# AJAE appendix for Rules versus discretion in food storage policies*

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## Characterization of the optimal state-contingent policy under discretion

To solve the optimal policy problem (18), we need to reformulate the complementarity equation (5) because it cannot be included directly as a constraint in a maximization problem. To restate this equation, we introduce a positive slack variable, $\phi$, with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta \operatorname{E}_t \left(P_{t+1}\right), \tag{S1}$$

$$S_t \phi_t = 0. \tag{S2}$$

$\zeta$ is taken to equal zero since there is no subsidy under a public storage policy.

As we focus on a Markovian equilibrium, the solution is constrained to depend only on the value of the current period's state, so price can be characterized by a function of the state variable: $P_t = \mathscr{P}\left(A_t^{\mathrm{T}}\right)$. Using this function $\mathscr{P}$, expectations of next-period price can be substituted by a function of current period variables, which makes the problem recursive. Associating the Lagrange multipliers $\lambda$, $\mu$, $\nu$, and $\chi$ to equations (S1), (S2), (7), and (11) allows us to write the optimal policy problem as the following dynamic programming problem using the value function $J(\cdot)$:

$$\begin{aligned} J\left(A_t^{\mathrm{T}}\right) = \min_{\Phi_t} \max_{\Omega_t} \Big( & v\left(P_t, Y\right)/w + P_t A_t^{\mathrm{T}} - \Psi\left(H_t\right) - \left(P_t + k\right)\left(S_t + S_t^{\mathrm{G}}\right) \\ & + \lambda_t \left\{\beta \operatorname{E}_t \left[\mathscr{P}\left(S_t + S_t^{\mathrm{G}} + H_t \varepsilon_{t+1}\right)\right] + \phi_t - P_t - k\right\} \\ & + \pi_t S_t \phi_t \\ & + \nu_t \left\{\beta \operatorname{E}_t \left[\mathscr{P}\left(S_t + S_t^{\mathrm{G}} + H_t \varepsilon_{t+1}\right)\varepsilon_{t+1}\right] - \Psi'\left(H_t\right)\right\} \\ & + \chi_t \left[A_t^{\mathrm{T}} - D\left(P_t\right) - S_t - S_t^{\mathrm{G}}\right] \\ & + \beta \operatorname{E}_t \left[J\left(S_t + S_t^{\mathrm{G}} + H_t \varepsilon_{t+1}\right)\right]\Big), \end{aligned} \tag{S3}$$

where $\Phi_t = \{\lambda_t, \pi_t, \nu_t, \chi_t\}$ and $\Omega_t = \{S_t \geq 0, H_t, P_t, S_t^{\mathrm{G}} \geq 0, \phi_t \geq 0\}$.

In this setting with occasionally binding constraints, we cannot assume $\mathscr{P}$ to be differentiable everywhere. Thus, in theory, it is not possible to characterize the first-order conditions of this problem since they would imply derivatives of $\mathscr{P}$. But since, in practice, $\mathscr{P}$ is approximated by a spline, which is differentiable

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everywhere, we can solve the dynamic programming problem numerically by solving the first-order conditions of the corresponding approximated problem:

(S4) $S_t : S_t \geq 0 \quad \perp \quad -w(P_t+k) - \chi_t + \beta\,\mathrm{E}_t\left[(\lambda_t + \nu_t \varepsilon_{t+1})\,\mathscr{P}'\left(A^{\mathrm{T}}_{t+1}\right) + wP_{t+1} + \chi_{t+1}\right] + \pi_t \phi_t \leq 0,$

(S5) $H_t : \beta\,\mathrm{E}_t\left[(\lambda_t + \nu_t \varepsilon_{t+1})\,\varepsilon_{t+1}\mathscr{P}'\left(A^{\mathrm{T}}_{t+1}\right) + \varepsilon_{t+1}\chi_{t+1}\right] = \nu_t \Psi''(H_t),$

