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Performance analysis and improvement of the Bike Sharing System Using Closed Queuing
Networks With Blocking Mechanism

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Nomenclature

- c_i The number of servers in node i
- M The number of nodes
- L The total number of bikes in the network
- L_i The virtual capacity of node i
- N_i The capacity of the Single Server queue i
- π_{ij} The blocking probability that a completer from the queue i is blocked by the queue j $i = 1, \dots, M, j = 1, \dots, M, j \neq i$
- λ_i The rate of the inter-arrival time of bikes to node $i, i = 1, \dots, M$
- $\tilde{\lambda}_i$ The rate of the effective (without rejection) inter-arrival time of bikes to node i
- C_{ai} The squared coefficient of variation (scv) of the inter-arrival time of bikes to node i
- $p_i(n_i)$ The marginal probability of the node i to contain n_i bikes
- $\langle n_i \rangle$ The mean number of bikes in node i
- X_i The throughput of node i
- A_b The availability of bikes
- A_d The availability of docks
- y_i The Lagrangian coefficients corresponding to the constraints $\phi_i, i = 1, \dots, M$
- \tilde{C}_{di} The squared coefficient of variation of the effective (without rejection) inter-departure time from queue i
- $\tilde{\mu}_i$ The rate of the effective (without rejection) service at the node i
- \tilde{C}_{si} The effective (without rejection) squared coefficient of variation of the service time of queue i

Introduction

The first bike sharing system (BSS) was launched in Amsterdam in 1965 (Shaheen, 2012). Such systems offer soft and flexible mode of transport and are environmental friendly. A BSS consists of stations distributed throughout the city. Every station has a limited number of docking space for bikes. A user picks up a bike from a non-empty station; uses it for a journey and brings it to a destination station if a free dock is available. Otherwise, he/she goes towards another station for docking. The central studied problem in such systems is to determine a satisfactory number of bikes and docks per station (Laporte, Meunier, & Wolfler Calvo, 2015; Nair, Miller-Hooks, Hampshire, & Bušić, 2013) to guarantee a high satisfaction rate. Relevant strategies for bikes re-distribution by BSS operators, fleet-sizing, finding the capacity of docking of the station and geographic dispatching of stations are among the most studied subjects in this field. Three sets of operations can be carried out to increase a BSS performances, required by users and operators:

- **Monitoring.** It corresponds to those real-time or very short/short time regulation activities targeting at freeing places in saturated stations and feeding the starving ones. This is often done manually by relocating bikes using trucks (Kadri, Kacem, & Labadi, 2016; Ho & Szeto, 2016). Another possible monitoring operation is to make consumers use the system differently by bringing bikes to some critical starving stations for instance via financial incentives (Preisler, Dethlefs, & Renz, 2016; Lin, Wang, & Feng, 2017).
- **Control.** Controlling means to modify the number of docks and bikes of the BSS. These actions are more expensive and require often transformations at stations and their control system (Nair & Miller-Hooks, 2014; García-Palomares, Gutiérrez, & Latorre, 2012) .
- **(re-)Design.** It is to define the very initial network for the first implementation of a

BSS or to redefine it, partially or totally, by adding new stations or eliminating useless ones. It corresponds therefore to the most complex decisions and include those ones regarding the capacity of (old and new) stations and the bike fleet size (Garcia-Gutierrez, Romero-Torres, & Gaytan-Iniestra, 2014; Mizuno, Iwamoto, Seki, & Yamaki, 2016)

All these activities are performed by the BSS operator. One rational way to make such decisions, would be to use a relevant BSS model, realistic enough to analyze the possible decisions' impacts on the global performance of the system before any real implementation. Such usable models have to deal with stochastic parameters and also scaling. By scaling we mean to be able to model (partially or totally) a BSS according to several detail levels to ease their analysis, performance measurement, control and (re-)design. This requirement is mainly due to the very large size of such systems. Together, these two characteristics make the modeling tasks hard and that is the reason why various modeling techniques have been used by researchers to test their relevancy and predictability. Hereafter, we will discuss the related works in BSS monitoring-control-design in order to show out what challenges still persist and to justify our motivations in this research. A BSS is a stochastic system due to the randomness of the parameters which qualify and quantify the users' behaviors (inter-arrivals and travel times for instance) (Gast, Massonet, Reijsbergen, & Tribastone, 2015; Montoliu, 2012). Browsing the literature, we found research works, dealing with this randomness, using discrete event simulation techniques or more formal models based on stochastic techniques. As well, it should been noticed that there are other techniques such as data mining (Bordagaray, Dell'Olio, Fonzone, & Ibeas, 2016; Jiménez, Nogal, Caulfield, & Pilla, 2016; Vogel, Greiser, & Christian, 2011) or mathematical programming (Hu & Liu, 2014; Ghosh, Varakantham, Adulyasak, & Jaillet, 2015; Pal & Zhang, 2017) that are excluded from this state-of-the-art. Hereafter, we present a critical analysis of some of the past research papers.

Simulation-based approaches

The simulation works are used either for BSS studies or more generally for Vehicle Sharing Systems, noted VSS, see for instance (Clemente, Fanti, Iacobellis, & Ukovich, 2013; Clemente, Fanti, Mangini, & Ukovich, 2013; M. Fanti, Mangini, Pedroncelli, & Ukovich, 2016; Labadi, Benarbia, Barbot, Hamaci, & Omari, 2015; Febbraro, Sacco, & Saeednia, 2012; Kaspi, Raviv, & Tzur, 2014; Barth & Todd, 1999). The main used approach is almost always the same. First, the model's components (users, bikes, stations) are identified and their parameters are put in the simulator. The behavior of each component is programmed using the possibilities offered by the simulation environment (i.e. possible travels of bikes). Running a discrete event simulation is based on the sequential execution of elementary events ordered in what so called sample paths (Fishman, 2001). Elementary events could be for instance the arrival to or the departure from the bike stations. Numerous sample paths are generated to characterize the behavior patterns of the target system. The system state evolves after the occurrence or activation of elementary events contained in those sample paths. At the end of the simulation runs, the pre-selected performance indicators and behaviors of various components are studied based on the collected data. The goals of simulation studies could be context-specific (vehicles usage rate or users' waiting time for instance) sticked with the case study, or they could be more generic (i.e. profit or service rate improvement). For instance, in (Clemente, Fanti, Iacobellis, & Ukovich, 2013), authors use a simulation model for relocation of vehicles (i.e. voluntary displacement of vehicles within the network of stations) to increase the system revenue and level of service. The service level is defined there as "the ratio of the average number of served users to the total number of users arrived at the station". The closed QN fits well to the VSS studies because of the fixed number of jobs, i.e. vehicles, within the network of stations. Among all types of closed QN, a specific class, referred to as BCMP (Baskett, Chandy, Muntz, & Palacios-Gomez, 1975), is quite promising. A BCMP-QN is an extension of the Jackson network and interestingly admits a *product form solution* for the joint distribution of the

jobs in the network. This made them very popular. A first use of the BCMP-QN model for VSS studies is due to George and Xia in (D. K. George & Xia, 2011). A bike station is considered there as a queuing node with one single server with a queue of bikes waiting for service. The server servicing time is the inter-arrival time of users to pick up bikes. The travel between two bike stations was modeled by a queuing node with an infinite server. In this work, George and Xia look for the optimal fleet size and propose a nonlinear optimization problem by defining the profit of the operator as a function of the fleet size. Authors proved that the stations should have the same availabilities to meet maximum user satisfaction. Authors were able to solve the CQN problem containing 100 stations and find out the optimal fleet size. In their study, Fanti et al. in (M. P. Fanti, Mangini, Pedroncelli, & Ukovich, 2014) modeled an electrical VSS to evaluate the operator revenue. They extended the queuing framework introduced by (D. K. George & Xia, 2011) by adding a multiple server queuing node to illustrate the recharging process.

The major limitation of these aforementioned models is that they consider both an infinite capacity for all stations.

Some other attempts were made by (Fricker & Gast, 2014; H. ter Beek, Gnesi, Latella, & Massink, 2015; Fricker & Bourdais, 2015) to model the system in an aggregated manner by assuming that all the stations of the VSS can be put into one cluster called a homogeneous system. This presupposes that all the stations have the same parameters: capacity, arrival rate of users and uniform routing of users among stations. These hypotheses are used to study large systems and look for evaluation of asymptotic performances. However, these hypotheses are patently far from the reality where a VSS contains simultaneously highly dynamic stations (close to transportation hubs with a high turn-over for instance) and starving and saturated stations. To deal with these shortcomings, the models were extended to study heterogeneous systems containing several clusters of stations, see (H. ter Beek et al., 2015; Fricker, Gast, & Mohamed, 2012) and (Fricker & Serval, 2016). Authors used the mean field technique for the analysis of the

models. This technique, originating from statistical physics (Oppen & Saad, 2001), provides the limiting steady-state queue length in the system as it gets large (number of stations and fleet size). This technique offers an alternative analytical method deriving steady state analysis even if a closed form expression for stationary state is out of reach.

The asymptotic studies provide a relevant answer to scaling for large size systems such as VSS or BSS. By considering homogeneous or heterogeneous clusters, these models handle the characteristics of individual stations (routing and inter-arrival of users) in an aggregated manner. Moreover, these models take account of the limited capacity of stations by a re-orientation strategy. This means that when a dock demand is rejected by a full station, the bike user is led towards one of the possible stations of the network randomly chosen from the entire network. Patently, this latter point does not model well the real behavior of users which would head towards the neighboring stations close to the rejection station (i.e. the destination station).

To sum up this presentation of the used models in the literature, the Table.1 gathers their advantages and drawbacks. Globally, some lessons may be learnt from the resolution techniques. The discrete event simulation offers the possibility to model every individual station but suffers from the scaling capability and do not offer rigorous insights to the studied system dynamics. The stochastic models offer a powerful alternative and by using specific techniques, they allow to deal well with scaling. Nevertheless, their major limitations come from their inability to take account of the whole reality; they need to use simplification hypotheses.

Our goal and methodology

We deal with fleet and dock sizing of a BSS. Our motivation is to suggest a more realistic model of a BSS considering:

- the limited capacity of stations,
- the behavior of users rejected by a full destination station.

