysical solutions (Smirnov: [94] Chap. II, § 2, pp. 35-37, pp. T-35 - T-37 (= P-39 - P-41) of the English translation, [133], p. 449, [137], pp. 47-48; Schrödinger: [138], Part I, Sec. 4, p. 87 (reprint [256], p. 319), around eq. (4.10)).

Finally, in 1943 J. McConnell (Dublin Institute for Advanced Studies, Ireland) published an analysis of photon-photon scattering in the framework of the Born-Infeld theory [140]. Again, the analysis is performed to leading nontrivial order in the constant b (the critical field strength) and can, therefore, be compared to the description of photon-photon scattering in the framework of the Euler-Kockel-Heisenberg Lagrangian [45, 72, 74] of QED. The quantization is performed on the level of an effective field theory technique (phrasing it in modern field theoretic terms) and the resulting cross section for photon-photon scattering is compared with the analogous QED result ([140], Sec. 4, pp. 170-171). McConnell finds that the angular dependence of the differential cross section for photon-photon scattering and the dependence on the photon frequency are the same to lowest nontrivial order in (quantized) Born-Infeld theory and QED.

2. The scattering of light by electromagnetic background fields

In the time period under review, the consideration of the scattering of light by electromagnetic background fields has been performed for two different situations: for constant homogeneous electric and magnetic fields and for the presence of a Coulomb field.

A first short note studying the case of a constant homogeneous electric background field is published by S. P. Shubin (Ural Physico-Technical Institute, Sverdlovsk, USSR) and Smirnov in 1936 [93]. Details of the calculation are contained in the candidates (Ph.D.) thesis of Smirnov ([94], Chap. II, § 2, pp. 33-48, pp. T-33 - T-48 (= P-37 - P-52) of the English translation) which also comprises an analogous investigation of the case of a magnetic background field (Chap. II, § 4, pp. 55-58, pp. T-55 - T-58 (= P-59 - P-62) of the English translation). Shubin and Smirnov primar-

ily find for a weak test wave that to leading order (in the critical field strength b) the electric background field causes a change in the phase velocity of the test wave, in dependence on the direction of its propagation and its polarization. They compare the situation found for the Lagrangian (3) to the birefringence found in an uniaxial crystal (also see [117], footnote * on p. 132 of the English version; this footnote does not exist in the Russian version). In his thesis ([94], Chap. II, § 3, pp. 48-55, pp. T-48 -T-55 (= P-52 - P-59) of the English translation.), Smirnov extends the analysis to the next to leading order in b . As additional effects of the electric background field he obtains higher harmonics generation in the forward and backward directions of the test wave and also a small component scattered backwards with the frequency of the test wave itself. A couple of years later, in 1942 Schrödinger [138], as one part of a broad analysis of the consequences of the Born-Infeld Lagrangian (4), also observes that an electric background field exerts a refractive effect on a weak test wave (p. 102; p. 334 of the reprint). In accordance with general insight obtained decades later [211, 212], in difference to Shubin and Smirnov who rely on the Lagrangian (3) he finds no birefringence.

Within nonlinear electrodynamics, the first study of the situation that corresponds to Delbrück scattering in QED, i.e., the scattering of light by a Coulomb field, has been published in 1937 by Shubin and Smirnov [117]. For a fairly general class of nonlinear electrodynamic Lagrangians, they analyze the scattering of light from a point charge and find the polarization averaged differential cross section (in a certain approximation). Then, they specify their result for the choice of the Born-Infeld Lagrangian (4). Furthermore, they also include a discussion of their results on a level where comparison with the corresponding results for the Euler-Kockel-Heisenberg Lagrangian is possible. In 1940, Smirnov publishes a follow-up paper [134] where instead of the Born-Infeld Lagrangian the Lagrangian proposed by Hoffmann and Infeld [108] is being used. The work by Shubin and Smirnov was followed in 1938 by a paper by S.I. Tomonaga and M. Kobayasi (both at RIKEN – Institute of Physical and Chemical Research, Tokyo, Japan) who along analogous lines analyzed the low-frequency scattering of light by a point charge in Born-Infeld theory. A couple of years later, in a comprehensive study Schrödinger [138] takes up the subject (Part II, pp. 100-116; pp. 332-348 of the reprint). In part he agrees with the results of Shubin/Smirnov and Tomonaga/Kobayasi for the cross section of the scattering of light from a charged point charge, in part he finds further terms (cf. eq. (10,10) on p. 109; p. 341 of the reprint).

3. The splitting of photons in electromagnetic background fields

Tomonaga and Kobayasi in their paper [129] also considered for the first time within Born-Infeld theory the process of photon splitting, i.e., the splitting of one incoming photon in the Coulomb field of a point charge into two outgoing photons. In the case of QED, the same process had been mentioned before by E. J. Williams (University of Copenhagen, Denmark) in 1935 [61], § 10, pp. 47-49 (cf. the subsec.

3.2). Tomonaga and Kobayasi calculate the differential cross section for the photon splitting process in some low-frequency approximation.

4. The Stefan-Boltzmann law

L. Rosenfeld and E. E. Witmer (University of Göttingen, Germany) [9] have pointed out in 1928 that interactions between photons will lead to changes in the characteristics of black body radiation. Their discussion amounts to a qualitative consideration without specifying details of the interactions among the photons being part of the cavity radiation. They only mention (p. 521) that possibly correction terms to the Maxwell equations could describe such interactions. Born-Infeld theory just specifies such correction terms to the Maxwell equations. Consequently, in 1936 B. Kwal and J. Solomon (France) [82] for the first time quantify the leading correction to the Stefan-Boltzmann law for the energy density u of black body radiation. They find (σ is some constant):

$$u = \sigma T^4 \left(1 + \frac{14\pi\sigma T^4}{b^2} \right). \quad (5)$$

Some time later, both authors present a derivation of the Stefan-Boltzmann law within a fairly large class of nonlinear electrodynamic theories [122]. Equations analogous to eq. (5) have later also been obtained by E. Milkutat (Berlin, Germany) [125] and Schrödinger [138], p. 100, eq. (7, 18) (p. 332 of the reprint [256]).

The early development of Born-Infeld electrodynamics in general has been described by few reviews. The most comprehensive one is the review written by Max Born himself in 1937 [101]. Less comprehensive are conference expositions of 1935 by Born [39] and of 1937 by Chrapływyj [106]. Finally, it also seems worth mentioning sections on Born-Infeld theory in the monographs by Frenkel' [33] (гл. [gl.]/[chap.] X, §11, §12), Sommerfeld [149] (Teil [part] IV, §37), and Ivanenko/Sokolov [151] (§32).

We conclude this section with two statements (dating from different times) by Albert Einstein and Max Born himself concerning Born-Infeld electrodynamics. In a private letter of March 22, 1934, Albert Einstein writes to Max Born ([213], letter 69, pp. 169/170, for the original German text see our App. A, p. 39; here we quote the English translation [214], p. 122.): “I am greatly interested in your attempt to attack the quantum problem of the field from a new angle, but I am not exactly convinced. I still believe that the probability interpretation does not represent a practicable possibility for the relativistic generalisation, in spite of its great success. Nor has the reasoning for the choice of a Hamiltonian function for the electromagnetic field, by analogy with the special theory of relativity, convinced me. I am afraid that none of us will live to see the solution of these difficult problems.”. Max Born comments this letter many years later (in 1969) by the following words ([213], pp. 170/171, for the original German text see our App. A, p. 39; here we quote

the English translation [214], pp. 122/123.): “Einstein’s objections to my ideas were twofold. The first was based on his rejection of the probability of quantum mechanics. This concerns a matter of principle. It did not really apply to the theory devised by Infeld and myself, because we ourselves did not in fact manage to make it fit in with quantum mechanics; he judged our efforts in this direction to be wrong in principle. Einstein’s second objection concerned our original classical field theory, which was complete in itself and free from inconsistencies. It was based on the following analogy: in the special theory of relativity the kinetic energy of a particle, which in classical mechanics is proportional to the square of its velocity, is represented by a rather complicated expression; for velocities which are small compared with that of light it tends to the classical expression, but deviates from it when the velocity approaches that of light. In Maxwell’s electrodynamics the energy density is a quadratic expression containing the field intensity. I replaced this with a general expression which approximates to the classical expression whenever the strength of the field is small compared with a certain field intensity, but diverges from it when this is not the case. From this it followed automatically that the total energy of the field of a point charge is finite, while it becomes infinite in the Maxwellian field. The absolute field has to be regarded as a new natural constant. Einstein did not find this analogical construction convincing. Infeld and I found it attractive for a long time. We abandoned the theory for completely different reasons, namely, because we did not succeed in reconciling it with the principles of the quantum field theory. In any case this constituted the first attempt to overcome the difficulties of microphysics by means of a non-linear theory. Heisenberg’s theory of elementary particles, which is much talked about today, is also non-linear. But I am guessing.”.

3.2 Quantum electrodynamics

As we have discussed in the Introduction (section 1), in the late 1920’s among physicists the idea developed that the emerging quantum theory should also allow and describe the scattering of photons by photons. However, the physical mechanism for such scattering events initially remained unclear. With the proposal by P. A. M. Dirac of the equation named after him (1928), the discovery of the positron (C. D. Anderson, 1932), and the first steps towards the theory of quantum electrodynamics elements of a possible explanation became available. O. Halpern (New York University, USA) [24] proposed in late 1933 in a short, qualitative note that the occurrence of virtual electron-positron pairs could be identified as the physical mechanism by means of which photon-photon scattering occurs (a more elaborate quantitative discussion announced by the author at the end of the note seems not to have emerged). The note of Halpern had been preceded by the discussion of two-photon production in “electron-proton” annihilation^[1] by Dirac [216], Oppenheimer [217], Tamm [218], the process of electron-positron pair creation by a photon in the

^[1]This process of “electron-proton” annihilation found its proper re-interpretation as electron-positron pair annihilation after the discovery of the positron in 1932 only.

field of an atomic nucleus by Oppenheimer and Plesset [219] (also see, published somewhat later, Heitler and Sauter [220], Bethe and Heitler [221], Nishina, Tomonaga, and Sakata [222, 223], Racah [224]), and the Breit-Wheeler process [225, 226] of electron-positron creation in two-photon scattering. Furthermore, also P. Debye is credited by W. Heisenberg to have mentioned similar ideas as Halpern in a private discussion with him^[sl]. In early 1935, D. D. Ivanenko (Physico-Technical Institute, Leningrad, USSR) [50], with reference to the article [34] (containing references to Halpern [24] and Debye), also discussed in a short qualitative note the possible role of the occurrence of virtual electron-photon pairs for photon-photon scattering. He points out that the existence of photon-photon scattering entails the violation of the superposition principle [characteristic for the (linear) Maxwell theory of electromagnetism], i.e., the phenomenon of photon-photon scattering is characteristic for a nonlinear electromagnetic theory. Consequently, as Ivanenko notes, the Dirac theory leads to some nonlinear electromagnetic theory which needs to be identified. Somewhat formal attempts to link the Dirac equation with nonlinear electrodynamics, specifically with the theory proposed by Born, have been published in 1935 by K. V. Nikolskiĭ (Steklov Mathematical Institute, Moscow, USSR) [53, 54].

A first answer to the task described by Ivanenko is given almost simultaneously in time: A completely new level in the discussion of photon-photon scattering is reached with the appearance in print in spring 1935 of a short article by H. Euler and B. Kockel (both Ph. D. students of Heisenberg at the University of Leipzig, Germany; for some reminiscences of W. Heisenberg concerning this research see our Appendix E) [45]. The article marks the transition from qualitative considerations concerning the phenomenon of photon-photon scattering to detailed quantitative calculations. The short article is followed in 1936 by the Ph.D. thesis of H. Euler [72]^[t] which contains a comprehensive exposition of the calculation used by Euler and Kockel to arrive at the results presented in [45] (For a detailed discussion of certain aspects of this Thesis see sec. 2.1, pp. 23-28 of [228]). Euler and Kockel determined the leading perturbative correction (for small field strengths and photon frequencies well below the electron-positron pair production threshold) to the Lagrange density \mathcal{L}_0 of the Maxwell field. They find (cf. eq. (10,6) on p. 444 of [72], written here in SI units):

$$\mathcal{L} = \mathcal{L}_0 + \frac{\alpha^2}{360} \frac{\epsilon_0^2 c^3 \lambda_e^4}{\hbar} \left[4 (F^{\mu\nu} F_{\mu\nu})^2 + 7 (F^{\mu\nu} F_{\mu\nu}^*)^2 \right] \quad (6)$$

$$= \mathcal{L}_0 + \frac{2\alpha^2}{45} \frac{\epsilon_0^2 \lambda_e^4}{\hbar c} \left[(\mathbf{E}^2 - c^2 \mathbf{B}^2)^2 + 7c^2 (\mathbf{E}\mathbf{B})^2 \right] \quad (7)$$

^[sl][34], p. 228, footnote 2 [also cf. [45], p. 246, footnote 5, [72], p. 398, footnote 3 (footnote 2 in the separate Thesis print)]

^[t]In his thesis review (University of Leipzig Archive, Philosophical Faculty, Prom. 769, reprinted in [227], pp. 125/126), Heisenberg credits Debye with having initiated this thesis research by posing him a question (quoted here after [227], p. 126: cf. our App. A, p. 39; English transl.: K.S.): “The subject of the present work originates from a question posed to me by colleague Debye.”

$$= \mathcal{L}_0 + \frac{2\alpha^2}{45} \frac{\lambda_e^4}{\hbar c} \left[4 \mathcal{L}_0^2 + \frac{7}{\mu_0^2} (\mathbf{E}\mathbf{B})^2 \right] \quad (8)$$

with the free field Lagrange density

$$\mathcal{L}_0 = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2} \epsilon_0 (\mathbf{E}^2 - c^2 \mathbf{B}^2) = \frac{1}{2} \left(\epsilon_0 \mathbf{E}^2 - \frac{1}{\mu_0} \mathbf{B}^2 \right). \quad (9)$$

The used symbols are: α is the fine structure constant, $\lambda_e = \hbar/(m_e c)$ is the (reduced) Compton wavelength of the electron with mass m_e , the velocity of light (squared) is given by $c^2 = 1/(\epsilon_0 \mu_0)$ with ϵ_0 , μ_0 being the dielectric permeability and the magnetic permittivity of the vacuum, respectively, $F_{\mu\nu}^* = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}/2$ is the dual of the electromagnetic field strength tensor $F^{\mu\nu}$. Using the relations given in the eqs. (D.4), (D.5) of Appendix D, one can write eq. (8) the following way

$$\mathcal{L} = \mathcal{L}_0 + \frac{2\alpha^2}{45} \frac{\lambda_e^4}{\hbar c} \left[-10 \mathcal{L}_0^2 + \frac{7}{4} T^{\mu\nu} T_{\mu\nu} \right] \quad (10)$$

$$= \mathcal{L}_0 + \frac{2\alpha^2}{45} \frac{\lambda_e^4}{\hbar c} \left[-10 \mathcal{L}_0^2 + 7 \left((T_{00})^2 - \frac{1}{c^2} \mathbf{S}^2 \right) \right] \quad (11)$$

where $T^{\mu\nu}$ (eq. (D.2)) is the stress-energy-momentum tensor of the free electromagnetic field, and the Poynting vector is given by

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}. \quad (12)$$

The found (Euler-Kockel-Heisenberg) Lagrangian (EKH Lagrangian) is a nonlinear Lagrangian for the electromagnetic field which leads to nonlinear corrections to the classical Maxwell equations. The resulting (effective) Maxwell equations do not respect the superposition principle and describe nonlinear electromagnetic phenomena such as photon-photon scattering. One can recognize from eq. (11) that the term quadratic in the Poynting vector provides us with an interaction term between electromagnetic (photon) energy currents which is relevant for the experimental proposal by Bauer [17] discussed at the end of subsec. 2.2.2. Euler and Kockel also give for the first time a quantitative estimate for the cross section σ of photon-photon scattering. They find (for photons of frequency $\hbar\omega \ll m_e c^2$):

$$\sigma \sim \alpha^4 \lambda_e^8 \left(\frac{\omega}{c} \right)^6 \quad (13)$$

entailing a size of the cross section of 10^{-30} cm^2 for γ -rays and of 10^{-70} cm^2 for visible light ([45], bottom of p. 247, [72], p. 446, eq. (10,10))^[u]. These cross sections are fairly small and Euler immediately points out ([72], p. 446, below of eq.

^[u]In a letter of June 17, 1934 quoted here from [200], p. 331 (cf. our App. A, p. 40; English transl.: K.S.), W. Heisenberg mentions to N. Bohr that: “Debye came up with the idea that the solar corona originates from this scattering of light; the above value $[(e^2/\hbar c)^4 (\hbar/mc)^2]$ for the cross

(10,10)) that this will make it difficult to experimentally prove the phenomenon of photon-photon scattering. This is a comment proving its significance even today after 80 years. Furthermore, the cross section estimate of Euler and Kockel explains the negative results of all experiments performed earlier (discussed in subsec. 2.1) to directly detect photon-photon scattering.

Roughly one year after the article by Euler and Kockel [45] an article by Heisenberg and Euler [74] appeared in print which considerably extended the result obtained by the former. While Euler and Kockel limited the perturbative calculation of the effective Maxwell Lagrangian to sufficiently small electromagnetic fields, Heisenberg and Euler solved the Dirac equation for constant, arbitrarily strong, parallel magnetic and electric fields and calculated on the basis of this information the effective Maxwell Lagrangian. This (in modern terms: 1-loop) effective Maxwell (Euler-Heisenberg) Lagrangian for constant electromagnetic fields together with its weak-field limit, the EKH Lagrangian, represents a milestone in the history of quantum field theory.

An independent confirmation of the results obtained by Euler and Kockel came in 1936 with a short note by N. Kemmer and V. F. Weisskopf (University of Zurich and ETH Zurich, Switzerland) [80]. Some aspects of this work, which cannot easily be read from this short note, are pointed out by V. F. Weisskopf in an oral history interview in 1965^[v]: “We connected the scattering of light by light with the Delbrück scattering. Today it’s a triviality; one light quantum is replaced by the Coulomb field^[w]^[footn. K.S.], but at that time it was not so trivial. . . . Euler and Kockel at that time, under Heisenberg, calculated the scattering of light by light, but had to do a lot of subtracting because there were a great many terms that were infinite. They did this in the usual clever way and got the result. And Kemmer and I showed that you can do the calculation without raking any subtractions, because you can show that it is equivalent to the Delbrück scattering, replacing one light quantum by the Coulomb field, and the Delbrück scattering doesn’t diverge.” The same year, a long paper by Weisskopf [98] presented a more efficient and improved re-derivation of the

section] would excellently fit this thesis. However, as already said, one still has to see whether all calculations are correct.” In a letter one day earlier ([200], letter [374] of June 16, 1934, pp. 331-333, specifically p. 332: cf. our App. A, p. 40; English transl.: K.S.), W. Heisenberg also wrote to W. Pauli: “This value seems to fit well to the idea of Debye that the solar corona emerges through this scattering of light by light.” S. I. Vavilov had discussed the possible role of the scattering of light by light for the emergence of the solar corona in his article [8] in 1928 (cf. subsec. 2.3) which is the written version of a talk delivered by him at the VI Congress of Russian Physicists in the same year (cf. [229], p. 47, item 80). P. Debye has been a participant to this congress (cf., e.g., [230], chap. 6, p. 226, English translation: chap. 4, p. 140). It is well possible that he had listened to the talk of S. I. Vavilov.

^[v]Cf. our Appendix G (quotation with kind permission of the American Institute of Physics), for some related historic recollections by Kemmer see our Appendix H.

^[w]K. S.: More precisely, they consider for the scattering electric field \mathbf{E} the condition $|\text{grad } \mathbf{E}| \ll |\mathbf{E}| mc/h$ (weakly varying fields at a length scale of the electron Compton wave length) which in the strict sense is not fulfilled for the Coulomb field: in a region close to the center of the nucleus.

Euler-Heisenberg Lagrangian (For some historic recollections by V. F. Weisskopf also concerning this article see our Appendix G.). In [98], Weisskopf in principle follows the path taken by Heisenberg and Euler [74] but solves the Dirac equation for a constant magnetic field accompanied by a parallel, spatially periodic electric field. While Heisenberg and Euler, considering a constant electric field, had to deal with the Klein paradox (any spatially constant electric field creates electron-positron pairs) Weisskopf was able to sail around this difficulty by considering a sufficiently weak, spatially periodic electric field for which no electron-positron pair creation occurs. The result of his calculation confirmed the expression obtained by Heisenberg and Euler for the effective Maxwell (Euler-Heisenberg) Lagrangian.

The theoretical study of photon-photon scattering was also in the focus of research interests elsewhere: L. D. Landau (Ukrainian Physico-Technical Institute, Kharkov, USSR) handed the task of studying certain aspects of photon-photon scattering to his doctoral student A. I. Akhiezer. As a first result a short note by Akhiezer, Landau, and I. Ya. Pomeranchuk (Ukrainian Physico-Technical Institute, Kharkov, USSR) appeared [62] treating for the photon-photon scattering cross section the high-frequency case $\hbar\omega \gg m_e c^2$. They found:

$$\sigma \sim \alpha^4 \left(\frac{c}{\omega}\right)^2 \quad (14)$$

noting that the proportionality factor is difficult to compute (In the low frequency case $\hbar\omega \ll m_e c^2$ studied by Euler and Kockel [45] the proportionality factor can be found from the EKH Lagrangian.). The result by Akhiezer, Landau, and Pomeranchuk showed that the cross section σ for photon-photon scattering must have “a maximum value in a region $\hbar\omega \sim mc^2$ ” ($m = m_e$) [62]. The kand. diss. (Ph.D. Thesis) research of A. I. Akhiezer giving the details of the calculation leading to eq. (14) is finally published in 1937 [99] (For historic recollections concerning the thesis research of A. I. Akhiezer see our Appendices I, J.). It should be mentioned that in 1935 also W. Pauli (University of Zurich, Switzerland) spent some time attacking the problem of high-frequency photon-photon scattering^[x]. However, the calculation proved to be difficult and in a letter of December 5, 1935^[y] Pauli informed Weisskopf: “Presently, the “Euler-Kockel” problem for short waves looks somewhat bleak to me.” A short time later, the *Nature* article by Akhiezer, Landau, and Pomeranchuk [62] appeared in print.

Besides the fundamental nonlinear quantum-electrodynamic process of photon-photon scattering related phenomena have found attention at about the same time. As discussed in subsec. 2.2.3, in 1933 Delbrück [23] had qualitatively considered the interaction of photons with the Coulomb field of nuclei via their impact on the

^[x]See his letter [421a], [231], pp. 769-771 (specifically p. 770) of September 27, 1935 to Weisskopf, the corresponding section is quoted in our Appendix F.

^[y]Reprinted as letter [423b] in [231], pp. 777/778, specifically p. 778: cf. our App. A, p. 40; English transl.: K.S..

surrounding QED vacuum^[z]. However, the cross section calculations related to this process Delbrück attempted in the time thereafter proved to be extremely difficult (cf. footnote [i] on p. 16). Weisskopf then independently took a look at the problem, but as W. Pauli reported to W. Heisenberg in June 1935^[aa]: “Weisskopf is on the Delbrück problem and the problems of subtraction physics occurring there are getting not very nice.” Finally, in 1936 the joint work of Weisskopf and Kemmer paid off and as a first result – as already mentioned above – they established a connection between the scattering of light by light in the low-frequency domain and Delbrück scattering (understood here in a somewhat generalised sense as scattering of photons from a sufficiently slowly varying electric field) and confirmed this way the expression for the EKH Lagrangian [80] (also cf. our Appendices G, H). Details of the calculation were given by Kemmer in 1937 in [111]^[bb]. A short follow-up article by Kemmer and Ludwig [112] explored the calculational aspect that occur if the assumption of a sufficiently slowly varying electric field is given up and the Coulomb field is considered instead. For this case, they derived qualitatively an expression for the low-frequency (differential) cross section [cf. eq. (1) at the end of subsec. 2.2.3] which, however, contained two unknown constants. Simultaneously with Kemmer and Ludwig, Akhiezer and Pomeranchuk had considered Delbrück scattering [100]. They obtained in the low- and high-frequency limits ($\hbar\omega \ll m_e c^2$ and $\hbar\omega \gg m_e c^2$, respectively; ω is the photon frequency) expressions for the total cross section whose size however remained undetermined up to a constant. Specifically, they found for $\hbar\omega \ll m_e c^2$ [in accordance with the above eq. (1) on page 17]:

$$\sigma = b Z^4 \left(\frac{e^2}{m_e c^2} \right)^6 \left(\frac{\omega}{c} \right)^4 \quad (15)$$

and for $\hbar\omega \gg m_e c^2$ (the absorptive contribution of real electron-positron pair creation is not included):

$$\sigma = a \alpha^2 Z^4 \frac{c^2}{\omega^2} \ln \frac{\hbar\omega}{m_e c^2} \quad (16)$$

where a and b are unknown constants. Akhiezer and Pomeranchuk conclude again^[cc]: “The effective cross section has a maximum at $\hbar\omega \sim m c^2$.” ($m = m_e$).

^[z]For another discussion of the articles to be considered in this paragraph on Delbrück scattering see [232], subsec. 3.1, pp. 84-87.

^[aa][200], letter [412] of June 15, 1935, pp. 402-405, specifically p. 402: cf. our App. A, p. 40; English transl.: K.S..

^[bb]For a rare glimpse into the communication between Kemmer and Weisskopf concerning this work see the letter of June 8, 1936 from Kemmer to Weisskopf (letter [430b] reprinted in [231], pp. 792-794). For the interest of Pauli in this problem see his letters of May 19, 1936 and of June 3, 1936 to Weisskopf (letters [427f] and [430a] reprinted in [231], pp. 788-790 (specifically p. 788) and pp. 790-792 (specifically p. 791), respectively.)

^[cc][100], p. 567 in the Russian original (cf. our App. A, p. 40), p. 7 of the reprint, p. 478 in the German transl.. English transl.: K.S..

In the period under consideration, besides photon-photon scattering and Delbrück scattering the possible phenomenon of splitting of photons has also been discussed. Halpern [24] had mentioned in 1933 photon splitting as a process conceivable in principle within quantum electrodynamics. Following Halpern, W. Heitler (University of Bristol, UK) in 1936 in his book also mentioned this possibility ([75], 1. and 2. eds., Chap. IV, § 19, pp. 193/194). Furthermore, he correctly notes that the probability for the vacuum decay of a photon into two photons vanishes – a modern consideration would refer to Furry’s theorem [233] to infer this – leaving as minimal possible case a decay into three photons. Relying on the formalism used by Heisenberg in [34], M. P. Bronshtein (Leningrad Physico-Technical Institute, USSR) in 1937 concludes [102] (preceded by a corresponding short note in 1936 [68]) that in free Minkowski space (spontaneous) photon splitting is not possible at all within the framework of quantum electrodynamics^[dd]. On the other hand, photon splitting in the presence of an external field is possible and Williams discussed in 1935 [61], § 10, pp. 47-49, for the first time the splitting of a photon in the presence of an electric particle (electric field).

Within the framework of quantum electrodynamics, the further investigation of such processes as photon-photon scattering, Delbrück scattering, and photon splitting continued in the 1950’s only and falls, therefore, beyond the time frame of the present exposition. In the 1930’s and 1940’s these subjects have been considered both within Born-Infeld electrodynamics and in quantum electrodynamics. Interestingly, however, the theoretical study of the propagation of light in constant homogeneous magnetic and electric fields has been performed in the period under review within Born-Infeld electrodynamics only. Corresponding studies within QED began in the 1950’s only (J. S. Toll).

Acknowledgements

I am grateful to I. A. Smirnova (Kiev) and S. V. Smirnov (Helsinki) for granting me access to [94] and to A. P. Nosov (Ekaterinburg) for providing me with a copy of [263]. Thanks also go to V. V. Nesterenko (Dubna) for communication concerning [65]. Kind hospitality at the Theoretical Particle Physics Group of the Department of Physics and Astronomy of the Vrije Universiteit Amsterdam is gratefully acknowledged.

^[dd]For a modern argument see [234], sec. III, p. 2343. Note that in eq. (31) the first sum sign should be read as a product sign.

Appendices

Appendix A

Original language versions of the quotations in the main text:

- P. 3: Kepler ([157], p. 23, reprint (1859) p. 142, reprint (1939) p. 32, proposition 26): “*Lucis radii se mutuò neque colorant, neque illustrant, neque impediunt ullo modo. . . . Sanè vt nec physicus motus alter alterum mouet.*”
- P. 3: Huygens ([163], p. 20): “Une autre, & des plus merueilleuses prepietez de la lumiere est que, quand il en vient de divers costez, ou mesme d’opposez, elles font leur effet l’une à travers l’autre sans aucun empêchement.”
- P. 4: Vavilov ([8], p. 555, reprint [165], p. 234): “ Упрек, который в XVIII. в. часто делали ньютоновской теории, заключался именно в том, что должны обнаруживаться соударения световых корпускул, т. е. нарушение суперпозиции. Выходом из затруднения было допущение крайней малости корпускул: «Знаю, – писал Ломоносов ^{3[K.S.: orig. footn.]} по адресу защитников корпускулярной гипотезы, – что вы разделяете материю света на толь малые частицы и толь редко одну по всемирному пространству поставляете, что все оное количество может сжаться и уместиться в порожних скважинах одной песчинки».”
- P. 8: Vavilov ([8], § 2, pp. 556/557, reprint [165], p. 236): “ Едва ли не самые большие мгновенные плотности радиации в лабораторных условиях получаются при помощи света конденсированной искры. Концентрируя этот свет линзой, легко добиться мгновенных значений плотности лучистой энергии, превышающих соответствующие значения у поверхности Солнца. Средняя плотность в этом случае мала вследствие краткой длительности и редкости искр, но гипотетический эффект «соударений» световых квантов должен быть пропорциональным к в а д р а т у мгновенной плотности, поэтому искра оказывается значительно выгоднее, чем, например, дуга. Наблюдения при этом производились с обычными предосторожностями, на фоне далеко отстоящих стенок сосуда, отклеенных черным бархатом; для контроля опыты повторялись со светом лампы накаливания, которая давала такую же среднюю плотность радиации; в обоих случаях результат был одинаково отрицательным.”
- P. 11: Kapitsa ([177], p. 31, [177], p. 40, reprints: 1. [178], p. 40. 2. [179], 3. ed. p. 375, 4. ed. p. 374): “ В 30-х годах в Кавендишской лаборатории я осуществил метод получения магнитных полей по силе на порядок выше, чем

³[K.S.: orig. footn.] М. В. Ломоносов, Слово о происхождении света. Собрание разных сочинений, ч. III, 1803, стр. 155. (K.S.: M. V. Lomonosov, Slovo o proiskhozhdenii sveta. Sobranie raznykh sochinenii, ch. (pt.) III, 1803, str. (p.) 155 [169])

до сих пор это было достигнуто. В одной беседе Эйнштейн пытался меня убедить экспериментально изучать влияние магнитного поля на скорость распространения света. Эти опыты уже делались, никакого эффекта не было обнаружено. В моих магнитных полях можно было бы поднять предел точности измерения порядка на два, поскольку эффект должен был зависеть от квадрата интенсивности магнитного поля. Я возражал Эйнштейну, что, согласно существующей картине электромагнитных явлений, не видно, откуда можно было бы ждать таково измеримого явления. Не находя возможности обосновать необходимость таких опытов, Эйнштейн, наконец, сказал: «Я думаю, что дорогой господь бог (*der liebe Gott*) не мог так создать мир, чтобы магнитное поле не влияло на скорость света». Конечно, это аргумент, с которым трудно спорить.”