(S6) $P_t : v_P(P_t, Y) + wD(P_t) - \lambda_t - \chi_t D'(P_t) = 0,$

(S7) $S^{\mathrm{G}}_t : S^{\mathrm{G}}_t \geq 0 \quad \perp \quad -w(P_t+k) - \chi_t + \beta\,\mathrm{E}_t\left[(\lambda_t + \nu_t \varepsilon_{t+1})\,\mathscr{P}'\left(A^{\mathrm{T}}_{t+1}\right) + wP_{t+1} + \chi_{t+1}\right] \leq 0,$

(S8) $\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t + \pi_t S_t \leq 0,$

(S9) $\lambda_t : \phi_t = P_t + k - \beta\,\mathrm{E}_t(P_{t+1}),$

(S10) $\pi_t : S_t \phi_t = 0,$

(S11) $\nu_t : \beta\,\mathrm{E}_t(P_{t+1}\varepsilon_{t+1}) = \Psi'(H_t),$

(S12) $\chi_t : A^{\mathrm{T}}_t = D(P_t) + S_t + S^{\mathrm{G}}_t.$

In equations (S4), (S5) and (S7) note the presence of derivatives of a policy function, $\mathscr{P}'\left(A^{\mathrm{T}}_{t+1}\right) = \partial P_{t+1}/\partial A^{\mathrm{T}}_{t+1}$. These Euler equations including functional terms are referred to in the literature as Generalized Euler equations (e.g., Krusell, Kurusçu, and Smith 2002; Klein, Krusell, and Ríos-Rull 2008). For a detailed interpretation of similar first-order conditions, see Gouel (2013a).

## Equations of a price-band program

For a rigorous mathematical characterization of the behavior of public stock under a price-band we need to introduce two variables: $\Delta S^{\mathrm{G}+}$ and $\Delta S^{\mathrm{G}-}$, which refer to increases and decreases in public stock. Both are positive and bounded from above. The increase in public stock is bounded from above by the remaining storage capacity, and the decrease in public stock by the level of existing stocks. To defend the price-band, public stocks are managed by the four conditions (19)–(22), which can be restated as two mixed complementarity equations:[1]

(S13) $0 \leq \Delta S^{\mathrm{G}+}_t \leq \bar{S}^{\mathrm{G}} - S^{\mathrm{G}}_{t-1} \quad \perp \quad P_t - P^{\mathrm{F}},$

(S14) $0 \leq \Delta S^{\mathrm{G}-}_t \leq S^{\mathrm{G}}_{t-1} \quad \perp \quad P^{\mathrm{C}} - P_t.$

Equation (S13) means that public stocks increase to prevent the price from decreasing below the floor price, $P^{\mathrm{F}}$. The floor is defended until public stocks reach the limit $\bar{S}^{\mathrm{G}}$. Equation (S14) governs the decrease in public stocks. They decrease to prevent the price from rising above the ceiling price, $P^{\mathrm{C}}$. The release of stocks is constrained by the existing level of stocks $S^{\mathrm{G}}_{t-1}$.

Market equilibrium and public stock transition are defined by

(S15) $A^{\mathrm{P}}_t = D(P_t) + S_t + \Delta S^{\mathrm{G}+}_t - \Delta S^{\mathrm{G}-}_t,$

(S16) $S^{\mathrm{G}}_t = S^{\mathrm{G}}_{t-1} + \Delta S^{\mathrm{G}+}_t - \Delta S^{\mathrm{G}-}_t.$

The recursive equilibrium under a price-band program is defined by the equilibrium equations (5), (7) and (S13)–(S15), and the transition equations (9) and (S16).

[1] Here the "perp" notation ($\perp$) is extended to situations with two complementarity constraints. The expression $a \leq X \leq b \perp F(X)$ is a compact formulation for $X = a \Rightarrow F(X) \geq 0$, $X \in (a,b) \Rightarrow F(X) = 0$, $X = b \Rightarrow F(X) \leq 0$.