We focus on a local study of a set of inter-connected stations within the entire network. The model does not deal with scaling but is able to model fine-tuned behavior of users. The methodology used throughout this study is as follows. To improve the modeling capability of the suggested QN models available in the literature, we introduced the blocking mechanism of full stations extending the original queueing model proposed by George and Xia. Moreover, the behavior of consumers in the case of station rejection was modeled by a relevant routing matrix, defined thanks to the treatment of data gathered from the Paris' BSS opendata. Combined as such, the obtained model is then solved and its performance indicators were defined. The resolution technique is the one initially set up by (Kouvatsos & Xenios, 1989) by using the Entropy Maximization (Jaynes, 1957) of the probability distribution of the queue lengths. Finally, in an iterative approach, we made a sensitivity study of two major performance indicators: the bike and dock availabilities.

The remainder of this paper is organized as follows. In section 2, we describe the closed queueing network for the BSS with blocking mechanisms. In section 3, the resolution of the corresponding framework is exposed; these techniques gather those ones suggested by (Kouvatsos & Xenios, 1989). In section 4, a case study of a system of 20 stations, extracted from the BSS of Paris called Velib, is presented. Several control studies are carried out such as fleet sizing, capacity sizing, inter-arrival of users and routing changes for stations. The results are then discussed. Finally, in the last section, we give an overview of the advantages and limitations of this model and highlight some future perspectives in terms of modeling and resolution technique improvements. Finally, in the appendix we provide the necessary details of the used resolution technique.

Queueing model with RS-RD blocking for BSS

Closed Queueing model with blocking

The closed network is composed of M nodes treating a fix number of jobs (here the bikes) L . The jobs are probabilistically routed between the nodes. These routings are

captured in a routing matrix $\alpha_{ij}, i, j = 1, \dots, M$. The bike stations have a limited capacity.

If a new job or bike looks to enter to a full waiting space of a node, the blocking mechanism impeaches it. Several blocking mechanisms are studied in the literature such as the transfer blocking, blocking-before-service or repetitive blocking, see (Bose, 2013).

We exploit the **repetitive service random destination** blocking type, noted RS-RD, for its convenience. Hereafter, we go through various components of the used model and interpret.

Model for the BSS

Three types of queuing nodes are used to model the whole structure of the BSS:

- **SS**, a limited capacity single server node. It represents a **real** bike station. A limited capacity waiting line is associated with this node. The queue capacity, noted N , corresponds to the number of docks of the bike station.
- **MSB**, a virtual blocking node. Associated always with a SS node, it models the blocking mechanism of the real bike station. An MSB node has a short and negligible serving time comparing to other service time of SS and MS nodes. The number of servers in a MSB, equals exactly the fleet size of the system which noted L meaning that even if all the BSS bikes come to the bike station, they can be treated by this blocking node.
- **MS**, a virtual travel node. It models the travel between two bike stations (i.e. two SS nodes).

Fig. 1 shows how these nodes are connected together.

The departure of bikes from stations. The departure of bikes from a bike station is supported by the SS node. As suggested by George and Xia in (D. K. George & Xia, 2011), the service time of the SS server equals to the inter-arrival time of users to this station.

The arrival of bikes to stations. The arrival of the bikes to a bike station i is modeled by the interactions between the SS_i node and its associated virtual blocking node MSB_i . The blocking behavior is ensured by the RS-RD blocking as follows:

The bike comes first to MSB_i . Two cases are possible:

(i) the SS_i queue is not full. The bike leaves MSB_i and arrives to SS_i , delayed only by the negligible time of the MSB node servicing time. The user locks the bike and the journey is ended.

(ii) the SS_i queue is full. The bike is sent back to MSB_i . After this first loop, the next bike station is selected based on the routing probabilities.

Here, the definition of the routing probabilities is fundamental. The routing to SS_i has the highest probability. The closest bike stations will have high probabilities while far away stations are practically excluded from this re-orientation by very low probabilities. This routing strategy models the behavior of a user who could wait for a while at the station for a free dock (departure of a bike).

At any new bike station, the user will have the same behavior pattern. The user will repeat the process again and again until the bike gets parked.

The travel between stations. There is no constraints for travels between bike stations. The MS node has therefore an infinite capacity to model the infinite travel ways. In the most extreme case, all the bikes of a BSS could be on the roads. Therefore, the travel between any couple of stations is modeled thanks to the virtual node MS containing L parallel servers. The serving time is the travel time of users.

The explicit model of two bike stations. Fig. 2 shows the explicit model of two interconnected bike stations 1 and 2. This figure expands the model of Geroge and Xia by introducing the blocking mechanism through the MSB nodes.

Nodes 1 and 2 in dark gray represent the two real bike stations (SS nodes). Each SS node is fed by bikes coming from its virtual blocking node represented by dotted lines stations (MSB stations). The light gray stations model the four possibilities of traveling

(MS stations) between the two stations.

Modeled as such, solving the closed queuing network model means to find out the probability of any state of any node (mainly the SS) in the permanent state and to deduce the required performance indicators describing the whole system. In the next section, we present the Maximum Entropy resolution approach used for this framework.

Resolution of the closed network under RS-RD blocking using Entropy Maximization Method

State space

The BSS is modeled by a closed network under RS-RD blocking mechanism. It consists of M (First Come First Served) nodes with general interarrival-time and service-time distributions. These nodes can either be with a limited capacity (i.e. N) single server (G/G/1/N) or a multiple servers with infinite capacity (G/G/L) where L represents the number of servers. For a given fleet size L , the state space of the network can be represented by:

$$S = \left\{ (n_1, \dots, n_M) : \sum_{i=1}^M n_i = L, 0 \leq n_i \leq L_i, i = 1, \dots, M \right\} \quad (1)$$

where $L_i = \min(N_i, L)$ is the virtual capacity of the node i of the network and N_i the capacity of the single server node i . The state vector (n_1, n_2, \dots, n_M) is noted from now on \mathbf{n} , and the equilibrium probability of the network to be in the state \mathbf{n} is noted $p(\mathbf{n})$. Let also $p_i(n_i)$ be the equilibrium marginal state probability of a queue i containing n_i jobs. From solving this network, we seek to calculate the probability of each state $p(\mathbf{n})$. These probabilities allow to compute various performance indicators generally associated with any nodes. Referring to (Kouvatsos & Xenios, 1989; Kouvatsos, 1994), an approximative product form for this network can be computed using the Entropy Maximization through an iterative approach. The product-form simplifies the resolution as the nodes of the network can be considered separately and solved in isolation (Bolch, Greiner, de Meer, & S.Trivedi, 2006).

Procedure for solving the closed network of the BSS model

Analytical formulations and details of the resolution are due to Kouvatsos, see (Kouvatsos, 1994; Kouvatsos & Xenios, 1989), and are shortly provided in appendix B. We give hereafter an overview of the resolution procedure. The closed network is solved in two phases: (1) the resolution of the pseudo-open network corresponding to the original closed network, (2) the deduction of the probability of the states $p(\mathbf{n})$ of the original closed network.

Phase 1-Solving the pseudo-open queuing network

In this part of the resolution, in an iterative manner, all the blocking probabilities, π_{ij} , and the scv (squared coefficient of variation) of the effective inter-departure time, \tilde{C}_{di} , are computed by initializing the iterations by some arbitrary values.

Initialization- The algorithm starts by setting some initial arbitrary values for π_{ij} and \tilde{C}_{di} , i.e. defining an arbitrary probability value for the bike rejection and the dispersion of the effective inter-departure time from the nodes.

Step 1- For every queue:

- The inter-arrival rate and its scv $\{\lambda_i, C_{ai}\}$ are computed.
- The effective service time and its dispersion $\{\tilde{\mu}_i, \tilde{C}_{si}\}$ are also calculated.

Step 2- The nodes of the network, $i, i=0, \dots, M$, are individually solved applying the Lagrange multipliers method for maximization of the Entropy function (10) subject to normalization (11) and marginal constraints (12)–(14). Therefore, for every queue, the Lagrange coefficients (i.e. $g_{(i,k)}$, x_i and y_i for $i = 1, \dots, M$ and $k = 1, \dots, c_i$) relating to the corresponding constraints (see Appendix B) will be then available.

- SS nodes are solved as censored nodes¹, $(\text{GE}(\lambda_i, C_{ai}) / \text{GE}(\tilde{\mu}_i, \tilde{C}_{si}) / 1/0; N_i)$.

¹ i.e. those nodes where the arriving customers are turned away when the buffer is full

- MSB and MS nodes are solved as stable nodes² ($GE(\lambda_i, C_{ai}) / GE(\tilde{\mu}_i, \tilde{C}_{si})/L$).

Step 3- New values of the blocking probabilities π_{ij} are calculated from the newly obtained values of $p_i(n_i)$, and \tilde{C}_{si} by using an iterative method to find the roots of (49) presented in the Appendix B.

Step 4- Return to step 1 until convergence of $\{\pi_{ij}\}$ $i=0, \dots, M, j=0, \dots, M$. The convergence value is fixed as a problem-solving parameter to find out a relatively quick solution determination. One can use for instance 0.01 to fix the idea.

Step 5- Find new values for $\{\tilde{C}_{di}\}$, $i=0, \dots, M$, (53), once the last set of π_{ij} is obtained from the step 4.

Step 6- Return to step 1 until convergence of $\{\tilde{C}_{di}\}$ $i=0, \dots, M$. In a same manner, the convergence criterion is chosen by user. According to (Kouvatsos, 1994) and (Kouvatsos & Xenios, 1989), the convergence is always guaranteed. In our case, the convergence criterion is 0.01.

Phase 2-Solving the closed queuing network

The resolution of the pseudo-open queuing network gives an estimation of the Lagrange coefficients for the closed queuing network. These parameters are implemented for the maximum entropy state probability $p(\mathbf{n})$ (33) of the original closed network. This is performed in the rest of the approach.

Step 7- Use a convolution method to compute the marginal probabilities, $p_i(n_i)$, $i \in 0, \dots, M$, for the original CQN.

Step 8- Find two parameters of the nodes which are the mean queue length $\langle n_i \rangle$, and throughput X_i for $i = 0, \dots, M$.