- P. 14: Delbrück [23]: “... daß es sich um einen *Photoeffekt* an einem der unendlich vielen Elektronen in Zuständen negativer Energie handelt, die nach Diracs Theorie den ganzen Raum mit unendlicher Dichte erfüllen und die zu einem solchen Absorptionsprozeß vermöge ihrer Wechselwirkung mit dem Kern wohl befähigt wären.”
- P. 14: Delbrück [23]: “Eine solche Auffassung zwingt dann auch zu der Folgerung, daß diese Elektronen negativer Energie γ -Strahlen zu *streuen* vermögen, und zwar *kohärent*, analog dem Phänomen der *unverschobenen* Comptonlinie.”
- P. 16: Heisenberg ([200], letter [374] of June 16, 1934, pp. 331-333, specifically p. 332): “Delbrück war neulich hier und erzählte von seinen vergeblichen Versuchen, die kohärente Streuung zu rechnen. Er wäre sicher froh, wenn er die Arbeit gemeinsam mit einem anderen z.B. Weisskopf machen könnte (Sauter ist jetzt in Göttingen).”
- P. 18: Ebert ([3], pp. 381-383):
- “Zum Schluss erlaube ich mir noch, das obige Resultat auf einige astrophysikalische Fragen anzuwenden.

Ist die Fortpflanzungsgeschwindigkeit des Lichtes von seiner Intensität in irgend einem merklichen Grade abhängig, so würde zu dem Umstande, dass die uns am Himmel in demselben Augenblicke entgegretenden Erscheinungen in Wirklichkeit sehr verschiedenen Zeiten angehören, noch die weitere Complication hinzutreten, dass diese Zeiten selbst bei gleicher räumlicher Entfernung für die verschiedenen Objecte sehr verschieden wären. Bei den grossen Intensitätsunterschieden, die sich hier vorfinden, und den grossen Entfernungen, welche die Lichtstrahlen zu durchlaufen haben, ehe sie zu uns gelangen, konnten sich selbst kleine Unterschiede in der Fortpflanzungsgeschwindigkeit in hohem Grade geltend machen. Dies müsste z. B. bei den physischen Doppelsternen eintreten; hier haben wir zwei Lichtquellen, die zwar sehr angenähert gleich weit von uns abstehen, deren Lichtintensität sich aber im allgemeinen sehr erheblich voneinander unterscheidet. Trotzdem finden wir, dass, wenn es

uns gelungen ist die Bahnen von Doppelsternen zu berechnen, die beiden Componenten wirklich zu gleicher Zeit an entsprechenden Punkten ihrer Bahnen erscheinen und mit ihrem gemeinsamen Schwerpunkte immer auf einer geraden Linien liegen. Diese Thatsache können wir also geradezu als neue Stütze für das von uns experimentell festgestellte Resultat heranziehen. Ein Beispiel möge übrigens noch zeigen, dass die von mir nachgewiesene Grenze für die Unabhängigkeit der beiden in Rede stehenden Grössen für die astronomische Praxis jedenfalls völlig ausreicht.

Der von Bessel berechnete, von Clark entdeckte Begleiter des Sirius besitzt nach den Berechnungen von Auwers und Peters eine ausserordentlich grosse Masse ($\frac{1}{2}$ bis $\frac{2}{3}$ der Siriusmasse selbst). Gleichwohl erscheint er uns nur als Stern etwa 9. Grösse, seine Helligkeit ist also gegen die seines Hauptsternes eine sehr geringe, nach Schönfeld ¹⁾^[K.S.: orig. footn.] nur ca. $\frac{1}{1000}$ von der des Sirius. Oben wurde für Strahlen mittlerer Wellenlänge (für das Natriumlicht) gefunden, dass bei Abschwächung des Lichtes von der Intensität 1 bis zur Intensität $\frac{1}{33}$ die Fortpflanzungsgeschwindigkeit sich noch nicht um $\frac{1}{500\,000}$ ändert. Gehen wir von diesem Grenzwert aus, d. h. nehmen wir an, dass sich bei Helligkeitsdifferenzen von 33:1 die Fortpflanzungsgeschwindigkeit gerade noch um diesen Betrag unterscheiden könne, und dass derselbe proportional mit dem Intensitätsunterschiede wachse, so würde das Licht des Begleiters in dem vorliegenden Beispiele in jeder Secunde um $30 \times 0,6$ oder 18 km hinter dem Lichte des Hauptsternes zurückbleiben. Die Entfernung des Siriuissystems beträgt in Lichtzeit etwa 30 Jahre. Nehmen wir an, es seien genau 30 Jahre, so würde das Licht des Begleiters:

$$\frac{300\,000}{299\,982} \times 30 = 30,0018$$

Jahre gebrauchen, um zu uns zu gelangen, d. h. von zwei gleichzeitig ausgehenden Lichtstrahlen würde der des Begleiters immer nur etwa 0,7 Tag später eintreffen, als der vom Sirius selbst ausgesandte. Da aber die Umlaufzeit beider Körper um ihren gemeinsamen Schwerpunkt etwa 50 Jahre beträgt, so ist jene Zeitdifferenz und die ihr entsprechende Ortdifferenz zu vernachlässigen. Die gemachten Annahmen sind jedenfalls sehr ungünstige; wir können also sicher bei allen Bahnbestimmungen von der etwa noch vorhandenen Abhängigkeit der Fortpflanzungsgeschwindigkeit von der Intensität, deren Betrag unter den durch das Experiment festgestellten Grenzen liegt, absehen.

Das erhaltene Resultat ist ferner bei den Anwendungen des Doppler'schen Principes von Bedeutung, wo die relativen Geschwindigkeiten der Gestirne in der Richtung der Sehlinie aus Aenderungen in der mittleren Brechbarkeit isolirter Spectrallinien bestimmt werden. Hier liegen in den einzelnen Fällen

¹⁾^[K.S.: orig. footn.] Schönfeld, Die dunklen Fixsternbegleiter. Mannheimer Verein für Naturkunde. 30. Jahresbericht. (K.S.: cf. [204])

oft sehr grosse Helligkeitsunterschiede vor, und es ist folglich für die Anwendbarkeit dieses Principes wichtig, direct nachgewiesen zu haben, dass diese Helligkeitsunterschiede nicht mit in Frage kommen.”

P. 28: Einstein ([213], letter 69, pp. 169/1970): “Dein Versuch, die Quantenfrage des Feldes von einer neuen Seite anzugreifen, hat mich sehr interessiert, aber nicht gerade überzeugt. Ich glaube immer noch, daß die Wahrscheinlichkeits-Interpretation trotz ihres großen Erfolges keine gangbare Möglichkeit für die relativistische Verallgemeinerung bildet. Auch die Begründung der Wahl einer Hamiltonfunktion für das elektromagnetische Feld mit der Analogie zur speziellen Relativitätstheorie hat mich nicht überzeugt. Ich fürchte, daß wir alle die wirkliche Lösung dieses harten Problems nicht erleben werden.”

P. 29: Born ([213], pp. 170/1971): “Zweierlei Einwände hatte Einstein gegen meine Ideen: Der erste beruhte auf seiner Ablehnung der Wahrscheinlichkeits-Interpretation der Quantenmechanik. Dabei geht es um eine Prinzipienfrage des Denkens, von der noch mehr die Rede sein wird. Er traf die von Infeld und mir entworfene Theorie eigentlich nicht, da wir selber mit der Einfügung derselben in die Quantenmechanik nicht fertig wurden; er verurteilte unsere Bemühungen in dieser Richtung als grundsätzlich falsch. Der zweite Einwand Einsteins bezog sich auf unsere ursprüngliche, ›klassische‹ Feldtheorie, die in sich geschlossen und widerspruchsfrei war. Sie beruhte auf folgender Analogie: In der speziellen Relativitätstheorie wird die kinetische Energie eines Teilchens, die in der klassischen Mechanik dem Quadrat der Geschwindigkeit proportional ist, durch einen etwas komplizierteren Ausdruck dargestellt; für Geschwindigkeiten, die klein gegen die des Lichtes sind, geht er in den klassischen Ausdruck über, weicht aber davon ab, wenn die Geschwindigkeit der des Lichtes nahe kommt. In der Maxwellschen Elektrodynamik ist die Energiedichte quadratisch in den Feldstärken; ich ersetzte sie durch einen allgemeinen Ausdruck, der in den klassischen übergeht, wenn die Feldstärken klein gegen eine ›absolute Feldstärke‹ sind, aber davon abweichen, wenn das nicht der Fall ist. Daher ergab sich ganz von selbst, daß die Gesamtenergie des Feldes einer Punktladung endlich ist, während sie im Maxwellschen Feld unendlich groß wird. Das absolute Feld muß als eine neue Naturkonstante angesehen werden.

Diese Analogie-Konstruktion fand Einstein nicht überzeugend. Infeld und mir schien sie lange anziehend. Wir gaben die Theorie aus ganz anderen Gründen auf, nämlich weil es uns nicht gelang, sie mit den Prinzipien der Quantenfeldtheorie in Einklang zu bringen.

Jedenfalls war dieser Ansatz der erste Versuch, die Schwierigkeiten in der Mikro-Physik durch eine nicht-lineare Theorie zu beheben. Heisenbergs Theorie der Elementarteilchen, die heute viel von sich reden macht, ist auch nicht-linear. Aber das gehört nicht hierher.”

- P. 30: Heisenberg ([227], p. 126): “Das Thema der vorliegenden Arbeit geht auf eine Frage zurück, die mir Kollege Debye stellte.”
- P. 31: Heisenberg ([200], p. 331, letter of June 17, 1934 to N. Bohr): “Debye hatte die Idee, daß die Sonnenkorona durch diese Streuung von Licht entsteht; zu dieser These würde der obige Wert $[(e^2/\hbar c)^4(\hbar/mc)^2]$ für den Wirkungsquerschnitt ausgezeichnet passen. Aber, wie gesagt, man muß noch abwarten, ob alle Rechnungen in Ordnung sind.”
- P. 32: Heisenberg ([200], letter [374] of June 16, 1934, pp. 331-333, specifically p. 332): “Dieser Wert scheint gut zu der Idee von Debye zu passen, daß die Sonnenkorona durch diese Streuung von Licht an Licht entsteht.”
- P. 33: Pauli ([231], letter [423b] of December 5, 1935, pp. 777/778, specifically p. 778): “Das „Euler-Kockel“-Problem für kurze Wellen sieht mir im Moment etwas trübe aus.”
- P. 34: Pauli ([200], letter [412] of June 15, 1935, pp. 402-405, specifically p. 402): “Weisskopf ist am Delbrückschen Problem und die dabei auftretenden Fragen der Subtraktionsphysik werden sehr unschön.”
- P. 34: Akhiezer ([100], p. 567): “Эффективный поперечник рассеяния имеет максимум при $\hbar\omega \sim mc^2$.”

Appendix B

Extract from the book *Ondes et Mouvements* (1926) by L. de Broglie ([7], Chap. XI, Sec. 2, pp. 96-98. English transl.: K.S.. The English translation is appended after the original French text.):

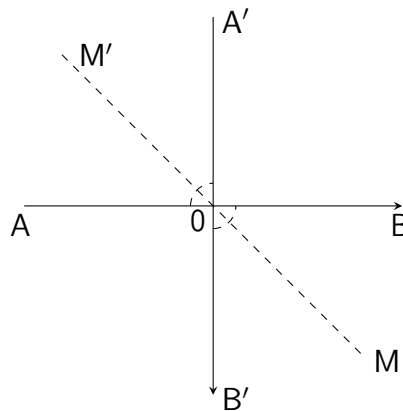
“**2. Chocs entre atomes de radiation.** – Examinons une autre question assez curieuse : deux quanta de lumière peuvent-ils échanger de l’énergie par choc, autrement dit peuvent-ils changer de fréquence à la suite d’une collision ? L’expérience n’a rien révélé de semblable et un tel phénomène est tout à fait étranger aux théories classiques. Cependant, il existe peut-être une raison de le croire possible au moins en principe. Dans la dernière partie de ce livre, je montrerai qu’à la suite des travaux de MM. Bose et Einstein et des miens, il est légitime de considérer le rayonnement noir comme un gaz d’atomes de lumière ; des raisonnements statistiques appliqués à nos conceptions ondulatoires nous conduiront à la loi Planck, mais elles ne nous montreront pas par quel mécanisme cette répartition des quanta entre les diverses valeurs de l’énergie se trouve réalisée et maintenue. Il paraît assez tentant de supposer que l’équilibre résulte des échanges d’énergie et de quantité de mouvement entre quanta dues à leur interaction mutuelle, à leurs chocs au sens le plus général du mot.

Peut-être est-il donc intéressant d’étudier un peu ce genre de chocs.

Il serait facile de résoudre le problème général de la rencontre de deux quanta de fréquences différentes comme nous l’avons fait pour l’effet Compton.

Pour ne pas multiplier les formules, je me contenterai d’envisager un cas très simple. Supposons qu’à l’aide de collimateurs, on fasse se croiser à angle droit deux faisceaux monochromatiques de même fréquence, aucune condition de cohérence n’étant d’ailleurs exigée. Représentons schématiquement ces deux faisceaux par des lignes droites AB et $A'B'$ (*fig. 9*) se croisant en O .

Fig. 9.



Observons, s’il en existe, la lumière diffusée au point O dans la direction OM bissectrice de l’angle $\widehat{BOB'}$. Si, par suite d’un choc, un quantum est diffusé suivant

OM, l'autre quantum sera diffusé suivant OM'. Soient ν la fréquence initiale, ν_1 et ν_2 les fréquences des quanta diffusés vers M et vers M'. Les équations de conservation s'écrivent

$$\begin{aligned} h\nu_1 + h\nu_2 &= 2 h\nu, \\ \frac{h\nu_1}{c} - \frac{h\nu_2}{c} &= 2 \frac{h\nu}{c} \cos 45^\circ ; \end{aligned}$$

d'où

$$\begin{aligned} \nu_1 &= \nu(1 + \cos 45^\circ) = 1,7 \nu, \\ \nu_2 &= 0,3 \nu. \end{aligned}$$

Si la longueur d'onde des faisceaux employés était $\lambda = 0,68$, la radiation observée en M correspondrait à $\lambda_1 = \frac{0,68}{1,7} = 0,4$. En visant le point de croisement de deux faisceaux rouges, on recueillerait de la lumière violette. Ce serait un beau phénomène ! Je ne sais s'il existe et, en ce cas, s'il serait observable, mais, si un jour il était décelé, sa place serait toute marquée dans l'ensemble de nos nouvelles vues théoriques sur les radiations."

English translation:

“2. Collisions between atoms of radiation. – We are considering another interesting problem: Can two light quanta exchange energy in a collision, or speaking differently, can they change the frequency in course of a collision? Experience does not reveal anything similar and such a phenomenon is rather alien to classical theories. However, there may be a reason for considering this to be possible, at least in principle. In the last part of this book, I will show, as a result of work done by Messrs. Bose and Einstein and myself, that it is legitimate to consider the black radiation in a gas of atoms of light; statistical reasoning applied to our wave conceptions leads us to the Planck law, but it does not explain by what mechanism this redistribution of quanta among the different energy levels is achieved and maintained. It seems quite tempting to assume that the equilibrium results from exchanges of energy and momentum among quanta as a result of their mutual interactions, their collisions in the most general sense of the word.

Consequently, it is perhaps interesting to somewhat investigate this kind of collisions.

It is easy to solve the problem of the encounter of two quanta with different frequencies the same way as we did for the case of the Compton effect. In order to not multiply formulae, I will only consider a very simple case.

Consider, using collimators, intersecting at a right angle two monochromatic beams of the same frequency, incidentally, no coherence condition is being required. We represent the two beams schematically by means of the two perpendicular lines AB and A'B' (*fig. 9*) intersecting each other in O.

Fig. 9. (K.S.: see above)

We observe whether any light scatters from the point O into the direction OM bisecting the angle $\widehat{BOB'}$. If, through a collision, a quantum is scattered along OM, the other quantum is scattered along OM'. Let ν be the initial frequency, and ν_1 and ν_2 the frequencies of the quanta scattered towards M and towards M'. The equations of conservation are

$$\begin{aligned} h\nu_1 + h\nu_2 &= 2 h\nu, \\ \frac{h\nu_1}{c} - \frac{h\nu_2}{c} &= 2 \frac{h\nu}{c} \cos 45^\circ ; \end{aligned}$$

wherefrom

$$\begin{aligned} \nu_1 &= \nu(1 + \cos 45^\circ) = 1,7 \nu, \\ \nu_2 &= 0,3 \nu. \end{aligned}$$

If the wavelength of the beams used is $\lambda = o^\mu, 68$, the radiation observed in M would correspond to $\lambda_1 = \frac{0,68}{1,7} = o^\mu, 4$. Aiming at the crossing point with two red beams, one would obtain violet light. This would be a beautiful phenomenon! I do not know if it exists, and in that case, whether it is observable, but if one day it was detected it would assume a distinguished place within the framework of our new theoretical ideas on radiation.”

Appendix C

On the experimental result of Banwell and Farr, Proc. Roy. Soc. London
A 175(1940)1 [132]:

Given the present-day interest in the type of experiment Banwell and Farr reported in 1940 [132] it seems to be interesting to contemplate the question what influence might have led to the non-null result (which Banwell and Farr themselves – see quote in subsec. 2.2.2 in the main text – considered as likely to be erroneous). In the following we will attempt to hypothesize about a possible reason for their non-null result for the refractive index of the vacuum in a magnetic field which is much larger than the present-day expectation based on the Euler-Kockel-Heisenberg effective Lagrangian of QED (and of opposite sign). Reading the carefully written article by Banwell and Farr one notices that they say very little about one important optical element used in their experimental set-up, namely, the mirrors employed in the Michelson interferometer. They are only characterized as optical flats, silvered or half-silvered (p. 1, begin of the second paragraph; pp. 3/4). The point is that two of the mirrors used by Banwell and Farr in the experiment are subject – at least – to magnetic stray fields: one mirror (mirror O , cf. fig. 1 on p. 3 of [132] redisplayed in our Fig. 2) at the end of the arm of the Michelson interferometer placed in the strong magnetic field, and a semi-transparent mirror (mirror N , cf. Fig. 2) used to split the initial light beam into the two arms of the Michelson interferometer. Both mirrors are just located a few decimeters away from the coils generating the strong magnetic field. This aspect of the experimental set-up is neither mentioned nor discussed by Banwell and Farr. Let us shortly consider the physical situation at the mirror (mirror O) at the end of the arm of the Michelson interferometer placed in the strong magnetic field (The semi-permeable mirror N is also subjected to stray fields but the light in both arms of the Michelson interferometer interacts with N in a fairly symmetric manner and the mirror N can, therefore, be left outside the current consideration.). The light from this arm falls perpendicularly onto the mirror and is perpendicularly being reflected at the silver layer (by the way, whose thickness is not given in the article by Banwell and Farr). In this metallic reflection (by a metallic medium with the complex refractive index $n = n' + i n'' = n'(1 + i\kappa)$) the reflected light beam experiences a phase shift φ relative to the incident light beam given by the equation^[ee]:

$$\tan \varphi = \frac{2 n' \kappa}{n'^2 + n'^2 \kappa^2 - 1} . \quad (\text{C.1})$$

For massive silver at room temperature and light of $\lambda = 546.1$ nm (equivalent to a photon energy of 2.27 eV) one has $n' = 0.17$, $n'\kappa = 3.31$ ([237], Table IX, p. 356, interpolated by means of [238]) resulting in a phase shift φ of about 0.59 rad (corresponding to 34°). Now, once the refractive index of the mirror material (in this

^[ee][235], p. 86, eq. (40); for a modern discussion see, eg.g., [236], sec. 2.4, p. 35, eq. (2.4.14); N.B. For simplicity we neglect in the present discussion the influence of any intermediate glass layer.

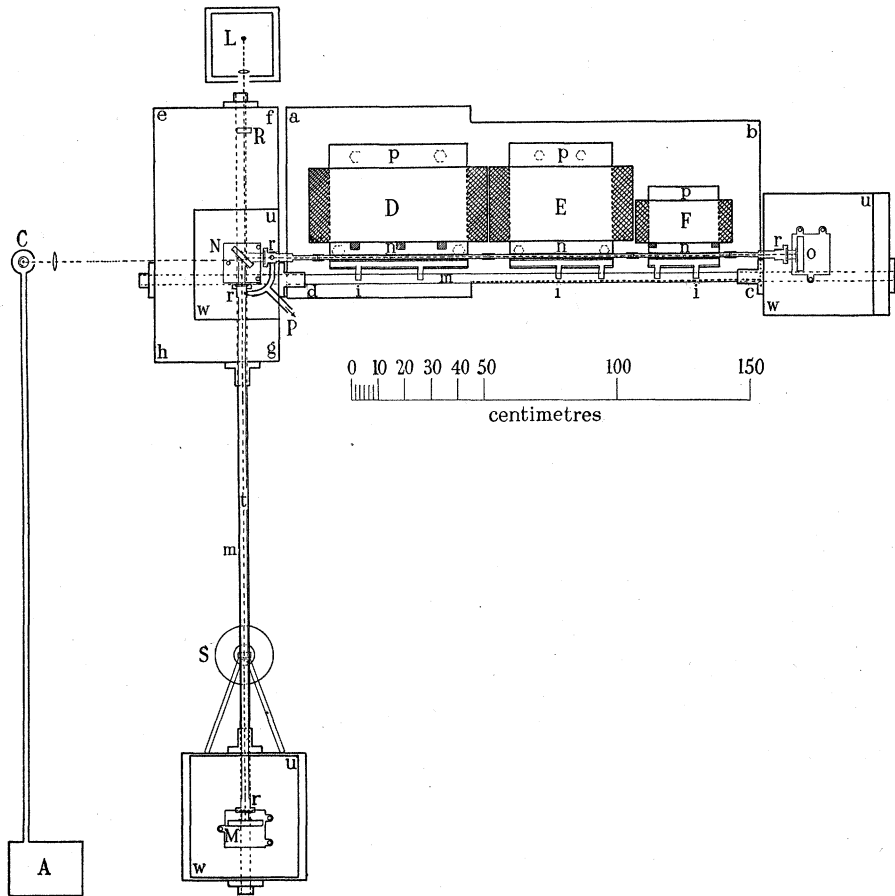


Figure 2: Top view of the Michelson interferometer used by Banwell and Farr (re-display of Fig. 1 of [132], p. 3). L is the light source while observation is made at C (photoelectric cell). D , E , F are the electromagnets of the arm subjected to the magnetic field, M , O are mirrors (silvered flats) while N is a half-silvered flat.

case, silver) changes the phase shift φ also experiences a corresponding change $\Delta\varphi$ to some $\bar{\varphi} = \varphi + \Delta\varphi$. Once this change in the mirror properties (and, consequently, the change in the phase shift at the mirror) occurs only for one of the mirrors at the end of the two arms of the Michelson interferometer the interferometric pattern observed in the experiment will also change. If this phenomenon is not properly taken care of the corresponding change in the interferometric pattern can lead to a misinterpretation of results. As we will detail further below it seems to be conceivable that the change in the mirror properties (mirror O) effected by the stray magnetic field may have led to the non-null result reported by Banwell and Farr.

In their Michelson interferometer experiment, Banwell and Farr have interpreted the observed change in the interference pattern once the magnetic field acting on one interferometer arm had been switched on as resulting from a changed vacuum

velocity of light $\bar{c} = f\bar{\lambda}$ (f is the frequency of the monochromatic light wave). Hypothesize now that the observed change in the interference pattern in fact completely originated from a change in the refractive index of the silver layer of the mirror O placed in the (stray) magnetic field. Then, according to the interpretation applied by Banwell and Farr the optical path of length l ($= 2 \times 1.17 \text{ m} = 2.34 \text{ m}$) placed in the magnetic field would be stretched out by n wave cycles of wave length $\bar{\lambda}$ (We also have taken into account the phase shift/delay under reflection.)

$$l = n \bar{\lambda} - \frac{\varphi}{2\pi} \bar{\lambda}. \quad (\text{C.2})$$

while in reality it should be written as

$$l = n \lambda - \frac{\bar{\varphi}}{2\pi} \lambda. \quad (\text{C.3})$$

From the equations (C.2), (C.3) follows that the assumed change in the vacuum velocity of light $\Delta c = \bar{c} - c$ can be expressed as a change in the phase shift under reflection according to the equation

$$\Delta\varphi = -2\pi \frac{l}{\lambda} \frac{\Delta c}{\bar{c}} \quad (\text{C.4})$$

$$\simeq -2\pi \frac{l}{\lambda} \frac{\Delta c}{c} \quad (\text{C.5})$$

where equation (C.5) represents the relation at leading order of the small quantity $\Delta c/c$. Inserting now the values given by Banwell and Farr into eq. (C.5) one finds the following. The change in the interference pattern observed in the Michelson interferometer which has been interpreted by Banwell and Farr as a change in the (vacuum) velocity of light in a magnetic field could have arisen equally well from a different source in the experimental apparatus. Let us assume that in the presence of the applied strong magnetic field stray fields might have modified the reflective properties of the silver mirror (mirror O) at the end of the arm of the Michelson interferometer placed in the magnetic field. Then, a corresponding change in the phase shift under reflection experienced by the light beam propagating in this arm of the Michelson interferometer of the size

$$\Delta\varphi \simeq -2\pi (4.7 \pm 2.6) \cdot 10^{-3}. \quad (\text{C.6})$$

might have modified the interference pattern equivalently. From eq. (C.1) it is clear that any change in the (complex) refractive index of the silver layer of the order of $\Delta\kappa/\kappa$, $\Delta n'/n' \sim 10^{-2}$ will result in the right order of magnitude of change in the phase shift under reflection given by eq. (C.6).

Let us ponder now the question if it is conceivable that a magnetorefractive effect might result in a change in the refractive index of the silver mirror that has the right magnitude (We primarily imagine that the magnetic stray field is parallel to

the mirror surface.). Unfortunately, there is apparently little information available in the literature in this respect. Gostishchev and Sobol' [239] estimate for low temperatures, however, and a magnetic field of the order of 10^5 oersted (corresponding to a magnetic induction of 10 T in vacuo) that for visible light the change in the reflection coefficient (compared to the situation with no magnetic field applied) for a metal with a closed Fermi surface is not larger than 10^{-3} (the same applies to the refractive index). On one hand side, the estimate of Gostishchev and Sobol' is somewhat smaller than required by eq. (C.6) and on the other hand, they consider the case of low temperatures while the experiment of Banwell and Farr has been performed at room temperature. To broaden the perspective, let us mention that the Drude model of metals (free gas of conduction electrons) relates the conductivity in a metal to its refractive index. Consequently, any study of the phenomenon of magnetoresistance in a metal has in principle its bearing on its magnetorefractive properties. Of course, this connection can only be of heuristic value as the reliability of the Drude model is restricted to the domain of infrared light, while the experiment of Banwell and Farr has been performed using visible light. Magnetoresistance effects are primarily related to two phenomena: the deviation of the Fermi surface from a spherical shape (cf., e.g., [240], Sec. 12.3, p. 490-495, specifically p. 494) and the influence of size effects as they occur in thin films. For experimental results see, e.g., [241, 242].

To conclude, in principle magnetorefractive effects in silver mirrors exist which might lead to a change in the phase shift under reflection in accordance with eq. (C.6). The present state of research reflected in the literature, however, is insufficient to decide the question whether these magnetorefractive effects might have the right order of magnitude to explain the non-null result by Banwell and Farr.