## Numerical algorithm

The numerical algorithm used to produce the results is inspired by Miranda and Fackler (2002, Ch. 8–9), Fackler (2005), and Miranda and Glauber (1995). This is a projection method with a collocation approach. Because several models are solved, we present a general method that can be applied to all of them. Following Fackler (2005), rational expectations problems can be expressed using three groups of equations. State variables $s$ are updated through a transition equation:

$$\dot{s} = g(s, x, \dot{e}), \tag{S17}$$

where $x$ are response variables, $e$ are stochastic shocks, and next-period variables are indicated by a dot over the character. Response variables are defined by solving a system of complementarity equilibrium equations:

$$\underline{x}(s) \leq x \leq \bar{x}(s) \quad \perp \quad f(s, x, z). \tag{S18}$$

Response variables can have lower and upper bounds, $\underline{x}$ and $\bar{x}$, which can themselves be functions of state variables.[2] For generality, equilibrium equations have been expressed as complementarity problems. In cases where response variables have no lower and upper bounds (e.g., $H$), equation (S18) simplifies to a traditional equation:

$$f(s, x, z) = 0. \tag{S19}$$

$z$ is a variable representing the expectations about next period and is defined by

$$z = \mathrm{E}\left[h(s, x, \dot{e}, \dot{s}, \dot{x})\right]. \tag{S20}$$

One way to solve this problem is to find a function that gives a good approximation of the behavior of the response variables. We consider a spline approximation of the behavior of response variables as a function of state variables:

$$x \approx \mathscr{X}\left(s, \theta\right), \tag{S21}$$

where $\theta$ are the parameters defining the spline approximation. To calculate this spline, we discretize the state space, and the spline must hold exactly for all points on the grid.

The expectations operator in equation (S20) is approximated through a Gaussian quadrature (with 7 points), which defines a set of pairs $\{e_l, w_l\}$ in which $e_l$ represents a possible realization of shocks and $w_l$ the associated probability. Using this discretization, and equations (S17) and (S20)–(S21), we can express the equilibrium equation (S18) as

$$\underline{x}(s) \leq x \leq \bar{x}(s) \quad \perp \quad f\left(s, x, \sum_l w_l h\left(s, x, e_l, g\left(s, x, e_l\right), \mathscr{X}\left(g\left(s, x, e_l\right), \theta\right)\right)\right). \tag{S22}$$

For a given spline approximation, $\theta$, and a given $s$, equation (S22) is a function of $x$ only and can be solved using a mixed complementarity solver.

Once all the above elements are defined, we can proceed to the algorithm, which runs as follows:

1. Initialize the spline approximation, $\theta^{(0)}$, based on a first-guess, $x^{(0)}$.

[2] It is the case in the price-band where upper bounds on $\Delta S_t^{G+}$ and $\Delta S_t^{G-}$ are functions of $S_{t-1}^{G}$.

2. For each point of the grid of state variables, $s_i$, solve for $x_i$ equation (S22) using the solver PATH (Dirkse and Ferris 1995):

$$\text{(S23)} \qquad \underline{x}(s_i) \le x_i \le \overline{x}(s_i) \quad \perp \quad f\left(s_i, x_i, \sum_l w_l h\left(s_i, x_i, e_l, g\left(s_i, x_i, e_l\right), \mathscr{X}\left(g\left(s_i, x_i, e_l\right), \theta^{(n)}\right)\right)\right).$$

3. Update the spline approximation using the new values of response variables, $x = \mathscr{X}\left(s, \theta^{(n+1)}\right)$.

4. If $\|\theta^{(n+1)} - \theta^{(n)}\|_2 \ge 10^{-8}$ then increment $n$ to $n+1$ and go to step 2.

Once the rational expectations equilibrium is identified, the spline approximation of the decision rules is used to simulate the model.