Step 9- Correct the Lagrangian Coefficients y_i corresponding to the constraint of the state probabilities of the full SS nodes (14). In fact, the Lagrange Coefficients obtained

² i.e. those nodes without capacity limitations

from the resolution of the pseudo-open network (noted by \tilde{y}_i), obtained in Step-2, should be revised by using:

$$y_i = \tilde{y}_i \tilde{\lambda}_i L \left[X_i \sum_{j=1}^M \frac{\lambda_j \langle n_j \rangle}{X_j} \right]^{-1} \quad (2)$$

Step 10- Return to step 7 till obtaining the same ratio of the rate of the effective interarrival-time to the throughput, (3) for all the nodes

$$H = \frac{\tilde{\lambda}_i}{X_i} \quad i = 1, \dots, M \quad (3)$$

Performance indicators of the BSS

The resolution technique presented in the past section allows us to compute the following performance indicators:

1. **Availability of bikes** for each bike station. This is the probability that a user finds at least one bike at the station i (4).
2. **Availability of docks** for each bike station. This is the probability that a user finds at least one free dock at the station i (5).
3. **Global performance of a station.** It combines both availabilities by using relevant weights coefficient a_i and b_i (6) to allow decision-makings regarding individual stations.

$$Ab_i = 1 - p_i(n_i = 0) \quad (4)$$

$$Ad_i = 1 - p_i(n_i = N_i) \quad (5)$$

$$a_i Ab_i + b_i Ad_i \quad (6)$$

$$\frac{1}{M} \sum_{i=1}^M a_i Ab_i + b_i Ad_i \quad (7)$$

where i refers to the SS node corresponding to a real station, and $a_i + b_i = 1, i = 1, \dots, M$.

Numerical Results

Data analysis and Hypothesis

The Velib system consists of almost 1700 stations and 23000 bikes (Steffan, 2012). The resolution algorithm was time consuming for the whole network. The computation complexity of the algorithm is $O(k_1(\Omega + 1)^3)$ for Phase 1 and of $O(k_2M^2L^2)$ for Phase 2 (Kouvatsos & Xenios, 1989), where k_1 is the number of iterations for steps 1-6, k_2 is the number of execution of the loop in steps 7-10 and Ω is the cardinality of $\{\pi_{ij} : i, j = 1, \dots, M; i \neq j\}$. Accordingly, it was decided to focus on a small geographic zone with a limited number of stations (20 stations).

We considered 3 stations in "Ile de la Cite" which is at the very center of Paris. Other 17 stations, i.e. the most visited destination stations from these three aforementioned stations complete the subnetwork of stations studied here. All these stations are located in Paris touristic zones and have a stable usage rate. In fact, analyzing the usage rate (Feng, Affonso, & Zolghadri, 2017), they represent a very similar pattern of usage during a day. The data were collected for a period of 30 days, between 5 September and 15 October 2016 excluding the weekends. This period was chosen because of the homogeneity in the weather conditions. Raw data was collected from the Open database of JCDecaux, the operator of Velib. The data was treated to derive the parameters of the network such as the rate and the scv of the service time of the nodes and the routing matrix. Mainly, we focused on the time points where bikes are picked up in one of these 20 stations bring to another station. The inter-arrival rate of users to stations is obtained from the treatment of the inter-departure of bikes from these stations. In fact, the inter-arrival of users to a station equals the inter-departure of bikes when there is at least one bike in the station. We treated the data by considering only those states where non-empty condition was met. We estimated the number of users arriving to pick up bikes at each station by an hour time slot for every day during the considered period. Then for every time slot, the mean rate and its dispersion were calculated over all the days of the studied period. Moreover, it was

observed that the values of rate and scv of two time slots (from 8am to 9am and from 9am to 10am) are very close, we decided then to study these two time slots together. The study is then focused on the network dynamic from 8 am to 10 am which corresponds to a rush hour of bike usage in these stations.

The duration of the travels between the stations were extracted from the raw data too. We have then calculated the rates and the squared coefficient of variations over this data. This gives the needed parameters of the MS nodes of the model. Regarding the routing probabilities, the raw data were treated by eliminating weekends and special events such as festival days. The probability of taking a certain rout to a destination station equals then to the ratio between the number of trips to this destination and the sum of all the trips departing from the origin station. By this way, the routing probabilities between the stations in the studied zone and from-and-to the exterior stations were computed. Selecting the sub-network, the aforementioned closed queuing model resolution technique was used even if we knew that the sub-network cannot be considered as a real closed network. However, by selecting the most visited stations from the three central stations limited greatly side effects of connection between the considered zones (the 20 stations) and the rest of the network. This issue is discussed in the conclusions and perspectives section of the paper.

Experiments and Results

The resolution method of the model of 20 stations was programed and implemented in Matlab. We seek to foresee the effects of the fleet size, the docking capacity, the inter-arrival rate of users picking up bikes and the flow of incoming bikes to a station on the system performance.

- **Fleet-sizing.** We would like to study the effect of fleet size change on the stations performance indicators: dock availability and bike availability. We present the fleet sizing of three different bike stations: 4002, 4003 and 4017 in Fig. 3. Having different

characteristics, the behavior of these stations is detailed hereafter.

It can be noticed that the bike availability of stations increases while the fleet size increases and tends towards 100%. This is normal because increasing the number of bikes makes their availability at the stations more probable. Conversely, the dock availability decreases from 100% as the fleet size increases. This is also quite normal, because a higher number of bikes in the system decreases the possibility of finding free docks at the stations. This opposite evolution of the two curves creates a **crossing point**. The crossing point appears in all the stations and it reflects an interesting zone of performance, called the best performance area and represented by a dotted rectangle in Fig. 3. In fact, both availabilities are higher than 0,95 in this zone. This behavior was observed for all of the other bike stations not reported in this paper. In Fig. 3, we show the curve representing the global performance of a station. In this case, it was decided to attribute the same importance to both indicators reflecting that it is always interesting to find out bikes and docks at stations. In a real situation, the system operator could attribute different coefficients according to the dynamic situation of stations (societal and geographic specificity of the station, see **Performance indicators of the BSS**). For instance, for a bike station close to a transport hub it can be judged that the presence of the bikes in the morning rush hours is more important than dock availability. By this way, the best performance areas can be found out easily on the figure. Obviously, every station reach the performance area for a different fleet size. For instance, for station 4002 the optimal fleet size would be 150 bikes and for station 4017 the best performance is noticed for a fleet of 650 bikes. Therefore, as the fleet size involves all the stations, it is necessary to compute the fleet size taking account of all the stations. Fig. 4 shows the evolution of the aggregated network performance indicator as a function of the fleet size. The two availabilities have equal importance, i.e.

$a_i = b_i = 0.5, i = 1, \dots, M, M = 20$. In the same figure, we show the curves of the

mean availability of bikes and the mean availability of docks of all the stations.

Very interestingly, the global performance curve shows a *flat performance area* between 380 and 520 bikes. This means the whole subnetwork has a very robust behavior regarding the fleet size; the global performance is about 88% for a fleet size varying between these two extreme values. The analysis can be completed by considering a "reference fleet size". In our case, we consider that 440 which represents the highest network performance corresponds to the reference fleet size. In (ITDP, 2013), authors reveal that in most of the bike sharing systems (Montreal, London, etc.), operators use an experimental ratio of docking capacity to fleet size of 2 - 2.5. Applying this rule in our case, since the total capacity of the 20 stations is 621 docks, the fleet size should be between 248 and 310 bikes. Our conclusion is therefore different from this experimental rule. This can be explained by the fact that operators are leveraging docking availabilities at expense of the availabilities of bikes following this rule; in our case we have given the same importance to both availabilities.

Let us now look at the stations 4002 and 4017 more precisely according to their local optimal performances which refer to their respective best fleet size and the reference fleet size: 150, 650 and 440, see Table.2.

With 150 bikes in the network, the station 4002 offers a high chance of finding an available bike and a free dock. At the reference fleet size, the station 4002 would have almost always available bikes but more rarely available docks. A bike-demanding user has therefore a very high chance of finding an available bike while a dock-demanding user has only about 52% of chance to find out a free dock. It seems that the station 4002 has a tendency to have more incoming bikes than outgoing ones. Finally, with 650 bikes, the same station 4002 becomes saturated offering almost always bikes but rarely free docks. Opposite situations can be found for the station 4017, even if the bike finding chance remains quite high for the reference fleet size.

To pursue the studies of the sub-network, we used the reference fleet size. For this fleet size, we focused on an arbitrary station (station 4003). This station has the bike and dock availabilities of $A_b = 1$, $A_d = 0.8$, cf. Fig. 3. As, the best performance zone is not reached for the reference fleet size, it would be interesting to find out the sensitivity of these performance indicators using other decisions such as modifying its capacity, the inter-arrival rate of users and the incoming flows of bikes.

- **Docking capacity.** To visualize the effect of docking capacity, we modified the number of docks of the station 4003 by adding or removing a proportion of its actual capacity (20 docks) as shown in Fig. 5. When the capacity is increased the availability of bikes tends to remain constant, which is quite normal. The chance of finding a free dock is also increased allowing more users to bring their bikes to this station. These results are rational and reasonable if we compare the state of the station at two special fleet sizes: the reference fleet size (440 bikes) and the optimal fleet size (320 bikes) for this station. With an initial capacity of 20 docks, our first analysis shows that the best performance is reached for a fleet of 320 bikes (see Fig.3). Since the reference fleet size (440 bikes) is greater than this station optimal fleet size, the number of bikes in the subnetwork is greater. We would see more bikes remaining at this station. Therefore, we tend to have a saturated station for this reference fleet size and therefore a lower availability of docks. So, to overcome the shortage of docks, the capacity of the station should be enlarged. Nevertheless, the 100% global performance may be reached by 140 more docks (700% of capacity increase) ! Obviously, this is not realistic to target at that level of performance for evident reasons (cost, place, etc.).