Appendix D

Some formulas for the electromagnetic field strength tensor:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{1}{c}E_1 & -\frac{1}{c}E_2 & -\frac{1}{c}E_3 \\ \frac{1}{c}E_1 & 0 & -B_3 & B_2 \\ \frac{1}{c}E_2 & B_3 & 0 & -B_1 \\ \frac{1}{c}E_3 & -B_2 & B_1 & 0 \end{pmatrix} \quad (\text{D.1})$$

The (symmetric) stress-energy-momentum tensor of the free electromagnetic field reads:

$$T^{\mu\nu} = \frac{1}{\mu_0} \left[F^{\mu\alpha} F^\nu{}_\alpha - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right] \quad (\text{D.2})$$

After some calculation one finds:

$$T^{\mu\nu} T_{\mu\nu} = 2 \epsilon_0^2 \left[(\mathbf{E}^2 - c^2 \mathbf{B}^2)^2 + 2 c^2 (\mathbf{E}\mathbf{B})^2 \right] \quad (\text{D.3})$$

$$= 8 \mathcal{L}_0^2 + \frac{4}{\mu_0^2} (\mathbf{E}\mathbf{B})^2 \quad (\text{D.4})$$

$$= 4 \left[(T_{00})^2 - \frac{1}{c^2} \mathbf{S}^2 \right] = -4 T^{0\nu} T_{0\nu} \quad (\text{D.5})$$

with the free field Lagrange density

$$\mathcal{L}_0 = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2} \epsilon_0 (\mathbf{E}^2 - c^2 \mathbf{B}^2) = \frac{1}{2} \left(\epsilon_0 \mathbf{E}^2 - \frac{1}{\mu_0} \mathbf{B}^2 \right) \quad (\text{D.6})$$

and

$$T_{00} = \frac{1}{2} \epsilon_0 (\mathbf{E}^2 + c^2 \mathbf{B}^2) = \frac{1}{2} \left(\epsilon_0 \mathbf{E}^2 + \frac{1}{\mu_0} \mathbf{B}^2 \right). \quad (\text{D.7})$$

One recognizes from the eqs. (D.4), (D.5), that the square of the Poynting vector $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ is related to the field invariant $(\mathbf{E}\mathbf{B})^2$ (By virtue of the Lagrange identity for three-dimensional vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$: $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{b} \cdot \mathbf{c})(\mathbf{a} \cdot \mathbf{d})$).

Appendix E

Reminiscences of W. Heisenberg onto his conversations with H. Euler concerning the scattering of light by light in advance of the thesis research of the latter [243], chap. 13, pp. 220-222, 225 (pp. 220-222, 225 of the reprint, pp. 160-162, 164 of the English translation. The English translation is appended after the original German text.):

“So kam ich oft mit Euler zusammen, und wir berieten daher über die möglichen Konsequenzen der Diracschen Entdeckung und der Umwandlung von Energie in Materie.

»Wir haben doch von Dirac gelernt«, so könnte Euler etwa gefragt haben, »daß ein Lichtquant, das an einem Atomkern vorbeifliegt, sich dabei in ein Paar von Teilchen, ein Elektron und ein Positron, verwandeln kann. Bedeutet das eigentlich, daß ein Lichtquant aus einem Elektron und einem Positron besteht? Dann wäre das Lichtquant so eine Art Doppelsternsystem, in dem Elektron und Positron umeinander kreisen. Oder ist das eine falsche anschauliche Vorstellung?«

»Ich glaube nicht, daß ein solches Bild viel Wahrheit enthält. Denn aus diesem Bild würde man doch schließen, daß die Masse eines solchen Doppelsterns nicht viel kleiner sein sollte als die Summe der Massen der beiden Teile, aus denen es besteht. Und man könnte auch nicht einsehen, warum dieses System sich immer mit Lichtgeschwindigkeit durch den Raum bewegen muß. Es könnte doch auch irgendwo zur Ruhe kommen.«

»Was soll man aber dann über das Lichtquant in diesem Zusammenhang sagen?«

»Man darf vielleicht sagen, daß das Lichtquant virtuell aus Elektron und Positron besteht. Das Wort ›virtuell‹ deutet an, daß es sich um eine Möglichkeit handelt. Der eben ausgesprochene Satz behauptet dann nur, daß das Lichtquant sich eben in gewissen Experimenten möglicherweise in Elektron und Positron zerlegen läßt. Mehr nicht.«

»Nun könnte in einem sehr energiereichen Stoß ein Lichtquant doch vielleicht auch in zwei Elektronen und zwei Positronen verwandelt werden. Würden Sie dann sagen, daß das Lichtquant virtuell auch aus diesen vier Teilchen besteht?«

»Ja, ich glaube, das wäre konsequent. Das Wort ›virtuell‹, das die Möglichkeit bezeichnet, erlaubt ja die Behauptung, daß das Lichtquant virtuell aus zwei oder vier Teilchen besteht. Zwei verschiedene Möglichkeiten schließen sich ja nicht aus.«

»Aber was gewinnt man dann noch mit einem solchen Satz?« wandte Euler ein. »Dann kann man doch gleich sagen, daß jedes Elementarteilchen virtuell aus irgendeiner beliebigen Zahl von anderen Elementarteilchen besteht. Denn bei sehr energiereichen Stoßprozessen wird schon irgendeine beliebige Zahl von Teilchen entstehen können. Das ist doch fast keine Aussage mehr.«

»Nein, so beliebig sind Zahl und Art der Teilchen denn doch nicht. Nur solche Konfigurationen von Teilchen werden als mögliche Beschreibung des einen darzustellenden Teilchens in Betracht kommen, die die gleiche Symmetrie haben wie das ursprüngliche Teilchen. Statt Symmetrie könnte man noch genauer sagen: Trans-

formationseigenschaft gegenüber solchen Operationen, unter denen die Naturgesetze unverändert bleiben. Wir haben doch schon aus der Quantenmechanik gelernt, daß die stationären Zustände eines Atoms durch ihre Symmetrieeigenschaften charakterisiert sind. So wird es eben auch bei den Elementarteilchen sein, die ja auch stationäre Zustände aus Materie sind.«

Euler war noch nicht so recht zufrieden. »Das wird doch reichlich abstrakt, was Sie jetzt sagen. Es käme wohl mehr darauf an, sich Experimente auszudenken, die anders ablaufen, als man bisher angenommen hätte, und zwar deshalb anders, weil die Lichtquanten virtuell aus Teilchenpaaren bestehen. Man würde doch vermuten, daß man wenigstens qualitativ vernünftige Resultate bekommt, wenn man das Bild vom Doppelsternsystem einen Moment ernst nimmt und fragt, was nach der früheren Physik daraus folgen sollte. Zum Beispiel könnte man sich für das Problem interessieren, ob zwei Lichtstrahlen, die sich im leeren Raum kreuzen, wirklich so ungehindert durcheinander hindurchgehen, wie man bisher immer angenommen hat und wie die alten Maxwellschen Gleichungen es fordern. Wenn in dem einen Lichtstrahl virtuell, das heißt als Möglichkeit, Paare von Elektronen und Positronen vorhanden sind, so könnte der andere Lichtstrahl doch an diesen Teilchen gestreut werden; also müßte es eine Streuung von Licht an Licht geben, eine gegenseitige Störung der beiden Lichtstrahlen, die man aus der Diracschen Theorie ausrechnen könnte und die auch experimentell zu beobachten wäre.«

»Ob man so etwas beobachten kann, hängt natürlich davon ab, wie groß diese gegenseitige Störung ist. Aber Sie sollten ihre Wirkung unbedingt ausrechnen. Vielleicht finden die Experimentalphysiker dann auch Mittel und Wege, sie nachzuweisen.«

»Eigentlich finde ich diese Philosophie des ›als ob‹, die hier betrieben wird, doch sehr merkwürdig. Das Lichtquant verhält sich in vielen Experimenten so, ›als ob‹ es aus einem Elektron und einem Positron bestünde. Es verhält sich auch manchmal so, ›als ob‹ es aus zwei oder noch mehr solchen Paaren bestünde. Scheinbar gerät man in eine ganz unbestimmte verwaschene Physik hinein. Aber man kann aus der Diracschen Theorie doch die Wahrscheinlichkeit dafür, daß ein bestimmtes Ereignis eintritt, mit großer Genauigkeit berechnen, und die Experimente werden das Ergebnis schon bestätigen.«

...

Euler berechnete zusammen mit einem anderen Mitglied meines Seminars, Kockel, die Streuung von Licht an Licht, und obwohl der experimentelle Nachweis hier nicht so direkt geführt werden konnte, besteht heute wohl kein Zweifel mehr daran, daß es die von Euler und Kockel behauptete Streuung wirklich gibt.”

English translation:

“And so we would talk about atomic physics instead and, in particular, about the possible consequences of Dirac’s discovery, and the transformation of energy into matter.

“Dirac has shown,” Euler said, “that when a light quantum flies past an atomic nucleus, it may change into a pair of particles –an electron and a positron. Does this

mean the light quantum itself consists of an electron and a positron? In that case, it would be a kind of double star, one in which the electron and positron revolve about each other. Or is this a false picture?"

"I don't think it's very convincing. You see, the mass of a double star cannot be much smaller than the sum of the masses of its constituent parts. Nor would it necessarily have to move through space with the velocity of light. There is no reason why it should never come to rest."

"But, what *can* we say about the light quantum in this context?"

"Perhaps that is virtually made up of an electron and a positron. The word 'virtually' means that we are dealing with a possibility. In that case, my assertion means no more than that the light quantum may, in certain experiments, split up into an electron and a positron—noting more."

"Well, in a very high-energy impact, a light quantum might easily be transformed into two electrons and two positrons. Does that mean that it is virtually made up of these four particles as well?"

"Yes, I believe that would be the consistent view. Since the term 'virtually' denotes possibilities, we are entitled to say that the light quantum is virtually made up of two of four particles. Two different possibilities do not necessarily exclude each other."

"But what is the advantage of this sort of assertion?" Euler asked. "We might equally well say that every elementary particle is virtually made up of any number of other particles. After all, any number of particles might be created during high-energy collisions. In that case our statement says very little indeed."

"I should not put it like that, for, you see, the number and type of particles are not as arbitrary as all that. Only such configurations may be considered possible descriptions of a particular particle as have the same symmetry as the original particle. Instead of 'symmetry,' we might say more precisely: transformation characteristics under operations that leave the physical laws unchanged. After all, quantum mechanics has taught us that the stationary states of an atom are characterized by their symmetries. Things are probably much the same with elementary particles, which, when all is said and done, are simply stationary states of matter."

Euler was still not fully satisfied. "The whole argument is a bit too abstract for my liking. What we probably ought to be doing is to think up experiments that would lead to unexpected results, and this precisely because light quanta are virtually made up of pairs of particles. It seems reasonable to assume that we should obtain at least qualitatively satisfactory results if we stuck to the model of the double star, and asked what conclusions orthodox physics would draw. For instance, we could investigate whether or not two light rays crossing in empty space really pass through each other with no interaction, as we have assumed until now, and as the old Maxwellian equations demand. If pairs of electrons and positrons are virtually present, i.e., contained as a possibility, in a light ray, then another light ray ought to be scattered by these particles; hence there would be deflection of light by light, that is, an interaction of the two light rays. We ought to be able to demonstrate its existence and to calculate its extent from Dirac's theory."

“Whether of not we would be able to observe it would, of course, depend on the intensity of the mutual perturbations. But by all means calculate the effect. Perhaps experimental physicists will then discover ways and means of corroborating your results.”

“I really think this whole ‘as if’ philosophy is terribly odd. The light quantum is said to behave in some experiments as if it consisted of an electron and a positron. But at other times it apparently behaves as if it consisted of two or more such pairs. The result is a wishy-washy kind of physics. And yet we can use Dirac’s theory to calculate the probability of a certain event with great precision, and find that experiments will confirm the results.”

...

Meanwhile Euler, together with another of my pupils, B. Kockel, determined the scattering of light by light, and although no experimental verification was possible here, there is little doubt today that the scattering effect they deduced is a fact.”

Appendix F

Extract from a letter of September 27, 1935 from W. Pauli to V. Weisskopf (Reprinted as letter [421a] in [231], pp. 769-771, specifically p. 770. English transl.: K.S.. The English translation is appended after the original German text.):

“...Pryce^[...] und ich haben beschlossen, wenigstens zu versuchen, die Häufigkeit des Streuprozesses zweier Lichtquanten aneinander *ohne* die Euler-Kockelsche Einschränkung hinsichtlich Kleinheit der Lichtfrequenzen gegen mc^2/h auszurechnen.

Ich bin mir darüber klar, daß das Problem nicht leicht ist. Nicht nur sind die auftretenden Integrale von einem komplizierteren Typus als das erwähnte Heisenberg-Serbersche Integral, sondern vor allem bekommt man zunächst die praktisch unendlich große Zahl von $6 \times 24 = 144$ Termen (nach Euler-Kockel). Dennoch scheint mir das Problem nicht so ganz hoffnungslos zu sein. Ich habe den Eindruck, daß die riesigen Koeffiziententabellen von Euler-Kockel nicht so sehr dem Subtraktionsformalismus als solchem zur Last zu legen sind als vielmehr dem Umstand, daß viel zu früh nach den g 's entwickelt wurde. Letzteres scheint mir nicht nur in formaler Hinsicht ungeschickt, sondern es wird auch in physikalischer Hinsicht durch die Beschränkung auf kleine v auch die Ähnlichkeit mit der Bornschen Theorie³[K.S.: orig. footn.] vorgetäuscht; aber diese liefert keine Paarerzeugung! Das ist so wie wenn man eine Theorie hätte, die (für lange Wellen) Dispersion liefert, aber keine Absorption! Nach meiner Meinung haben Euler und Kockel in ihrer publizierten Note⁴[K.S.: orig. footn.] ein viel zu starkes Gewicht gelegt auf den Vergleich mit diesem Ungetüm von einer Pseudotheorie!

Nun aber zurück zur mathematischen Seite des Problems. Was mir eine gewisse Hoffnung gibt, durchzukommen ist der Umstand, daß das Problem sich sehr vereinfachen läßt durch Einführung eines speziellen Bezugssystems. Abgesehen von dem singulären Sonderfall, daß die beiden Lichtquanten im Anfangszustand exakt in derselben Richtung laufen (in welchem Sonderfall aus Ihnen sehr bekannten Gründen die Streuwahrscheinlichkeit ohnehin exakt verschwinden dürfte), kann man immer ein Bezugssystem einführen, wo $g_1 + g_2 = 0$ (d. h. die Lichtquanten laufen in entgegengesetzter Richtung aufeinander zu und ihre Frequenzen sind gleich). Und in diesem Normalkoordinatensystem werden die Formeln viel einfacher. Vielleicht gelingt es da, die Zahl 6×24 der Terme so weit zu reduzieren, daß man die Streuwahrscheinlichkeit wirklich ausrechnen kann (für beliebige Frequenz der Lichtquanten). – Zunächst rechnet Pryce allgemein die Spur aus (womit er fast fertig ist) und dann werden wir weiter sehen, ob wir durchkommen. Princeton ist ein Ort, der sehr geeignet ist, um komplizierte Integrale auszurechnen. ...”

English translation:

“...Pryce^[...] and I have decided to at least try to calculate the probability of the scattering process of two quanta of light off each other *without* the Euler-Kockel

³[K.S.: orig. footn.] Vgl. Born und Infeld (1934). (K.S.: cf. [28])

⁴[K.S.: orig. footn.] Euler und Kockel (1934). (K.S.: cf. [45])

restriction with respect to the smallness of the light frequency in comparison with mc^2/h .

I am aware of the fact that the problem is not an easy one. Not only the integrals occurring are of a more complicated type than the Heisenberg-Serber integral mentioned but primarily one gets the practically infinitely large number of $6 \times 24 = 144$ terms (according to Euler-Kockel). Despite this it seems to me that the problem is not that hopeless. It is my impression that the huge coefficient tables of Euler-Kockel are not that much due to the subtraction formalism as such as rather due to the problem that much too early an expansion in the g 's has been applied. To me, the latter does not only seem to be awkward but also in physical respect by virtue of the restriction to small v a similarity to the Born theory³[*K.S.: orig. footn.*] is being pretended to exist; but it does not yield any pair creation! This is the same as if one would have a theory that (for long waves) yields dispersion but not any absorption! In my view, in their published note⁴[*K.S.: orig. footn.*] Euler and Kockel have put much to strong emphasis on the comparison with this monstrosity of a pseudo-theory.

But now back to the mathematical side of the problem. What provides me with some hope to get through is the fact that the problem can considerably be simplified by introducing a special reference system. Disregarding the singular special case that two quanta of light in the initial state propagate in exactly the same direction (in which special case for reasons very well known to you the scattering probability should vanish exactly anyway), one can always introduce a reference system where $g_1 + g_2 = 0$ (i.e., the quanta of light propagate in directions opposite to each other and their frequencies are the same). And in this system of normal coordinates the formulas are becoming much easier. Perhaps one can succeed in reducing the number 6×24 of terms that much that one can really calculate the scattering probability (for arbitrary frequency of the quanta of light). – First Pryce calculates the trace in general (what he almost is done with) and then we will see further if we get through. Princeton is a place which is very much suited for calculating complicated integrals. ...”

³[*K.S.: orig. footn.*] K.S.: see footnote 3 on page 53.

⁴[*K.S.: orig. footn.*] K.S.: see footnote 4 on page 53.

Appendix G

Historic recollections by V. F. Weisskopf related to the articles [80, 98] (The passages are quoted from [244] with kind permission of the American Institute of Physics.):

Weisskopf: ... Now I come to the work about the vacuum polarization, which I published in the Danish Academy^[ff]*[footn. K.S.]*. I very rarely complain about not getting enough recognition because I think I've received in my life more recognition than I really deserved, but for this paper I don't get enough recognition. In my opinion, this paper is really the beginning of re-normalization; and when you read it you'll find, it. The first purpose of the work, and the one for which it is perhaps best known, was the recalculation in a very much simpler way of the Euler-Heisenberg vacuum polarization for slowly varying fields, which Wick mentioned today. It was really only a recalculation, although with very nice methods suggested by Pauli. This is why I didn't publish it in *Zeitschrift für Physik*; I thought I should have something in the Danish Academy, and so I published this there because it was really only a simplification.

However, this same paper contains a study in which I was not very sure of myself, which is also why I published it in the Danish Academy, but which excited me very, very much. The study was to show that all the infinities that come about in calculation are in fact infinities that you cannot measure, namely, infinities of charge, infinities of mass, and infinities of what I called there the "dielectric constant of the vacuum." As it says there explicitly, one could assume that the total result is given by nature and one can forget about these infinities. What is given there is the recipe for re-normalization. Again I say that if I had had my Sommerfeld training I could have done much more with this. In fact I used rather primitive methods there to prove my point and perhaps that is another reason that the work was not too well known. But you could directly quote from there a recipe for re-normalization. There is a paper by Dirac in which he says the same thing for the mass, I think, but I knew that already. I directly say there are three magnitudes which are essentially nowadays the three "Z's", the three infinities; and I say in there that these are the three infinities; but is characteristic that were they finite, you wouldn't be able to notice them.

This is why you can forget them, and I say this explicitly in this paper. I did this work rather independently; Pauli was of course interested in it and he advised me in many things. By the way, I was always a sloppy man, and this paper was one that has the greatest number of calculating mistakes of any paper ever written; it's terrific what's wrong in there, but in principle it was right. ...

...

Heilbron: There's just one paper, isn't there, in the Danish Academy?

Kuhn: Let's get that chronology right on the tape now.

^[ff]K.S.: see ref. [98].

Weisskopf: The papers are, in order, as follows: '34, the self energy paper ordered by Pauli^[gg]^[footn. K.S.]; then the boson paper, the Klein-Gordon equation with Pauli^[hh]^[footn. K.S.]; and in '36, the polarization of the vacuum and the renormalization proposal^[ii]^[footn. K.S.], which was done partially with Pauli but was really written in Copenhagen. That's why it was published here.

...

Weisskopf: ... I did another paper with Kemmer on the scattering of light by light. I think this was in a letter to Nature^[jj]^[footn. K.S.]. We connected the scattering of light by light with the Delbrück scattering. Today it's a triviality; one light quantum is replaced by the Coulomb field, but at that time it was not so trivial. That thing was in fact the beginning of this later paper on the vacuum polarization where very similar problems are treated. That just shows that these were the things one was worrying about. No, I know what it was about; it had deeper significance, that letter to the editor. Euler and Kockel at that time, under Heisenberg, calculated the scattering of light by light, but had to do a lot of subtracting because there were a great many terms that were infinite. They did this in the usual clever way and got the result. And Kemmer and I showed that you can do the calculation without raking any subtractions, because you can show that it is equivalent to the Delbrück scattering, replacing one light quantum by the Coulomb field, and the Delbrück scattering doesn't diverge. This is a special case of what one does now every day if one calculates these things. ... ”

^[gg]K.S.: see ref. [245].

^[hh]K.S.: see ref. [247].

^[ii]K.S.: see ref. [98].

^[jj]K.S.: see ref. [80].

Appendix H

Historic recollections by N. Kemmer [249], pp. 171-172 (English transl.: K.S.. The English translation is appended after the original German text.):

“Weißkopf war damals auch in Zürich und arbeitete am Selbstenergieproblem in diesem neuen Rahmen^[kk]^[footn. K.S.]. Er fand, daß die neue Theorie immer noch an Divergenzen litt, die aber nur schwach (logarithmisch) waren [11]^[ll]^[footn. K.S.]. Ich beteiligte mich ein wenig an diesen Rechnungen und erlebte es mit, als Nachricht von Heisenberg ankam, daß zwei seiner Schüler, Euler und Kockel, aufgrund dieser Theorie die „Streuung von Licht an Licht“ berechnet hatten [12]^[mmm]^[footn. K.S.]. Dies war ein wohldefinierter, kleiner, aber im Prinzip beobachtbarer Effekt, der als Abweichung von den Maxwellschen Gleichungen beschrieben werden konnte. Die Autoren mußten bis zur vierten Ordnung in der Störungstheorie rechnen und ohne vollständige Rechtfertigung gewisse divergente Integrale vernachlässigen. Die Berechnung war enorm lang, obwohl sie nur für einige Spezialfälle durchgeführt wurde. Heisenberg war aber überzeugt, daß trotz der Unvollkommenheit der Theorie das Endresultat relativistisch invariant sein müsse. Unter dieser Annahme konnten Euler und Kockel ihr Ergebnis in der Form eines Zusatzes zur Lagrange-Funktion des Maxwell-Feldes angeben

$$L = \frac{1}{8\pi} (E^2 - B^2) + \frac{C}{360\pi^2} \left[(E^2 - B^2)^2 + 7 (\vec{E}\vec{B})^2 \right] \quad (2)$$

(in den damals üblichen Einheiten), wo

$$C = \left(\frac{e^2}{\hbar c} \right)^2 \cdot \left(\frac{\hbar}{mc} \right)^3 \cdot \frac{1}{mc^2} .$$

Unter Weißkopfs Leitung und mit Dr. Guido Ludwig als Mitarbeiter berechnete ich einen verwandten Effekt [13]^[nnn]^[footn. K.S.], nämlich die (Delbrück-) Streuung von Licht an einem elektrostatischen Potential. In der Abwesenheit invarianter Rechenmethoden war es nicht offensichtlich, daß unser Resultat mit dem Euler-Kockelschen übereinstimmen würde. Ich fand aber genau dieselben Koeffizienten wie in (2). So lernte ich, daß man Invarianzargumenten trauen konnte – auch für eine ganz unvollkommene Theorie!”

^[kk]K.S.: i.e., hole theory, electron-positron theory [249], p. 171.

^[ll]K.S.: Reference in the original article [249]: [11] *Weisskopf, V.*, Z. Phys. **98**, 27 (1934), Berichtigung [correction] **90**, 817 (1934). (K.S.: cf. our ref. [245])

^[mmm]K.S.: Reference in the original article [249]: [12] *Euler, H.*, u. *B. Kockel*, Naturwissenschaften **23**, 246 (1935). (K.S.: cf. our ref. [45]. For the letters of W. Heisenberg to W. Pauli reporting about the progress of this calculation see letter [374], pp. 331-333, and letter [393], pp. 358-360, in [200].)

^[nnn]K.S.: Reference in the original article [249]: [13] *Kemmer, N.*, Helv. Phys. Acta **10**, 112 (1937) u. *G. Ludwig*, Helv. Phys. Acta **10**, 182 (1937). (K.S.: cf. our refs. [111, 112].)

English translation:

“At that time, Weiskopf also stayed in Zurich and worked on the self-energy problem within this new framework^[oo]*[footn. K.S.]*. He discovered that the new theory still suffered from divergencies, however weak (logarithmic) ones [11]^[pp]*[footn. K.S.]*. I took some part in these calculations and witnessed when news from Heisenberg arrived that two of his students, Euler and Kockel, had calculated the “scattering of light by light” on the basis of this theory [12]^[qq]*[footn. K.S.]*. This was a well-defined, small, but observable in principle effect which could be described as a deviation from the Maxwell equations. The authors had to perform calculations up to the fourth order of perturbation theory and to neglect certain integrals without complete justification for doing so. The calculation was enormously long although it was done for some special cases only. However, Heisenberg was convinced that the final result had to be relativistically invariant despite the imperfection of the theory. Relying on this assumption, Euler and Kockel could present their result in terms of an addition to the Lagrange function of the Maxwell field

$$L = \frac{1}{8\pi} (E^2 - B^2) + \frac{C}{360\pi^2} \left[(E^2 - B^2)^2 + 7 (\vec{E}\vec{B})^2 \right] \quad (2)$$

(in the units customary then) where

$$C = \left(\frac{e^2}{\hbar c} \right)^2 \cdot \left(\frac{\hbar}{mc} \right)^3 \cdot \frac{1}{mc^2} .$$

Under the guidance of Weiskopf and with Dr. Guido Ludwig as collaborator I calculated a related effect [13]^[rr]*[footn. K.S.]*, namely the (Delbrück-) scattering of light by an electrostatic potential. In the absence of invariant calculational methods it was not obvious that our result would agree with the one by Euler-Kockel. However, I obtained the same coefficients as in (2). This way I learned that one can trust invariance arguments – also for a fairly imperfect theory!”

^[oo]K.S.: see footnote [kk] on page 57.

^[pp]K.S.: see footnote [ll] on page 57.

^[qq]K.S.: see footnote [mm] on page 57.

^[rr]K.S.: see footnote [nn] on page 57.

Appendix I

Historic recollections by A. I. Akhiezer about his kand. diss. (Ph.D. Thesis) research leading to the publications [62], [99] (The following passage is quoted from [250], pp. 36-38; for a somewhat shorter version also see [251], pp. 107-109, pp. 77-78 of the English translation.):

“Photon-photon scattering

Alter I passed the *theorminimum* Landau gave me a research subject. I was to study the scattering of light by light. Landau had earlier given this subject to Rosenkevitch, who was another of his students. He and Landau were going to investigate the scattering of light by light in the low-frequency domain, where the photon energy is much less than the mass of the electron. (Electron-positron pair production makes the electron mass an important parameter here.) But Rosenkevitch failed to solve this problem, and besides, there soon appeared a remarkable paper by Werner Heisenberg and his student Hans Euler that gave a complete solution. Landau was upset that the problem “got away” from him.

That happened just when I was passing the *theorminimum*. So Landau decided to test my abilities with this difficult problem of photon-photon scattering, but this time in the high-frequency domain where the photon energy exceeds the electron mass. The phenomenon in question was a fourth-order effect in perturbation theory. To calculate the scattering probability in that approximation, one should really use Dirac’s relativistic theory of the electron. But at that time, perturbation theory had been worked out only in nonrelativistic quantum mechanics. It was a hard task to calculate the scattering amplitude, because one had to take into account numerous intermediate states without overlooking any.

Eventually I had the amplitude written down and showed it to Landau. And that’s when the first, and last, blowup in my relations with Landau erupted. He didn’t like the nonrelativistic form of the probability amplitude. Besides, it was written in terms of photon vector potentials rather than the electromagnetic fields themselves. Therefore the expression was not gauge invariant. So Landau started getting angry, but he couldn’t make my expression relativistically and gauge invariant either. Nonetheless I strongly objected to his assertion that it couldn’t be done within the existing perturbation theory.

This conversation had become rather unpleasant by the time Rosenkevitch came in. Sizing up the situation, Rosenkevitch took two candlesticks standing on the desk, gave one to me and the other to Landau, and said, “Now fight it out.” Landau burst out laughing and said: “The hell with you. Do the calculations the way you want.” I understood that he couldn’t deny my general formula. So I decided that since the formula was correct, it should lead eventually to a properly invariant expression.

‘The best gammists’

It was then that Pomeranchuk began to work with me. The calculations were horrifying exercises in the manipulation of gamma matrices. Evgenii Lifshitz, coauthor of the famous Landau textbooks, joked that Pomeranchuk and I were “the best gammists in the Soviet Union.” (The joke, which loses something in translation, depends on the similarity of “gamma” to an indecorous Russian word for excrement.)

Landau was insistent that we check the gauge invariance of our result. To that end we replaced the vector potential with the field itself. We got 144 terms, which had to sum to zero. Chuk (that’s what we used to call Pomeranchuk) and I held our breaths as we did the sum. I can’t remember any other moment when I’ve been as happy as I was when I finally saw that the sum did indeed vanish. We immediately ran to Landau. He was happy too. Soon we had completed the calculation, getting a relativistically invariant expression for the scattering cross section in the high-frequency domain. We also succeeded in removing possible divergences simply by exploiting the gauge invariance of the amplitude.

We described our work to Victor Weisskopf, who had recently arrived in Kharkov. He was very pleased with it. At Landau’s seminar Weisskopf reported on his own work on the nonlinear electrodynamics of the vacuum, and he gave me the galley proofs of his paper. Though he obtained the same result Heisenberg and Euler had gotten earlier, Weisskopf’s method was beautiful. Landau was full of praise.

Landau suggested that we publish our results in the British journal *Nature*. Pomeranchuk and I wrote a brief paper, and Fritz Houtermans, a German émigré at our institute, promptly translated it into English. (For more on the peripatetic Houtermans see the letter by Victor Frenkel on page 104 and the article by Iosif Khriplovich in *PHYSICS TODAY*, July 1992, page 29.) Without bothering to get official permission, we took the article to the post office, and soon it was published under our three names.

Then Pomeranchuk and I concerned ourselves with the problem of the coherent scattering of gamma rays in the Coulomb field of the nucleus. This was another problem to be solved in the framework of Dirac theory. In 1937 there was a nuclear physics conference in Moscow. Pauli came, and Landau introduced Pomeranchuk and me to him. We familiarized Pauli with our work on light-light scattering and gamma scattering off nuclei. He approved. It was all a brilliant success for Landau’s “school,” because it indicated that we were dealing with the most important theoretical problems of the day.”