For the discretionary problem, whose equations are presented above, this method can be applied without significant modifications. The model equations include, for the discretionary problem, derivatives of $\mathscr{P}$, $\mathscr{P}$ being an approximation of the behavior of price with respect to total availability. Calculation of this approximation has already been carried out in the above method in step 3. Since derivatives of $\mathscr{P}$ appear only in expectations terms, to apply the method we need to ensure that definition of the function $h$ includes these terms, which leads to a new equation (S20):

$$\text{(S24)} \qquad z = \mathrm{E}\left[h(s, x, \dot{e}, \dot{s}, \dot{x}, \mathscr{X}_s\left(\dot{s}, \theta\right))\right],$$

where $\mathscr{X}_s\left(\dot{s}, \theta\right)$ is the derivative of $\mathscr{X}$ with respect to the next-period state variables.

This is only a sketch of the solution method. We use several methods in this article to solve the models, depending on which is the most efficient. For example, instead of using the simple updating rule in step 3, a Newton, or inexact Newton updating is used when feasible. For more precisions, see the MATLAB program files and the RECS solver (Gouel 2013b), with which the models are solved.

## Accuracy of the equivalent variation approximation

To make the social welfare function tractable, the equivalent variation has been approximated to the first-order around the path followed without intervention in equation (16). By focusing on the asymptotic distribution, it can be shown (following Wright and Williams 1988, Appendix) that the equivalent variation is of the same order of magnitude as the change in the variance of price, so the square of the equivalent variation is of a smaller order for a small price stabilization and second-order terms in the approximation can be neglected. We confirm here numerically that the first-order approximation is accurate enough for our analysis.

Figure S1 represents on the left panel the true equivalent variation, from equation (12), as a function of the storage subsidy and on the right panel the approximation error caused by the first-order approximation. The approximation error is presented in a base-10 logarithm so that a result of $-4$ for $\log_{10}|EV_{\text{true}} - EV_{\text{approx}}|$ should be read as a precision of 4 digits after the decimal point.

As expected, the precision of the first-order approximation deteriorates with the subsidy level and so with the extent of price stabilization. However, for the equivalent variation of the optimal policies (around or below 0.35 in table 2), the precision is close to 4 digits after the decimal points or better. Such a precision error represents an error of 1E$-$4 percent of the steady-state commodity budget share, which would not matter in the policy design.

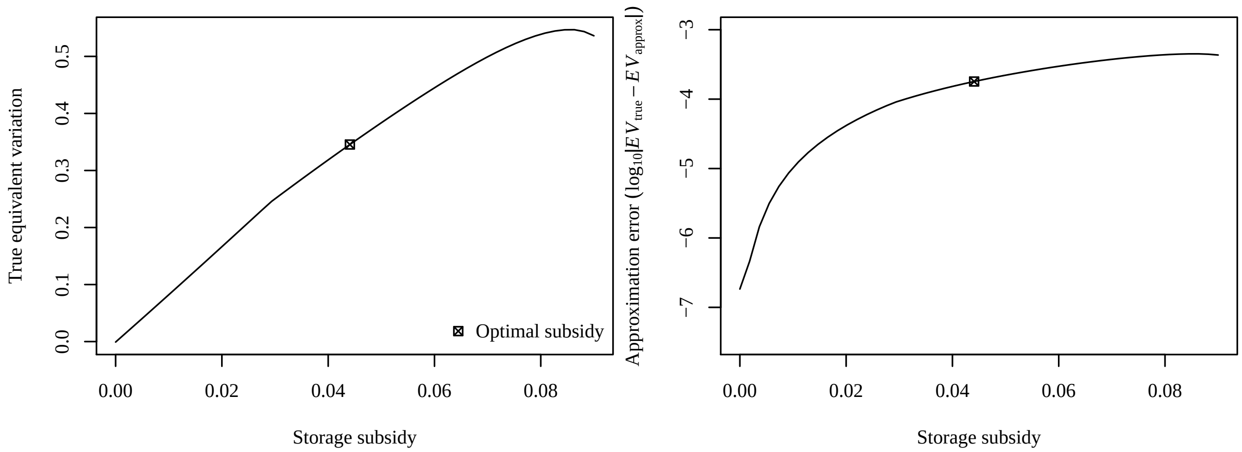


Figure S1: Equivalent variation and approximation error for different storage subsidies

# Sensitivity analysis

Here, we analyze the sensitivity of results to the parameters defining consumers' and producers' behavior, and to the shape of the yield distribution. Because of the combination of grid search and optimization from multiple starting points, the optimal price-band is difficult to find numerically, so it is not considered in this sensitivity analysis.