- **Incoming flow variation.**

It is possible to modify the incoming flow of bikes to stations by an economic incentive for instance. In this experiment, we would like to find out how the

performance of the station 4003 evolves as a function of the incoming flow. This effect is represented in Fig. 6. For the initial incoming flow rate, the availability of bikes is 100% but the chance of finding a free dock is lower, i.e. 80%. We would like to know whether any change in the incoming flow could contribute to increase the dock availability without serious deterioration of the bikes availability. By focusing on the global performance of the station (same importance of bike and dock availabilities) in Fig. 6, it can be seen that the maximum rate is located at the incoming flow of -20%. This allows to have very good bike and dock availabilities (about 96%). This result is also coherent. As a matter of fact, as previously mentioned, this station has a tendency to be full. So, it is reasonable to reduce its incoming flow of bikes (here by 20%) to make it "less" full while allowing a good bike availability.

- **Demands for bikes variation.**

It is also possible to increase the demand of bikes (to drain the bike from saturated or closely saturated stations) by economic incentives again for instance. Therefore, we would like to know then, whether a BSS operator may launch such incentives to improve the performance of stations. Fig. 7 shows the result of these experiments where both the bike and dock availabilities and the global performance of the station are computed as a function of the variation of the arrival rate of users to the station 4003. For the initial value of arrival rate, the dock availability is 80% while the bike availability is very good (close to 100%). The highest global performance value is obtained for an increase of 27% of the arrival rate. In this situation, both availabilities are very good (around 97%). This result is consistent with the aforementioned fact that the station 4003 tends to be full and increasing the demand rate of bikes allow to increase the dock availability while reducing slightly the bike availability.

Discussion of the obtained results

By monitoring or controlling the BSS, an operator seeks to get the best performances for the stations and for the entire system. This target would be for the whole day or at least for the peak hours. In these experiments, we focused on a the performance of a subnetwork during the morning rush hours, from 8am to 10am. Feng et al. in (Feng et al., 2017) shows that there are two peaks of usage of the system: in the morning and at the evening. So, the same experiments can be conducted to the other hours in the evening. Our experiments have shown that modifying the used parameters do modify the bike and dock availabilities, but they do evolve in the opposite direction. So, trade-offs should be found between them. To this respect, the global performance indicator seems to authors as a better indicator on which the operator can rely. It is simple and can reflect the local target (for every studied station taking account of the station's specificities) or for the whole (sub)network.

Control-oriented decisions: Robustness of the global performance regarding the fleet size. Practically, the BSS operator has to do some control actions by adding the number of bikes and/or docks to increase the stakeholders satisfaction in a more permanent way. Studying the fleet size, it was noticed that every station has a different optimal fleet size regarding its global performance. This suggests clearly that finding out the best size of the fleet for the whole considered sub-network should go through trade-offs among the stations where some stations could be more critical than others. In our case study, the stations which are at the center of Paris (very close to the more touristic sites) could be considered as critical. In other cases, the criticality would be based on satisfaction of station geographical position i.e. close to working areas or transport hubs (close to train stations, etc.). In a top-down analysis, the operator can first find out the "best" fleet size which maximizes the global performance of the whole subnetwork. The very good news is that this global performance is not very sensitive to the fleet size for a large size window (between 380 and 500 bikes). The global performance is quite robust regarding this parameter. It is therefore possible to dig out the exact fleet size by considering the size

that increases the performances for critical stations in a second step.

Control-oriented decisions: The global performance not highly sensitive to capacity sizing. As the second control action, we wanted to evaluate the capacity change of a station, here the station 4003, which tends to be saturated. It was observed that the increase of the capacity improves the availability of docks with holding the same availability of bikes. But, by decreasing the capacity, a deterioration of both availabilities is observed. However, the capacity modifications are expensive and do not procure too much performances. More studies are still necessary to this respect but our first set of results tend to tell that the global performance is not highly sensitive to capacity sizing.

Monitoring decisions: The global performance relatively robust to the departure and arrival of bikes to stations. In terms of monitoring actions, there are instantaneous corrective measures that can be taken by the operator to gain further performances. Acting locally on a station by changing its attractiveness seems to be very interesting.

Curves in Fig. 6 and Fig. 7 show that locally, modifications of the arrival and departure rates of bikes to stations impact directly their global performances. However, the global performance remains again relatively robust around the determined best rates. For instance, changing the incoming flow of bikes or the bikes demand rate by 10% induces only 1% of performance deterioration. The monitoring actions are performed now by trucks, but our experiments show that economic incentives are very effective. Lowering the service price during the rush hours for instance can contribute to compensate the normal tendency of saturation and starving.

Conclusions and further research works

The conclusions of this research can be shown according to the following points:

1. Modeling and resolution. In this work, a closed queuing network supporting a blocking mechanism was used to model and to assess the performance of a BSS.

Accordingly, a resolution approach based on the Entropy maximization was applied to solve the obtained model. The originality of this work resides in its ability to model in a more realistic way the dynamic of stations and the network. We introduced the limited capacity of stations to the initial model developed by (D. K. George & Xia, 2011). Moreover, by exploiting the reality of Paris' BSS, we introduced within the model, a more realistic behavior of bike returns from the destination stations.

2. Methodological issues. The performance of a BSS was determined by bikes and docks availability. These two indicators were aggregated together into a global performance which allows to take account of local preferences for station management. These performance indicators were computed for the model against decisions subdivided into three categories: monitoring, control and (re-)design. Focused on the two first ones, the monitoring decisions target at improving the system performance acting on real or short time. Regulation of bikes arrival (incoming flow) and departure (demand) are the two monitoring decisions. Control decisions are more expensive and tend to procure more durable performances dealing with changing the fleet size or the capacity size.
3. Experiments and discussions. As the resolution is quite complex and very time-consuming, it was decided to model a subnetwork of Paris BSS composed of 20 stations. Following a restrict experimentation protocol, real data were gathered, pre-treated, and used for experiments. Two sets of experiments were conducted following the monitoring and control decision possibilities. The results allowed to draw conclusions:
 - Control decisions: fleet-sizing. They have positive influences on the bike availability but will patently degrade the dock availability if the fleet is too large. Locally, every station points at a fleet size that maximizes its optimal global performance. From the subnetwork point of view, the global performance

is quite robust for a relatively "large" window of fleet size.

- Control decisions: capacity-sizing. Increasing the capacity has no effect on the availability of bikes but enhance the availability of docks; its reduction deteriorates both availabilities. More expensive than the fleet-sizing decisions, the global performance evolves slowly by a drastic increase of the station's capacity (700% for station 4003).

The BSS operator has to improve first the size of the fleet before taking actions for modifying the stations capacity.

4. Monitoring decisions: bikes arrival and departure. Locally modification of bikes arrival to and departure from a station seems to be very effective with a concrete impact of the global performance, once the exact arrival and departure rates identified thanks to the computed results. They may be cheaper than the used techniques now by operators (bikes displacements by trucks). Their effectiveness needs however a deeper economical study.

Despite these insights to the BSS monitoring and control decisions, there are some shortcomings that should be improved in the future. As the resolution of the model is tedious and time consuming, a subnetwork of the real BSS was chosen for the experiments. Extracting such subnetwork needs further research to evaluate the side effects of exchanges between the subnetwork stations and those stations outside the subnetwork. However, hopefully, there is no need to take account of the whole network when focusing on a subnetwork, because according to the statistics of the bike usage in Paris, almost no travel is registered farther than 30 minutes from a given station. In this case, to consider the most statistically representative ring around the subnetwork, the ring should contain all those stations 30 minutes far from the frontier stations. This allows to study more in detail the subnetwork. However, the second tough point to work it out is the efficiency of the programs used in this research. They should be optimized to reduce the computation

times, make them reasonable and feasible. The last perspective of research corresponds to the extension of the study to other time windows during a typical day.

Table 1

State of the art.

	Simulation-based techniques	Stochastic approaches
Regulation	(Clemente, Fanti, Iacobellis, & Ukovich, 2013; Clemente, Fanti, Mangini, & Ukovich, 2013; Labadi et al., 2015; Febbraro et al., 2012; Barth & Todd, 1999)	(D. George, 2012; Fricker & Gast, 2014)
Design/re-design	(M. Fanti et al., 2016; Kaspi et al., 2014)	(D. K. George & Xia, 2011; Fricker & Gast, 2014; H. ter Beek et al., 2015; Fricker & Bourdais, 2015; Fricker et al., 2012; Fricker & Servel, 2016)
Main Advantages	<ul style="list-style-type: none"> • Can deal with detailed model. • Are flexible and allow to model every single station. • Are easy to use for partial VSS. 	<ul style="list-style-type: none"> • Are not time-consuming to populate the model with stations. • Give insights about the performing system.
Main Drawbacks	<ul style="list-style-type: none"> • There is no formal way to validate the model and to verify the results. • Are time consuming and hard to implement for large systems. • Hardly allow scaling. 	<ul style="list-style-type: none"> • Modeling basics and resolution are complex. • Their resolution complexity imposes the use of approximate solutions.

Table 2

Stations 4002 and 4017

Fleet	4002	4017
150		
bike finding chance	97%: almost always	42%: low chance
dock finding chance	97%: almost always	100%: almost always
440		
bike finding chance	100%: almost always	80%: not good chance
dock finding chance	52%: not good chance	100%: almost always
650		
bike finding chance	100%: almost always	95%: almost always
dock finding chance	35%: low chance	95%: almost always

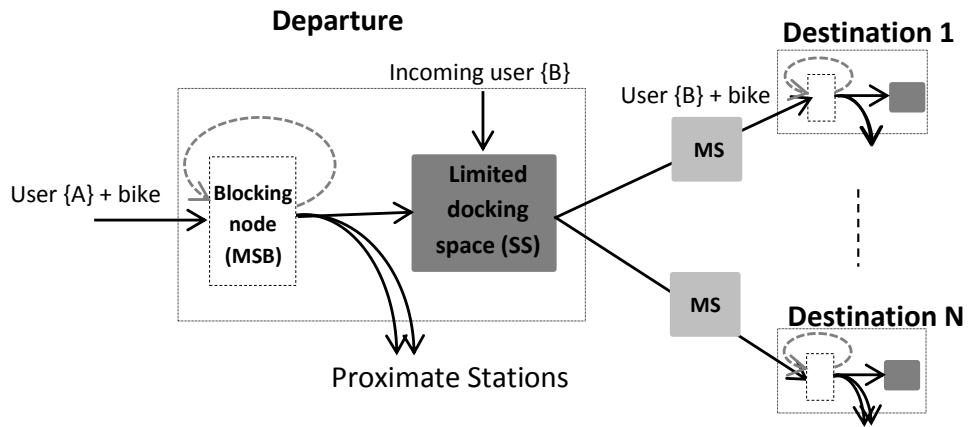


Figure 1. The structure of the queuing network model.