Appendix J

Historic recollections by László Tisza (MIT, Cambridge, MA, USA) about the kand. diss. (Ph.D. Thesis) research by A. I. Akhiezer leading to the publications [62], [99] (The following passage is quoted from [252], pp. 307-308.):

“III. SCATTERING OF LIGHT BY LIGHT

P. A. M. Dirac published in 1928 a remarkable paper in which he established a relativistically invariant form for quantum mechanics. This theory was at first plagued by a curious difficulty, it called for negative energy states for the electron. Dirac later showed that instead of reversing the sign of the energy, one might reverse the sign of its charge. At first this did not improve the situation much until the discovery of the positron in 1932 changed the difficulty into a most remarkable prediction of any theory. The use of the Dirac equation still was not obvious. The 4×4 gamma matrices involved in this equation made manipulations difficult. There was an International Theoretical Conference in May 1934 at the UFTI^[ss]^[footn. K.S.] where these questions were discussed. By 1935 pair production problems were rather standard although they remained labor-intensive.

Shura^[tt]^[footn. K.S.] and myself completed about the same time our theoretical minimum. We turned to Landau to start us off in research. The Dirac perturbation theory was ready for use, mainly as a result of the so-called Casimir method for handling gamma matrices; we all started along this line. Shura got the assignment of the «scattering of light by light». This was a very difficult problem of fourth-order perturbation calculus. After a while he and Chuk^[uu]^[footn. K.S.] decided to join forces. This was a fortunate decision. I vividly remember the two sitting side by side at two desks, working through long sequences of calculations. They were doing the same step independently and proceeded to the next step only after their results checked. They reminded me of the famous cartoon characters: Max and Moritz by Wilhelm Busch, two mischievous boys, one of them with a funny hairdo. This one was clearly Chuk. He was always full of ideas, be it something funny, or some important physics. Landau said that Chuk reminded him of his younger self. They had both striking ironical faces, but Dau^[vv]^[footn. K.S.] was tall and Chuk was short and boyish. Shura has his own benign sense of humor. The two were sitting at their desks, constantly joking and cursing while doing their ghastly calculations.

Eventually they finished and convinced Dau that all was right. (See Akhiezer’s paper in *Physics Today*^[ww]^[footn. K.S.] of 1994.) At the time the foremost expert Victor Weisskopf was visiting and he endorsed the work as well. When the secretary received the manuscript for typing, the title «Scattering of light by light» provoked

^[ss]K.S.: UFTI = Ukrainian Physico-Technical Institute.

^[tt]K.S.: Shura = A. I. Akhiezer.

^[uu]K.S.: Chuk = I. Ya. Pomeranchuk.

^[vv]K.S.: Dau = L. D. Landau.

^[ww]K.S.: see ref. [250] (and our App. I).

her perceptive remark: «do chevo zhe dumalis!» In free translation: «What will they dream up next?»

His hard work paid off, Akhiezer passed his grueling test to be ready to become Landau's successor in due time.”

Main list of references

[For references in Cyrillic letters, we apply the (new) *Mathematical Reviews* transliteration (transcription) scheme (to be found at the end of index issues of *Mathematical Reviews*). Georgian references are transliterated on the basis of ISO 9984. Chinese references are transcribed using Hanyu Pinyin with tone marks. For Japanese references the modified Hepburn transcription system is used.]

1871

- [1] J. J. Müller: Über die Fortpflanzung des Lichtes [On the propagation of light]. *Annalen der Physik und Chemie*, 2. Series, **145** (**221** of all series altogether):1(1871)86-132 (DOI: [10.1002/andp.18722210106](https://doi.org/10.1002/andp.18722210106)). [in German] – Cited at pp.: [10](#), [91](#)

1875

- [2] F. Lippich: Über die behauptete Abhängigkeit der Lichtwellenlänge von der Intensität [On the asserted dependence of the wave length of light on its intensity]. *Sitzungsberichte der Mathematisch-Naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften, II. Abtheilung (Wien)* **72**:2(1875)355-365 (The volume is freely available online at the Internet Archive site: <http://www.archive.org/details/sitzungsbericht461klasgoog>). [in German] – Cited at pp.: [10](#), [90](#)

1887

- [3] H. Ebert: Ueber die Abhängigkeit der Wellenlänge des Lichtes von seiner Intensität [On the dependence of the wave length of light on its intensity]. *Annalen der Physik und Chemie*, 3. Series, **32** (**268** of all series altogether):11(1887)337-383 (DOI: [10.1002/andp.18872681102](https://doi.org/10.1002/andp.18872681102)). [in German] Also separately issued as: H. Ebert: *Über die Abhängigkeit der Wellenlänge des Lichtes von seiner Intensität. Inaugural-Dissertation zur Erlangung der Doctorwürde der hohen Philosophischen Facultät der Universität Erlangen eingereicht von Hermann Ebert aus Leipzig*. Druck von Metzger & Wittig, Leipzig, 1887. (contains a short CV at the end) – Cited at pp.: [10](#), [17](#), [18](#), [37](#), [88](#)

1904

- [4] Th. E. Doubt: The effect of the intensity upon the velocity of light. *Physical Review*, 1. Series, **18**:3(1904)129-134 (DOI: [10.1103/PhysRevSeriesI.18.129](https://doi.org/10.1103/PhysRevSeriesI.18.129)). Also separately issued as: Th. E. Doubt: *The Effect of the Intensity Upon the Velocity of Light*. Press of the New Era Printing Company, Lancaster, PA, 1904 (Ph.D. Thesis, University of Chicago, Chicago, IL, 1904; the volume is freely available online at the Haithi Trust Digital Library site: <http://hdl.handle.net/2027/mdp.39015067268766>). German translation: Die Wirkung der Lichtintensität auf die Lichtgeschwindigkeit. *Physikalische*

Zeitschrift 5:15(1904)457-460 (The volume is freely available online at the Internet Archive site:

<http://www.archive.org/details/physikalischeze00borngoog>). – Cited at pp.: 10, 88

1912

- [5] G. Mie: Grundlagen einer Theorie der Materie. Erste Mitteilung: *Annalen der Physik*, 4. Series, **37** (**342** of all series altogether):3(1912)511-534 (DOI: [10.1002/andp.19123420306](https://doi.org/10.1002/andp.19123420306)), Zweite Mitteilung: *ibid.*, 4. Series, **39** (**344** of all series altogether):11(1912)1-40 (DOI: [10.1002/andp.19123441102](https://doi.org/10.1002/andp.19123441102)), Dritte Mitteilung: *ibid.*, 4. Series, **40** (**345** of all series altogether):1(1913)1-66 (DOI: [10.1002/andp.19133450102](https://doi.org/10.1002/andp.19133450102)). [in German] Partial English translation: Foundations of a theory of matter (excerpts). In: J. Renn, M. Schemmel (Eds.): *Gravitation in the Twilight of Classical Physics. The Promise of Mathematics*. Volume 4 of: J. Renn (Ed.): *The Genesis of General Relativity*. Boston Studies in the Philosophy of Science, Vol. 250. Springer, Dordrecht, 2007 (DOI: [10.1007/978-1-4020-4000-9](https://doi.org/10.1007/978-1-4020-4000-9)), pp. (vol. 4) 633-697 (DOI: [10.1007/978-1-4020-4000-9_36](https://doi.org/10.1007/978-1-4020-4000-9_36)). – Cited at pp.: 21, 22, 91

1925

- [6] K. Schaposchnikow: Über Zusammenstöße von Lichtquanten [On collisions of light quanta]. *Zeitschrift für Physik* **33**:1(1925)706-709 (DOI: [10.1007/BF01328356](https://doi.org/10.1007/BF01328356)). [in German] – Cited at pp.: 5, 92

1926

- [7] L. de Broglie: *Ondes et Mouvements*. Collection de Physique Mathématique, Vol. 1. Gauthier-Villars et C^{ie} Éditeurs, Paris, 1926. Reprinted: Éditions Jacques Gabay, Sceaux, 1988. [in French] – Cited at pp.: 5, 41, 87

1928

- [8] С. И. Вавилов [S. I. Vavilov]: Замечания об эмпирической точности оптического принципа суперпозиции [Zamechaniya ob èmpiricheskoï tochnosti opticheskogo printsipa superpozitsii]. *Журнал Русского Физико-Химического Общества при Ленинградском Университете, Часть Физическая* [*Zhurnal Russkogo Fiziko-Khimicheskogo Obshchestva pri Leningradskom Universitete, Chast' Fizicheskaya*] **60**:6(1928)555-563. [in Russian, German summary] The article is the written version of a talk delivered by S. I. Vavilov at the VI. Congress of Russian Physicists. For the conference abstract see [229], p. 47, item 80. The article is reprinted in (changed footnote numbering, German summary omitted): [165], item 24, pp. 234-241. English translation (by K.S.): S. I. Vavilov: *Remarks on the empirical accuracy of the optical superposition principle* [Engl. transl. of *Zh. Russk. Fiz.-Khim. Obshch., Ch. Fiz.* **60**(1928)555].

arXiv:1708.06817 (<https://arxiv.org/abs/1708.06817>),
<hal-01599214> (<https://hal.archives-ouvertes.fr/hal-01599214>),
2017. – Cited at pp.: 3, 4, 5, 8, 9, 19, 32, 36, 93

- [9] L. Rosenfeld, E. E. Witmer: Über die Hohlraumstrahlung und die Lichtquantenhypothese [On cavity radiation and the light quantum hypothesis]. *Zeitschrift für Physik* **47**:7-8(1928)517-521 (DOI: 10.1007/BF01340337). [in German] – Cited at pp.: 5, 28, 92, 94

1929

- [10] A. L. Hughes, G. E. M. Jauncey: Collisions and coalescence of light quanta. *Physical Review* **33**:2(1929)290. The abstract is item 51 of: Proceedings of the American Physical Society, Minutes of the New York Meeting, December 27, 28, 29 and 31, 1928. *Physical Review* **33**:2(1929)275-296 (DOI: 10.1103/PhysRev.33.275). – Cited at pp.: 5, 89
- [11] К. Шапошников [K. Shaposhnikov]: По поводу статьи С. И. Вавилова: «Замечания об эмпирической точности оптического принципа суперпозиции» [Po povodu stat'i S. I. Vavilova: «Zamechaniya ob èmpiričeskoj točnosti optičeskogo printsipa superpozitsii»]. *Журнал Русского Физико-Химического Общества при Ленинградском Университете, Часть Физическая* [Zhurnal Russkogo Fiziko-Khimicheskogo Obshchestva pri Leningradskom Universitete, Chast' Fizicheskaya] **61**:3(1929)215-217. [in Russian] English translation (by K.S.): K. Shaposhnikov: *On the article by S. I. Vavilov: "Remarks on the empirical accuracy of the optical superposition principle"*. [Engl. transl. of Zh. Russk. Fiz.-Khim. Obshch., Ch. Fiz. **61**(1929)215]. <hal-01620890> (<https://hal.archives-ouvertes.fr/hal-01620890>), 2017. – Cited at pp.: 5, 92
- [12] С. И. Вавилов [S. I. Vavilov]: Ответ на замечания К. Н. Шапошниковой [Otvet na zamechaniya K. N. Shaposhnikova]. *Журнал Русского Физико-Химического Общества при Ленинградском Университете, Часть Физическая* [Zhurnal Russkogo Fiziko-Khimicheskogo Obshchestva pri Leningradskom Universitete, Chast' Fizicheskaya] **61**:4(1929)393-395. [in Russian] English translation (by K.S.): S. I. Vavilov: *Reply to the remarks by K. N. Shaposhnikov* [Engl. transl. of Zh. Russk. Fiz.-Khim. Obshch., Ch. Fiz. **61**(1929)393]. <hal-01631223> (<https://hal.archives-ouvertes.fr/hal-01631223>), 2017. – Cited at pp.: 5, 93
- [13] W. H. Watson: The effect of a transverse magnetic field on the propagation of light in vacuo. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **125**:797(1929)345-351 (DOI: 10.1098/rspa.1929.0172, stable JSTOR URL: <http://www.jstor.org/stable/95423>). – Cited at pp.: 11, 93

1930

- [14] G. E. M. Jauncey, A. L. Hughes: An attempt to detect collisions between photons. *Physical Review* **35**:11(1930)1439. The article is item 93 of: Proceedings of the American Physical Society, Minutes of the Washington Meeting, April 24-26, 1930. *Physical Review* **35**:11(1930)1415-1446 (DOI: [10.1103/PhysRev.35.1415](https://doi.org/10.1103/PhysRev.35.1415)). – Cited at pp.: 8, 89
- [15] A. L. Hughes, G. E. M. Jauncey: An attempt to detect collisions of photons. *Physical Review* **36**:4(1930)773-777 (DOI: [10.1103/PhysRev.36.773](https://doi.org/10.1103/PhysRev.36.773)). – Cited at pp.: 8, 9, 19, 89
- [16] S. Vavilov: On the attempt to detect collisions of photons. *Physical Review* **36**:10(1930)1590 (DOI: [10.1103/PhysRev.36.1590](https://doi.org/10.1103/PhysRev.36.1590)). Reprinted in Russian: О попытке обнаружить соударения фотонов [О попытке обнаружить соударения фотонов]. In: [165], item 32, p. 302. – Cited at pp.: 8, 9, 19, 93

1931

- [17] H. Bauer: Betreffs Nachweis der Möglichkeit des Impuls- und Energieaustausches zwischen Lichtquanten [Concerning proof of possibility of momentum and energy exchange between quanta of light]. *Zeitschrift für Physik* **70**:1-2(1931)38-42 (DOI: [10.1007/BF01391029](https://doi.org/10.1007/BF01391029)). [in German] – Cited at pp.: 13, 31, 87
- [18] A. K. Das: On collisions of photons. *Physical Review* **37**:1(1931)94-95 (DOI: [10.1103/PhysRev.37.94](https://doi.org/10.1103/PhysRev.37.94)). – Cited at pp.: 9, 88

1932

- [19] C. C. Farr, C. J. Banwell: Velocity of propagation of light in vacuo in a transverse magnetic field. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **137**:832(1932)275-282 (DOI: [10.1098/rspa.1932.0135](https://doi.org/10.1098/rspa.1932.0135), stable JSTOR URL: <http://www.jstor.org/stable/95908>). – Cited at pp.: 12, 87, 88

1933

- [20] M. Born: Modified field equations with a finite radius of the electron. *Nature* **132**:3329(1933)282 (DOI: [10.1038/132282a0](https://doi.org/10.1038/132282a0)). – Cited at pp.: 22, 87
- [21] M. Born, L. Infeld: Electromagnetic mass. *Nature* **132**:3347(1933)970 (DOI: [10.1038/132970a0](https://doi.org/10.1038/132970a0)). – Cited at pp.: 23, 87, 89
- [22] M. Born, L. Infeld: Foundations of the new field theory. *Nature* **132**:3348(1933)1004 (DOI: [10.1038/1321004b0](https://doi.org/10.1038/1321004b0)). – Cited at pp.: 23, 87, 89

- [23] M. Delbrück: Zusatz bei der Korrektur von M. Delbrück [Note added in proof by M. Delbrück]. *Zeitschrift für Physik* **84**(1933)144. Addendum to the article: L. Meitner, H. Kösters: Über die Streuung kurzwelliger γ -Strahlen [On the scattering of shortwave γ rays]. *Zeitschrift für Physik* **84**:3-4(1933)137-144 (DOI: [10.1007/BF01333827](https://doi.org/10.1007/BF01333827)). [in German] English translation in [198], Appendix, p. 135. – Cited at pp.: [14](#), [16](#), [17](#), [33](#), [37](#), [88](#)
- [24] O. Halpern: Scattering processes produced by electrons in negative energy states. *Physical Review* **44**:10(1933)855-856 (DOI: [10.1103/PhysRev.44.855.2](https://doi.org/10.1103/PhysRev.44.855.2)). – Cited at pp.: [5](#), [29](#), [30](#), [35](#), [89](#)
- [25] F. L. Mohler: Experimental evidence for the absence of scattering of light by light. *Journal of the Optical Society of America* **23**:11(1933)386-387 (DOI: [10.1364/JOSA.23.000386](https://doi.org/10.1364/JOSA.23.000386)). – Cited at pp.: [9](#), [91](#)

1934

- [26] M. Born: On the quantum theory of the electromagnetic field. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **143**:849(1934)410-437 (DOI: [10.1098/rspa.1934.0010](https://doi.org/10.1098/rspa.1934.0010), stable JSTOR URL: <http://www.jstor.org/stable/96101>). Reprinted in: [209], vol. 2, item 73, pp. 486-513. – Cited at pp.: [22](#), [23](#), [24](#), [25](#), [87](#)
- [27] M. Born, L. Infeld: Principes de la nouvelle électrodynamique quantique [Principles of the new quantum electrodynamics]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **199**:23(1934)1297-1299 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3151h/f1297>). [in French] – Cited at pp.: [24](#), [87](#), [89](#)
- [28] M. Born, L. Infeld: Foundations of the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **144**:852(1934)425-451 (DOI: [10.1098/rspa.1934.0059](https://doi.org/10.1098/rspa.1934.0059), stable JSTOR URL: <http://www.jstor.org/stable/2935568>). Reprinted in: 1. [209], vol. 2, item 74, pp. 514-540. 2. [253], pp. 55-78. For the annotation of some misprints see [59], footnote on p. 472. Japanese translation: 日本數學物理學會誌 / ニホンスウガクブツリガツカイシ [*Nihon sūgaku butsuri gakkaiishi*]/[*Journal of the Physical Society of Japan*] **8**:8(1934)307-324 (The article is freely available online at the J-Stage site given by the DOI: [10.11429/subutsukaishi1927.8.307](https://doi.org/10.11429/subutsukaishi1927.8.307)). – Cited at pp.: [23](#), [24](#), [53](#), [87](#), [89](#)
- [29] M. Born, L. Infeld: Remarks on the paper by Frenkel on Born's theory of the electron. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **146**:859(1934)935 (DOI: [10.1098/rspa.1934.0199](https://doi.org/10.1098/rspa.1934.0199), stable JSTOR URL: <http://www.jstor.org/stable/2935507>). – Cited at pp.: [24](#), [87](#), [89](#)

- [30] M. Born, L. Infeld: On the quantization of the new field equations. I. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **147**:862(1934)522-546 (DOI: [10.1098/rspa.1934.0234](https://doi.org/10.1098/rspa.1934.0234), stable JSTOR URL: <http://www.jstor.org/stable/96309>). Reprinted in: 1. [209], vol. 2, item 75, pp. 541-565. 2. [253], pp. 79-100. – Cited at pp.: 24, 87, 89
- [31] M. Born, L. Infeld: Nowa elektrodynamika kwantowa [A new quantum electrodynamics]. Abstract in: [254], p. 15, item 2. [in Polish] – Cited at pp.: 24, 87, 89
- [32] J. Frenkel: On Born's theory of the electron. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **146**:859(1934)930-934 (DOI: [10.1098/rspa.1934.0198](https://doi.org/10.1098/rspa.1934.0198), stable JSTOR URL: <http://www.jstor.org/stable/2935506>). – Cited at pp.: 24, 88
- [33] Я. И. Френкель [Ya. I. Frenkel']: *Электродинамика. Том 1: Общая теория электричества* [*Ėlektrodinamika. Tom 1: Obshchaya teoriya èlektrichestva*]. ОНТИ, Государственное технико-теоретическое издательство [ONTI, Gosudarstvennoe tekhniko-teoreticheskoe izdatel'stvo], Leningrad, 1934 (The book is freely available online at the EqWorld site: http://eqworld.ipmnet.ru/ru/library/books/Frenkel_t1_1934ru.djvu and at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=44641607>). [in Russian] Reprinted in: Я. И. Френкель [Ya. I. Frenkel']: *Собрание избранных трудов. Том 1: Электродинамика (Общая теория электричества)* [*Sobranie izbrannykh trudov. Tom 1: Ėlektrodinamika (Obshchaya teoriya èlektrichestva)*]. Издательство Академии наук СССР [Izdatel'stvo Akademii nauk SSSR], Moscow, 1956 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=45101358>). – Cited at pp.: 23, 28, 88
- [34] W. Heisenberg: Bemerkungen zur Diracschen Theorie des Positrons. *Zeitschrift für Physik* **90**:3-4(1934)209-231 (DOI: [10.1007/BF01333516](https://doi.org/10.1007/BF01333516)), erratum *ibid.* **92**(1934)692 (DOI: [10.1007/BF01340782](https://doi.org/10.1007/BF01340782)). [in German] Reprint of the original article in: [255], pp. 132-154, erratum p. 161. English translation (by D. H. Delphenich): *Remarks on the Dirac theory of positron* (The translation is freely available online at the URL: http://neo-classical-physics.info/uploads/3/0/6/5/3065888/heisenberg-remarks_on_the_dirac_theory_of_positrons.pdf). – Cited at pp.: 30, 35, 89
- [35] G. Heller, L. Motz: Averages over portions of configuration space. *Physical Review* **46**:6(1934)502-505 (DOI: [10.1103/PhysRev.46.502](https://doi.org/10.1103/PhysRev.46.502)). – Cited at pp.: 24, 89, 91

- [36] B. Hoffmann: The new field theory. *Nature* **134**:3383(1934)322 (DOI: [10.1038/134322a0](https://doi.org/10.1038/134322a0)). – Cited at pp.: 25, 89
- [37] A. L. Hughes, G. E. M. Jauncey: On collisions between photons. *Physical Review* **45**:3(1934)217 (DOI: [10.1103/PhysRev.45.217](https://doi.org/10.1103/PhysRev.45.217)). – Cited at pp.: 9, 89

1935

- [38] T. Awano: On Born's new field theory. *Proceedings of the Physico-Mathematical Society of Japan*, 3. Series, **17**(1935)127-136 (The article is freely available online at the J-Stage site given by the DOI: [10.11429/ppmsj1919.17.0_127](https://doi.org/10.11429/ppmsj1919.17.0_127)). – Cited at pp.: 25, 87
- [39] M. Born: Quantum electrodynamics. In: J. H. Awbery (Ed.): *International Conference on Physics, London 1934, a Joint Conference Organized by the International Union of Pure and Applied Physics and the Physical Society. Papers & Discussions in Two Volumes. Vol. I. Nuclear Physics*. The Physical Society, London, 1935, pp. 19-27. A considerably abbreviated French translation has appeared under the title: *Electrodynamique quantique*, in: *Congrès International de Physique organisé par l'Union Internationale de Physique et la Physical Society, Londres 1934. II: Transmutations. Actualités Scientifiques et Industrielles* **341**(1936)15-21. – Cited at pp.: 28, 87
- [40] M. Born, L. Infeld: On the quantization of the new field theory. II. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **150**:869(1935)141-166 (DOI: [10.1098/rspa.1935.0093](https://doi.org/10.1098/rspa.1935.0093), stable JSTOR URL: <http://www.jstor.org/stable/96328>). Reprinted in: [253], pp. 101-122. – Cited at pp.: 24, 87, 89
- [41] M. Born, E. Schrödinger: The absolute field constant in the new field theory. *Nature* **135**:3409(1935)342 (DOI: [10.1038/135342a0](https://doi.org/10.1038/135342a0)). Reprinted in: [256], p. 195. – Cited at pp.: 24, 87, 92
- [42] Z. Chrapływyj: On the Lorentz equation of motion in the new electrodynamics. O równaniu ruchu Lorentza w nowej elektrodynamice. *Acta Physica Polonica* **4**:4(1935)395-404. [Polish abstract] – Cited at pp.: 24, 87
- [43] W. H. Erskine: *On the Lagrangians of the New Field Theory*. Ph. D. thesis, Johns Hopkins University, Baltimore, 1935. – Cited at pp.: 23, 88
- [44] W. H. Erskine: The Born-Infeld theory of electromagnetism. *Bulletin of the American Mathematical Society* **41**:1(1935)28, abstract no. 37 (DOI: [10.1090/S0002-9904-1935-06031-8](https://doi.org/10.1090/S0002-9904-1935-06031-8), the abstract is also freely available online at the Project Euclid site: <http://projecteuclid.org/euclid.bams/1183497961>). – Cited at pp.: 23, 88

- [45] H. Euler, B. Kockel: Über die Streuung von Licht an Licht nach der Diracschen Theorie. *Die Naturwissenschaften* **23**:15(1935)246-247 (DOI: [10.1007/BF01493898](https://doi.org/10.1007/BF01493898)). [in German] English translation (by D. H. Delphenich): *The scattering of light by light in Dirac's theory* (The translation is freely available online at the URL: http://neo-classical-physics.info/uploads/3/0/6/5/3065888/euler-koeckel_-_scattering_of_light_by_light.pdf). – Cited at pp.: 5, 10, 20, 25, 26, 30, 31, 32, 33, 53, 57, 88, 90
- [46] E. Feenberg: On the Born-Infeld field theory of the electron. *Physical Review* **47**:2(1935)148-157 (DOI: [10.1103/PhysRev.47.148](https://doi.org/10.1103/PhysRev.47.148)). – Cited at pp.: 24, 88
- [47] B. Hoffmann: Gravitational and electromagnetic mass in the Born-Infeld electrodynamics *Physical Review* **47**:11(1935)877-880 (DOI: [10.1103/PhysRev.47.877](https://doi.org/10.1103/PhysRev.47.877)). – Cited at pp.: 25, 89
- [48] B. Hoffmann: On the spherically symmetric field in relativity (III). *Quarterly Journal of Mathematics* **6**:1(1935)143-160 (DOI: [10.1093/qmath/os-6.1.149](https://doi.org/10.1093/qmath/os-6.1.149)). – Cited at pp.: 25, 89
- [49] B. Hoffmann: On the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **148**:864(1935)353-364 (DOI: [10.1098/rspa.1935.0023](https://doi.org/10.1098/rspa.1935.0023), stable JSTOR URL: <http://www.jstor.org/stable/96422>). – Cited at pp.: 25, 89
- [50] Д. Д. Иваненко [D. D. Ivanenko]: Электродинамика и теория дырок Дирака [Èlektrodinamika i teoriya dyrok Diraka]/[Electrodynamics and Dirac's theory of holes]. *Доклады Академии Наук СССР, Новая Серия [Doklady Akademii Nauk SSSR, Novaya Seriya]* – *Comptes Rendus de l'Académie des Sciences de l'URSS, Nouvelle Série* **1**:7-8(1935)464-466. [in Russian, English abstract (author name transcription: D. Iwanenko)] – Cited at pp.: 30, 89
- [51] B. Kwal: Quelques remarques sur l'électrodynamique de Born e Infeld [Some remarks on the electrodynamics of Born and Infeld]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **200**:20(1935)1656-1657 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3152t/f1656>). [in French] – Cited at pp.: 24, 90
- [52] J. Meixner: Über einige Folgerungen aus dem Born-Schrödingerschen Elektronenradius [On some consequences of the electron radius of Born-Schrödinger]. *Annalen der Physik*, 5. Series, **23** (**415** of all series altogether):5(1935)371-379 (DOI: [10.1002/andp.19354150407](https://doi.org/10.1002/andp.19354150407)). [in German] – Cited at pp.: 24, 91
- [53] K. Nikolsky: Sur le champ électromagnétique de l'électron de Dirac [On the electromagnetic field of the Dirac electron]. *Comptes Rendus Hebdomadaires*

des Séances de l'Académie des Sciences (Paris) **200**:13(1935)1086-1088 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3152t/f1086>). [in French] – Cited at pp.: 30, 91

- [54] К. В. Никольский [K. V. Nikol'skii]: О соотношении между уравнениями поля теории Борна-Инфельда и квантовым уравнением Дирака [O sootnoshenii mezhd u uravneniyami polya teorii Borna-Infel'da i kvantovym uravneniem Diraka]/[On the relation between the field equations of the Born-Infeld theory and the quantum equation of Dirac]. *Доклады Академии Наук СССР, Новая Серия [Doklady Akademii Nauk SSSR, Novaya Seriya]*– *Comptes Rendus de l'Académie des Sciences de l'URSS, Nouvelle Série* **1**:4(1935)210-212. [in Russian] German version: K. Nikolskij: Über die Beziehung zwischen den Born-Infeldschen Feldgleichungen und der Diracschen Quantengleichung. *Доклады Академии Наук СССР, Новая Серия [Doklady Akademii Nauk SSSR, Novaya Seriya]* – *Comptes Rendus de l'Académie des Sciences de l'URSS, Nouvelle Série* **1**:4(1935)212-213. – Cited at pp.: 30, 91
- [55] M. H. L. Pryce: The two-dimensional electrostatic solutions of Born's new field equations. *Proceedings of the Cambridge Philosophical Society* **31**:1(1935)50-68 (DOI: [10.1017/S0305004100012937](https://doi.org/10.1017/S0305004100012937)). – Cited at pp.: 24, 92
- [56] M. H. L. Pryce: On a uniqueness theorem. *Proceedings of the Cambridge Philosophical Society* **31**:4(1935)625-628 (DOI: [10.1017/S0305004100013645](https://doi.org/10.1017/S0305004100013645)). – Cited at pp.: 24, 92
- [57] M. H. L. Pryce: Commuting co-ordinates in the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **150**:864(1935)166-172 (DOI: [10.1098/rspa.1935.0094](https://doi.org/10.1098/rspa.1935.0094), stable JSTOR URL: <http://www.jstor.org/stable/96329>). – Cited at pp.: 24, 92
- [58] E. Schrödinger: ¿Son lineales las verdaderas ecuaciones del campo electromagnético [Are the true equations of the electromagnetic field linear]? *Anales de la Sociedad Española de Física y Química* **33**:324(1935)511-517 (For further material on this publication see the Phaidra website: <http://phaidra.univie.ac.at/o:166267>). [in Spanish, French abstract] Reprinted in: [256], pp. 209-215. – Cited at pp.: 7, 92
- [59] E. Schrödinger: Contributions to Born's new theory of the electromagnetic field. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **150**:870(1935)465-477 (DOI: [10.1098/rspa.1935.0116](https://doi.org/10.1098/rspa.1935.0116), stable JSTOR URL: <http://www.jstor.org/stable/96399>). Reprinted in: [256], pp. 196-208. – Cited at pp.: 24, 67, 92