## Sensitivity to risk aversion and demand elasticity

The risk aversion and demand elasticity are two parameters that have important implications for the quantitative findings. These two parameters govern the importance of the welfare losses from price instability. Consumers enjoy welfare gains from a stabilization at mean price if their indirect utility function is concave in price. From Turnovsky, Shalit, and Schmitz (1980), we know that this is the case if $\gamma(\eta-\rho)-\alpha<0$, which illustrates the role of these parameters.

Table S1 presents the sensitivity of results to price elasticity and relative risk aversion. Price elasticity affects the results in two converging ways: (i) lower elasticity implies more volatility and so higher potential gains from stabilization, and (ii) according to Turnovsky, Shalit, and Schmitz (1980) for the same reduction in variance lower elasticity also increases the equivalent variation from stabilization. As risk aversion is increased, the stabilization policies become more aggressive and achieve lower levels of price instability as well as higher welfare gains, which can reach as much as 2.8% of steady-state commodity budget share for $\rho=4$ and $\alpha=-0.15$.

Table S1 shows that the ability of a constant subsidy to achieve results close to a discretionary policy holds well for all parameters (and even more in tables S2 and S3). Small welfare differences appear for high risk aversion and low price elasticity, with the subsidy leading to higher welfare gains. This situation can arise because the optimal policy under discretion is the one followed by a government that would reoptimize at each period, while the optimal subsidy is chosen to maximize welfare at the initial period, which is the welfare that matters eventually. The lower the price elasticity, the larger is the difference. Indeed, as can be seen above in the first-order conditions, in the discretionary problem, government has to take into account producers' expectations of its behavior and this feedback is all the more important when price elasticity is low, because producers' reaction matters more when price is more reactive.

**Table S1. Sensitivity to risk aversion and demand elasticity**

| Relative risk aversion ($\rho$): | 1 | | | | **2** | | | | 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price elasticity ($\alpha$): | −0.15 | −0.30 | −0.40 | −0.60 | −0.15 | −0.30 | **−0.40** | −0.60 | −0.15 | −0.30 | −0.40 | −0.60 |
| Total gains | | | | | | | | | | | | |
| Discretion | 0.123 | 0.024 | 0.010 | 0.003 | 0.746 | 0.166 | **0.084** | 0.026 | 2.744 | 0.645 | 0.354 | 0.122 |
| Subsidy | 0.121 | 0.024 | 0.010 | 0.003 | 0.745 | 0.166 | **0.084** | 0.026 | 2.821 | 0.649 | 0.355 | 0.122 |
| CV of price | | | | | | | | | | | | |
| Without policy | 0.378 | 0.248 | 0.206 | 0.157 | 0.378 | 0.248 | **0.206** | 0.157 | 0.378 | 0.248 | 0.206 | 0.157 |
| Discretion | 0.289 | 0.222 | 0.192 | 0.151 | 0.204 | 0.186 | **0.169** | 0.140 | 0.122 | 0.146 | 0.139 | 0.124 |
| Subsidy | 0.294 | 0.221 | 0.192 | 0.151 | 0.225 | 0.190 | **0.170** | 0.140 | 0.167 | 0.157 | 0.144 | 0.125 |

Note: Benchmark in bold. Welfare gains as a percentage of the steady-state commodity budget share.

## Sensitivity to supply elasticity

Supply elasticity affects the volatility without public policy (table S2). A higher supply elasticity implies more stable prices, since planned production reacts more to changes in expected prices. Supply elasticity affects also the importance of time-consistency, that is how much an optimal discretionary policy is distinct from an optimal policy under commitment. As private storers are crowded out by public storage, the issue of time-consistency lies in the expectations of producers with regard to the policy. For a low supply elasticity, these expectations do not represent a constraint for the government as producers have limited reactions to changes in price expectations. So the optimal policy under discretion would become arbitrarily close to a policy under commitment with a decrease in supply elasticity, which explains why the discretionary policy achieves higher welfare gains than the subsidy for low elasticity values.