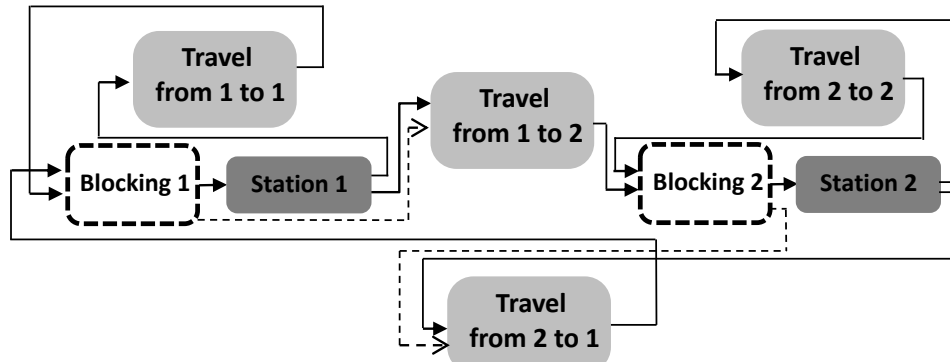


Figure 2. Closed network for two stations BSS, extension of the original model of (D. K. George & Xia, 2011)

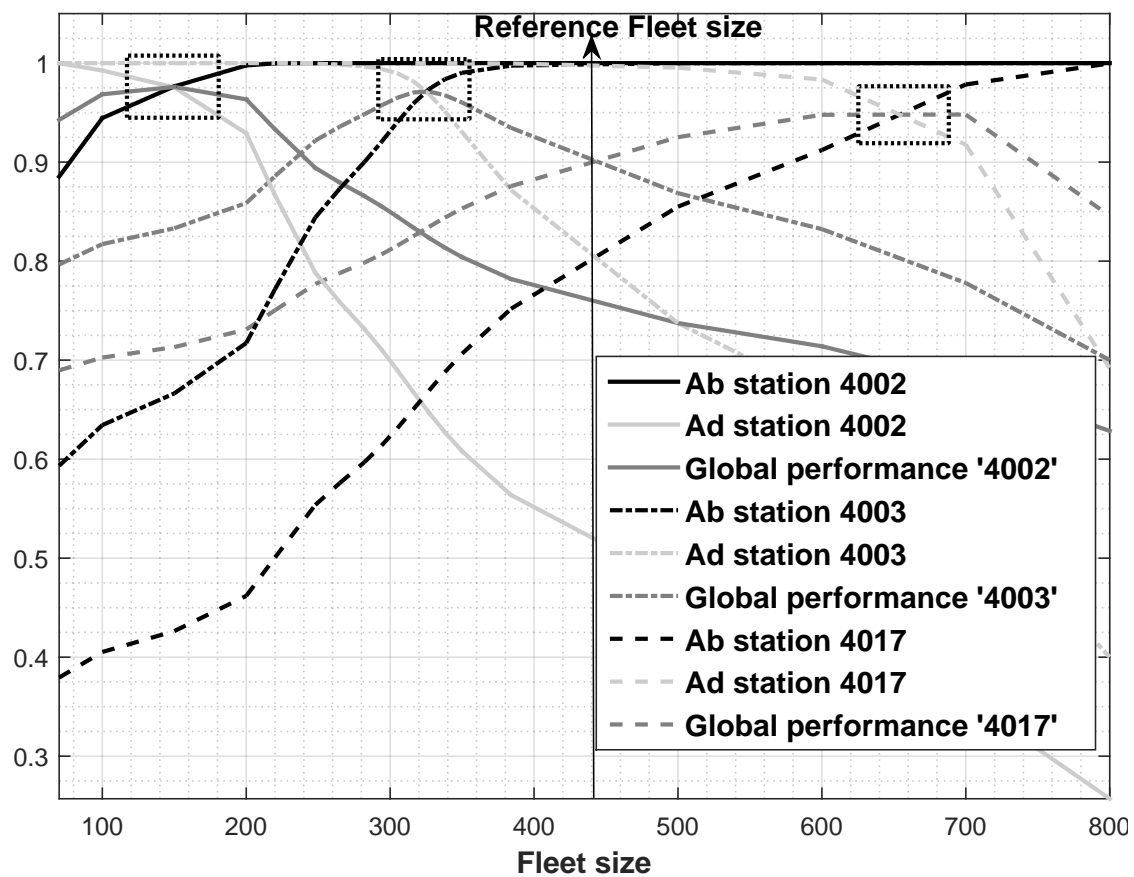


Figure 3. Availabilities as function of the fleet size

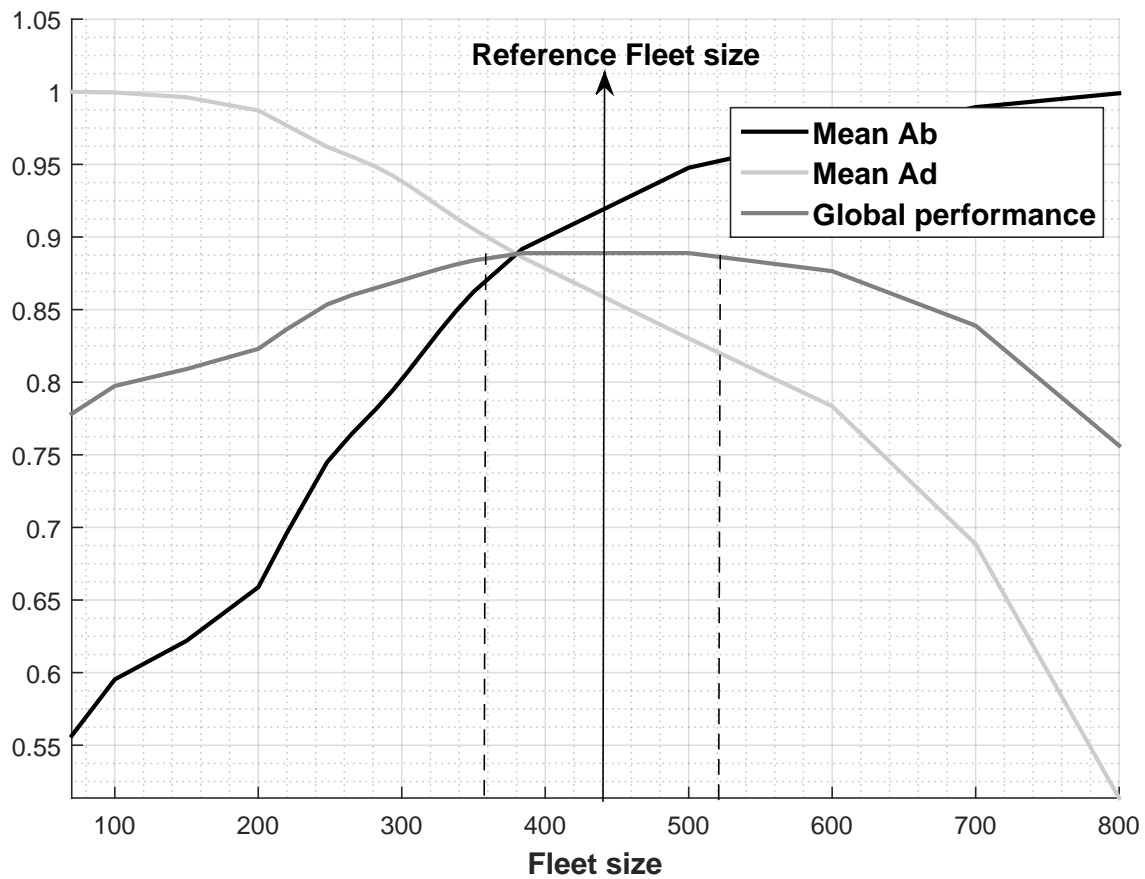


Figure 4. The 20 stations performance

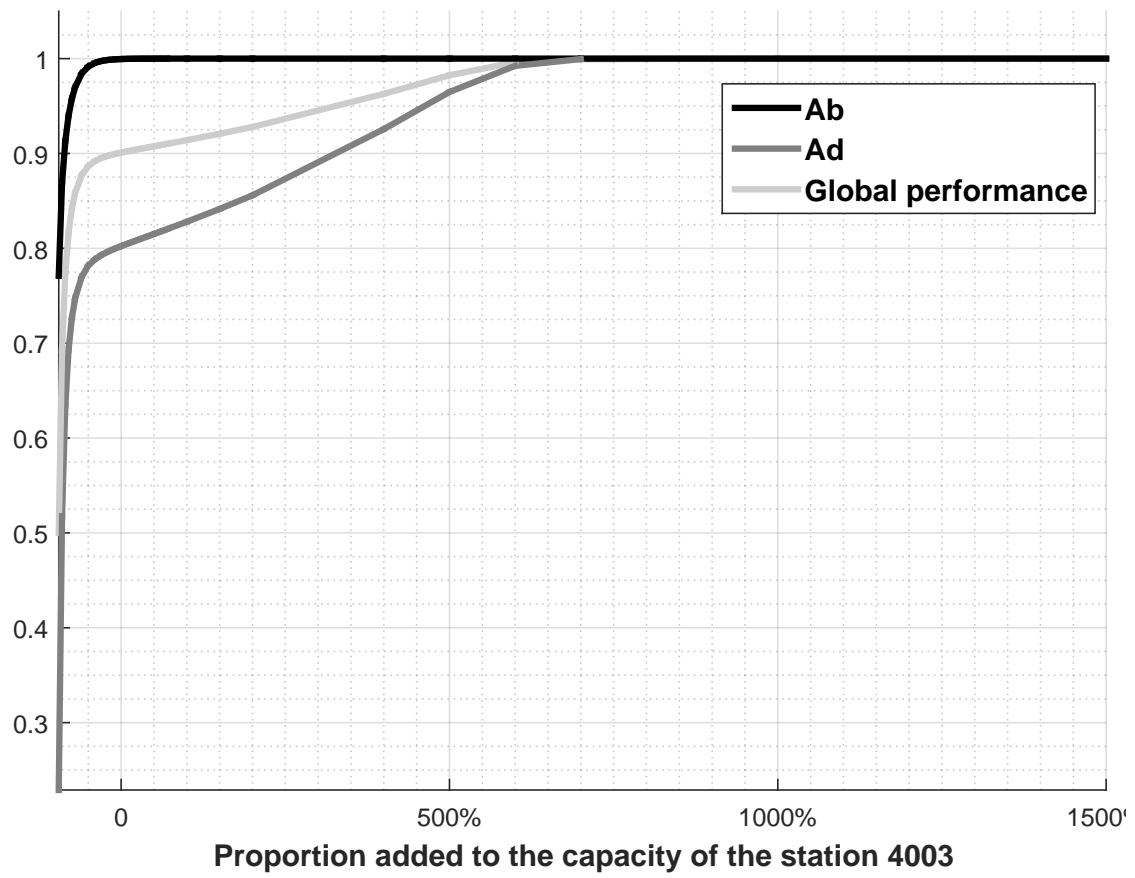


Figure 5. The effect of the capacity change

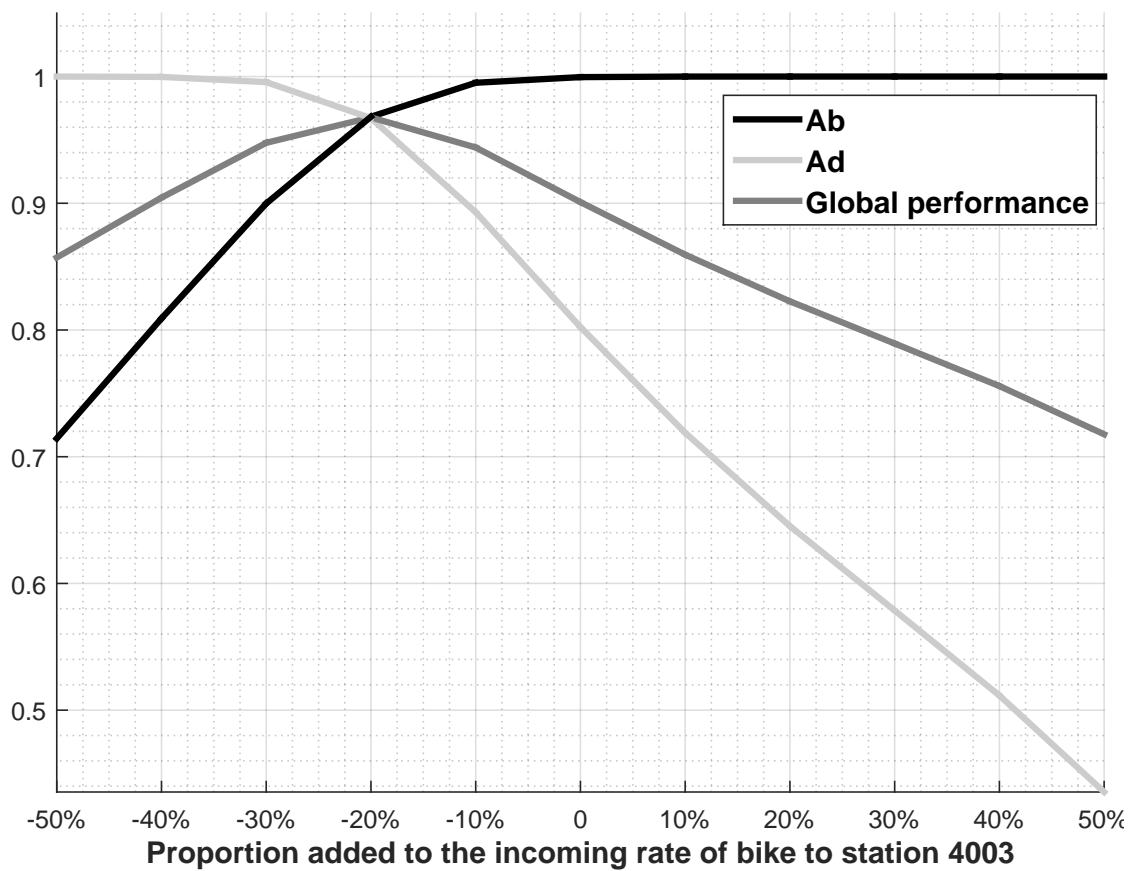


Figure 6. The effect of the incoming flow of bike change

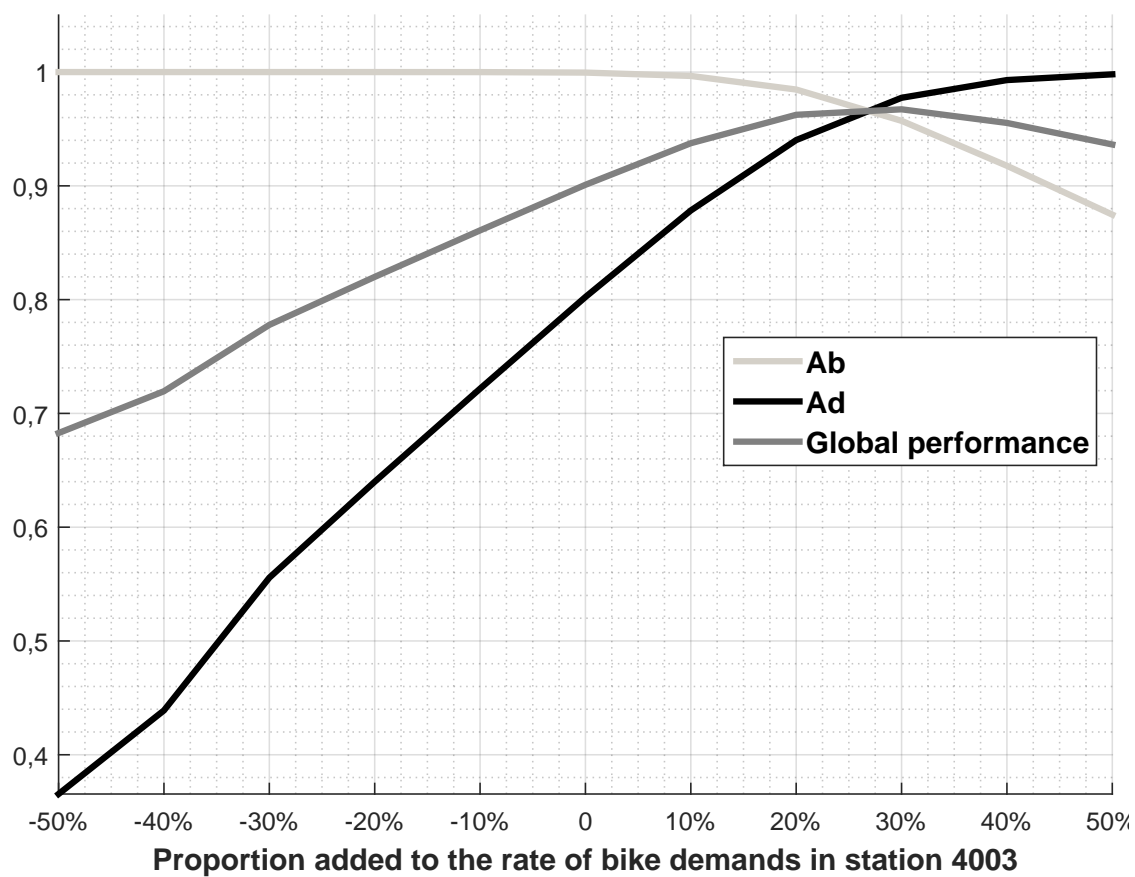


Figure 7. The effect of the demand for bikes change

Appendix A

Queuing Networks

An elementary queuing node is composed of a server and a waiting line of finite or infinite size. Jobs arrive at the queue waiting there to be served by the server. Such queuing node is defined by several parameters that allow to model its dynamic behavior (transient and stationary). The service time is the amount of time necessary to fulfill an operation on jobs. The arrival of jobs to the queuing node follows a dynamic pattern. The inter-arrival time and the service time are random variables modeled by a probability distribution law, very often the exponential distribution. The service station could have more than a server. All the servers are supposed to have the same service time and a job will choose the first free server. The waiting jobs are selected for service according to a scheduling discipline such as First-In-First-Out. The parameters of a node (inter-arrival time, queuing capacity, server number, server operating time and the queuing schedule) are represented by Kendall's notation. For example, in our case, we use G/G/1/K; N censored nodes and stable G/G/L nodes. The former type queuing node is a single server having a finite capacity N and a minimum queue length K. In this case the number of jobs cannot neither less than K nor bigger than N jobs. The latter type queue is composed by L parallel servers and have no capacity limitation. The arrival and service process for both kinds of queuing nodes follow the general distribution, noted G, which is defined by its mean and SCV. A QN is composed of several interconnected nodes. Jobs flow from node to node to perform a predefined 'routings'. A routing defines the possible nodes that would be visited by a job after finishing its service at a given node. The routing matrix will define the probability p_{ij} of heading towards the queue j from the queue i, where $\sum_j p_{ij} = 1$. If the number of jobs in a QN is fix (no departure nor arrival) the QN is called closed QN, otherwise it is open.

Appendix B

Resolution Approach

In what follows, we expose the resolution of the closed network with RS-RD blocking mechanism, the queuing framework for the BSS, as suggested by Kouvatsos in (Kouvatsos & Xenios, 1989; Kouvatsos, 1994).