- [60] W. H. Watson: Note on γ -transformation of electromagnetic fields. *Physical Review* **48**:9(1935)776 (DOI: [10.1103/PhysRev.48.776](https://doi.org/10.1103/PhysRev.48.776)). – Cited at pp.: 24, 93
- [61] E. J. Williams: Correlation of certain collision problems with radiation theory. *Mathematisk-Fysiske Meddelelser, Det Kongelige Danske Videnskabernes Selskab* **13**:4(1935)1-50 (The article is freely available online at the University of Southern Denmark library site: <http://www.sdu.dk/Bibliotek/matfys>). – Cited at pp.: 27, 35, 94

1936

- [62] A. Akhiezer, L. Landau, I. Pomeranchuk: Scattering of light by light. *Nature* **138**:3483(1936)206 (DOI: [10.1038/138206a0](https://doi.org/10.1038/138206a0)). Reprint: [257], item 26, p. 184 (DOI: [10.1016/B978-0-08-010586-4.50031-6](https://doi.org/10.1016/B978-0-08-010586-4.50031-6)). Reprinted in Russian: Рассеяние света на свете [Rasseyanie sveta na svete]. In: 1. [258], item 25, pp. 222-223. 2. [259], item 33, pp. 5-6. – Cited at pp.: 33, 59, 61, 86, 90, 92
- [63] M.-A. Baudot: Sur les électrodynamiques nouvelles [On the new electro-dynamical theories]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **202**:11(1936)919-921 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3154f/f919>). [in French] – Cited at pp.: 24, 93
- [64] M.-A. Baudot: Remarques sur la forme d'une fonction d'action [Remarks on the form of an action function]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **202**:13(1936)1158-1159 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3154f/f1158>). [in French] – Cited at pp.: 23, 93
- [65] Д. И. Блохинцев [D. I. Blokhintsev]: К электродинамике Борна-Инфельда [K èlektroinamike Borna-Infel'da]/[On the electrodynamics of Born-Infeld]. *Труды Физического Института [Trudy Fizicheskogo Instituta]* **1**:1(1936)136-137 (Abstract only, printed under the heading: Работы, сданные в печать сотрудникам физического института в 1935 г. [Raboty, sdannye v pechat' sotrudnikam fizicheskogo instituta v 1935 g.]/[Works, sent for publication by the staff of the institute in 1935]. No corresponding published work exists.). [in Russian] – Cited at pp.: 7, 35, 87
- [66] M. Born: On the linearization of the energy density of the electromagnetic radiation. *Proceedings of the Cambridge Philosophical Society* **32**:1(1936)102-107 (DOI: [10.1017/S0305004100018892](https://doi.org/10.1017/S0305004100018892)). – Cited at pp.: 24, 87
- [67] A. Bramley: The forces acting on high energy protons at collisions. Bartol Research Foundation Communication No. 93. *Journal of the Franklin Institute*

- 222:2(1936)141** (DOI: [10.1016/S0016-0032\(36\)90720-2](https://doi.org/10.1016/S0016-0032(36)90720-2)). – Cited at pp.: [24](#), [87](#)
- [68] M. Bronstein: Über den spontanen Zerfall der Photonen [On the spontaneous decay of photons]. *Physikalische Zeitschrift der Sowjetunion* **10:5(1936)686-688**. [in German] Reprinted in: [260], Appendix 2, pp. 184-186. – Cited at pp.: [35](#), [87](#)
- [69] Z. Chraplywy: Sur les équations de mouvement de l'électrodynamique nouvelle [On the equations of motion of the new electrodynamics]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **202:5(1936)396-397** (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3154f/f396>). [in French] – Cited at pp.: [24](#), [87](#)
- [70] Z. Chraplywy: Równanie ruchu Lorentza w nowej elektrodynamice [The Lorentz equation of motion in the new electrodynamics]. Abstract in: [261], p. 35, item 62. [in Polish] – Cited at pp.: [24](#), [87](#)
- [71] Z. Chraplywy: Atom wodoru a jednolita teoria pola [The hydrogen atom and the unitary field theory]. Abstract in: [261], p. 35, item 63. [in Polish] – Cited at pp.: [24](#), [87](#)
- [72] H. Euler: Über die Streuung von Licht an Licht nach der Diracschen Theorie [On the scattering of light by light according to Dirac's theory]. *Annalen der Physik*, 5. Series, **26 (418 of all series altogether):5(1936)398-448** (DOI: [10.1002/andp.19364180503](https://doi.org/10.1002/andp.19364180503)). [in German] Also separately issued as: H. Euler: *Über die Streuung von Licht an Licht nach der Diracschen Theorie. Inaugural-Dissertation genehmigt von der mathematisch-naturwissenschaftlichen Abteilung der Philosophischen Fakultät der Universität Leipzig*. Johann Ambrosius Barth, Leipzig, 1936 (contains a short CV at the end). The thesis reviews by W. Heisenberg and F. Hund (University of Leipzig Archive, Philosophical Faculty, Prom. 769) are reprinted in [227], pp. 125-127. – Cited at pp.: [26](#), [30](#), [31](#), [88](#)
- [73] J. Géhéniau: Sur la masse propre du photon et le tenseur électromagnétique [On the proper mass of the photon and the electromagnetic tensor]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **202:7(1936)559-560** (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3154f/f559>). [in French] – Cited at pp.: [24](#), [88](#)
- [74] W. Heisenberg, H. Euler: Folgerungen aus der Diracschen Theorie des Positrons. *Zeitschrift für Physik* **98:11-12(1936)714-732** (DOI: [10.1007/BF01397189](https://doi.org/10.1007/BF01397189)). [in German] Reprinted in: [255], pp. 162-180. English translation (by W. Korolevski, H. Kleinert): *Consequences of Dirac*

theory of the positron. [arXiv:physics/0605038](https://arxiv.org/abs/physics/0605038) (<https://arxiv.org/abs/physics/0605038>). – Cited at pp.: 26, 32, 33, 88, 89

- [75] W. Heitler: *The Quantum Theory of Radiation*. The International Series of Monographs on Physics. Oxford University Press, Oxford, 1. ed. 1936, 2. corr. ed. 1944, 3. rev. and enlarged ed. 1954. Reprint of the 3. ed.: Dover Classics of Science and Mathematics. Dover Publications, New York, NY, 1984. – Cited at pp.: 35, 89
- [76] L. Infeld: The new electrodynamics and the fine structure constant. *Nature* **137**:3468(1936)658 (DOI: [10.1038/137658b0](https://doi.org/10.1038/137658b0)). – Cited at pp.: 23, 89
- [77] L. Infeld: The new action function and the unitary field theory. *Proceedings of the Cambridge Philosophical Society* **32**:1(1936)127-137 (DOI: [10.1017/S0305004100018922](https://doi.org/10.1017/S0305004100018922)). – Cited at pp.: 23, 89
- [78] L. Infeld: Nowe funkcje działania w jednolitej teorii pola [New action function in the unitary field theory]. Abstract in: [261], p. 35, item 60. [in Polish] – Cited at pp.: 23, 89
- [79] L. Infeld: Stała 137, pole protonu, a jednolita teoria pola [The number 137, the proton field, and the unitary field theory]. Abstract in: [261], p. 35, item 61. [in Polish] – Cited at pp.: 23, 89
- [80] N. Kemmer, V. Weisskopf: Deviations from the Maxwell equations resulting from the theory of the positron. *Nature* **137**:3468(1936)659 (DOI: [10.1038/137659a0](https://doi.org/10.1038/137659a0)). – Cited at pp.: 32, 34, 55, 56, 89, 94
- [81] J. Kreisler: Uwagi o Schrödingerowskiej postaci jednolitej teorii pola Born-Infelda [Notes on Schrödinger's formulation of the unitary field theory of Born-Infeld]. Abstract in: [261], p. 37, item 68. [in Polish] – Cited at pp.: 24, 90
- [82] B. Kwal, J. Solomon: Sur une conséquence de la nouvelle électrodynamique non linéaire [On a consequence of the new nonlinear electrodynamics]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **202**:11(1936)933-934 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3154f/f933>). [in French] – Cited at pp.: 28, 90, 93
- [83] B. S. Madhava Rao: Ring-singularity in Born's unitary theory – I. *Proceedings of the Indian Academy of Sciences A* **4**:3(1936)355-376 (DOI: [10.1007/BF03045360](https://doi.org/10.1007/BF03045360), the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/4/3/355-376/viewpage.html). – Cited at pp.: 24, 90

- [84] B. S. Madhava Rao: A theorem on action functions in Born's field theory. *Proceedings of the Indian Academy of Sciences A* 4:3(1936)377-381 (DOI: 10.1007/BF03045361, the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/4/3/377-381/viewpage.html). – Cited at pp.: 23, 90
- [85] B. S. Madhava Rao: Semi-vectors in Born's field theory. *Proceedings of the Indian Academy of Sciences A* 4:4(1936)436-451 (DOI: 10.1007/BF03036386, the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/4/4/377-381/viewpage.html). – Cited at pp.: 24, 90
- [86] B. S. Madhava Rao: Complex representation in Born's field theory. *Proceedings of the Indian Academy of Sciences A* 4:5(1936)575-589 (DOI: 10.1007/BF03045716, the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/4/5/575-589/viewpage.html). – Cited at pp.: 24, 90
- [87] ი. მირცხულავა [i. mirc'xulava]: ორგანზომილებიანი პრობლემის გადაწყვეტის მეთოდი ბორნის რელატიურ ელექტროდინამიკაში [organzomilebiani problemis gadacqvets met'odi bornis relatiur elek'trodinamikaši]/[A method for the solution of the two-dimensional problem in the Born relativistic electrodynamics]. ტფილისის სახელმწიფო უნივერსიტეტის შრომები [tp'ilisis saxelmcip'o universitetis šromebi] = *Труды Тифлисского Государственного Университета* [Trudy Tiflisskogo Gosudarstvennogo Universiteta] = *Travaux de l' Université d'État de Tiflis* 2(1936)290-308 (The article is freely available online at the Tbilisi State University library site: http://digitallibrary.tsu.ge/book/!2016/shromebi/oragvelidze_shromebi_2/index.html#/302). [in Georgian] – Cited at pp.: 24, 91
- [88] W. Pauli: Remarks on the new field theory. Seminar contribution, in: [262], p. 137. – Cited at pp.: 24, 92
- [89] W. Pauli: Remarks on the quantization of the new field theory. Seminar contribution, in: [262], pp. 143-147. – Cited at pp.: 24, 92
- [90] M. H. L. Pryce: On the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* 155:886(1936)597-613 (DOI: 10.1098/rspa.1936.0122, stable JSTOR URL: <http://www.jstor.org/stable/96769>). – Cited at pp.: 24, 92
- [91] M. H. L. Pryce: The new field theory. Seminar contribution, in: [262], pp. 117-136. – Cited at pp.: 24, 92

- [92] M. H. L. Pryce: Quantization of the new field theory. Seminar contribution, in: [262], pp. 137-143. – Cited at pp.: 24, 92
- [93] С. Шубин [S. Shubin], А. Смирнов [A. Smirnov]: Простой пример из электродинамики Борна [Prostoĭ primer iz èlektrodinamiki Borna]/[A simple example from the electrodynamics of Born]. Доклады Академии Наук СССР, Новая Серия [Doklady Akademii Nauk SSSR, Novaya Seriya] **1 (X)**:2(79)(1936)65-68. [in Russian] Slightly extended reprint: [263], pp. 243-246. German translation: S. Šubin (Schubin), A. Smirnow: Ein einfaches Beispiel aus der Bornschen Elektrodynamik. *Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS, Nouvelle Série* **1 (X)**:2(79)(1936)69-72. – Cited at pp.: 26, 92, 93
- [94] А. А. Смирнов [A. A. Smirnov]: Применение электродинамики Борна к теории распространения света в электромагнитных полях [Primenenie èlektrodinamiki Borna k teorii rasprostraneniya sveta v èlektromagnitnykh polyakh]. Кандидатская диссертация [Kandidat-skaya dissertatsiya]/[Ph. D. thesis], Московский государственный университет [Moskovskii gosudarstvennyi universitet]/[Moscow State University], Moscow, 1936 ([arXiv:1708.06823](https://arxiv.org/abs/1708.06823), [hal-01620893](https://hal.archives-ouvertes.fr/hal-01620893)), a copy of the thesis is filed at the Institute of Archival Studies of the National Library of Ukraine, архівний фонд [arkhivniĭ fond]/[archival collection] no. 167, розділ [rozdil]/[section] 1, item no. 3, see http://www.irbis-nbuv.gov.ua/cgi-bin/irbis_arch/cgiirbis_64.exe?C21COM=2&I21DBN=ARHIV&P21DBN=ARHIV&Image_file_name=OPYS/167_1.pdf&IMAGE_FILE_DOWNLOAD=1). [in Russian] English translation (by K.S.): A. A. Smirnov: *The application of the electrodynamics of Born to the theory of the propagation of light in electromagnetic fields [Engl. transl. of kand. diss. (Ph.D. thesis), 1936]*. [arXiv:1708.06823](https://arxiv.org/abs/1708.06823) (<https://arxiv.org/abs/1708.06823>), [hal-01620893](https://hal.archives-ouvertes.fr/hal-01620893) (<https://hal.archives-ouvertes.fr/hal-01620893>), 2017. – Cited at pp.: 25, 26, 27, 35, 93
- [95] C. D. Thomas: The scattering of light by light according to the Born-Infeld theory. *Physical Review* **50**:11(1936)1046-1049 (DOI: [10.1103/PhysRev.50.1046](https://doi.org/10.1103/PhysRev.50.1046)). Also separately issued as: C. D. Thomas: *The Scattering of Light by Light According to the Born-Infeld Theory*. Lancaster Press, Lancaster, PA, 1936 (Ph.D. Thesis, University of Chicago, Chicago, IL, 1937). – Cited at pp.: 26, 93
- [96] M.-A. Tonnelat-Baudot: Relation entre la fonction d'action et la force qui agit sur l'électron [A relation between the action function and the force acting on an electron]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **203**:24(1936)1332-1333 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k3155r/f1332>). [in French] – Cited at pp.: 24, 93

- [97] W. H. Watson: Note on the representation of electromagnetic fields by biquaternions. *Transactions of the Royal Society of Canada, Section III, Mathematical, Physical and Chemical Sciences*, 3. Series, **30**(1936)105-113. – Cited at pp.: [24](#), [93](#)
- [98] V. Weisskopf: Über die Elektrodynamik des Vakuums auf Grund der Quantentheorie des Elektrons. *Mathematisk-Fysiske Meddelelser, Det Kongelige Danske Videnskabernes Selskab* **14**:6(1936)1-39 (The article is freely available online at the University of Southern Denmark library site: <http://www.sdu.dk/Bibliotek/matfys>). [in German] Reprinted in: [264], paper 8, pp. 92-128. English translation (by W. Grant): The electrodynamics of the vacuum based on the quantum theory of the electron. In: [246], pp. 206-226. – Cited at pp.: [32](#), [33](#), [55](#), [56](#), [94](#)

1937

- [99] A. Achieser: Über die Streuung von Licht an Licht [On the scattering of light by light]. *Physikalische Zeitschrift der Sowjetunion* **11**:3(1937)263-283. [in German] – Cited at pp.: [33](#), [59](#), [61](#), [86](#)
- [100] А. Ахиезер [A. Akhiezer], И. Померанчук [I. Pomeranchuk]: Когерентное рассеяние γ -лучей ядрами [Kogerentnoe rasseyanie gamma-luchey yadrami]/[Coherent scattering of γ -rays by nuclei]. *Журнал Экспериментальной и Теоретической Физики [Zhurnal Èksperimental'noï i Teoreticheskoy Fiziki]* **7**:5(1937)567-578. [in Russian] Reprinted in: [259], item 34, pp. 7-20. German translation: A. Achieser, I. Pomerantschuk: Kohärente Streuung von γ -Strahlen an Kernen. *Physikalische Zeitschrift der Sowjetunion* **11**:4(1937)478-497. – Cited at pp.: [17](#), [34](#), [40](#), [86](#), [92](#)
- [101] M. Born: Théorie non-linéaire du champ électromagnétique. *Annales de l'Institut Henri Poincaré* **7**:4(1937)155-265 (The article is freely available online at the Numdam site, stable URL: http://www.numdam.org/item?id=AIHP_1937__7_4_155_0). [in French] English translation (by D. H. Delphenich): *Nonlinear theory of the electromagnetic field* (The translation is freely available at the URL: http://neo-classical-physics.info/uploads/3/0/6/5/3065888/born_-_nonlinear_field_theory.pdf). – Cited at pp.: [28](#), [87](#)
- [102] М. П. Бронштейн [M. P. Bronshtein]: О возможности спонтанного расщепления фотонов [O vozmozhnosti spontannogo rasshchepleniya fotonov]/[On the possibility of the spontaneous splitting of photons]. *Журнал Экспериментальной и Теоретической Физики [Zhurnal Èksperimental'noï i Teoreticheskoy Fiziki]* **7**:3(1937)335-356. [in Russian] Partial reprint (sec. 1, pp. 335-340): [265], pp. 283-290. – Cited at pp.: [35](#), [87](#)
- [103] B. Castaldi: *Collision of Photons with Photons*. Master Thesis. Clark University, Worcester, MA, 1937. An abstract is printed in: *Clark University Bulletin*

No. 135 (1937), *Abstracts of Dissertations and Theses* **9**(1937)126-128. – Cited at pp.: [9](#), [87](#)

- [104] Z. Chrapływyj: On the Lorentz equation of motion in the new electrodynamics II. O równaniu ruchu Lorentza w nowej elektrodynamice II. *Acta Physica Polonica* **6**:1(1937)31-39 (also note Errata slip). [Polish abstract] – Cited at pp.: [24](#), [87](#)
- [105] Z. Chrapływyj: O pojęciu potencjału w nowej elektrodynamice. – Zum Potentialbegriff in der neuen Elektrodynamik [On the notion of the potential in the new electrodynamics]. *Bulletin International de l'Académie Polonaise des Sciences et de Lettres, Classe des Sciences Mathématiques et Naturelles, Série A: Sciences Mathématiques* (1937) No. 9-10, 509-520. [in German] – Cited at pp.: [24](#), [87](#)
- [106] З. Храпливий [Z. Khraplivii]: Основні поняття електродинаміки а унітарна теорія поля [Osnovnī ponyattya eľektrodinamiki a unītarna teoriya polya]/[The basic concepts of electrodynamics and the unitary field theory]. *Збірник Математично-Природничо-Лікарської Секції Науково Товариства Імені Шевченка* [Zbīrnik Matematichno-Pririodopisno-Likars'koї Sektsii Naukogo Tovaristva Imeni Shevchenka]/*Sammelschrift der Mathematisch-Naturwissenschaftlich-Ärztlichen Sektion der Ševčenko-Gesellschaft der Wissenschaften in Lwiv (Lemberg)* **31**(1937)51-56. [in Ukrainian] – Cited at pp.: [28](#), [87](#)
- [107] G. Darrieus: Nouvelle forme sans singularités de électrodynamique de Born [A new form of the electrodynamics of Born without singularities]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **204**:26(1937)1923-1925 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k31562/f1923>). [in French] – Cited at pp.: [23](#), [87](#)
- [108] B. Hoffmann, L. Infeld: On the choice of the action function in the new field theory. *Physical Review* **51**:9(1937)765-773 (DOI: [10.1103/PhysRev.51.765](https://doi.org/10.1103/PhysRev.51.765)). – Cited at pp.: [23](#), [27](#), [89](#)
- [109] L. Infeld: A new group of action functions in the unitary field theory. II. *Proceedings of the Cambridge Philosophical Society* **33**:1(1937)71-78 (DOI: [10.1017/S0305004100016790](https://doi.org/10.1017/S0305004100016790)). – Cited at pp.: [23](#), [89](#)
- [110] L. Infeld: The Lorentz transformations in the new quantum electrodynamics. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **158**:894(1937)368-371 (DOI: [10.1098/rspa.1937.0025](https://doi.org/10.1098/rspa.1937.0025), stable JSTOR URL: <http://www.jstor.org/stable/96823>). – Cited at pp.: [24](#), [89](#)

- [111] N. Kemmer: Über die Lichtstreuung an elektrischen Feldern nach der Theorie des Positrons [On the scattering of light by electric fields according to the theory of the positron]. *Helvetica Physica Acta* **10**:2(1937)112-122 (The article is freely available online at the ETH Zurich Library site given by the DOI: [10.5169/seals-110734](https://doi.org/10.5169/seals-110734)). [in German] Correction note as footnote 1 on p. 182 of [112]. – Cited at pp.: [34](#), [57](#), [89](#)
- [112] N. Kemmer, G. Ludwig: Über die Lichtstreuung an elektrischen Feldern nach der Theorie des Positrons. (II) [On the scattering of light by electric fields according to the theory of the positron. (II)]. *Helvetica Physica Acta* **10**:3(1937) 182-184 (The article is freely available online at the ETH Zurich Library site given by the DOI: [10.5169/seals-110741](https://doi.org/10.5169/seals-110741)). [in German] – Cited at pp.: [17](#), [34](#), [57](#), [79](#), [89](#), [90](#)
- [113] B. S. Madhava Rao: On the fine structure of the Balmer lines. *Proceedings of the Indian Academy of Sciences A* **5**:1(1937)56-62 (DOI: [10.1007/BF03045519](https://doi.org/10.1007/BF03045519), the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/5/1/56-62/viewpage.html). – Cited at pp.: [24](#), [90](#)
- [114] B. S. Madhava Rao: Ring-singularity in Born's unitary theory – II. *Proceedings of the Indian Academy of Sciences A* **6**:2(1937)129-134 (DOI: [10.1007/BF03051248](https://doi.org/10.1007/BF03051248), the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/6/2/129-134/viewpage.html). – Cited at pp.: [24](#), [90](#)
- [115] B. S. Madhava Rao: Generalized action functions in Born's electro-dynamics. *Proceedings of the Indian Academy of Sciences A* **6**:3(1937)158-173 (DOI: [10.1007/BF03051499](https://doi.org/10.1007/BF03051499), the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/6/3/158-173/viewpage.html). – Cited at pp.: [23](#), [90](#)
- [116] M. H. L. Pryce: On the new field theory. II. Quantum theory of field and charges. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **159**:898(1937)355-382 (DOI: [10.1098/rspa.1937.0077](https://doi.org/10.1098/rspa.1937.0077), stable JSTOR URL: <http://www.jstor.org/stable/96883>). – Cited at pp.: [24](#), [92](#)
- [117] С. Шубин [S. Shubin], А. Смирнов [A. Smirnov]: О рассеянии света электростатическим полем точечного заряда по нелинейной квантовой электродинамике [O rasseyanii sveta èlektrostaticheskim polem tochechnogo zaryada po nelineinoi kvantovoï èlektrodinamike]. *Доклады Академии Наук СССР, Новая Серия [Doklady Akademii Nauk SSSR, Novaya Seriya]* **15**:3(1937)133-136. [in Russian] Slightly extended reprint: [[263](#)], pp. 247-252.

Slightly extended English translation: S. Šubin, A. Smirnov: On the scattering of light by the electrostatic field of a point charge on the non-linear quantum electrodynamics. *Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS, Nouvelle Série* **15**:3(1937)131-135. – Cited at pp.: 27, 92, 93

- [118] M.-A. Tonnelat-Baudot: Linéarisation de la densité d'énergie et de la fonction d'action à l'aide de vecteurs complexes [Linearisation of the energy density and the action function by means of complex vectors]. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences (Paris)* **204**:2(1937)110-112 (The article is freely available online at the Gallica site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k31562/f110>). [in French] – Cited at pp.: 24, 93
- [119] P. Weiss: Born's electrodynamics in complex form. *Proceedings of the Cambridge Philosophical Society* **33**:1(1937)79-93 (DOI: 10.1017/S0305004100016807). – Cited at pp.: 24, 94
- [120] F. Zwicky: Intrinsic properties of light and corpuscles from distant sources. *Proceedings of the National Academy of Sciences of the United States of America* **23**:2(1937)106-110 (DOI: 10.1073/pnas.23.2.106, stable JSTOR URL: <http://www.jstor.org/stable/86455>). – Cited at pp.: 17, 20, 94

1938

- [121] T. Iwatsuki, Y. Mimura, K. Morinaga: Born's electrodynamics in terms of wave geometry. Wave Geometry No. 24. *Journal of Science of Hiroshima University, Series A (Mathematics, Physics, Chemistry)* **8**(1938)43-50. – Cited at pp.: 24, 89, 91
- [122] B. Kwal, J. Solomon: La loi de Stefan et l'électrodynamique non linéaire [The Stefan law and nonlinear electrodynamics]. *Le Journal de Physique et le Radium*, 7. Series, **9**:5(1938)205-208 (DOI: 10.1051/jphysrad:0193800905020500, the article is freely available online at the HAL site: <http://hal.archives-ouvertes.fr/jpa-00233580>). [in French] – Cited at pp.: 28, 90, 93
- [123] B. S. Madhava Rao: Biquaternions in Born's electrodynamics. *Proceedings of the Indian Academy of Sciences A* **7**:5(1938)333-338 (DOI: 10.1007/BF03045494, the article is freely available online at the journal website: http://www.ias.ac.in/j_archive/proca/7/5/333-338/viewpage.html). – Cited at pp.: 24, 90
- [124] Г. Мандель [G. Mandel']: Электродинамика Борна и космология [Èlektrodinamika Borna i kosmologiya]. *Доклады Академии Наук СССР, Новая Серия [Doklady Akademii Nauk SSSR, Novaya Seriya]* **18**:8(1938)543-547. [in Russian] German translation: H. Mandel: Bornsche Elektrodynamik

und Kosmologie. *Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS, Nouvelle Série* **18**:8(1938)543-547. – Cited at pp.: 25, 90

- [125] E. Milkutat: Das *Stefan-Boltzmanns*che Strahlungsgesetz bei extrem hohen Temperaturen nach der nichtlinearen Elektrodynamik [The *Stefan-Boltzmann* radiation law at extremely high temperatures according to nonlinear electrodynamics]. *Astronomische Nachrichten* **265**:6360(1938)379-380. [in German] (DOI: [10.1002/asna.19382652406](https://doi.org/10.1002/asna.19382652406), the article is freely available online at the SAO/NASA Astrophysics Data System site: <http://adsbit.harvard.edu/full/seri/AN.../0265//0000202.000.html>). – Cited at pp.: 28, 91
- [126] И. А. Мирцхулава [I. A. Mirtskhulava]: Решение двух- и трехмерной задачи в электродинамике Борна-Инфельда [Reshenie dvukh- i trekhmernoĭ zadachi v èlektrodinamike Borna-Infel'da]/[Solution of the two- and three-dimensional problem in the Born-Infeld electrodynamics]. *Журнал Экспериментальной и Теоретической Физики [Zhurnal Èksperimental'noĭ i Teoreticheskoi Fiziki]* **8**:4(1938)377-396. [in Russian] – Cited at pp.: 24, 91
- [127] T. Sibata: Wave geometry unifying Einstein's law of gravitation and Born's theory of electrodynamics. Wave Geometry No. 25. *Journal of Science of Hiroshima University, Series A (Mathematics, Physics, Chemistry)* **8**(1938)51-79. – Cited at pp.: 24, 25, 92
- [128] C. D. Thomas: The static interaction of charged particles. *Physical Review* **54**:5(1938)367-370 (DOI: [10.1103/PhysRev.54.367](https://doi.org/10.1103/PhysRev.54.367)). – Cited at pp.: 24, 93
- [129] S. Tomonaga, M. Kobayashi: Scattering and splitting of photons on the nonlinear field theory of Born and Infeld. Paper 852. *Scientific Papers of the Institute of Physical and Chemical Research* **34**:852(1938)1643-1649. Reprinted in: [266], item 12, pp. 125-131. – Cited at pp.: 27, 90, 93

1939

- [130] ი. მირცხულავა [i. mirc'xulava]: ბორნის ელექტრო-დინამიკის ორ-განზომილებიანი ამოცანის გადაწყვეტის ახალი მეთოდი [bornis elek'tro-dinamikis or-ganzomilebiani amoc'anis gadacqvetsis axali met'odi]/[A new method for the solution of the two-dimensional problem in the Born electrodynamics]. სტალინის სახელმწიფო უნივერსიტეტის შრომები [*stalinis saxelobis t'bilisis saxelmcip'o universitetis šromebi*] = *Труды Тбилисского Государственного Университета имени Сталина [Trudy Tbilisskogo Gosudarstvennogo Universiteta imeni Stalina]* = *Travaux de l' Université Staline à Tbilissi (Tiflis)* **8**(1939)89-97 (The article is freely available online at the Tbilisi State University library site: http://digitallibrary.tsu.ge/book/!2016/shromebi/bolkvadze_shromebi_8/index.html#/92). [in Georgian, English and extended Russian abstracts] – Cited at pp.: 24, 91

- [131] И. А. Мирцхулава [I. A. Mirtskhulava]: Взаимодействие двух источников поля в теории Борна-Инфельда [Vzaimodeĭstvie dvukh istochnikov polya v teorii Borna-Infel'da]/[Interaction of two sources in Born-Infeld theory]. სტალინის სახელობის თბილისის სახელმწიფო უნივერსიტეტის შრომები [*stalinis saxelobis t'bilisis saxelmcip'o universitetis šromebi*] = Труды Тбилисского Государственного Университета имени Сталина [*Trudy Tbilisskogo Gosudarstvennogo Universiteta imeni Stalina*] = *Travaux de l' Université Staline à Tbilissi (Tiflis)* **9**(1939)9-14 (The article is freely available online at the Tbilisi State University library site: http://digitallibrary.tsu.ge/book/!2016/shromebi/bolkvadze_shromebi_9/index.html#/14). [in Russian] – Cited at pp.: 24, 91

1940

- [132] C. J. Banwell, C. C. Farr: Further investigation of the velocity of propagation of light *in vacuo* in a transverse magnetic field. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **175**:960(1940)1-25 (DOI: [10.1098/rspa.1940.0040](https://doi.org/10.1098/rspa.1940.0040), stable JSTOR URL: <http://www.jstor.org/stable/97422>). – Cited at pp.: 12, 44, 45, 87, 88
- [133] A. A. Smirnov: The problem of two plane waves in classical non-linear electrodynamics. *Journal of Physics (Academy of Sciences of the USSR)/Физический Журнал (Академия Наук Союза ССР) [Fizicheskĭ Zhurnal (Akademiya Nauk Soyuzs SSR)]* **3**:6(1940)447-453 (The article is a slightly revised English version of [137].). – Cited at pp.: 25, 26, 83, 93
- [134] А. А. Смирнов [A. A. Smirnov]: Рассеяние света электростатическим полем заряда в электродинамике Гофмана-Инфельда [Rasseyanie sveta èlektrostaticheskim polem zaryada v èlektrodinamike Gofmana-Infel'da]/[The scattering of light by the electrostatic field of a charge in the non-linear electrodynamics of Hoffmann-Infeld]. *Журнал Экспериментальной и Теоретической Физики [Zhurnal Èksperimental'noĭ i Teoreticheskoi Fiziki]* **10**:3(1940)257-262. [in Russian] – Cited at pp.: 27, 93
- [135] R. V. Wiegand: *Collision of Photons by the Use of X-Rays*. Master Thesis. Montana State College, Bozeman, MT, 1940 (The thesis is freely available online at the Montana State College library site: <https://scholarworks.montana.edu/xmlui/handle/1/6379>). – Cited at pp.: 10, 94

1941

- [136] L. Infeld: *Quest. The Evolution of a Scientist*. Doubleday, Doran & Co., Inc., New York; Victor Gollancz Limited, London, 1941. Reprints: 1. Readers Union Limited, London, 1942, 1945. 2. Scientific Book Club, London, 1941, 1942. 3. Macmillan Co., New York, 1949. 4. Transaction Publishers, New Brunswick,

NJ, 1989. 2nd expanded ed.: *Quest. An Autobiography*. Chelsea Publishing Company, New York, NY, 1980. AMS Chelsea Publishing, Providence, RI, 2006. Translations:

1. Japanese: 真実の探求. 一科学者の生長 [*Shinjitsu no tankyū. Ichi kagaku-sha no seichō*]. 日本評論社 [Nihonhyōronsha], Tokyo, 1950.
2. Portuguese: *A evolução de um cientista*. Biblioteca do Espírito Moderno, Série 3, História e Biografia, Vol. 20. Companhia Editôra Nacional, São Paulo, 1942.