**Table S2. Sensitivity to Supply Elasticity**

| Supply elasticity ($1/\mu$): | 0.01 | 0.05 | 0.10 | **0.50** | 1.00 | 2.00 |
|---|---|---|---|---|---|---|
| Total gains | | | | | | |
| Discretion | 0.085 | 0.085 | 0.084 | **0.084** | 0.090 | 0.096 |
| Subsidy | 0.083 | 0.083 | 0.083 | **0.084** | 0.090 | 0.097 |
| Coefficient of variation of price | | | | | | |
| Without policy | 0.230 | 0.227 | 0.223 | **0.206** | 0.197 | 0.188 |
| Discretion | 0.200 | 0.196 | 0.191 | **0.169** | 0.156 | 0.144 |
| Subsidy | 0.200 | 0.195 | 0.191 | **0.170** | 0.157 | 0.144 |

Note: Benchmark in bold. Welfare gains as a percentage of the steady-state commodity budget share.

On welfare, supply elasticity has two opposite effects. On the one hand, a higher supply elasticity decreases potential welfare gains by decreasing price volatility without policy. On the other hand, an elastic supply is complementary to storage (Gouel 2013a): the higher the elasticity, the more planned production is reduced when stocks are accumulated, thus preventing over-accumulation from public storage and the more planned production is increased when stocks reach zero. This complementarity between supply and storage dominates at high supply elasticity the first effect of reduction in potential gains as welfare gains increase with supply elasticity when this latter exceeds 0.5.

## Sensitivity to the yield distribution

Next we analyze the sensitivity to the shape of the yield distribution. To do so, we fix the mean and standard deviation of the distribution at their benchmark values, 1 and 0.112, and adjust its shape. The shape of a beta distribution is determined by its two shape parameters, but changing them also affects the mean and the standard deviation. To keep the mean and standard deviation constant, we adjust the lower and upper bounds of the distribution accordingly. We consider three alternative situations: a beta mimicking a normal distribution (skewness equal to 0 and kurtosis close to 3 in table S3, shape parameters: (50,50)), a right-skewed beta (parameters: (2,4)) and a left-skewed beta (parameters: (4,2)).

**Table S3. Sensitivity to the shape of the beta distribution**

| Shape parameters Beta$(a,b)$: | **(2,2)** | (50,50) | (2,4) | (4,2) |
|---|---|---|---|---|
| Total gains | | | | |
| Discretion | **0.084** | 0.104 | 0.058 | 0.110 |
| Subsidy | **0.084** | 0.103 | 0.058 | 0.109 |
| Coefficient of variation of price | | | | |
| Without policy | **0.206** | 0.227 | 0.188 | 0.241 |
| Discretion | **0.169** | 0.184 | 0.157 | 0.194 |
| Subsidy | **0.170** | 0.184 | 0.159 | 0.196 |
| Features of the beta distribution | | | | |
| Lower bound | **0.750** | −0.124 | 0.791 | 0.582 |
| Upper bound | **1.250** | 2.124 | 1.418 | 1.209 |
| Skewness | **0.000** | 0.000 | 0.468 | −0.468 |
| Kurtosis | **2.143** | 2.942 | 2.625 | 2.625 |

Note: Benchmark in bold. Welfare gains as a percentage of the steady-state commodity budget share.

When the distribution allows the occurrence of more negative outcomes (in the approximately normal and the left-skewed beta), price volatility without policy tends to increase. Private storage is better able to alleviate low prices due to high productivity shocks than high prices because of the non-negativity constraint. Hence, when negative shocks become more likely, storage provides less stabilization. For a higher volatility without intervention, there are higher welfare gains from stabilization (as in table S1). Except for this effect, there is no qualitative sensitivity to the yield distribution.

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