In this approach, we use the open network to approximate a solution for the closed network through a pseudo-open network. A pseudo-open network is a closed network which is represented as an open network with no external arrival streams and no external departures (Bose, 2013). The pseudo-open network should have the same characteristics as the original closed network (same number of queues and servers, service-time characteristics and transition probabilities). To solve the pseudo-open network we consider (i) the set (8) which describes the job flow balance at every node with $\tilde{\alpha}_{ji}$ shows the effective (without rejection) transition probability from node j to node i , and (ii) the fixed number of bikes represented by (9).

$$\tilde{\lambda}_i = \sum_{j=1}^M \tilde{\alpha}_{ji} \tilde{\lambda}_j, \quad i \in \{1, \dots, M\} \quad (8)$$

$$L = \sum_{i=1}^M \langle n_i \rangle \quad (9)$$

The Entropy Maximization of the open network with RS-RD blocking admits an approximate product form solution. The resolution of the pseudo-open network requires then the decomposition and the resolution of the individual nodes in isolation using the Maximum Entropy Method. Further in this appendix, we present prior information of the ME resolution of individual nodes of two types: the G/G/1/0;N censored queue and the stable G/G/L queue which are the building blocks of the network model. Afterward, we present the resolution technique of the closed queuing network then the approximate ME product-form solution of the pseudo-open network. We expose then the analytical decomposition of the network into individual queues. The resolution steps are resumed in

Fig. B1.

Entropy Maximization for censored and stable queues

Two types of queues G/G/1/0;N censored queue and stable G/G/L queue have to be solved. The generalized exponential distribution, GE, is used as an approximation of the generalized distribution to solve this queues. This means that the resolution concerns the GE/GE/1/0;N censored queue and stable GE/GE/L queue. The state probability $\{p(n), n = 0, \dots, L_i\}$ is determined by maximizing the Entropy function (10) for every queue when i indexes a queue.

$$H_p(n) = - \sum_{n=0}^{L_i} p(n) \log p(n) \quad (10)$$

The Entropy function (10) for censored and stable queues are solved under the normalization (11) and marginal constraints (12)–(14).

1- The normalization. It looks for having a normalized measure of probabilities of the queue length.

$$\sum_{n=0}^{L_i} p(n) = 1 \quad (11)$$

Hereafter, the values in the marginal constraints are supposed to be known. They are probabilities and mean queue lengths.

2- The probabilities $u_{(i,k)} : i = 1, \dots, M. k = 1, \dots, c_i$.

$$\left(\sum_{n \geq k} p_i(n) = u_{(i,k)} \right) \quad (12)$$

with $0 \leq u_{(i,k)} \leq 1$.

3- The mean queues lengths excluding c_i jobs, $\langle n - c_i \rangle, i \in \{1, \dots, M\}$

$$\sum_{n=0}^{L_i} h_i(n) p_i(n) = \langle n - c_i \rangle \quad (13)$$

$\langle n - c_i \rangle \in [0, L_i]$ and $h_i(n) = \max(0, n - c_i)$.

4- The probabilities of the full queues, $p_i(N_i)$, when i indexes a SS.

$$\sum_{n=0}^{N_i} f_i(n) p_i(n) = \phi_i \quad (14)$$

$$0 \leq \phi_i \leq 1 \text{ and } f_i(n) = \max(0, n - N_i + 1)$$

The resolution is presented in (Kouvatsos & Xenios, 1989) by applying Lagrange's method. It determines the expressions of the queue length probabilities as a function of the Lagrange coefficients $\{g_{(i,k)}, k = 1, \dots, c_i\}$, x_i and y_i corresponding respectively to constraints $\{u_{(i,k)}, k = 1, \dots, c_i\}$, $\langle n_i - c_i \rangle$ and ϕ_i , $i \in \{1, \dots, M\}$. The final solutions are given by (23) and (29). We consider:

$$\tilde{\sigma}_i = \frac{2}{\tilde{C}_{si} + 1}, \quad i \in \{1, \dots, M\} \quad (15)$$

$$\rho_i = \frac{\lambda_i}{\tilde{\mu}_i} \quad (16)$$

$$\tau_i = \frac{2}{C_{ai} + 1} \quad (17)$$

GE/GE/1/0;N censored queue resolution

The Lagrange coefficients are obtained as follows:

$$x_i = \frac{\tau_i \rho_i + \sigma_i (1 - \tau_i)}{\tau_i \rho_i (1 - \sigma_i) + \sigma_i} \quad (18)$$

$$y_i = \frac{1}{1 - (1 - \sigma_i)x_i} \quad (19)$$

$$g_{(i,1)} = \frac{\tau_i \sigma_i \rho_i}{\tau_i \rho_i (1 - \sigma_i) + \sigma_i} \quad (20)$$

$$f_i(n) = \max(0, n - N_i + 1) \quad (21)$$

$$p_i(0) = \begin{cases} (1 + g_{(i,1)}(N_i - 1 + y_i))^{-1} & \text{if } \rho_i = 1 \\ (1 + g_{(i,1)} \frac{1 - x_i^{N_i - 1}}{1 - x_i} + y_i x_i^{N_i - 1})^{-1} & \text{if } \rho_i \neq 1 \end{cases} \quad (22)$$

$$p_i(n) = p_i(0) g_{i,1} x_i^{n-1} y_i^{f_i(n)} \quad (23)$$

GE/GE/L stable queue resolution

The Lagrange coefficients are obtained as follows:

$$x_i = \frac{\lambda_i \tau_i + L \tilde{\sigma}_i \tilde{\mu}_i (1 - \tau_i)}{\lambda_i \tau_i (1 - \tilde{\sigma}_i) + L \tilde{\sigma}_i \tilde{\mu}_i} \quad (24)$$

$$g_{(i,k)} = \frac{(\lambda_i \tau_i + (k-1) \tilde{\sigma}_i \tilde{\mu}_i (1 - \tau_i) \tilde{\sigma}_i}{k \tilde{\sigma}_i \tilde{\mu}_i (1 - (1 - \tilde{\sigma}_i)(1 - \tau_i))} \quad k \in \{1, \dots, L-1\} \quad (25)$$

$$g_{i,L} = \frac{(\lambda_i \tau_i + (L-1) \tilde{\sigma}_i \tilde{\mu}_i (1 - \tau_i) \tilde{\sigma}_i)}{\lambda_i \tau_i (1 - \tilde{\sigma}_i) + L \tilde{\sigma}_i \tilde{\mu}_i} \quad (26)$$

$$G_n^{(i)} = \prod_{k=1}^n g_{(i,k)} \quad (27)$$

$$p_i(0) = \left(1 + \sum_{n=1}^{L-1} G_n^{(i)} + \frac{G_L^{(i)}}{1 - x_i} \right)^{-1} \quad (28)$$

$$p_i(n) = \left(\prod_{k=1}^L g_{(i,k)}^{h_k(n)} \right) x^{Lq(n)} \quad n \in \{1, \dots, L\} \quad (29)$$

with

$$\begin{aligned} h_k(n) &= 1 && \text{if } n \geq k \text{ or } 0 \text{ otherwise} \\ L_q(n) &= n - L && \text{if } n \geq L \text{ or } 0 \text{ otherwise} \end{aligned} \quad (30)$$

Resolution of the closed network

We consider a closed queuing network under RS-RD blocking mechanism. It consists of M First Come First Serve multiple server queues with general inter-arrival time and service time distributions. The state space of such network is the set of tuple of integers $\mathbf{n} = (n_1, n_2, \dots, n_M)$, where n_i is the number of bikes in node i , $i \in \{1, \dots, M\}$. Let $p(\mathbf{n})$ be the equilibrium probability that the network is in state \mathbf{n} and $p_i(n_i)$ is the equilibrium marginal state probability of queue i , $i \in \{1, \dots, M\}$. The maximum entropy solution $p(\mathbf{n})$ of the closed queuing network is determined by Maximizing the Entropy functional defined by:

$$H_p(\mathbf{n}) = - \sum_{\mathbf{n}} p(\mathbf{n}) \log p(\mathbf{n}) \quad (31)$$

For the resolution of the network, we assume that we have prior information about the state probabilities $p(\mathbf{n})$ through the normalization

$$\sum_{\mathbf{n}} p(\mathbf{n}) = 1 \quad (32)$$

and the marginal constraints $\{u_{(i,k)} : k = 1, \dots, c_i\}$ (12), $\langle n_i - c_i \rangle$ (13) and ϕ_i (14), $i \in \{1, \dots, M\}$.

The form of the maximized state probability, $p(\mathbf{n})$, subject to the normalization and the aforementioned marginal constraints is given by

$$p(\mathbf{n}) = \frac{1}{Z(L, M)} \prod_{i=1}^M w_i(n_i) \quad (33)$$

where $Z(L, M)$ is the normalizing constant.

For single server node (SS), we have:

$$w_i(n_i) = \left\{ \begin{array}{ll} 1 & \text{if } n_i = 0 \\ g_{(i,1)} x_i^{n_i-1} y_i^{f_i(n_i)} & \text{if } 0 < n_i \leq N_i \\ 0 & \text{otherwise,} \end{array} \right\} \quad (34)$$

For the multiple servers nodes (MSB or MS):

$$w_i(n_i) = \left\{ \begin{array}{ll} 1 & \text{if } n_i = 0, \\ \left(\prod_{k=1}^L g_{(i,k)}^{h_k(n_i)} \right) x_i^{Lq(n_i)} & \text{if } n_i \in \{1, \dots, L\} \end{array} \right\} \quad (35)$$

for $k = 1, \dots, L$ and $i = 1, \dots, M$:

- $g_{(i,k)}$ are the Lagrange coefficients corresponding to the constraint $u_{(i,k)}$,
- x_i are the Lagrange coefficients corresponding to the constraint $\langle n_i - 1 \rangle$ for the SS nodes and $\langle n_i - L \rangle$ for the MS and MBS,
- y_i are the Lagrange coefficients corresponding to the constraints $\phi_i, i = 1, \dots, M$.

Moreover:

- $f_i(n_i) = \max(0, n_i - N_i + 1)$ and $h_k(n_i) = 1$ if $n_i \geq k$, or 0 otherwise.
- $L_q(n_i) = n_i - L$ if $n_i \geq L$, or 0 otherwise.

The measures of the Lagrange coefficients have no closed form expressions in terms of raw system data. Therefore, they are approximated from those of the pseudo-open network. The use of this approximation is justified by the fact that "the state probability of a closed queueing network with population size L can be viewed as the conditional one of an open network sampled at intervals during which L jobs are enqueued" (Reiser & Kobayashi, 1975).