– Cited at pp.: 23, 89

- [137] A. A. Смирнов [A. A. Smirnov]: Задача двух плоских волн в классической нелинейной электродинамике [Zadacha dvukh ploskikh voln v klassicheskoi nelineinoi elektrodinamike]/[The problem of two plane waves in classical non-linear electrodynamics]. *Ученые Записки Свердловского Государственного Университета им. А. М. Горького* [*Uchenye Zapiski Sverdlovskogo Gosudarstvennogo Universiteta im. A. M. Gor'kogo*] No. 3 (1941)44-56 (For a slightly revised English translation of the article see [133]). [in Russian] – Cited at pp.: 25, 26, 82, 93

1942

- [138] E. Schrödinger: Non-linear optics. *Proceedings of the Royal Irish Academy of Sciences, Section A, Mathematical, Astronomical, and Physical Science* 47(1941-1942)77-117 (stable JSTOR URL: <http://www.jstor.org/stable/20488437>). Reprinted in: [256], pp. 309-349. – Cited at pp.: 25, 26, 27, 28, 92
- [139] E. Schrödinger: Dynamics and scattering-power of Born's electron. *Proceedings of the Royal Irish Academy of Sciences, Section A, Mathematical, Astronomical, and Physical Science* 48(1942-1943)91-122 (stable JSTOR URL: <http://www.jstor.org/stable/20488443>). Reprinted in: [256], pp. 350-381. – Cited at pp.: 24, 92

1943

- [140] J. McConnell: Non-linear quantum electrodynamics of the vacuum. *Proceedings of the Royal Irish Academy of Sciences, Section A, Mathematical, Astronomical, and Physical Science* 49(1943-1944)149-176 (stable JSTOR URL: <http://www.jstor.org/stable/20488456>). – Cited at pp.: 24, 26, 90
- [141] E. Schrödinger: A new exact solution in non-linear optics (two-wave-system). *Proceedings of the Royal Irish Academy of Sciences, Section A, Mathematical, Astronomical, and Physical Science* 49(1943-1944)59-66 (stable JSTOR URL: <http://www.jstor.org/stable/20488451>). Reprinted in: [256], pp. 398-405. – Cited at pp.: 24, 26, 92

1946

- [142] J. Stark: *Unknown title*. Private print, Traunstein, October 30, 1946 (No known copy, cited in [145]. A copy of the second private print (1 leaf, 2 pp.), of November 24, 1946, mentioned in [145] is filed as part of the Nachlaß of J. Stark at the Staatsbibliothek zu Berlin, Preußischer Kulturbesitz, Handschriftenabteilung, Nachlaß Stark, Kasten 10, Sondermappe III, 6.). [in German] – Cited at pp.: 13, 93

1947

- [143] М. Ф. Широков [M. F. Shirokov]: О роли гравитации при построении элементарных частиц [O roli gravitatsii pri postroenii èlementarnykh chastits]/[On the role of gravitation in the structure of elementary particles]. *Вестник Московского Университета* [Vestnik Moskovskogo Universiteta] – *Bulletin of Moscow University* No. 4 (1947)67-75. [in Russian, English abstract] 25, 92
- [144] J. Stark: *Experimentelle Untersuchungen über die Natur des Lichtes* [Experimental investigations on the nature of light]. Private print, 1947, 8 pp. (mentioned on p. 42 of [185] and on p. 504 of [186], no known copy, reprinted in [185], secs. 14-19, pp. 44-61). [in German] – Cited at pp.: 13, 93
- [145] S.a.: Die Versuche von J. Stark über den Einfluß des elektrostatischen Feldes auf Licht [The experiments of J. Stark on the influence of the electrostatic field on light]. *Physikalische Blätter* 3:5(1947)162-163 (The article is freely available online at the journal website given by the DOI: 10.1002/phbl.19470030510 .). [in German] First part (p. 162): S.a.: Versuche von J. Stark [The experiments of J. Stark]; second part (p. 163): E. Kappler: Bemerkung zu den Stark'schen Versuchen von E. Kappler [Remark on the Stark experiments by E. Kappler]. – Cited at pp.: 13, 84

1948

- [146] И. З. Фишер [I. Z. Fisher]: Учет гравитационных эффектов в теории Борна-Инфельда [Uchet gravitatsionnykh èffektov v teorii Borna-Infel'da]/[The consideration of gravitational effects in Born-Infeld theory]. *Журнал Экспериментальной и Теоретической Физики* [Zhurnal Èksperimental'noï i Teoreticheskoi Fiziki] 18:7(1948)668-669. [in Russian] – Cited at pp.: 25, 88
- [147] L. Hohl: *Über die von J. Stark behauptete Wirkung inhomogener elektrischer Felder auf Licht* [On the influence, asserted by J. Stark, of inhomogeneous electric fields on light]. Ph. D. thesis, University of Rostock, Rostock, 1948. [in German] – Cited at pp.: 13, 89
- [148] P. Kunze, L. Hohl: *Über den Einfluß elektrostatischer Felder auf Licht nach J. Stark* [On the influence of electrostatic fields on light according to J. Stark].

Annalen der Physik, 6. Series, **2** (437 of all series altogether):5-6(1948)270-272 (DOI: 10.1002/andp.19484370506). [in German] – Cited at pp.: 13, 89, 90

[149] A. Sommerfeld: *Vorlesungen über theoretische Physik, Band III: Elektrodynamik*. Dieterich'sche Verlagbuchhandlung, Wiesbaden, 1948; Geest & Portig K.-G., Leipzig, 1. ed. 1949, 2. ed. 1954, 3. rev. and ext. ed. 1961, 4. ed. 1964, 5. ed. 1967. Reprints of the 4. (rev. and ext.) ed.: Verlag Harri Deutsch, Thun, Frankfurt/M., 1977, 1988, 2001. [in German] Translations:

1. English: *Electrodynamics*. Lectures on Theoretical Physics, Vol. III. Academic Press, New York, 1952, reprinted: 1956, 1959, 1963, 1964, 1971 (DOI: 10.1016/C2013-0-07600-1).

2. Russian: А. Зоммерфельд [A. Zommerfel'd]: *Электродинамика [Èlektrodinamika]*. Издательство иностранной литературы [Izdatel'svto inostrannoï literatury], Moscow, 1958 (The book is freely available online at the EqWorld site: <http://eqworld.ipmnet.ru/ru/library/books/Zommerfeld1958ru.djvu>).

– Cited at pp.: 28, 93

[150] J. Sperling: Bemerkung zu den Stark'schen Versuchen über den Einfluß des elektrostatischen Feldes auf Licht [Comment on Stark's experiments on the influence of the electrostatic field on light]. *Physikalische Blätter* 4:2(1948)71 (The article is freely available online at the journal website given by the DOI: 10.1002/phbl.19480040206 .). [in German] – Cited at pp.: 13, 93

1949

[151] Д. Иваненко [D. Ivanenko], А. Соколов [A. Sokolov]: *Классическая теория поля (Новые проблемы) [Klassicheskaya teoriya polya (Novye problemy)]/[Classical Field Theory (New Problems)]*. Государственное издательство технико-теоретической литературы [Gosudarstvennoe izdatel'svto tekhniko-teoreticheskoi literatury], Moscow, Leningrad, 1. ed. 1949, 2. ext. ed. 1951, in particular §32. [in Russian] Translations of the 2. ed. (no English translation):

1. Chinese: 伊万宁科 [Yī wàn níng kē], 索科洛夫 [Suǒ kē luò fu]: 經典場論. 新的問題 [*Jīngdiǎn chǎng lùn. Xīn de wèntí*]. 科出版社 [Kēxué chūbǎn shè]/[Science Publishers], Beijing, 1958.

2. German: D. Iwanenko, A. Sokolow: *Klassische Feldtheorie*. Akademie-Verlag, Berlin, 1953.

3. Hungarian: D. Ivanenko, A. Sokolov: *Klasszikus Tételmelet*. Akadémiai Kiadó, Budapest, 1955.

4. Romanian: D. Ivanenko, A. Sokolov: *Teoria Clasică a Câmpului*. Editura Tehnică, Bucarest, 1955.

– Cited at pp.: 28, 89, 93

- [152] P. Kunze: Die Linearität der elektrodynamischen Gleichungen von Maxwell [The linearity of the electrodynamic equations of Maxwell]. *Die Naturwissenschaften* **36**:12(1949)356-359 (DOI: 10.1007/BF00627171). [in German] – Cited at pp.: 6, 90

Miscellaneous years

- [153] E. Schrödinger: *Born & Infeld. I.* Notebook, started January 1935. Nachlaß Erwin Schrödinger, Österreichische Zentralbibliothek für Physik, Vienna (The notebook is freely available online at the Phaidra website: <http://phaidra.univie.ac.at/o:159934>). [in German] – Cited at pp.: 7, 92
- [154] E. Schrödinger: *Born und Infeld II. (Stromlinien) „Schallanalogie“ [Born and Infeld II. (Flux Lines) “Sound Analogy”]*. Notebook, no date. Nachlaß Erwin Schrödinger, Österreichische Zentralbibliothek für Physik, Vienna (The notebook is freely available online at the Phaidra website: <http://phaidra.univie.ac.at/o:159997>). [in German] – Cited at pp.: 7, 92
- [155] E. Schrödinger: *Born-Infeld. Quantisierungsfragen - Kugelmittel - Abelsche Integralgleichung [Born-Infeld. Quantization Problems - Spherical Average - Abel Integral Equation]*. Notebook, started 1935. Nachlaß Erwin Schrödinger, Österreichische Zentralbibliothek für Physik, Vienna (The notebook is freely available online at the Phaidra website: <http://phaidra.univie.ac.at/o:160040>). [in German] 7, 92
- [156] A. A. Смирнов [A. A. Smirnov]: Lecture material on nonlinear quantum electrodynamics, 1933-1947. Filed at the Institute of Archival Studies of the National Library of Ukraine, архівний фонд [arkhivnii fond]/[archival collection] no. 167, розділ [rosdil]/[section] II.B [II.V], items no. 90-92, see http://www.irbis-nbuv.gov.ua/cgi-bin/irbis_arch/cgiirbis_64.exe?C21COM=2&I21DBN=ARHIV&P21DBN=ARHIV&Image_file_name=OPYS/167_1.pdf&IMAGE_FILE_DOWNLOAD=1 . [in Russian] – Cited at pp.: 7, 93

List of full author names

(Main line: author name [refs. from 'Main list'] / [refs. from 'Further literature'])

Achieser, A. = see Akhiezer, A.

Akhiezer, A. (Achieser, A.) [62, 99, 100] / [250, 251]

= Akhiezer, Aleksandr Il'ich [31.10.(O.S. 18.10.)1911-4.5.2000]

= Ахиезер, Александр Ильич

https://en.wikipedia.org/wiki/Aleksander_Akhiezer

- Awano, T. [38]
 = Awano, Tamotsu (1908-1991) = 粟野, 保 / アワノ, タモツ
<http://viaf.org/viaf/254941549>
- Banwell, C. J. [19, 132]
 = Banwell, Congreve John [1908-1983 (?)]
- Baudot, M.-A. = see Tonnelat-Baudot, M.-A.
- Bauer, H. [17]
 = Bauer, Heinrich [1905-(?)]
<http://d-nb.info/gnd/138041903>
- Blokhintsev, D. I. [65]
 = Blokhintsev, Dmitriï Ivanovich [29.12.1907(O.S. 11.1.1908)-27.1.1979]
 = Блохинцев, Дмитрий Иванович
https://ru.wikipedia.org/wiki/Блохинцев,_Дмитрий_Иванович
- Born, M. [20–22, 26–31, 39–41, 66, 101] / [207–209]
 = Born, Max (11.12.1882-5.1.1970), Nobel Prize in Physics 1954
https://en.wikipedia.org/wiki/Max_Born
- Bramley, A. [67]
 = Bramley, Arthur [1900-3.6.1971]
<http://viaf.org/viaf/232445954>
 See [267].
- Bronstein, M. [68, 102]
 = Bronshtein, Matvei Petrovich [2.12.(O.S. 19.11.)1906-18.2.1938]
 = Бронштейн, Матвей Петрович
https://en.wikipedia.org/wiki/Matvei_Petrovich_Bronstein
- Castaldi, B. [103]
 = Castaldi, Basilio [17.2.1915-(?)]
- Chrapływyj, Z. [42, 69–71, 104–106]
 = Chrapływyj, Zenon Wasilovich (15.3.1904-1.10.1983)
 = Храпливий, Зіновій (Зенон) Васильович
 [Khraplivii, Zinovii (Zenon) Vasil'ovich]
https://uk.wikipedia.org/wiki/Храпливий_Зіновій_Васильович
- Darrieus, G. [107]
 = Darrieus, Georges (Jean Marie) (24.9.1888-15.7.1979)
https://en.wikipedia.org/wiki/Georges_Jean_Marie_Darrieus
- de Broglie, L. [7]
 = de Broglie, Louis (15.8.1892-19.3.1987), Nobel Prize in Physics 1929
https://en.wikipedia.org/wiki/Louis_de_Broglie

Das, A. K. [18]
 = Das, Anil Kumar (1.2.1902-18.2.1961)
<http://www.insaindia.org/deceaseddetail.php?id=N430180>

Delbrück, M. [23] / [199]
 = Delbrück, Max (4.9.1906-9.3.1981),
 Nobel Prize in Physiology and Medicine 1969
https://en.wikipedia.org/wiki/Max_Delbrück

Doubt, Th. E. [4]
 = Doubt, Thomas Eaton (15.2.1870-31.12.1947)
http://mypages.iit.edu/~segre/iit_physics_bios/doubt_t.html

Ebert, H. [3]
 = Ebert, (Cäsar) Hermann (Robert) (20.6.1861-12.2.1913)
https://en.wikipedia.org/wiki/Hermann_Ebert

Erskine, W. H. [43, 44]
 = Erskine, William Henry [6.12.1908-(?)]

Euler, H. [45, 72, 74]
 = Euler, Hans (Heinrich) (6.10.1909-1941)
https://en.wikipedia.org/wiki/Hans_Heinrich_Euler

Farr, C. C. [19, 132]
 = Farr, Clinton Coleridge (22.5.1866-27.1.1943)
https://en.wikipedia.org/wiki/Coleridge_Farr

Feenberg, E. [46]
 = Feenberg, Eugene (6.10.1906-7.11.1977)
https://en.wikipedia.org/wiki/Eugene_Feenberg

Fisher, I. Z. [146]
 = Fisher, Iosif Zal'manovich (24.2.1919-15.5.1995)
 = Фишер, Иосиф Зальманович
https://ru.wikipedia.org/wiki/Фишер,_Иосиф_Зальманович

Frenkel, J. = see Frenkel', Ya. I.

Frenkel', Ya. I. (Frenkel, J.) [32, 33]
 = Frenkel', Yakov Il'ich [10.2.(O.S. 29.1.)1894-23.1.1952]
 = Френкель, Яков Ильич
https://en.wikipedia.org/wiki/Yakov_Frenkel

Géhéniau, J. [73]
 = Géhéniau, Jules (31.3.1909-11.6.1991)
https://fr.wikipedia.org/wiki/Jules_Geheniau

- Halpern, O. [24]
 = Halpern, Otto (25.4.1899-28.10.1982)
https://de.wikipedia.org/wiki/Otto_Halpern
- Heitler, W. [75] / [220]
 = Heitler, Walter (2.1.1904-15.11.1981)
https://en.wikipedia.org/wiki/Walter_Heitler
- Heisenberg, W. [34, 74] / [243, 255]
 = Heisenberg, Werner (5.12.1901-1.1.1976), Nobel Prize in Physics 1932
https://en.wikipedia.org/wiki/Werner_Heisenberg
- Heller, G. [35]
 = Heller, Gabriel (dates of birth and death not known)
- Hoffmann, B. [36, 47–49, 108] / [262]
 = Hoffmann, Banesh (6.9.1906-5.8.1986)
https://en.wikipedia.org/wiki/Banesh_Hoffmann
- Hohl, L. [147, 148]
 = Hohl (married: Knacke), Lieselotte (* 14.8.1923)
 See [268], p. 2, item 1.
- Hughes, A. L. [10, 15, 27, 37]
 = Hughes, Arthur Llewelyn (18.12.1883-25.6.1978)
 See [269].
- Infeld, L. [21, 22, 28–31, 40, 76–79, 108–110, 136] / [253]
 = Infeld, Leopold (20.8.1898-15.1.1968)
https://en.wikipedia.org/wiki/Leopold_Infeld
- Ivanenko, D. D. [50, 151]
 = Ivanenko, Dmitriï Dmitrievich (29.7.1904-30.12.1994)
 = Иваненко, Дмитрий Дмитриевич
https://en.wikipedia.org/wiki/Dmitri_Ivanenko
- Iwatsuki, T. [121]
 = Iwatsuki, Toranosuke (1894-1945)
 = 岩付, 寅之助 / イワツキ, トラノスケ
 See [270].
- Jauncey, G. E. M. [10, 14, 15, 37]
 = Jauncey, George Eric Macdonnell (21.9.1888-19.5.1947)
<http://trove.nla.gov.au/people/823408?c=people>
- Kemmer, N. [80, 111, 112] / [249]
 = Kemmer, Nicholas (7.12.1911-21.10.1998)
 = Кеммер, Николас (Павлович) [Kemmer, Nikolas (Pavlovich)]

- https://en.wikipedia.org/wiki/Nicholas_Kemmer
Khrapliviĭ, Z. = Храпливий, З. = see Chrapływyj, Z.
- Kobayashi, M. [129]
= Kobayashi, Minoru (1908-2001) = 小林, 稔 / コバヤシ, ミノル
See [271].
- Kockel, B. [45]
= Kockel, (Wilhelm Paul) Bernhard (3.9.1909-27.5.1987)
https://de.wikipedia.org/wiki/Bernhard_Kockel
- Kreisler, J. [81]
= Kreisler, Juda [12.10.1904-1940s(?)]
See [272].
- Kunze, P. [148, 152]
= Kunze, Paul (2.11.1897-6.10.1986)
https://de.wikipedia.org/wiki/Paul_Kunze
- Kwal, B. [51, 82, 122]
= Kwal, Bernard [(?)-1952]
<http://viaf.org/viaf/161085053/>
- Landau, L. [62] / [257, 258]
= Landau, Lev (Davidovich) [22.1.(O.S. 9.1.)1908-1.4.1968],
Nobel Prize in Physics 1962
= Ландау, Лев (Давидович)
https://en.wikipedia.org/wiki/Lev_Landau
- Lippich, F. [2]
= Lippich, Ferdinand (Franz) (4.10.1838-18.10.1913)
https://de.wikipedia.org/wiki/Ferdinand_Lippich
- Ludwig, G. [112]
= Ludwig, Guido [1907-(?)]
- Madhava Rao, B. S. [83–86, 113–115, 123]
= Madhava Rao, Bangalore Srinivasa Rao (29.5.1900-6.11.1987)
<http://insaindia.org/deceaseddetail.php?id=N530418>
- Mandel', G. [124]
= Mandel', Genrikh (Aleksandrovich) [(?)-1946]
= Мандель, Генрих (Александрович)
- McConnell, J. [140]
= McConnell, James (Robert) (25.2.1915-13.2.1999)
https://en.wikipedia.org/wiki/James_Robert_McConnell

- Meixner, J. [52]
 = Meixner, Josef (24.4.1908-19.3.1994)
https://en.wikipedia.org/wiki/Josef_Meixner
- Mie, G. [5]
 = Mie, Gustav (Adolf Feodor Wilhelm Ludwig) (29.9.1868-13.2.1957)
https://en.wikipedia.org/wiki/Gustav_Mie
- Milkutat, E. [125]
 = Milkutat, Ernst (dates of birth and death not known)
<http://d-nb.info/gnd/10517615X>
- Mimura, Y. [121]
 = Mimura, Yositaka (1.3.1898-26.10.1965)
 = 三村, 剛昂 / ミムラ, ヨシタカ [Mimura, Yoshitaka]
 See: literature for Iwatsuki, T.
- mirc'xulava, i. = see Mirtskhulava, I. A.
- Mirtskhulava, I. A. (mirc'xulava, i. a.) [87, 126, 130, 131]
 = Mirtskhulava, Ivan A. (dates of birth and death not known)
 = Мирцхулава, Иван А.
 = მირც'ხულავა, ივანე ა. [mirc'xulava, ivane a.]
- Mohler, F. L. [25]
 = Mohler, Fred Loomis (23.8.1893-2.12.1974)
<http://viaf.org/viaf/56996589>
- Morinaga, K. [121]
 = Morinaga, Kakutarô (23.8.1905-20.11.1970)
 = 盛永, 恪太郎 / モリナガ, カクタロウ
 See [273].
- Motz, L. [35]
 = Motz, Lloyd (5.6.1909-14.3.2004)
https://en.wikipedia.org/wiki/Lloyd_Motz
- Müller, J. J. [1]
 = Müller, Johann Jakob (4.3.1846-14.1.1875)
https://en.wikipedia.org/wiki/Johann_Jakob_Müller
- Nikolsky, K. = see Nikol'skiĭ, K. V.
- Nicol'skiĭ, K. V. (Nikolsky, K.) [53, 54]
 = Nikol'skiĭ, Konstantin Vyacheslavovich (1905-1978)
 = Никольский, Константин Вячеславович
<http://nasledie.enip.ras.ru/ras/view/person/history.html?id=44116438>

- Pauli, W. [88, 89] / [206, 247, 262]
 = Pauli, Wolfgang (Ernst) (25.4.1900-15.12.1958),
 Nobel Prize in Physics 1945
https://en.wikipedia.org/wiki/Wolfgang_Pauli
- Pomeranchuk, I. = see Pomeranchuk, I.
- Pomeranchuk, I. (Pomeranchuk, I., Pomerantschuk, I.) [62, 100] / [259]
 = Pomeranchuk, Isaak (Yakovlevich) [20.5.(O.S. 7.5.)1913-14.12.1966]
 = Померанчук, Исаак (Яковлевич)
https://en.wikipedia.org/wiki/Isaak_Pomeranchuk
- Pomerantschuk, I. = see Pomeranchuk, I.
- Pryce, M. H. L. [55–57, 90–92, 116]
 = Pryce, Maurice Henry Lecombe (24.1.1913-24.7.2003)
https://en.wikipedia.org/wiki/Maurice_Pryce
- Rosenfeld, L. [9]
 = Rosenfeld, Léon (14.8.1904-23.3.1974)
https://en.wikipedia.org/wiki/Léon_Rosenfeld
- Schaposchnikow, K. = see Shaposhnikov, K.
- Schrödinger, E. [41, 58, 59, 138, 139, 141, 153–155] / [256]
 = Schrödinger, Erwin (Rudolf Josef Alexander) (12.8.1887-4.1.1961),
 Nobel Prize in Physics 1933
https://en.wikipedia.org/wiki/Erwin_Schrödinger
- Shaposhnikov, K. (Schaposchnikow, K.) [6, 11]
 = Shaposhnikov, Konstantin (Nikolaevich) [24.4.1880-(?)]
 = Шапошников, Константин (Николаевич)
<http://ispu.ru/book/export/html/1338>
- Shirokov, M. F. [143]
 = Shirokov, Mikhail Fedorovich (1901-24.10.1982)
 = Широков, Михаил Федорович
 See [274].
- Shubin, S. [93, 117] / [263]
 = Shubin, Semën (Petrovich) [31.7.(O.S. 18.7.)1908-28.11.1938]
 = Шубин, Семён (Петрович)
https://ru.wikipedia.org/wiki/Шубин,_Семён_Петрович
- Sibata, T. [127]
 = Sibata, Takasi (dates of birth and death not known)
 = 柴田, 孝 / シバタ, タカシ [Shibata, Takashi]

- Smirnov, A. A. [93, 94, 117, 133, 134, 137, 156]
 = Smirnov, Adrian Anatol'evich [16.11.(O.S. 3.11.)1908-6.12.1992]
 = Смирнов, Андриан Анатольевич
https://ru.wikipedia.org/wiki/Смирнов,_Андриан_Анатольевич
- Sokolov, A. [151]
 = Sokolov, Arseniĭ (Aleksandrovich) [19.3.1910-19.10.1986]
 = Соколов, Арсений (Александрович)
https://en.wikipedia.org/wiki/Arseny_Sokolov
- Solomon, J. [82, 122]
 = Solomon, Jacques (4.2.1908-23.5.1942)
https://en.wikipedia.org/wiki/Jacques_Solomon
- Sommerfeld, A. [149]
 = Sommerfeld, Arnold (5.12.1868-26.4.1951)
https://en.wikipedia.org/wiki/Arnold_Sommerfeld
- Sperling, J. [150]
 = Sperling, Joachim (dates of birth and death not known)
<http://d-nb.info/gnd/106504460>
- Stark, J. [142, 144] / [185, 186]
 = Stark, Johannes (15.4.1874-21.7.1957), Nobel Prize in Physics 1919
https://en.wikipedia.org/wiki/Johannes_Stark
- Thomas, C. D. [95, 128]
 = Thomas, Charles Danser [5.11.1908-(?)]
 See [275], Chap. 6, pp. 131-146.
- Tomonaga, S. [129] / [222, 223, 266]
 = Tomonaga, Sin-Itiro (31.3.1906-8.7.1979), Nobel Prize in Physics 1965
 = 朝永, 振一郎 / トモナガ, シンイチロウ [Tomonaga Shin'ichirō]
https://en.wikipedia.org/wiki/Sin-Itiro_Tomonaga
- Tonnelat-Baudot, M.-A. (Baudot, M.-A.) [63, 64, 96, 118]
 = Tonnelat-Baudot, Marie-Antoinette (5.3.1912-3.12.1980)
https://en.wikipedia.org/wiki/Marie-Antoinette_Tonnelat
- Vavilov, S. I. [8, 12, 16] / [165-167]
 = Vavilov, Sergeĭ Ivanovich [24.3.(O.S. 12.3.)1891-25.1.1951]
 = Вавилов, Сергей Иванович
https://en.wikipedia.org/wiki/Sergey_Ivanovich_Vavilov
- Watson, W. H. [13, 60, 97]
 = Watson, William Heriot (12.12.1899-9.11.1987)
<http://viaf.org/viaf/118253342>

- Weiss, P. [119]
 = Weiss, Paul (9.4.1911-19.1.1991)
 See [276].
- Weisskopf, V. [80, 98] / [244, 245, 248]
 = Weisskopf, Viktor (Friedrich) (19.7.1908-21.4.2002)
 = Weisskopf, Victor (Frederick)
https://en.wikipedia.org/wiki/Victor_Weisskopf
- Wiegand, R. V. [135]
 = Wiegand, Roy Vernon (13.5.1914-18.12.2000)
- Williams, E. J. [61]
 = Williams, Evan James (8.8.1903-19.9.1945)
https://en.wikipedia.org/wiki/Evan_James_Williams
- Witmer, E. E. [9]
 = Witmer, Enos Eby (28.11.1898-29.10.1987)
<http://viaf.org/viaf/45946401>
- Zwicky, F. [120]
 = Zwicky, Fritz (14.2.1898-8.2.1974)
https://en.wikipedia.org/wiki/Fritz_Zwicky

Further literature

- [157] J. Kepler: *Ad Vitellionem Paralipomena, Quibus Astronomiae Pars Optica Traditur*. C. Marnius & heirs of J. Aubrius, Frankfurt/M., 1604
 (The book is freely available online at the ETH Zurich Library site given by the DOI: [10.3931/e-rara-322](https://doi.org/10.3931/e-rara-322)). [in Latin] Reprints: 1. [158], pp. 119-398. 2. [159], pp. 5-391. 3. Culture et Civilisation, Brussels, 1968. Translations:
1. Catalans: J. Kepler: *Paralipòmens a Vitel·lió. Els Orígens de l'Òptica Moderna*. Pòrtic-EUMO-IEC, Barcelona, 2010.
 2. English: [160]
 3. French: J. Kepler: *Paralipomènes à Vitellion. Les Fondements de l'Optique Moderne*. L'Histoire des Sciences, Textes et Études, Vol. 21. J. Vrin, Paris, 1980.
 4. German: 1. J. Kepler: *J. Keplers Grundlagen der geometrischen Optik (im Anschluß an die Optik des Witelo)*. Ostwalds Klassiker der Exakten Wissenschaften, Vol. 198. Akademische Verlags-Gesellschaft, Leipzig, 1922 (The book is freely available online at the Deutsche Digitale Bibliothek website: <https://www.deutsche-digitale-bibliothek.de/item/DJMDW5WCXU7TFBJLOCBGHGF2FHWQP3D>).

2. J. Kepler: *Schriften zur Optik, 1604-1611. Eingeführt und ergänzt durch historische Beiträge zur Optik- und Fernrohrgeschichte von Rolf Riekher*. Ostwalds Klassiker der Exakten Wissenschaften, Vol. 198 (new edition of vols. 144 and 198 in one volume). Verlag Harri Deutsch, Frankfurt/M., 2008.

– Cited at pp.: 3, 36

- [158] J. Kepler: *Joannis Kepleri Astronomi Opera Omnia*. Vol. II. Heyder & Zimmer, Frankfurt/M., 1859 (The book is freely available online from the Sächsische Landesbibliothek – Staats- und Universitätsbibliothek Dresden site: <http://digital.slub-dresden.de/id338583289>). – Cited at pp.: 94
- [159] J. Kepler: *Gesammelte Werke. Band II, Astronomiae Pars Optica*. C. H. Beck'sche Verlagsbuchhandlung, Munich, 1939. – Cited at pp.: 94
- [160] J. Kepler: *Optics. Paralipomena to Witelo & Optical Part of Astronomy*. Translated by W. H. Donahue. Green Lion Press, Santa Fe, NM, 2000. – Cited at pp.: 3, 94
- [161] Witelo [G. Tannstetter, P. Apianus (Eds.)]: *Vitellonis Mathematici Doctissimi Περι Οπτικῆσ [Peri Optikēs], id est de natura, ratione, & proiectione radiorum visus, luminum, colorum atq[ue] formarum, quam vulgo Perspectivam vocant, libri X*. J. Petreius, Nuremberg, 1535, 2. ed. 1551 (The edition of 1535 is freely available online from the Sächsische Landesbibliothek - Staats- und Universitätsbibliothek Dresden at the URL: <http://digital.slub-dresden.de/id274466821> . The edition of 1551 is freely available online from the Universidad de Sevilla digital archive at the URL: <http://fondosdigitales.us.es/fondos/libros/978/10/vitellonis-mathematici-doctissimi-peri-optikes-id-est-de-natura-ratione-proiectione-radorum-uisus-luminumcolorum-atque-formarum-quam-uulgo-perspectiuam-uocant-libri-x/>). [in Latin] Reprinted in: 1. F. Risner (Ed.): *Opticae Thesaurus. Alhazeni Arabis Libri Septem, nunc primum editi: eiusdem liber de crepusculis et nubium ascensionibus. item Vitelloni Thuringopoloni Opticae Libri X / omnes instaurati, figuris illustrati & aucti, adiectis etiam in Alhazenum commentariis*. Episcopios, Basel, 1572 (The book is freely available online at the ETH Zurich Library site given by the DOI: [10.3931/e-rara-9817](https://doi.org/10.3931/e-rara-9817) .). Reprint: D. C. Lindberg (Ed.), Johnson Reprint Corp., New York, 1972. 2. Incomplete: Various volumes of the series *Studia Copernicana*, Ossolineum, The Polish Academy of Sciences Press, Wroclaw (The series also contains English and Polish translations.). – Cited at pp.: 3
- [162] S. Unguru: *Witelonis Perspectivae Liber Secundus et Liber Tertius. Books II and III of Witelo's Perspectiva – A Critical Latin Edition and English Translation with Introduction, Notes and Commentaries*. Studia Copernicana, Vol. XXVIII. Ossolineum, The Polish Academy of Sciences Press, Wroclaw, 1991. – Cited at pp.: 3

- [163] Ch. Huygens: *Traité de la Lumière où sont expliquées les causes de ce qui luy arrive dans la reflexion, & dans la refraction. Et particulièrement dans l'etrange refraction du cristal d'Islande*. Pierre van der Aa, Leiden, 1690 (The book is freely available online at the ETH Zurich Library site given by the DOI: [10.3931/e-rara-3766](https://doi.org/10.3931/e-rara-3766)). 3, 36

Reprints:

1. In: Ch. Huygens: *Œuvres Complètes. Tome XIX. Mécanique Théorique et Physique de 1666 à 1695*. Martinus Nijhoff, The Hague, 1937, pp. 451-548 (The text is freely available online from the Digitale Bibliotheek voor de Nederlandse Letteren (Digital Library of Dutch Literature) site: http://www.dbnl.org/tekst/huyg003oeuv19_01/downloads.php . The book is freely available online at the Gallica Site of the Bibliothèque Nationale de France, stable URL: <http://gallica.bnf.fr/ark:/12148/bpt6k77868x>).
2. Gressner und Schramm, Leipzig, 1885, 1887, [1913?].
3. Les Maîtres de la Pensée Scientifique. Gauthier-Villars, Paris, 1920.
4. Dawsons, London, 1966.
5. Culture et Civilisation, Brussels, 1967.
6. Philips' Gloeilampenfabrieken Eindhoven Lighting Design and Engineering Centre, Eindhoven, 1980.
7. Dunod, Paris, 1992.

Translations:

1. Chinese: 克里斯蒂安·惠更斯 [Kèlǐsīdì ān. Huì gèng sī]: 光 [Lùn guāng]. 科名著文 [Kēxué míngzhù wénkù]. 武出版社 [Wúhàn chūbǎn shè], Wuhan, 1993.
2. Dutch: Ch. Huygens: *Verhandeling over het licht*. Epsilon Uitgaven, Utrecht, 1990.
3. English: [164].
4. German: Ch. Huygens: *Abhandlung über das Licht: worin die Ursachen der Vorgänge bei seiner Zurückwerfung und Brechung und besonders bei der eigentümlichen Brechung des isländischen Spates dargelegt sind*. Ostwalds Klassiker der exakten Wissenschaften, Vol. 20. W. Engelmann, Leipzig, 1. ed. 1890, 2. ed. 1903, 3. ed. 1913; Wissenschaftliche Buchgesellschaft, Darmstadt, 1964; H. Deutsch, Thun, 1996.
5. Italian: 1. Trattato sulla luce. In: Ch. Huygens, C. Pighetti (Ed.) *Horologium Oscillatorium. Traité de la lumière / L'orologio a pendolo: dimostrazioni geometriche sul moto dei pendoli applicati agli orologi. Trattato sulla luce*. G. Barbèra, Florence, 1963. 2. Ch. Huygens: Trattato sulla luce. *Atti della Fondazione Giorgio Ronchi e Contributi dell'Istituto Nazionale di Ottica* **18:3**(1963)257-291, **18:4**(1963)361-415.

6. Latin: Tractatus de lumine. In: Ch. Huygens: *Christiani Hugenii Zuilichemii, dum viveret Zelhemii Toparchae, opuscula reliqua*. Vol. 1. Jansson-Waesberg, Amsterdam, 1728 (The book is freely available online at the ETH Zurich Library site given by the DOI: [10.3931/e-rara-4462](https://doi.org/10.3931/e-rara-4462) .).
7. Portuguese: Ch. Huygens: *Tratado sobre a Luz: onde são explicadas as causas daquilo que lhe acontece ce na reflexão e na refração e particularmente na estranha refração do cristal da Islândia*. Cadernos de História e Filosofia da Ciência (Suppl. 4, 1986). UNICAM, São Paulo, 1986.
8. Russian: Х. Гюйгенс [Kh. Gyuĭgens]: *Трактат о свете, в котором объяснены причины того, что с ним происходит при отражении и преломлении, в частности при странном преломлении исландского кристалла* [Traktat o svete, v kotorom ob"yasneny prichiny того, chto s nim proiskhodit pri otrazhenii i prelomlenii, v chastnosti pri strannom prelomlenii islandskogo kristalla]. Классики естествознания [Klassiki estestvoznaniya]. 1. ed.: ОНТИ [ONTI], Moscow-Leningrad, 1935; 2. ed.: Классики науки [Klassiki nauki]. URSS, Moscow, 2010.

– Cited at pp.:

- [164] Ch. Huygens: *Treatise on Light in which are explained the cause of that which occurs in reflexion, & in refraction. And particularly in the strange refraction of Iceland crystal*. Translated by S. P. Thompson. University of Chicago Press, Chicago, Macmillan, London, 1912 (The text is freely available online as eBook from The Project Gutenberg, <http://www.gutenberg.org/dirs/1/4/7/2/14725> . The book is freely available online at the Internet Archive site: <http://www.archive.org/details/treatiseonlight031310mbp>). Reprint: Dover Books on History of Science and Classics of Science, Vol. 179. Dover Publications, New York, 1962. – Cited at pp.: 3, 96
- [165] С. И. Вавилов [S. I. Vavilov]: *Собрание сочинений. Том I - Работы по физике 1914-1936* [Sobranie sochineniy. Tom I - Raboty po fizike 1914-1936]. Издательство Академии наук СССР [Izdatel'stvo Akademii nauk SSSR], Moscow, 1954 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=43932907>). [in Russian] – Cited at pp.: 4, 8, 36, 64, 66, 93
- [166] С. И. Вавилов [S. I. Vavilov]: *Микроструктура света (Исследования и очерки)* [Mikrostruktura sveta (Issledovaniya i ocherki)]. Серия «Итоги и проблемы современной науки» [Seriya «Itogi i problemy sovremennoy nauki»]. Издательство Академии наук СССР [Izdatel'stvo Akademii nauk SSSR], Moscow, 1950 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/>

[view/publication/general.html?id=46540521](#)). [in Russian] Reprinted in: [167], pp. 381-544. Translations (no English transl.):

1. Chinese: 瓦维洛夫 [Wǎ wéi luò fu]: 光的微观结构. 研究和概略 [*Guāng de wéiguān jiégòu. Yánjiū hé gàilüé*]. 中国科学院 [Zhōngguó kēxuéyuàn]/[Chinese Academy of Sciences], Beijing, 1954.
2. French: S. I. Vavilov: *Micro-structure de la lumière*. Editions en Langues Étrangères, Moscow, s.d..
3. German: S. I. Wawilow: *Die Mikrostruktur des Lichtes. Untersuchungen und Grundgedanken*. Akademie-Verlag, Berlin, 1953.
4. Hungarian: Sz. I. Vavilov: *A fény mikrostruktúrája*. Akadémiai Kiadó, Budapest, 1955.
5. Polish: S. I. Wawilow: *Mikrostruktura światła. Badania i szkice*. Państwowe Wydawnictwo Naukowe, Warsaw, 1953.
6. Romanian: S. I. Vavilov: *Mikrostruktura Lumini (Cercetări și Studii)*. Editura Academiei Republicii Populare Române, Bucarest, 1953.
7. Ukrainian: С. И. Вавилов [S. I. Vavilov]: *Мікроструктура світла* [*Mikrostruktura svītla*]. Радянська школа [Radyans'ka shkola], Kiev, 1956.

– Cited at pp.: 4, 93

[167] С. И. Вавилов [S. I. Vavilov]: *Собрание сочинений. Том II - Работы по физике 1937-1951* [*Sobranie sochineniy. Tom II - Raboty po fizike 1937-1951*]. Издательство Академии наук СССР [Izdatel'stvo Akademii nauk SSSR], Moscow, 1954 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=43460755>). [in Russian] – Cited at pp.: 93, 98

[168] N. Kipnis: *History of the Principle of Interference of Light*. Science Networks · Historical Studies, Vol. 5. Birkhäuser Verlag, Basel, 1991 (DOI: [10.1007/978-3-0348-8652-9](https://doi.org/10.1007/978-3-0348-8652-9)). – Cited at pp.: 3

[169] М. В. Ломоносов [M. V. Lomonosov]: *Слово о происхождении свѣта, новую теорію о цвѣтахъ представляющее, въ публичномъ собраніи императорской Академіи наукъ іюля 1 дня 1756 года говоренное Михайломъ Ломоносовымъ* [*Slovo o proiskhozhdenii svĕta, novuyu teoriyu o tsvĕtakh' predstavlyayushchee, v' publichnom' sobranii imperatorskoĭ Akademii nauk' iyulya 1 dnuya 1756 goda govorennoe Mikhailom' Lomonosovym'*]. Императорская Академія наукъ [Imperatorskaya Akademīya nauk'], St. Petersburg, 1757. [in Russian] Reprinted in: 1. *Полное собрание сочинений Михаила Васильевича Ломоносова. Часть третья* [*Polnoe sobranie sochineniy Mikhaila Vasil'evicha Lomonosova. Chast' tret'ya*]. Императорская Академія наукъ

[Imperatorskaya Akademīya nauk’], St. Petersburg, 1803, pp. 105-142 (The book is freely available online at the site:

<http://imwerden.de/cat/modules.php?name=books&pa=showbook&pid=2666>).

2. *Сочинения М. В. Ломоносова съ объяснительными примѣчаніями академика М. И. Сухомлинова* [*Sochineniya M. V. Lomonosova s’ ob’yasnitel’nyimi priměchaniyami akademika M. I. Sukhomlinova*]. Vol. 4. Императорская Академія наукъ [Imperatorskaya Akademīya nauk’], St. Petersburg, 1898, item XIII, p. 392-424 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=42070256>). 3. М. В. Ломоносов [M. V. Lomonosov]: Слово о происхождении света, новую теорию о цвѣтахъ представляющее, в публичномъ собраніи императорской Академіи Наукъ июля 1 дня 1756 года говоренное Михайломъ Ломоносовымъ [Slovo o proiskhozhdenii sveta, novuyu teoriyu o tsvětakh predstavlyayushchee, v publichnom sobranii imperatorskoj Akademii Nauk iyulya 1 dnya 1756 goda govorennoe Mikhaïlom Lomonosovym]. In: *Полное собрание сочинений. Том Третій. Труды по физике 1753-1765 гг.* [*Polnoe sobranie sochineniy. Tom tretiy. Trudy po fizike 1753-1765 gg.*]. Издательство Академіи наук СССР [Izdatel’stvo Akademii nauk SSSR], Moscow, 1952, item 15, pp. 315-344 (The book is freely available online at the site:

<http://lomonosov300.ru/6158.htm>). Translations:

1. English: Oration on the origin of light. A new theory of color, presented in a public meeting of the Imperial Academy of Sciences, July 1, 1756. In: *Mikhail Vasil’evich Lomonosov on the Corpuscular Theory*. Translated, with an Introduction, by H. M. Leicester. Harvard University Press, Cambridge, MA, 1970, pp. 247-269 (The book is freely available online at the Internet Archive site: <http://www.archive.org/details/mikhailvasilevic017733mbp>).
2. German: Rede über das Entstehen des Lichtes, die eine neue Farbenlehre darstellt. In: M. W. Lomonossow: *Ausgewählte Schriften in zwei Bänden. Band 1: Naturwissenschaften*. Akademie-Verlag, Berlin, 1961, pp. 329-360.
3. Hungarian: Beséd a fény keletkezéséről, amely a színek új elméletét is bemutatja. Elmondta Mihajlo Lomonoszov a Cári Tudományos Akadémia 1756. július 1-én tartott nyilvános ülésén. In: [M. Lomonoszov]: *Lomonoszov válogatott írásai*. Európa Könyvkiadó, Helikon Kiadó, Budapest, 1982, pp. 195-236.
4. Latin: *Oratio de origine lucis: sistens novam theoriam colorum, in publico conventu Academiae Scientiarum Imperialis Petropolitanae propter nominis festivitatem serenissimis principis magni ducis Pauli Petridae habita calendis Iuliis anni MDCCLVI a Michaele Lomonosow, consiliario academico*. Typis Academiae Scientiarum, St. Petersburg, 1756.

– Cited at pp.: 4, 36

- [170] E. D. Palik, B. W. Hennis: A bibliography of magneto-optics of solids. *Applied Optics* **6**:4(1967)603-630 (DOI: [10.1364/AO.6.000603](https://doi.org/10.1364/AO.6.000603)). – Cited at pp.: 4
- [171] J. W. Beams: Electric and magnetic double refraction. *Reviews of Modern Physics* **4**:1(1932)133-172. (DOI: [10.1103/RevModPhys.4.133](https://doi.org/10.1103/RevModPhys.4.133)). – Cited at pp.: 4
- [172] A. I. Milstein, M. Schumacher: Present status of Delbrück scattering. *Physics Reports* **243**:4(1994)183-214 (DOI: [10.1016/0370-1573\(94\)00058-1](https://doi.org/10.1016/0370-1573(94)00058-1)). – Cited at pp.: 5
- [173] R. N. Lee, A. L. Maslennikov, A. I. Milstein, V. M. Strakhovenko, Yu. A. Tikhonov: Photon splitting in atomic fields. *Physics Reports* **373**:3(2003)213-246 (DOI: [10.1016/S0370-1573\(02\)00030-3](https://doi.org/10.1016/S0370-1573(02)00030-3)) [[arXiv:hep-ph/0111447](https://arxiv.org/abs/hep-ph/0111447) (<https://arxiv.org/abs/hep-ph/0111447>)]. – Cited at pp.: 5
- [174] R. P. Mignani, V. Testa, D. González Caniulef, R. Taverna, R. Turolla, S. Zane, K. Wu: Evidence for vacuum birefringence from the first optical-polarimetry measurement of isolated neutron star RX J1856.5-3754. *Monthly Notices of the Royal Astronomical Society* **465**:1(2017)492-500 (DOI: [10.1093/mnras/stw2798](https://doi.org/10.1093/mnras/stw2798)) [[arXiv:1610.08323](https://arxiv.org/abs/1610.08323) (<https://arxiv.org/abs/1610.08323>)]. – Cited at pp.: 5
- [175] ATLAS Collaboration (Morad Aaboud (Oujda U.) et al.): Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC. *Nature Physics* **13**:9(2017)852-558 (DOI: [10.1038/nphys4208](https://doi.org/10.1038/nphys4208)) [CERN Preprint CERN-EP-2016-316, [arXiv:1702.01625](https://arxiv.org/abs/1702.01625) (<https://arxiv.org/abs/1702.01625>)]. – Cited at pp.: 5
- [176] R. Battesti, C. Rizzo: Magnetic and electric properties of a quantum vacuum. *Reports on Progress in Physics* **76**:1(2013)016401, 23 pp. (DOI: [10.1088/0034-4885/76/1/016401](https://doi.org/10.1088/0034-4885/76/1/016401)) [[arXiv:1211.1933](https://arxiv.org/abs/1211.1933) (<https://arxiv.org/abs/1211.1933>)]. – Cited at pp.: 11
- [177] П. Л. Капица [P. L. Kapitsa]: Альберт Эйнштейн [Al'bert Ėinshteĭn]. *Вопросы Философии* [*Voprosy Filosofii*] No. 6 (1980)29-31. Reprinted in: 1. [178], 2. [179], 3. ed. pp. 372-375, 4. ed. pp. 370-375. English translation: [180], reprinted in [181], item 29, pp. 315-317. – Cited at pp.: 11, 36
- [178] П. Л. Капица [P. L. Kapitsa]: Альберт Эйнштейн [Al'bert Ėinshteĭn]/[Albert Einstein]. *Вестник Академии Наук СССР* [*Vestnik Akademii Nauk SSSR*] No. 7 (1980)37-40. – Cited at pp.: 11, 36, 100

- [179] П. Л. Капица [P. L. Kapitza]: *Эксперимент. Теория. Практика. Статьи, выступления* [*Èksperiment. Teoriya. Praktika. Stat'i, vystupleniya*]. Наука [Nauka], Moscow, 3. ext. ed.: 1981, 4. corr. and ext. ed.: 1987. Translations of the 3. ed.:
1. German: P. L. Kapica: *Experiment, Theorie, Praxis. Aufsätze und Reden.* Beiträge zur Forschungstechnologie, Sonderband No. 3. Akademie-Verlag, Berlin, 1984.
 2. Polish: P. L. Kapica: *Eksperyment. Teoria. Praktyka. Artykuły, wystąpienia.* Państwowe Wydawnictwo Naukowe, Warsaw, 1987.
 3. Spanish: P. L. Kapitza: *Experimento. Teoría. Práctica. Artículos y conferencias.* Mir, Moscow, 1985.
- Cited at pp.: [11](#), [36](#), [100](#)
- [180] P. L. Kapitza: Albert Einstein. *Soviet Studies in Philosophy* **19**:3(1980)3-10 (DOI: [10.2753/RSP1061-196719033](https://doi.org/10.2753/RSP1061-196719033)). Reprinted in: [181], item 29, pp. 315-317.– Cited at pp.: [11](#), [100](#)
- [181] D. ter Haar (Ed.): *Collected Papers of P. L. Kapitza. Volume 4: 1965-1983.* Pergamon Press, Oxford, 1986. – Cited at pp.: [11](#), [100](#), [101](#)
- [182] Yu. M. Poluektov: *On the dependence of the speed of light in vacuum on temperature.* [arXiv:1706.01319](https://arxiv.org/abs/1706.01319) (<https://arxiv.org/abs/1706.01319>). – Cited at pp.: [11](#)
- [183] A. D. Beyerchen: *Scientists under Hitler. Politics and the Physics Community in the Third Reich.* Yale University Press, New Haven, 1977. – Cited at pp.: [12](#)
- [184] W. Stöcker: *Der Nobelpreisträger Johannes Stark, 1874-1957. Eine politische Biographie.* Medien Verlag Köhler, Tübingen, 2001. [in German] – Cited at pp.: [12](#)
- [185] J. Stark: *Erfahrungen und Theorien über Licht und Elektron.* Verlag G. H. Stifel, Traunstein Obb., 1950. [in German] – Cited at pp.: [13](#), [84](#), [93](#)
- [186] J. Stark: Weitere experimentelle Untersuchungen über die Natur des Lichts [Further experimental investigations on the nature of light]. *Zeitschrift für Physik* **133**:4(1952)504-512 (DOI: [10.1007/BF01336862](https://doi.org/10.1007/BF01336862)). [in German] – Cited at pp.: [13](#), [84](#), [93](#)
- [187] M. Planck: *Einführung in die Theorie der Elektrizität und des Magnetismus. Zum Gebrauch bei Vorträgen, sowie zum Selbstunterricht.* Verlag von S. Hirzel, Leipzig, 1. ed. 1922, 2. ed. 1928, 3. ed. 1937 (= Vol. 3 of: *Einführung in die Theoretische Physik*; the volume, 1. ed., is freely available online at the Internet Archive site:

- <http://www.archive.org/details/einfhrungindiet00plangoog>). English translation: *Theory of Electricity and Magnetism. Being Volume III of "Introduction to Theoretical Physics"*. Macmillan, London, pr. 1932, 1949, 1957. – Cited at pp.: 13
- [188] R. P. Feynman, R. B. Leighton, M. Sands: *The Feynman Lectures on Physics, Mainly Elektromagnetism and Matter*. Addison-Wesley Publishing Company, Inc., Reading, MA, 1964 (= Vol. 2 of: *The Feynman Lectures on Physics*; various further editions). – Cited at pp.: 13
- [189] W. Gentner: Die Absorption, Streuung und Sekundärstrahlung harter γ -Strahlung [The absorption, scattering, and secondary radiation of hard γ -radiation]. *Physikalische Zeitschrift* **38**:21(No. 985)(1937)836-853. [in German] Reprinted in: W. Gentner: *Schriften und Vorträge zur Kernphysik bis 1976*. Max-Planck-Institut für Kernphysik, Heidelberg, 1976, item 31, pp. 251-268. – Cited at pp.: 14
- [190] Ch. M. Davisson, R. D. Evans: Gamma-ray absorption coefficients. *Reviews of Modern Physics* **24**:2(1952)79-107 (DOI: [10.1103/RevModPhys.24.79](https://doi.org/10.1103/RevModPhys.24.79)). – Cited at pp.: 14
- [191] W. Gentner: Einiges aus der frühen Geschichte der Gamma-Strahlen [From the early history of gamma rays]. [in German] In: O. R. Frisch, F. A. Paneth, F. Laves, P. Rosbaud (Eds.): *Beiträge zur Physik und Chemie des 20. Jahrhunderts. Lise Meitner, Otto Hahn, Max von Laue zum 80. Geburtstag*. Friedrich Vieweg & Sohn, Braunschweig, 1959, pp. 28-44. Parallel edition: O. R. Frisch, F. A. Paneth, F. Laves, P. Rosbaud (Eds.): *Trends in Atomic Physics. Essays dedicated to Lise Meitner, Otto Hahn, Max von Laue on the occasion of their 80th birthday*. Interscience Publishers, New York, 1959. Reprinted in: W. Gentner: *Schriften und Vorträge zur Kernphysik bis 1976*. Max-Planck-Institut für Kernphysik, Heidelberg, 1976, item 57, pp. 466-482. – Cited at pp.: 14
- [192] L. Meitner, H. H. Hupfeld: Über die Prüfung der Streuungsformel von Klein und Nishina an kurzwelliger γ -Strahlung [On the verification of the scattering formula by Klein and Nishina on short-wave γ -radiation]. *Die Naturwissenschaften* **18**:22(1930)352-353 (DOI: [10.1007/BF01513428](https://doi.org/10.1007/BF01513428)). [in German] – Cited at pp.: 14
- [193] L. Meitner, H. H. Hupfeld: Prüfung der Streuungsformel von Klein und Nishina an kurzwelliger γ -Strahlung [Verification of the scattering formula by Klein and Nishina on short-wave γ -radiation]. *Physikalische Zeitschrift* **31**:21(No. 817)(1930)947-948. [in German] – Cited at pp.: 14
- [194] L. Meitner, H. H. Hupfeld: Über das Absorptionsgesetz für kurzwellige γ -Strahlung [On the law of absorption for short-wave γ -radiation]. *Zeitschrift für Physik* **67**:3-4(1931)147-168 (DOI: [10.1007/BF01394595](https://doi.org/10.1007/BF01394595)). [in German] – Cited at pp.: 14

- [195] L. Meitner, H. H. Hupfeld: Über das Streugesetz kurzwelliger γ -Strahlen [On the scattering law of short-wave γ -rays]. *Die Naturwissenschaften* **19**:37(1931)775-776 (DOI: [10.1007/BF01520525](https://doi.org/10.1007/BF01520525)). [in German] – Cited at pp.: [14](#)
- [196] L. Meitner, H. H. Hupfeld: Über die Streuung kurzwelliger γ -Strahlung an schweren Elementen [On the scattering of shortwave γ radiation off heavy elements]. *Zeitschrift für Physik* **75**:11-12(1932)705-715 (DOI: [10.1007/BF01341611](https://doi.org/10.1007/BF01341611)). [in German] – Cited at pp.: [14](#)
- [197] L. Meitner: Über die Streuung kurzwelliger γ -Strahlen (nach gemeinsamen Versuchen mit H. H. Hupfeld und H. Kösters) [On the scattering of short-wave γ rays (according to joint experiments with H. H. Hupfeld and H. Kösters)]. *Helvetica Physica Acta* **6**:6(1933)445-450 (The article is freely available online at the ETH Zurich Library site: <http://www.e-periodica.ch/digbib/view?pid=hpa-001:1933:6::447>). [in German] – Cited at pp.: [14](#)
- [198] L. M. Brown, D. F. Moyer: Lady or tiger? – The Meitner-Hupfeld effect and Heisenberg's neutron theory. *American Journal of Physics* **52**:2(1984)130-136 (DOI: [10.1119/1.13920](https://doi.org/10.1119/1.13920)). – Cited at pp.: [14](#), [67](#)
- [199] *Max Delbrück: Interview by Carolyn Harding, Pasadena, California, July 14-September 11, 1978*. Oral History Project, California Institute of Technology Archives, Pasadena, CA (The transcript - retrieved June 24, 2017 - is freely available online from the Caltech Archives site, persistent URL: http://resolver.caltech.edu/CaltechOH:OH_Delbruck_M). – Cited at pp.: [15](#), [88](#)
- [200] K. von Meyenn (Ed.): *Wolfgang Pauli. Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg u.a.. Band II: 1930-1939. Scientific Correspondence with Bohr, Einstein, Heisenberg a.o.. Volume II: 1930-1939*. Sources in the History of Mathematics and Physical Sciences, Vol. 6. Springer-Verlag, Berlin, 1985 (DOI: [10.1007/978-3-540-78801-0](https://doi.org/10.1007/978-3-540-78801-0)). – Cited at pp.: [16](#), [31](#), [32](#), [34](#), [37](#), [40](#), [57](#)
- [201] F. Rohrlich, R. L. Gluckstern: Forward scattering of light by a Coulomb field. *Physical Review* **86**:1(1952)1-9 (DOI: [10.1103/PhysRev.86.1](https://doi.org/10.1103/PhysRev.86.1)). – Cited at pp.: [16](#)
- [202] H. A. Bethe, F. Rohrlich: Small angle scattering of light by a Coulomb field. *Physical Review* **86**:1(1952)10-16 (DOI: [10.1103/PhysRev.86.10](https://doi.org/10.1103/PhysRev.86.10)). – Cited at pp.: [16](#)
- [203] L. Meitner: Die Streuung harter γ -Strahlen [The scattering of hard γ -rays]. *Die Naturwissenschaften* **22**:11(1934)174 (DOI: [10.1007/BF01496259](https://doi.org/10.1007/BF01496259)). [in German] – Cited at pp.: [17](#)