Resolution of the pseudo-open network

In this section we sketch a pseudo-open network and its approximate product form solution using the Entropy Maximization. The aforementioned notations and the similar topology to the CQN model are considered. The form of the ME state probability, $p(\mathbf{n})$ with $\mathbf{n} \in S$, is determined by

$$p(\mathbf{n}) = \frac{1}{Z(L, M)} \prod_{i=1}^M w_i(n_i) \quad (36)$$

with $Z(L, M)$ is the normalizing constant. and

$$w_i(n_i) = \left\{ \begin{array}{ll} g_{(i,1)} x_i^{n_i-1} y_i^{f_i(n_i)} & \text{for SS} \\ \left(\prod_{k=1}^L g_{(i,k)}^{h_k(n_i)} \right) x^{Lq(n_i)} & \text{for MSB or MS} \end{array} \right\} \quad (37)$$

From the resolution of the individual queues (23) and (29) and since it is verified (Kouvatsos & Xenios, 1989) that $Z(L, M)$ can be expressed as a product of $p_i(0)$ it can be deduced that $p(\mathbf{n})$ could have this form:

$$p(\mathbf{n}) = \prod_{i=1}^M p_i(n_i) \quad (38)$$

where $p_i(n_i)$ is the approximate marginal ME solution of a stable G/G/L queue for MS and MSB nodes, or a G/G/1/0;N queue under censored arrival process and a revised service time distribution for SS nodes. Obviously the ME state probability, $p(\mathbf{n})$, suggests a decomposition of the open network into individual queues under a censored arrival process and a revised service time, afterward it goes through the resolution of these individual queues which is already presented.

The decomposition of the network into individual queues

We first present a decomposition of the network into individual queues and deduce the rate and the scv of the actual inter-arrival time and the effective service time. We note by π_{ci} the probability that a completer from queue i is blocked under RS-RD blocking and by α_{ij} the transition probability that a completer of queue i attempts to join queue j .

The effective service-time is the service time in the servers after the consideration of the limited capacity of the queues and the blocking mechanism. The rate and the scv of the effective service time distribution are determined by:

$$\tilde{\mu}_i = \mu_i(1 - \pi_{ci}) \quad (39)$$

$$\tilde{C}_{si} = \pi_{ci} + C_{si}(1 - \pi_{ci}) \quad (40)$$

with

$$\pi_{ci} = \sum_{j=1}^M \alpha_{ij} \pi_{ij} \quad (41)$$

The effective transition probability, which is the transition probability of the network after consideration of the capacity limitation of the queues and the blocking mechanism, is determined by:

$$\tilde{\alpha}_{ji} = \frac{\alpha_{ji}(1 - \pi_{ji})}{(1 - \pi_{cj})}. \quad (42)$$

The rate of the effective inter-arrival time distribution of a queue i noted by $\tilde{\lambda}_i$ is calculated by solving the flow balance equations (8) and satisfying constraints on the fixed number of bikes(9).

The departing sub-stream from a queue j to a queue i , λ_{ji} is given by:

$$\lambda_{ji} = \frac{\tilde{\lambda}_j \tilde{\alpha}_{ji}}{(1 - \pi_{ji})} \quad (43)$$

The scv of the effective arriving stream at queue i generated from queue j is given by:

$$\tilde{C}_{dji} = 1 - \tilde{\alpha}_{ji} + \tilde{\alpha}_{ji} \tilde{C}_{dj} \quad (44)$$

and the blocking probability entering the node i :

$$\pi_i = \frac{\sum_{j \in A_i} \lambda_{ji} \pi_{ji}}{\sum_{j \in A_i} \lambda_{ji}} \quad (45)$$

\tilde{C}_{ai} the scv of the effective inter-arrival time to node i is defined as in (Kouvatsos, 1994; Kouvatsos & Xenios, 1989).

$$\tilde{C}_{ai} = -1 + \left[\sum_{j=1}^M \frac{\tilde{\lambda}_j \tilde{\alpha}_{ji}}{\tilde{\lambda}_i} (\tilde{C}_{dji} + 1)^{-1} \right]^{-1} \quad (46)$$

The rate and the scv of the inter-arrival time to a node i ; λ_i and C_{ai} are given by:

$$\lambda_i = \frac{\tilde{\lambda}_i}{(1 - \pi_i)}. \quad (47)$$

$$C_{ai} = \frac{\tilde{C}_{ai} - \pi_i}{(1 - \pi_i)}. \quad (48)$$

In (Kouvatsos & Xenios, 1989), π_{ij} the probability that a completer from queue i is blocked by queue j ($j \neq i$) have been demonstrated to have the form:

$$\begin{aligned} \pi_{ij} = & (1 - \tau_{ij})^{N_j} \frac{\tilde{\sigma}_j}{\tilde{\sigma}_j(1 - \tau_{ij}) + \tau_{ij}} p_j(0) \\ & + \sum_{n_j=1}^{N_j} (1 - \tau_{ij})^{(N_j - n_j)} p_j(n_j) \end{aligned} \quad (49)$$

with

$$\tau_{ij} = \frac{2}{C_{dij} + 1}, \quad i \in \{1, \dots, M\}, \quad j \in \{1, \dots, M\} \quad (50)$$

and

$$\tilde{\sigma}_j = \frac{2}{\tilde{C}_{sj} + 1}, \quad j \in \{1, \dots, M\} \quad (51)$$

The SCV of the overall arriving stream at queue j generated from queue i

$$C_{dij} = \frac{(\tilde{C}_{dij} - \pi_{ij})}{(1 - \pi_{ij})} \quad (52)$$

Approximation of the scv of the effective inter-departure time \tilde{C}_{di} from queue i can be analytically approximated at heavy traffic as mentioned in (Kouvatsos & Xenios, 1989) by the relation:

$$\tilde{C}_{di} = \tilde{\rho}_i(1 - \tilde{\rho}_i) + (1 - \tilde{\rho}_i)\tilde{C}_{ai} + \tilde{\rho}_i^2\tilde{C}_{si} \quad (53)$$

with

$$\tilde{\rho}_i = \frac{\tilde{\lambda}_i}{L\tilde{\mu}_i} \quad i \in MS \cup MSB \quad (54)$$

$$\tilde{\rho}_i = \frac{\tilde{\lambda}_i}{\tilde{\mu}_i} \quad i \in SS \quad (55)$$

The resolution method at a glance

In the following figure, the steps for the resolution of the CQN with RS-RD blocking are shown in synthetic way.

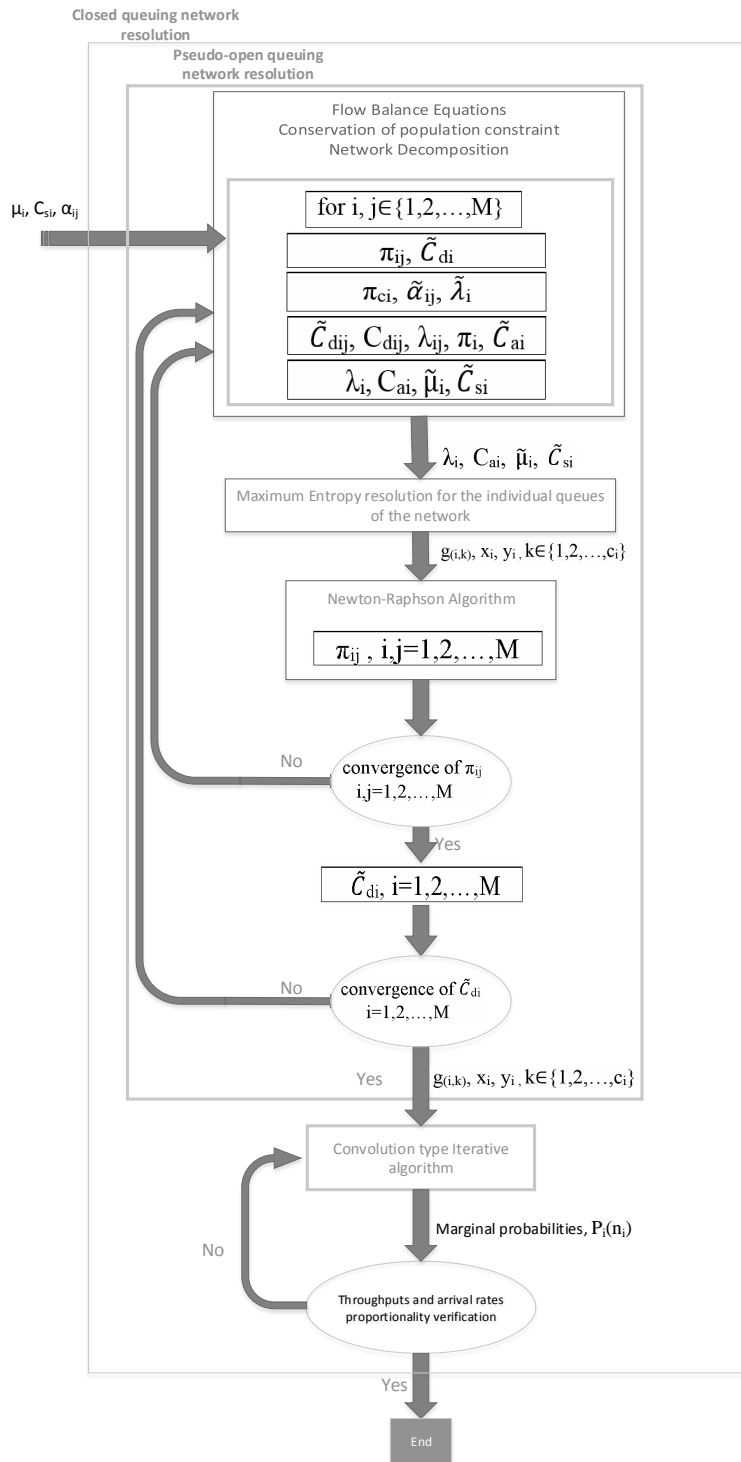


Figure B1. Resolution approach of the CQN with RS-RD blocking

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