- [204] E. Schönfeld: Die dunkeln Fixstern-Begleiter [The dark fixed star companions]. *Jahresbericht des Mannheimer Vereins für Naturkunde* **30**(1864)91-114. [in German] – Cited at pp.: [18](#), [38](#)
- [205] M. J. Aschwanden: *Physics of the Solar Corona. An Introduction*. Springer-Praxis Books in Geophysical Science. Praxis Publishing Ltd., Chichester; Springer-Verlag, Berlin, 2004. 2. ed. with corrections, problems, and solutions: *Physics of the Solar Corona. An Introduction with Problems and Solutions*. Springer-Praxis Books in Astronomy and Planetary Sciences. Praxis Publishing Ltd., Chichester; Springer-Verlag, Berlin, 2005, 2006 (DOI: [10.1007/3-540-30766-4](#)). – Cited at pp.: [20](#)
- [206] W. Pauli jun.: Relativitätstheorie. *Encyclopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Fünfter Band in drei Teilen. Zweiter Teil*. Verlag und Druck von B. G. Teubner, Leipzig, 1904-1922, sec. 19 (1920), pp. 539-775 (The article is freely available online at the GDZ Document Server: <http://gdz.sub.uni-goettingen.de/dms/load/toc/?PPN=PPN360709672>). Separately issued: *Relativitätstheorie. Sonderabdruck aus der Encyclopädie der Mathematischen Wissenschaften*. B. G. Teubner, Leipzig, 1921. Extended reprints: *Relativitätstheorie*. Boringhieri, Turin, 1963. *Relativitätstheorie. Neu herausgegeben und kommentiert von Domenico Giulini*. Springer, Berlin, 2000 (DOI: [10.1007/978-3-642-58355-1](#)). [in German] English translation: *Theory of Relativity*. Pergamon Press, New York, NY, 1958. Reprinted: Dover Books on Physics. Dover Publications, New York, NY, 1981. – Cited at pp.: [21](#), [92](#)
- [207] M. Born: *My Life. Recollections of a Nobel Laureate*. Charles Scribner's Son, New York, NY; Taylor & Francis, London, 1978. Reprint: Routledge Library Editions: 20th Century Science. Routledge, Abingdon, UK, New York, NY, 2014. German translation: *Mein Leben. Die Erinnerungen des Nobelpreisträgers*. Nymphenburger Verlagshandlung, Munich, 1975. – Cited at pp.: [21](#), [22](#), [23](#), [87](#)
- [208] M. Born: Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips [The theory of the rigid electron in the kinematics of the relativity principle]. *Annalen der Physik*, 4. Series, **30** (**335** of all series altogether):11(1909)1-56 (DOI: [10.1002/andp.19093351102](#)), *Erratum ibid.* 4. Series, **30** (**335** of all series altogether):14(1909)840 (DOI: [10.1002/andp.19093351410](#)). [in German] Also separately issued as: M. Born: *Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips*. J. A. Barth, Leipzig, 1909. The article is reprinted in: [209], vol. 1, item 2, pp. 105-160. – Cited at pp.: [21](#), [87](#)
- [209] M. Born: *Ausgewählte Abhandlungen. Mit einem Verzeichnis der wissenschaftlichen Schriften*. 2 Volumes. Vandenhoeck & Ruprecht, Göttingen, 1963. – Cited at pp.: [67](#), [68](#), [87](#), [104](#)

- [210] A. S. Eddington: *The Mathematical Theory of Relativity*. Cambridge University Press, 1. ed.: 1923, 2. ed.: 1924, various reprints (The first edition of the book is freely available online at the Internet Archive site, URL: <https://archive.org/details/mathematicaltheo00eddiuo>). – Cited at pp.: 23
- [211] G. Boillat: Nonlinear electrodynamics: Lagrangians and equations of motion. *Journal of Mathematical Physics* **11**:3(1970)941-951 (DOI: [10.1063/1.1665231](https://doi.org/10.1063/1.1665231)). – Cited at pp.: 27
- [212] J. Plebański: *Lectures on Non-Linear Electrodynamics*. Nordita, Copenhagen, 1970. – Cited at pp.: 27
- [213] A. Einstein, H. Born, M. Born: *Albert Einstein. Hedwig und Max Born. Briefwechsel 1916-1955, kommentiert von Max Born*. Nymphenburger Verlagshandlung, Munich, 1969. Translations:
1. Chinese (translation of [215]): 玻恩-爱因斯坦书信集 (1916-1955). 动荡时代的友谊, 政治和物理学 [Bō ēn-ài yīn sītǎn shūxìn jí (1916-1955). Dòngdàng shídài de yǒuyì, zhèngzhì hé wùlǐ xué]. 上海科技教育出版社 [Shànghǎi kējì jiàoyù chūbǎn shè]/ [Shanghai Science and Technology Education Press], Shanghai, 2010.
 2. English: [214]. Modified reprint: [215]
 3. French: *Albert Einstein. Max Born. Correspondance 1916-1955*. Seuil, Paris, 1972.
 4. Japanese: アインシュタインボルン往集 1916-1955 [Ainshutain borun ōfuku shokan-shū 1916-1955]. 三修社 [Sanshūsha], Tokyo, 1976.
 5. Norwegian: *Albert Einstein. Max Born. Brevveksling 1916-1955*. Aschehoug, Oslo, 1971.
 6. Spanish: 1. *Correspondencia (1916-1955). Albert Einstein. Max y Hedwig Born*. Siglo XXI, Madrid, Mexico City, 1. ed.: 1973, 2. ed.: 1999. 2. *Albert Einstein - Max Born. Cartas (1916-1955). (Y algunos aledaños)*. S.T.I. ediciones, Zaragoza, 2015.
- Cited at pp.: 28, 39
- [214] A. Einstein, H. Born, M. Born: *The Born-Einstein Letters. Correspondence between Albert Einstein and Max and Hedwig Born from 1961 to 1955 with commentaries by Max Born*. Macmillan, London; Walker, New York, 1971 (The book is freely available online at the Internet Archive site, URL: <https://archive.org/details/TheBornEinsteinLetters>). Modified reprint: [215]. – Cited at pp.: 28, 29, 105
- [215] M. Born: *The Born-Einstein Letters 1916-1955. Friendship, Politics and Physics in Uncertain Times*. Macmillan, Basingstoke, 2005. – Cited at pp.: 105

- [216] P. A. M. Dirac: On the annihilation of electrons and protons. *Proceedings of the Cambridge Philosophical Society* **26**:3(1930)361-375 (DOI: [10.1017/S0305004100016091](https://doi.org/10.1017/S0305004100016091)). Reprinted in: R. H. Dalitz (Ed.): *The Collected Works of P. A. M. Dirac 1924-1948*. Cambridge University Press, Cambridge, 1995, paper 1930:2, pp. 419-435. – Cited at pp.: 29
- [217] J. R. Oppenheimer: Two notes on the probability of radiative transitions. *Physical Review* **35**:8(1930)939-947 (DOI: [10.1103/PhysRev.35.939](https://doi.org/10.1103/PhysRev.35.939)). – Cited at pp.: 29
- [218] I. Tamm: Über die Wechselwirkung der freien Elektronen mit der Strahlung nach der Diracschen Theorie des Elektrons und nach der Quantelektrodynamik [On the interaction of free electrons with radiation according to Dirac's theory of the electron and according to quantum electrodynamics]. *Zeitschrift für Physik* **62**:7-8(1930)545-568 (DOI: [10.1007/BF01339679](https://doi.org/10.1007/BF01339679)). [in German] Reprinted in Russian: О взаимодействии свободных электронов с излучением по дираковской теории электрона и по квантовой электродинамике [O vzaimodeistvii svobodnykh elektronov s izlucheniem po dirakovskoi teorii elektrona i po kvantovoi elektrodinamike]. In: И. Е. Тамм [I. E. Tamm]: *Собрание научных трудов в двух томах [Sobranie nauchnykh trudov v dvukh tomakh], Том [Tom] II: Теория элементарных частиц и взаимодействий при высоких энергиях. Фундаментальные проблемы. Разное [Teoriya elementarnykh chastits i vzaimodeistvii pri vysokikh energiyakh. Fundamental'nye problemy. Raznoe]*. Наука [Nauka], Moscow, 1975, item 35, pp. 24-45 (The book is freely available online at the EqWorld site: http://eqworld.ipmnet.ru/ru/library/books/Tamm_t2_1975ru.djvu). – Cited at pp.: 29
- [219] J. R. Oppenheimer, M. S. Plesset: On the production of the positive electron. *Physical Review* **44**:1(1933)53-55 (DOI: [10.1103/PhysRev.44.53.2](https://doi.org/10.1103/PhysRev.44.53.2)). – Cited at pp.: 30
- [220] W. Heitler, F. Sauter: Stopping of fast particles with emission of radiation and the birth of positive electron. *Nature* **132**:3345(1933)892 (DOI: [10.1038/132892a0](https://doi.org/10.1038/132892a0)). – Cited at pp.: 30, 89
- [221] H. Bethe, W. Heitler: On the stopping of fast particles and on the creation of positive electrons. *Proceedings of the Royal Society of London. Series A, Containing Papers of Mathematical and Physical Character* **146**:856(1934)83-112 (DOI: [10.1098/rspa.1934.0140](https://doi.org/10.1098/rspa.1934.0140), stable JSTOR URL: <http://www.jstor.org/stable/2935479>). – Cited at pp.: 30
- [222] Y. Nishina, S. Tomonaga: On the creation of positive and negative electrons. *Proceedings of the Physico-Mathematical Society of Japan*, 3. Series, **15**:3-6(1933)248-249 (The article is freely available online at the J-Stage site given by the DOI: [10.11429/ppmsj1919.15.3-6_246](https://doi.org/10.11429/ppmsj1919.15.3-6_246)), *Japanese Journal of Physics*,

- Abstracts* **9**:1(1933)(21), item 56 (only text without references). – Cited at pp.: **30, 93**
- [223] Y. Nishina, S. Tomonaga, S. Sakata: On the photo-electric creation of positive and negative electrons. *Scientific Papers of the Institute of Physical and Chemical Research* **24**:Suppl. No. 17(1934)1-5. Reprinted in: [266], item 2, pp. 2-6. – Cited at pp.: **30, 93**
- [224] G. Racah: Sulla nascita degli elettroni positivi [On the creation of positive electrons]. *Il Nuovo Cimento*, 8. Ser., **11**:7(1934)477-481 (DOI: [10.1007/BF02959919](https://doi.org/10.1007/BF02959919)). – Cited at pp.: **30**
- [225] G. Breit, J. A. Wheeler: Collisions of two quanta. *Physical Review* **45**:10(1934)766 (DOI: [10.1103/PhysRev.45.739](https://doi.org/10.1103/PhysRev.45.739)), *Bulletin of the American Physical Society* **9**:2(1934)34 (erratum in footnote 2 on p. 1090 of [226]). The article is item 134 of: Program of the Washington Meeting at the Bureau of Standards and the National Academy of Sciences, April 26-28, 1934. *Bulletin of the American Physical Society* **9**:2(1934)1-36, Proceedings of the American Physical Society, Minutes of the Washington Meeting, April 26-28, 1934. *Physical Review* **45**:10(1934)739-771. – Cited at pp.: **30**
- [226] G. Breit, J. A. Wheeler: Collisions of two light quanta. *Physical Review* **46**:12(1934)1087-1091 (DOI: [10.1103/PhysRev.46.1087](https://doi.org/10.1103/PhysRev.46.1087)). – Cited at pp.: **30, 107**
- [227] H. Rechenberg, G. Wiemers (Eds.): *Werner Heisenberg: Gutachten- und Prüfungsprotokolle für Promotionen und Habilitationen (1929-1942)*. BBGNT - Berliner Beiträge zur Geschichte der Naturwissenschaften und Technik, Vol. 29. ERS-Verlag, Berlin, 2001. – Cited at pp.: **30, 40, 73**
- [228] A. Wüthrich: *The Genesis of Feynman Diagrams*. Archimedes, New Studies in the History of Science and Technology, Vol. 26. Springer, Dordrecht, 2010 (DOI: [10.1007/978-90-481-9228-1](https://doi.org/10.1007/978-90-481-9228-1)). – Cited at pp.: **30**
- [229] *VI Съезд русских физиков, Москва, Нижний-Новгород, Казань, Саратов (5-16 августа 1928 года). Перечень докладов, представленных на съезд, с кратким их содержанием* [VI S"eszd russkikh fizikov, Moskva, Nizhniĭ-Novgorod, Kazan', Saratov (5-16 avgusta 1928 goda). Perechen' dokladov, prestavlennykh na s"eszd, s kratkim ikh sodержaniem]. Государственное издательство [Gosudarstvennoe izdatel'stvo], Moscow, Leningrad, 1928 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=48235066>). – Cited at pp.: **32, 64**
- [230] В. Я. Френкель [V. Ya. Frenkel']: *Яков Ильич Френкель [Yakov Il'ich Frenkel']*. Наука [Nauka], Moscow, Leningrad, 1966. English translation: V. Ya. Frenkel: *Yakov Ilich Frenkel. His work, life and letters*. Springer Basel AG

- (Birkhäuser), Basel, 1996 (DOI: [10.1007/978-3-0348-8490-7](https://doi.org/10.1007/978-3-0348-8490-7)). – Cited at pp.: 32
- [231] K. von Meyenn (Ed.): *Wolfgang Pauli. Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg u.a.. Band III: 1940-1949. Scientific Correspondence with Bohr, Einstein, Heisenberg a.o.. Volume III: 1940-1949*. Sources in the History of Mathematics and Physical Sciences, Vol. 11. Springer-Verlag, Berlin, 1993 (DOI: [10.1007/978-3-540-78802-7](https://doi.org/10.1007/978-3-540-78802-7)). – Cited at pp.: 33, 34, 40, 53
- [232] P. Papatzacos, K. Mork: Delbrück scattering. *Physics Report* **21**:2(1975)81-118 (DOI: [10.1016/0370-1573\(75\)90048-4](https://doi.org/10.1016/0370-1573(75)90048-4), preprint version: *Arkiv for det Fysiske Seminar i Trondheim* No. 3 (1975)). – Cited at pp.: 34
- [233] W. H. Furry: A symmetry theorem in the positron theory. *Physical Review* **51**:2(1937)125-129 (DOI: [10.1103/PhysRev.51.125](https://doi.org/10.1103/PhysRev.51.125)). – Cited at pp.: 35
- [234] Z. Bialynicka-Birula, I. Bialynicki-Birula: Nonlinear effects in quantum electrodynamics. Photon propagation and photon splitting in an external field. *Physical Review D* **2**:10(1970)2341-2345 (DOI: [10.1103/PhysRevD.2.2341](https://doi.org/10.1103/PhysRevD.2.2341)). – Cited at pp.: 35
- [235] P. Drude: Ueber die Phasenänderung des Lichtes bei der Reflexion an Metallen [On the phase shift of light under reflection by metals]. *Annalen der Physik und Chemie*, 3. Series, **51** (287 of all series altogether):1(1894)86-104 (DOI: [10.1002/andp.18942870105](https://doi.org/10.1002/andp.18942870105)). [in German] – Cited at pp.: 44
- [236] M. Dressel, G. Grüner: *Electrodynamics of Solids. Optical Properties of Electrons in Matter*. Cambridge University Press, Cambridge, 2002. – Cited at pp.: 44
- [237] D. W. Lynch, W. R. Hunter: Comments on the optical constants of metals and an introduction to the data for several metals. In: E. D. Palik (Ed.): *Handbook of Optical Constants of Solids*, Vol. 1. Academic Press Handbook Series. Academic Press, Boston, 1985, pp. 275-367 (<http://www.sciencedirect.com/science/book/9780125444156>). – Cited at pp.: 44
- [238] *Refractive Index Database* (URL: <http://refractiveindex.info>). Retrieved on May 30, 2013. – Cited at pp.: 44
- [239] В. И. Гостищев [V. I. Gostishchev], В. Р. Соболев [V. R. Sobol']: Влияние магнитного поля на отражение света металлом в предположении изотропного закона дисперсии [Vliyanie magnitnogo polya na otrazhenie sveta metallom v predpolozhenii izotropnogo zakona disperzii]/[The influence of a magnetic field onto the reflection of light by metal under the assumption of an isotropic dispersion relation]. *Доклады Академии Наук БССР [Doklady Akademii Nauk BSSR]* **27**:4(1983)333-336 (The article is freely available online from the Byelorussian State

University repository site: <http://elib.bspu.by/handle/doc/23082> at the URL: [http://elib.bspu.by/bitstream/doc/23082/1/13-Доклады АН БССР-1983-27-4-CBP.pdf](http://elib.bspu.by/bitstream/doc/23082/1/13-Доклады_АН_БССР-1983-27-4-CBP.pdf)). [in Russian, English abstract] – Cited at pp.: 47

- [240] J. M. Ziman: *Electrons and Phonons. The Theory of Transport Phenomena in Solids*. International Series of Monographs on Physics. Clarendon Press, Oxford, 1960. Reprinted: Oxford Classic Texts in the Physical Sciences, 2001. – Cited at pp.: 47
- [241] H. J. Fink: Magnetoresistance of silver. *Physics Letters* **13**:2(1964)105-106 (DOI: 10.1016/0031-9163(64)90674-2). – Cited at pp.: 47
- [242] K. L. Chopra: Size effects in the longitudinal magnetoresistance of thin silver films. *Physical Review* **155**:3(1967)660-662 (DOI: 10.1103/PhysRev.155.660). – Cited at pp.: 47
- [243] W. Heisenberg: *Der Teil und das Ganze. Gespräche im Umkreis der Atomphysik*. Piper, Munich, 1969 (various further German editions). Reprinted in: W. Blum, H.-P. Dürr, H. Rechenberg (Eds.): *Werner Heisenberg – Gesammelte Werke/Collected Works. Abteilung C: Allgemeinverständliche Schriften/Philosophical and Popular Writings, Band III: Physik und Erkenntnis 1969-1976: Der Teil und das Ganze, Die Bedeutung des Schönen, Naturwissenschaftliche und religiöse Wahrheit, Elementarteilchen u.a.*. Piper, Munich, 1985, item 1, pp. 3-334. English translation: *Physics and Beyond. Encounters and Conversations*. World Perspectives, Vol. XLII. Harper & Row, Publishers, Inc., New York, 1971; Allen & Unwin, London, 1981. Numerous editions of translations into other languages. – Cited at pp.: 49, 89
- [244] *Interview of Victor Frederick Weisskopf by Thomas S. Kuhn and John L. Heilbron on July 10, 1963*. Niels Bohr Library & Archives, American Institute of Physics, College Park, MD (The transcript - retrieved June 24, 2017 - is freely available online at the following site of the American Institute of Physics: <https://www.aip.org/history-programs/niels-bohr-library/oral-histories/4944>). – Cited at pp.: 55, 94
- [245] V. Weisskopf: Über die Selbstenergie des Elektrons [On the self-energy of the electron]. *Zeitschrift für Physik* **89**:1-2(1934)27-39 (DOI: 10.1007/BF01333228), Erratum *ibid.* **90**:11-12(1934)817-818 (DOI: 10.1007/BF01340744). [in German] English translation (by W. Grant): [246], paper 5, pp. 157-168. – Cited at pp.: 56, 57, 94
- [246] A. I. Miller: *Early Quantum Electrodynamics: A Source Book*. Cambridge University Press, Cambridge, 1994. – Cited at pp.: 77, 109, 110
- [247] W. Pauli, V. F. Weisskopf: Über die Quantisierung der skalaren relativistischen Wellengleichung [On the quantization of the scalar relativistic wave equation]. *Helvetica Physica Acta* **7**:7(1934)709-731 (The article is freely available

- online at the ETH Zurich Library site given by the DOI: [10.5169/seals-110395](https://doi.org/10.5169/seals-110395) .). [in German] Reprinted in: [248], vol. II, pp. 701-731. English translation (by W. Grant): [246], paper 7, pp. 188-205. – Cited at pp.: 56, 92
- [248] R. Kronig, V. F. Weisskopf (Eds.): *Collected Scientific Papers by Wolfgang Pauli*. 2 Vols.. Interscience, New York, 1964. – Cited at pp.: 94, 110
- [249] N. Kemmer: Die Anfänge der Mesonentheorie und des verallgemeinerten Isospins [The beginnings of meson theory and of generalized isospin]. *Physikalische Blätter* **39**:7(1983)170-175 (DOI: [10.1002/phbl.19830390704](https://doi.org/10.1002/phbl.19830390704)). [in German] – Cited at pp.: 57, 89
- [250] A. I. Akhiezer: Recollections of Lev Davidovich Landau. *Physics Today* **47**:6(1994)35-42 (DOI: [10.1063/1.881434](https://doi.org/10.1063/1.881434)). – Cited at pp.: 59, 61, 86
- [251] А. И. Ахиезер [A. I. Akhiezer]: Воспоминания о Льве Давидовиче Ландау [Vospominaniya o L've Davidoviche Landau]. *Физика Низких Температур* [*Fizika Nizkikh Temperatur*] **19**:1(1993)106-117 (The article is freely available online at the journal website: <http://fnte.ilt.kharkov.ua>). [in Russian] English translation: A. I. Akhiezer: Reminiscences about Lev Davidovich Landau (on the 85th anniversary of his birth). *Low Temperature Physics* **19**:1(1993)76-85. – Cited at pp.: 59, 86
- [252] L. Tisza: Recollections about Alexander Pich Akhiezer. In: Национальная Академия наук Украины [Natsional'naya Akademiya nauk Ukrainy], Министерство образования и науки Украины [Ministerstvo obrazovaniya i nauki Ukrainy], Национальный научный центр «Харьковский физико-технический институт» [Natsional'nyĭ nauchnyĭ tsentr «Kharkovskiy fiziko-tekhnicheskii institut»]: А. И. Ахиезер. Очерки и воспоминания [A. I. Akhiezer. Ocherki i vospominaniya]. Факт [Fakt], Kharkov, 2003, pp. 304-311, Russian translation pp. 311-319. – Cited at pp.: 61
- [253] E. Infeld, I. Białyński-Birula, A. Trautman (Eds.): *Leopold Infeld, his Life and Scientific Work*. Polish Men of Science. Polish Scientific Publishers, Warsaw, 1978. – Cited at pp.: 67, 68, 69, 89
- [254] *Program VII Zjazdu Fizyków Polskich w Krakowie, 27.IX – 29.IX.1934*. S.n., s.l., 1934 (The booklet is freely available online at the Polona Digital Library site: <https://polona.pl/item/program-vii-zjazdu-fizykow-polskich-w-krakowie-27-ix-29-ix-1934,Njg30DU4MjM/0>). [in Polish] – Cited at pp.: 68
- [255] W. Blum, H.-P. Dürr, H. Rechenberg (Eds.): *Werner Heisenberg – Gesammelte Werke/Collected Works, Series A/Part II: Original Scientific Papers – Wissenschaftliche Originalarbeiten*. Springer-Verlag, Berlin, 1989 (DOI: [10.1007/978-3-642-70078-1](https://doi.org/10.1007/978-3-642-70078-1)). – Cited at pp.: 68, 73, 89

- [256] E. Schrödinger: *Collected Papers. Volume 2: Contributions to Field Theory/Gesammelte Abhandlungen. Band 2: Beiträge zur Feldtheorie*. Verlag der Österreichischen Akademie der Wissenschaften, Friedrich Vieweg & Sohn Braunschweig/Wiesbaden, Wien, 1984. – Cited at pp.: 25, 26, 28, 69, 71, 83, 92
- [257] D. ter Haar (Ed.): *Collected Papers of L. D. Landau*. Pergamon Press, Oxford, 1965 (DOI: 10.1016/B978-0-08-010586-4.50001-8). – Cited at pp.: 72, 90
- [258] Л. Д. Ландау [L. D. Landau]: *Собрание трудов [Sobranie trudov]*, Vol. 1. Наука [Nauka], Moscow, 1969 (The book is freely available online at the following site of the Russian Academy of Sciences: <http://nasledie.enip.ras.ru/ras/view/publication/general.html?id=45221810>). – Cited at pp.: 72, 90
- [259] И. Я. Померанчук [I. Ya. Pomeranchuk]: *Собрание научных трудов II. Физика элементарных частиц. Электромагнитные и слабые взаимодействия [Sobranie nauchnykh trudov. II. Fizika èlementarnykh chastits. Èlektromagnitnye i slabye vzaimodeïstviya]*. Наука [Nauka], Moscow, 1972. – Cited at pp.: 72, 77, 92
- [260] G. E. Gorelik, V. Ya. Frenkel: *Matvei Petrovich Bronstein and Soviet Theoretical Physics in the Thirties*. Science Networks, Historical Studies, Vol. 12. Birkhäuser, Basel, 1994 (DOI: 10.1007/978-3-0348-0200-0). – Cited at pp.: 73
- [261] *Program VIII Zjazdu Fizyków Polskich, Lwów, 28.IX – 2.X.1936*. Nakładem Komitetu organizacyjnego, Lwów, 1936 (The booklet is freely available online at the Kujawsko-Pomorska Digital Library site: <http://kpbc.umk.pl/publication/1763>). [in Polish] – Cited at pp.: 73, 74
- [262] W. Pauli, notes by B. Hoffmann: *The Theory of the Positron and Related Topics. (Report of a Seminar)*. The Institute for Advanced Study, Princeton, NJ, 1935/1936 (The seminar report is freely available online from the Wolfgang Pauli Archive website at CERN: <http://cds.cern.ch/record/788821>). – Cited at pp.: 75, 76, 89, 92
- [263] С. П. Шубин [S. P. Shubin]: *Избранные труды по теоретической физике. Очерки жизни. Воспоминания. Статьи [Izbrannye trudy po teoreticheskoi fizike. Ocherki zhizni. Vospominaniya. Stat'i]*. Edited by С. В. Вонсовский [S. V. Vonsovskii], М. И. Кацнельсон [M. I. Katsnel'son]. Институт физики металлов [Institut fiziki metallov]/[Institute of Metal Physics], Уральское отделение Академии наук СССР [Ural'skoe otdelenie Akademii nauk SSSR]/[Ural Branch of the Academy of Sciences of the USSR], Sverdlovsk, 1991. – Cited at pp.: 35, 76, 79, 92
- [264] J. Schwinger (Ed.): *Selected Papers on Quantum Electrodynamics*. Dover Publications, New York, 1958. – Cited at pp.: 77

- [265] *Эйнштейновский сборник 1980-1981 [Eĭnshĕĭnovskĭĭ sbornik 1980-1981]*. Наука [Nauka], Moscow, 1985. – Cited at pp.: 77
- [266] T. Miyazima (Ed.): *Scientific Papers of Tomonaga, Volume 1*. Misuzu Shobo Publishing Co., Tokyo, 1971. – Cited at pp.: 81, 93, 107
- [267] H. A. Barton: Arthur Bramley. *Physics Today* **24**:9(1971)73 (DOI: [10.1063/1.3022957](https://doi.org/10.1063/1.3022957)). – Cited at pp.: 87
- [268] R. Mahnke, R. Nareyka: *Liste der an der Rostocker Physik verteidigten Promotionen und Habilitationen im Zeitraum 1945 – 1999*. Universität Rostock, Rostock, 2014 (The list is freely available online at the following site of the University of Rostock: http://www.physik.uni-rostock.de/fileadmin/Physik/Mahnke/Geschichte/Promotionen_ab1945_bis_1962_ListePhysik_03072014.pdf). – Cited at pp.: 89
- [269] B. W. Sargent: Arthur Llewelyn Hughes, 1883-1978. *Proceedings of the Royal Society of Canada*, 4. Series, **23**(1985)109-114. – Cited at pp.: 89
- [270] 小長谷, 大介 / コナガヤ, ダイスケ [Konagaya, Daisuke]: 広大理論研設置をめぐる三村剛昂とその周辺 [Kōdai riron-ken setchi o meguru Mimura Yoshitaka to sono shūhen] / Yoshitaka Mimura and his coworkers on the establishment of the Research Institute of Theoretical Physics in Hiroshima. 龍谷紀要 [Ryūkoku kiyō] / *The Ryūkoku Journal of Humanities and Sciences* **35**:1(2013)37-45 (The article is freely available online at the journal website: <http://hdl.handle.net/10519/5166>). [in Japanese, English abstract] – Cited at pp.: 89
- [271] D. Konagaya: Yukawa Institute connecting prewar, wartime, and postwar science in Japan. *The Korean Journal for the History of Science* **36**:2(2014)205-215 (The article is freely available online at the journal website: <http://www.khss.or.kr/kjhs/5593>). – Cited at pp.: 90
- [272] A. Rovenchak: Bibliography of the Department for Theoretical Physics, University of Lviv, in 1914-1939. *Журнал Фізичних Досліджень [Zhurnal Fizichnikh Doslidzhen'] = Journal of Physical Studies* **17**:3(2013)3002, 13 pp., p. 3002-4 (The article is freely available online at the journal website: <http://physics.lnu.edu.ua/jps/2013/3/pdf/3002-13.pdf>). – Cited at pp.: 90
- [273] A. Kawaguchi: Professor Kakutaro Morinaga (23rd Augustus 1905-20th November 1970). *Tensor, N.S.* **22**:1(1971) page without pagination. – Cited at pp.: 91

- [274] В. Б. Брагинский [V. B. Braginskiĭ] et al.: Памяти Михаила Федоровича Широкова [Pamyati Mikhaila Fedorovicha Shirokova]. *Успехи Физических Наук* [*Uspekhi Fizicheskikh Nauk*] **141**:12(1983)719–720 (The article is freely available online at the journal website given by the DOI: [10.3367/UFNr.0141.198312i.0719](https://doi.org/10.3367/UFNr.0141.198312i.0719)). [in Russian]
English translation: V. B. Braginskiĭ, et al.: Mikhail Fedorovich Shirokov (Obituary). *Soviet Physics Uspekhi* **26**:12(1983)1114–1115 (DOI: [10.1070/PU1983v026n12ABEH004595](https://doi.org/10.1070/PU1983v026n12ABEH004595)). – Cited at pp.: 92
- [275] Eric S. Wright: *Experimentalism, Relativity and Quantum Mechanics at a Land Grant Institution: West Virginia University 1920-1960*. Master’s thesis, West Virginia University, Morgantown, 2008 (The thesis is freely available online at the Proquest website: <http://gradworks.umi.com/1458789.pdf>). – Cited at pp.: 93
- [276] L. Brown: Obituaries: Paul Weiss. *SIAM News* **24**:3(1991)2, 6. – Cited at pp.: 94